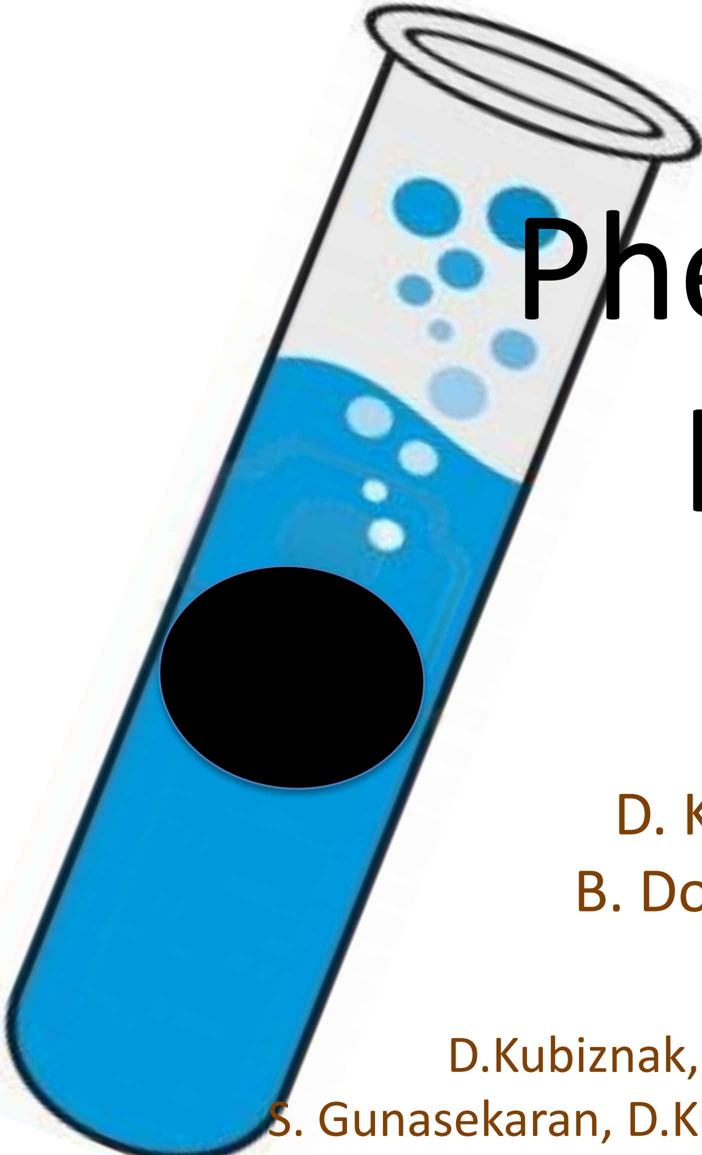


The Everyday Phenomena of Black Hole Chemistry

Robert Mann

D. Kubiznak, N. Altimirano, S. Gunasekaran,
B. Dolan, D. Kastor, J. Traschen, Z. Sherkatgnad

D.Kubiznak, R.B. Mann JHEP 1207 (2012) 033

S. Gunasekaran, D.Kubiznak, R.B. Mann JHEP 1211 (2012) 110

B. Dolan, D. Kastor, D.Kubiznak, R.B. Mann, J. Traschen Phys. Rev. D87 (2013) 104017

N. Altimirano, D.Kubiznak, R.B. Mann Phys. Rev. D88 (2013) 101502

N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann CQG 31 (2014) 042001

N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann Galaxies 2 (2014) 89

Black Hole Thermodynamics

Thermodynamics | Gravity

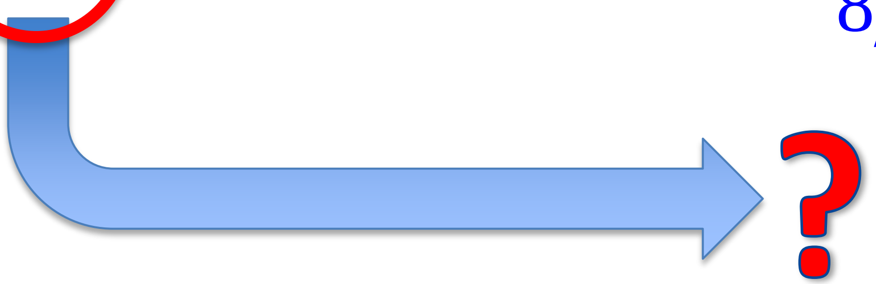
Energy $E \leftrightarrow M$ Mass

Temperature $T \leftrightarrow \frac{\hbar k}{2p}$ Surface gravity

Entropy $S \leftrightarrow \frac{A}{4\hbar}$ Horizon Area

$$dE = TdS + VdP + \text{work terms} \leftrightarrow dM = \frac{k}{8p} dA + W dJ + F dQ$$

First Law | First Law



Smarr Formula

L. Smarr PRL 30, 71 (1973)
[Err. 30, 521 (1973)].

$$ds^2 = -V dt^2 + \frac{dr^2}{V} + r^2 d\Omega_2^2$$

Schwarzschild Black hole $V = 1 - \frac{2M}{r}$

$$E = M = \frac{r_+}{2} \qquad T = \frac{1}{4\pi r_+} \qquad S = \pi r_+^2$$

$\Rightarrow$ $M = 2TS$

Smarr ✓

Schwarzschild-AdS Black hole $V = 1 - \frac{2M}{r} + \frac{r^2}{l^2}$

$$E = M = \frac{l^2 + r_+^2}{2l^2} r_+ \qquad T = \frac{l^2 + 3r_+^2}{4\pi r_+ l^2}$$

$$S = \pi r_+^2$$

$\Rightarrow$ $M \neq 2TS$

~~Smarr~~ ?

Scaling Arguments

Caldarelli/Cognola/Klemm, CQG 17, 399 (2000)

Suppose

$$f\left(a^{p}x, a^{q}y\right) = a^{r} f(x,y) \quad rf(x,y) = p\frac{¶f}{¶x}x + q\frac{¶f}{¶y}y$$

S-AdS Black Hole $\quad M \mu L^{D-3} \quad A \mu L^{D-2} \quad \mathrm{L} \mu L^{-2}$

$$M = M(A, \mathrm{L}) \quad (D-3)M = (D-2)\frac{¶M}{¶A}A - 2\frac{¶M}{¶\mathrm{L}}\mathrm{L}$$

$$S = \frac{A}{4G} \qquad T = \frac{k}{2p} = 4G\frac{¶M}{¶A}$$

$$P = -\frac{\mathrm{L}}{8p} = \frac{(D-2)(D-1)}{16pl^{2}}$$

$$M = \frac{(D-2)}{(D-3)}TS - \frac{2}{(D-3)}VP$$

$$V = -8p\frac{¶M}{¶\mathrm{L}}$$

Pressure from the Vacuum?

Dolan CQG 28 (2011) 125020; 235017

Schwarzschild-AdS Black hole

$$E = M = \frac{l^2 + r_+^2}{2l^2} r_+ \qquad T = \frac{l^2 + 3r_+^2}{4\pi r_+ l^2} \qquad S = \pi r_+^2 \qquad (D = 4)$$

$$M = 2(TS - VP)$$

Smarr

$$dE = TdS + VdP$$

First Law

Provided

$$P = -\frac{1}{8\pi}\Lambda = \frac{3}{8\pi}\frac{1}{l^2}$$

Thermodynamic Pressure

$$V = -8\pi \frac{\partial M}{\partial \Lambda} = \frac{4\pi}{3} r_+^3$$

Thermodynamic Volume

The Chemistry of AdS Black Holes

Include gauge charges:

First Law

$$dM = T_h dS_h + \sum_i (W_h^i - W_\infty^i) dJ^i + F_h dQ + V_h dP$$

Smarr Relation

$$\frac{D-3}{D-2} M = T_h S_h + \sum_i (W_h^i - W_\infty^i) J^i + \frac{D-3}{D-2} F_h Q - \frac{2}{D-2} P V_h$$

Thermodynamic Potential: Gibbs Free Energy

$$G = M - TS = G(T, P, J_i, Q)$$

- Equilibrium: Global minimum of Gibbs Free Energy
- Local Stability: Positivity of the Specific Heat

$$C_P = T \left(\frac{\partial S}{\partial T} \right)_{P, J_i, Q} > 0$$

Mass as Enthalpy

Thermodynamics | Gravity

$$\text{Enthalpy} \quad H \leftrightarrow M \quad \text{Mass}$$

$$\text{Temperature} \quad T \leftrightarrow \frac{\hbar k}{2p} \quad \text{Surface gravity}$$

$$\text{Entropy} \quad S \leftrightarrow \frac{A}{4\hbar} \quad \text{Horizon Area}$$

$$dH = TdS + VdP + \cdots \leftrightarrow dM = \frac{k}{8p} dA + VdP + \cdots$$

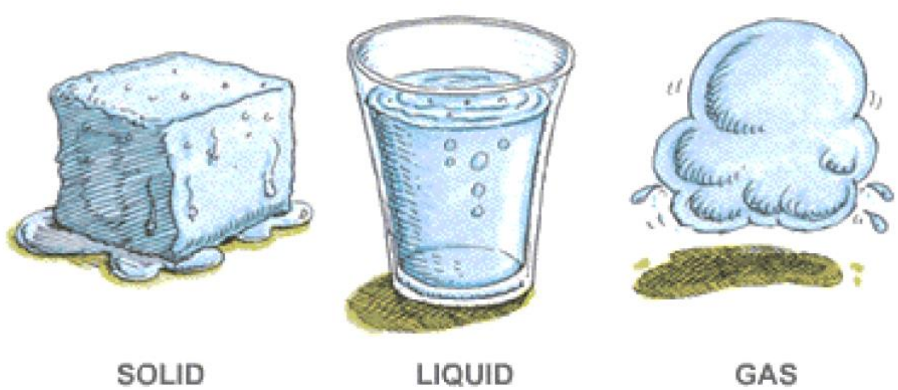
First Law | First Law

$$H = E + PV + \cdots \leftrightarrow M = E - rV$$

Mass
= Total Energy
- Vacuum
Contribution
(infinite)

Everyday AdS Black Hole Thermodynamics

- Hawking Page Transition
- Van der Waals Fluid and Charged AdS Black Holes
- Reentrant Phase Transitions
- Black Hole Triple Points ←→ Solid/Liquid/Gas



Hawking-Page Transition

S.W. Hawking & D.N. Page
Comm Math. Phys. 87 (1983) 577

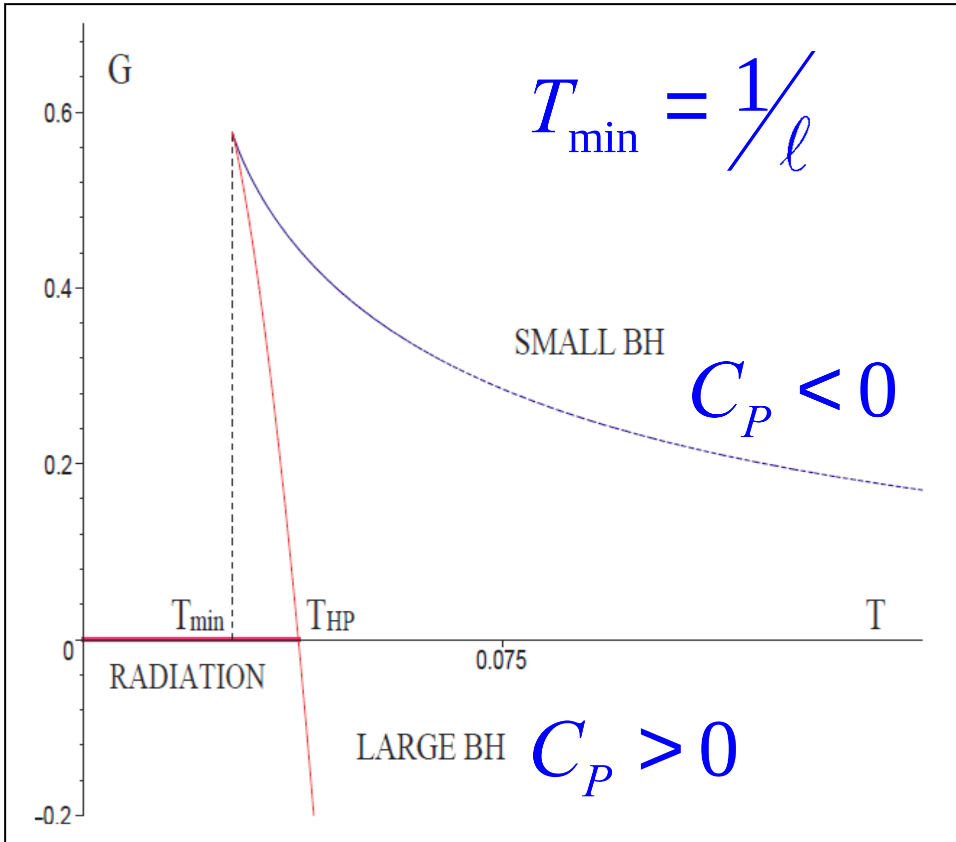
D-dim'l Schwarzschild-AdS Black hole

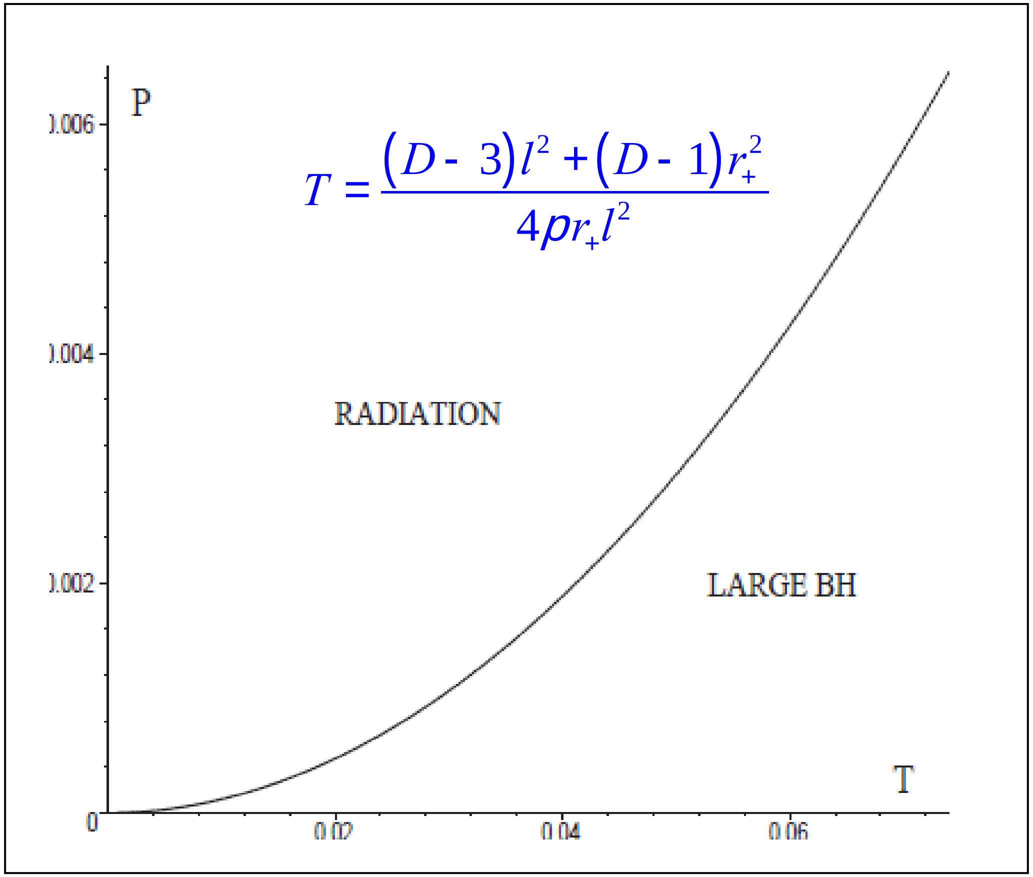
$$ds^2 = -V dt^2 + \frac{dr^2}{V} + r^2 d\Omega^2_{k,D-2} \qquad V = k - \frac{\tilde{M}}{r^{D-3}} + \frac{r^2}{\ell^2} \qquad k = \begin{cases} 1 & \text{spherical} \\ 0 & \text{planar} \\ -1 & \text{hyperbolic} \end{cases}$$

• AF black holes evaporate by Hawking radiation
• AdS is like a confining box
→ static black holes in thermal equilibrium

$$T < T_{min} \Rightarrow \quad \text{gas of particles}$$

1st order transition
between gas of particles and large black holes at Tc





$$T = \frac{(D-3)l^2 + (D-1)r_+^2}{4p r_+ l^2}$$

Phase transition in dual CFT (quark-gluon plasma)

Witten (1998)

Fluid interpretation:
solid/liquid PT
(infinite coexistence line)

D. Kubiznak/RBM arXiv [1404.2126]

Equation of state

$$Pv = T - \frac{k}{2pv}$$

depends on the horizon topology

$$v = 2l_P^2 r_+ = 2\left(\frac{3V}{4p}\right)^{1/3} = 6\frac{V}{N}$$

$$N = \frac{A}{l_P^2}$$

Planar black holes ←→ ideal gas

Van der Waals Fluids

$$Pv^3 - (kT + bP)v^2 + a(v - b) = 0$$

Critical Point

$$\frac{\partial P}{\partial v} = 0, \quad \frac{\partial^2 P}{\partial v} = 0$$

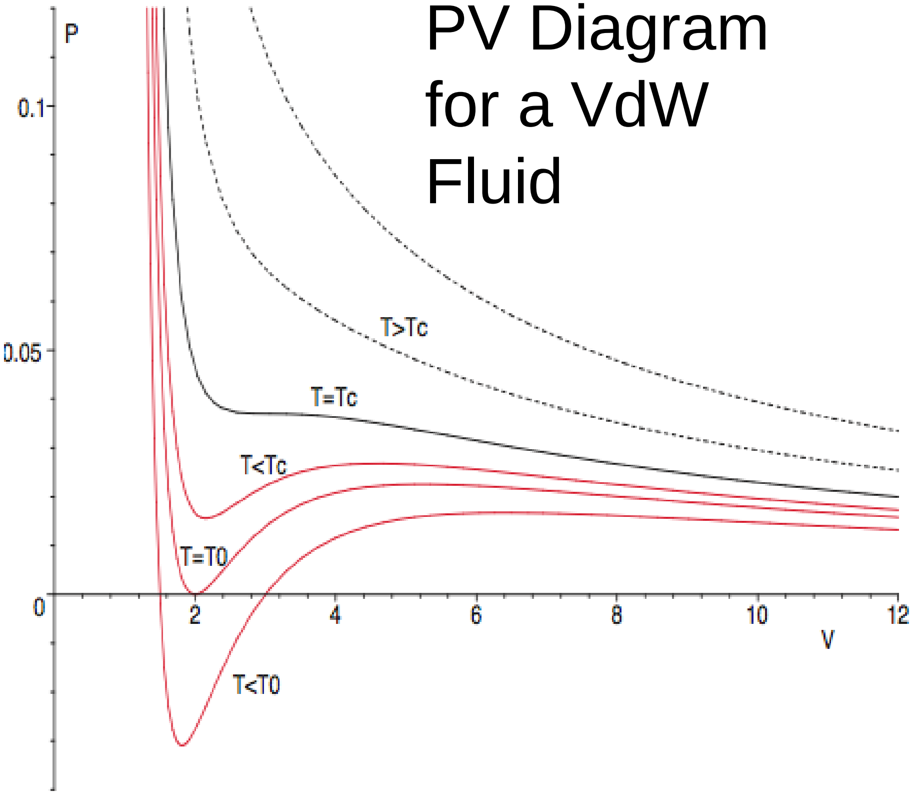
$$kT_c = \frac{8a}{27b}, \quad v_c = 3b, \quad P_c = \frac{a}{27b^2}$$

$$p = \frac{P}{P_c}, \quad n = \frac{v}{v_c}, \quad t = \frac{T}{T_c}$$

$$8t = (3n - 1)\left(p + \frac{3}{n^2}\right)$$

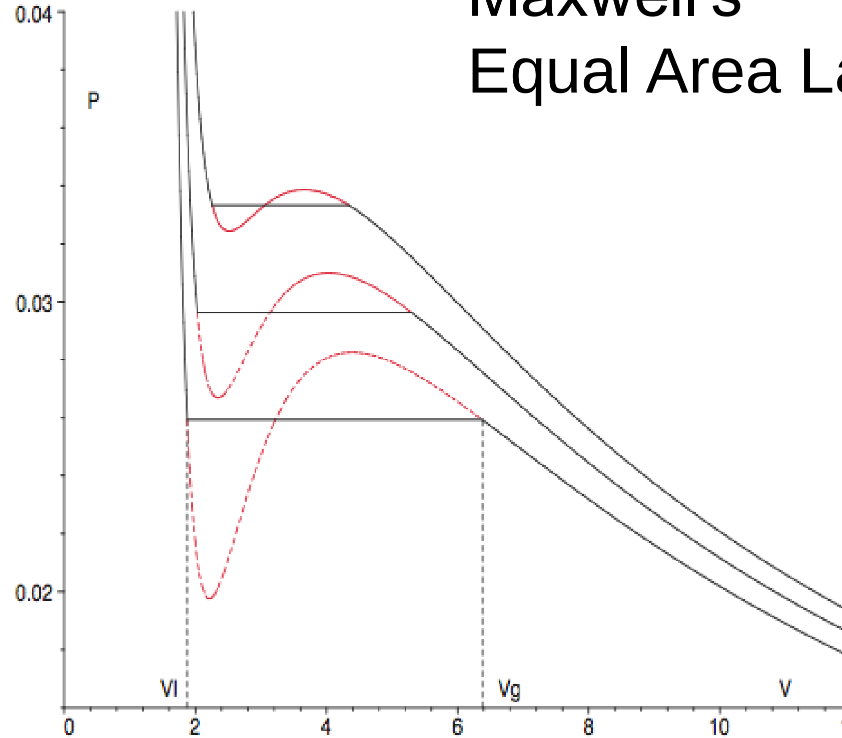
law of correspondin g states

PV Diagram for a VdW Fluid



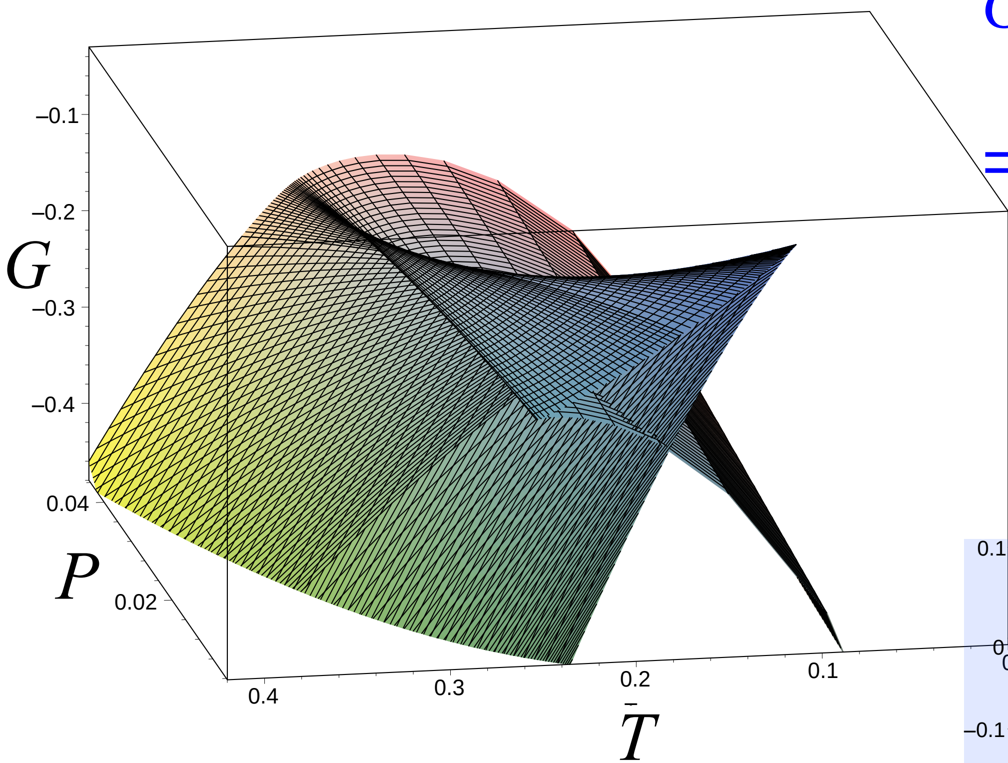
$$\frac{P_c v_c}{kT_c} = \frac{3}{8}$$

Maxwell's Equal Area Law



$$\oint v\,dP = 0$$

Gibbs Free Energy



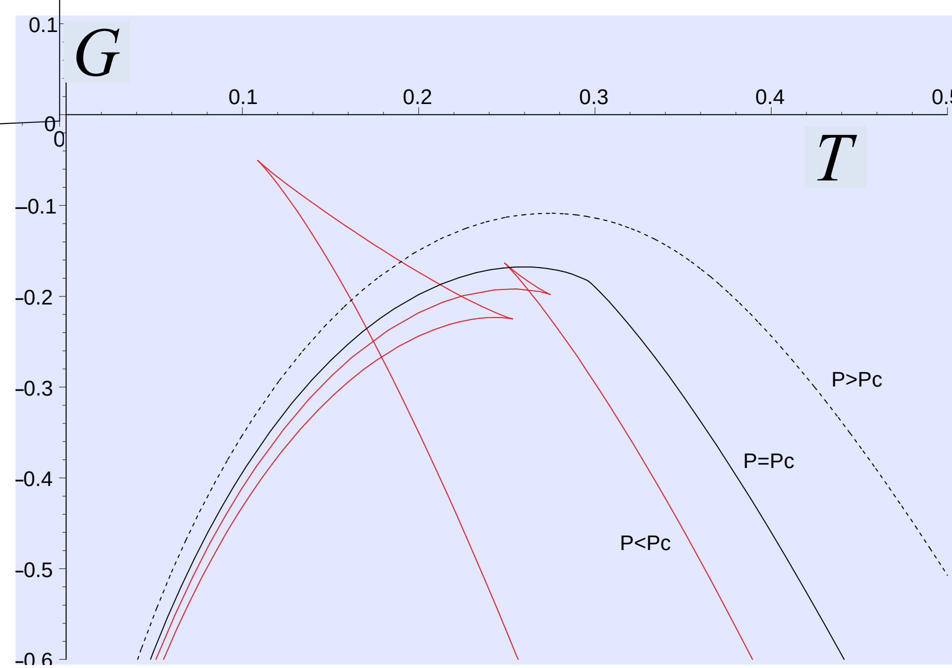
$$G = G(T,P)$$

$$= -kT\left(1 + \ln\left[\frac{(v-b)T^{3/2}}{F}\right]\right) - \frac{a}{v} + Pv$$

characteristic of the gas

First Law

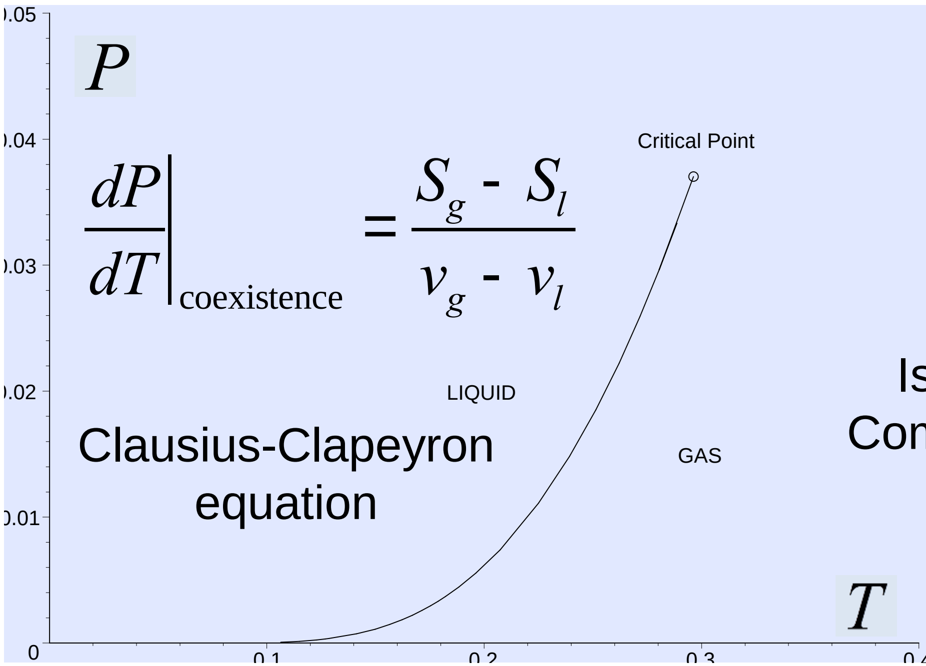
$$dG = -SdT + v\,dP$$



Critical Exponents

$$\pi = \frac{P}{P_c}, \quad \nu = \frac{v}{v_c}, \quad \tau = \frac{T}{T_c}$$

Line of Coexistence



$$\left.\frac{dP}{dT}\right|_{\text{coexistence}} = \frac{S_g - S_l}{v_g - v_l}$$

Clausius-Clapeyron equation

Specific Heat $\quad C_v = T \left.\frac{\partial S}{\partial T}\right|_v \propto |\tau - 1|^{-\alpha}$

Order Parameter $\quad \eta = v_g - v_l \propto |\tau - 1|^{\beta}$

Isothermal Compressibility $\quad \kappa_T = -\frac{1}{v}\left.\frac{\partial v}{\partial P}\right|_T \propto |\tau - 1|^{-\gamma}$

Critical Isotherm $\quad |P - P_c| \propto |v - v_c|^{\delta}$

For a VdW Fluid $\quad \alpha = 0 \quad \beta = \frac{1}{2} \quad \gamma = 1 \quad \delta = 3$

Charged AdS Black Holes as Van der Waals Fluids

Kubiznak/Mann JHEP 1207 (2012) 033

$$ds^2 = -V dt^2 + \frac{dr^2}{V} + r^2 d\Omega_2^2 \qquad F = dA \begin{cases} V = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2} \\ A = -\frac{Q}{r} dt \end{cases}$$

Temperature $T = \frac{1}{\beta} = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right)$

Entropy $S = \frac{A}{4} = \pi r_+^2$ Pressure $P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi}\frac{1}{l^2}$

Potential $\Phi = \frac{Q}{r_+}$ Volume $V = \frac{4}{3}\pi r_+^3$

$$M = 2\left(TS - PV\right) + \Phi Q$$

Smarr Relation

$$dM = TdS + VdP + \Phi dQ$$

First Law

Equation of State

$$r_+ = \left(\frac{3V}{4p}\right)^{1/3}$$

$$T = \frac{1}{4pr_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right)$$

$$P = -\frac{L}{8p} = \frac{3}{8p}\frac{1}{l^2}$$

$$P = \frac{T}{2r_+} - \frac{1}{8pr_+^2} + \frac{Q^2}{8pr_+^4}$$

Physical Equation of State

Thermodynamic Volume

$$\text{Press} = \frac{\hbar c}{l_P^2}P \quad \text{Temp} = \frac{\hbar c}{k}T$$

$$\text{Press} = \frac{k\text{Temp}}{2l_P^2 r_+} + \tfrac{1}{4}$$

$$v = 2l_P^2 r_+$$

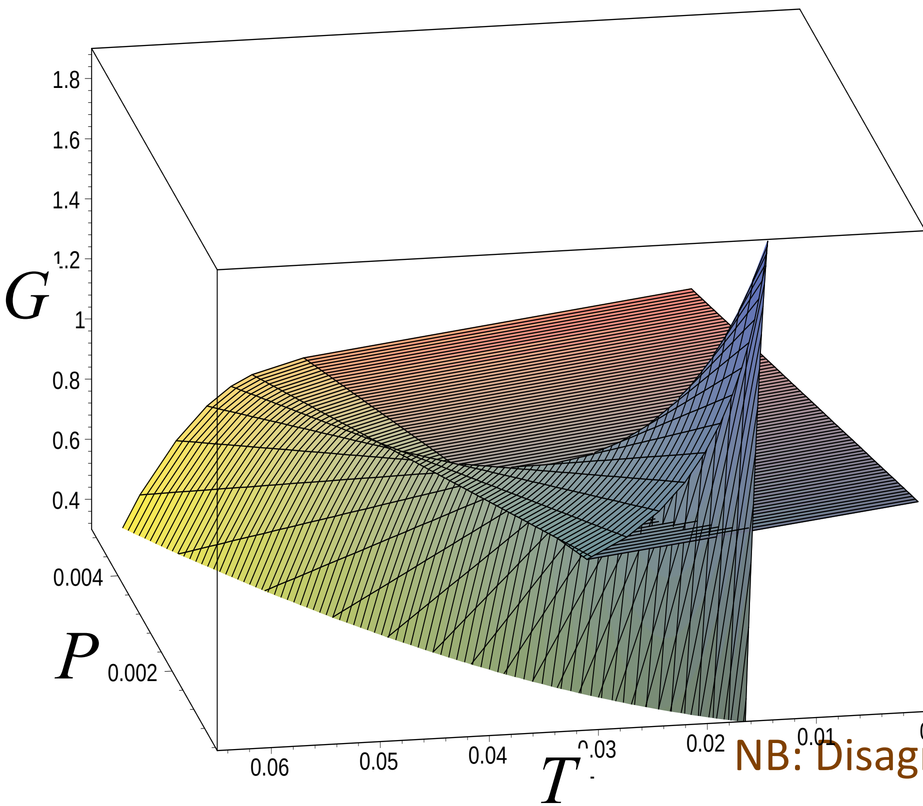
Van der Waal's Equation

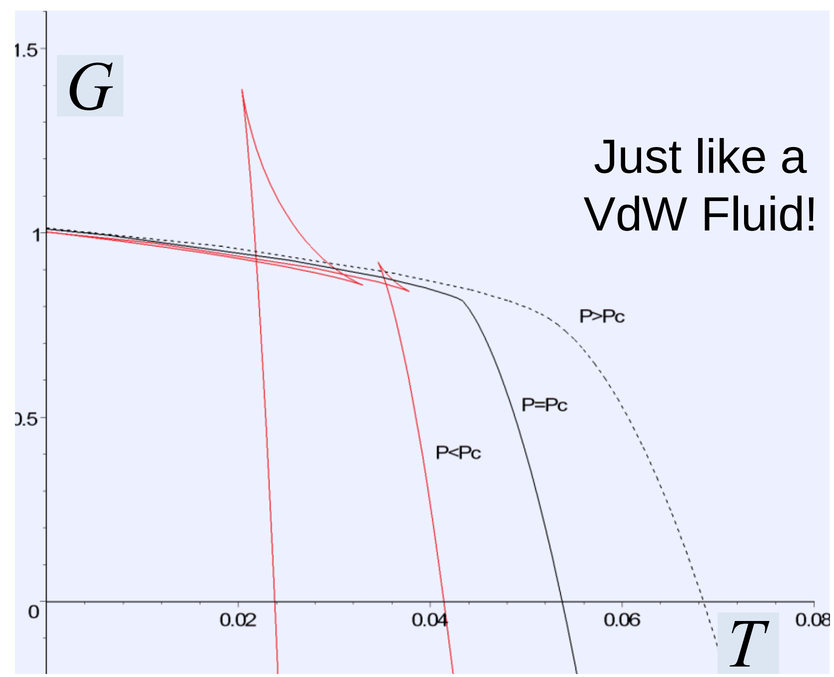
$$P = \frac{T}{v} - \frac{1}{2pv^2} + \frac{2Q^2}{pv^4}$$

Gibbs Free Energy of AdS RN BH

$$I = -\frac{1}{16\pi}\int_M \sqrt{-g}\left(R - F^2 + \frac{6}{l^2}\right) - \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{h}K - \underbrace{\frac{1}{4\pi}\int_{\partial M} d^3x\sqrt{h}\,n_a F^{ab}A_b}_{\text{Fixed Charge}} + I_c$$

$$G = G(T,P) = \frac{1}{4}\left(r_+ - \frac{8\pi}{3}Pr_+^3 + \frac{3Q^2}{r_+}\right)$$





NB: Disagree with
Chamblin et.al. PRDD60 (1999) 064018; 104026

Critical Behaviour

$$P = \frac{T}{v} - \frac{1}{2pv^2} + \frac{2Q^2}{pv^4}$$

$$\frac{\P P}{\P v} = 0 \qquad \frac{\P^2 P}{\P v} = 0$$

$$T_c = \frac{\sqrt{6}}{18pQ}$$

$$v_c = 2\sqrt{6}Q$$

$$P_c = \frac{1}{96pQ^2}$$

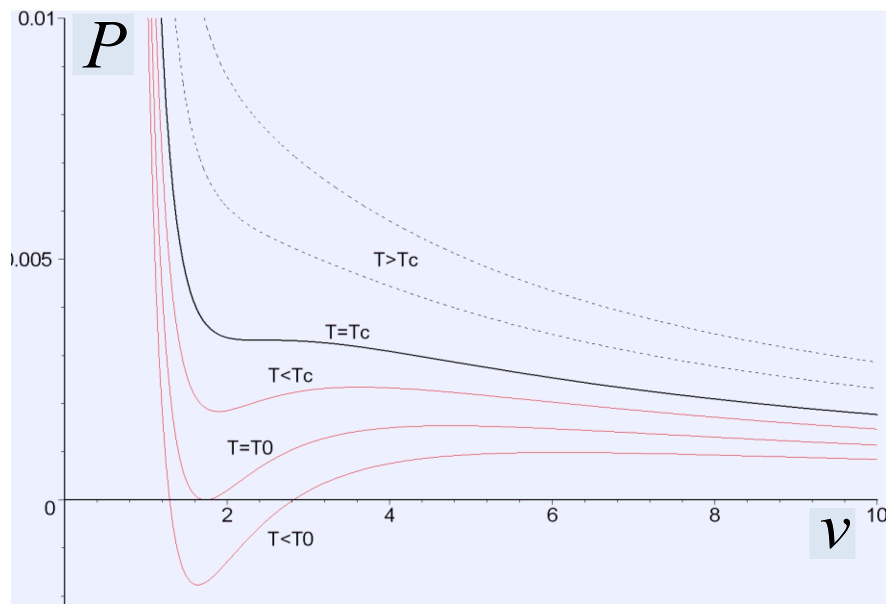
$$\frac{P_c v_c}{kT_c} = \frac{3}{8}$$

Just like a VdW Fluid!

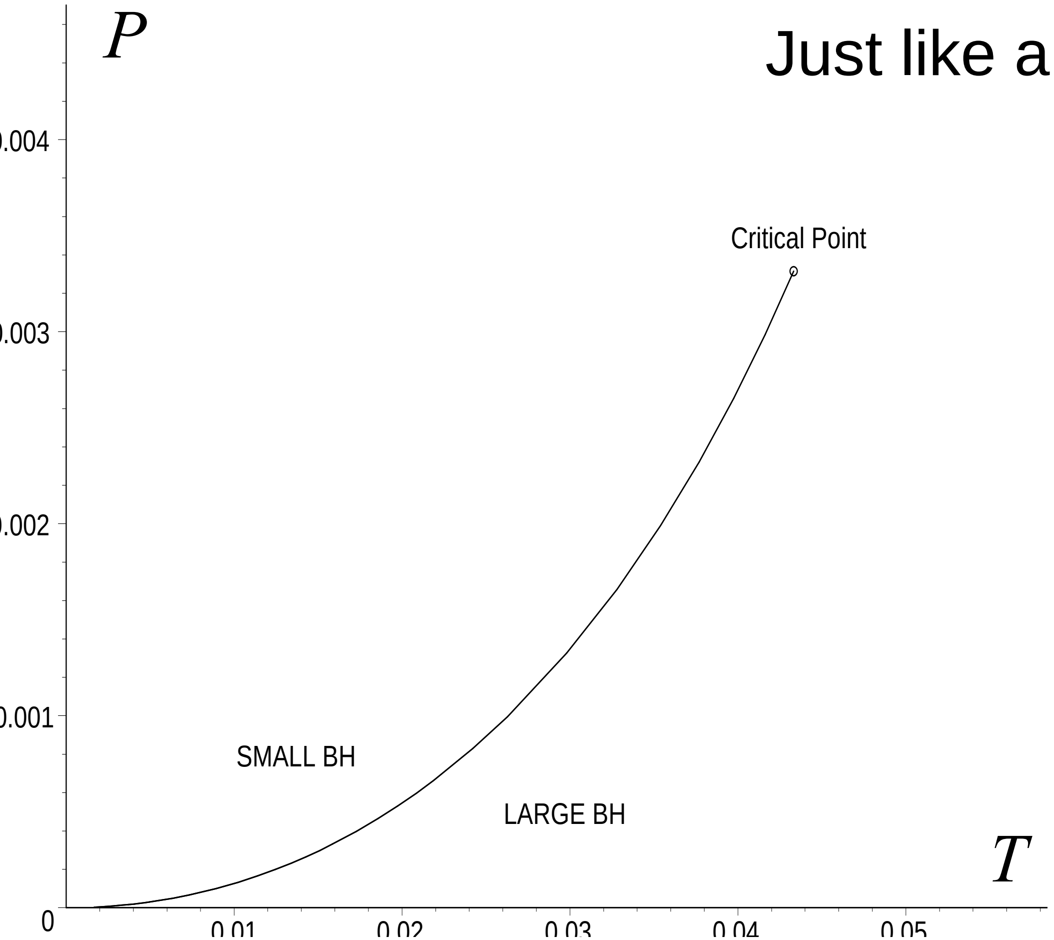
$$p = \frac{P}{P_c}, \quad n = \frac{v}{v_c}, \quad t = \frac{T}{T_c}$$

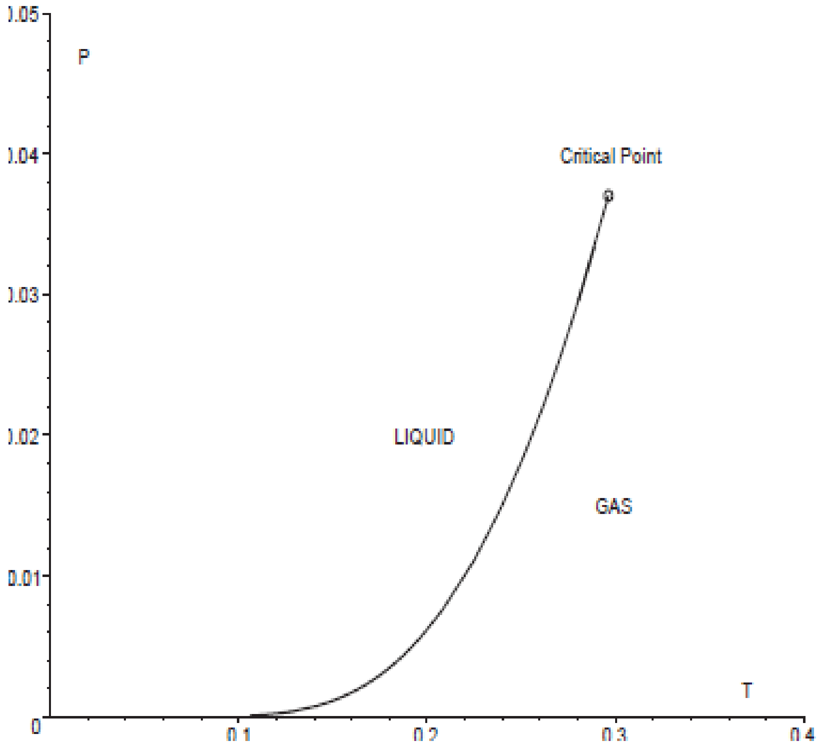
$$8t = 3n\left(p + \frac{2}{n^2}\right) - \frac{1}{n^3}$$

law of correspondin g states



Just like a VdW Fluid!





$$a = 0 \quad b = \frac{1}{2} \quad g = 1 \quad d = 3$$

govern volume, compressibility, specific heat, and pressure near the critical point

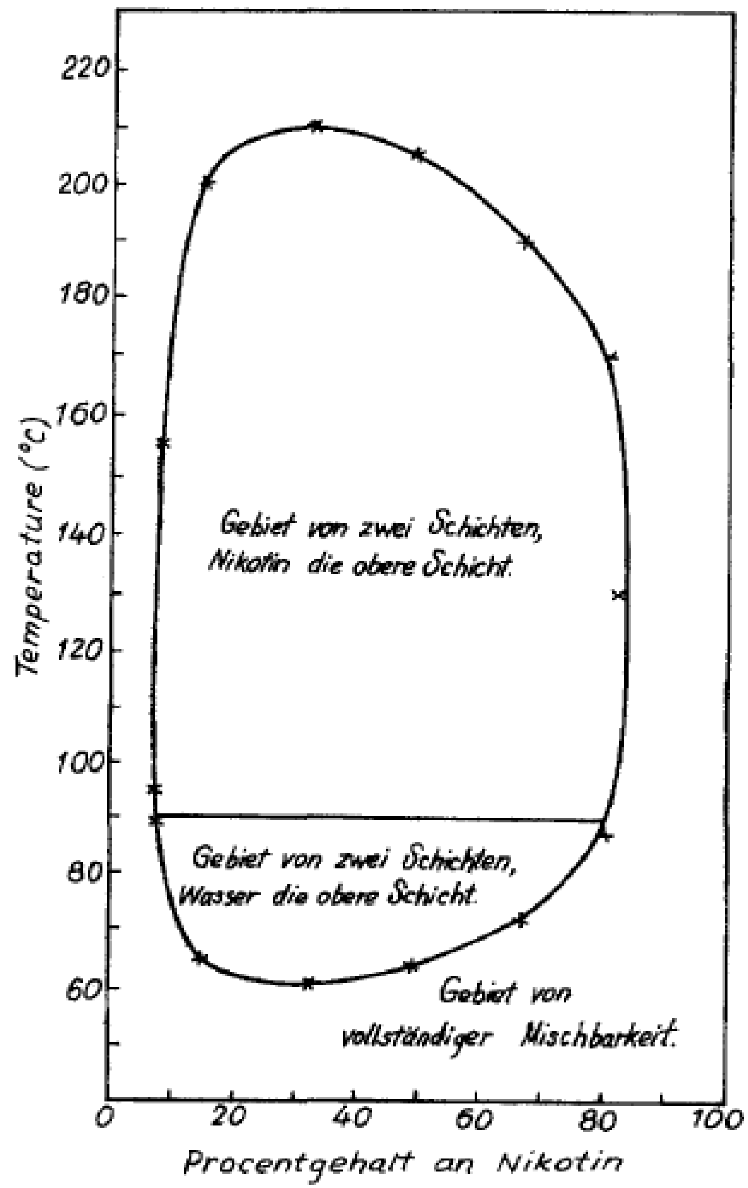
Reentrant Phase Transitions

RPT: If a **monotonic** variation of any thermodynamic quantity that results in two (or more) phase transitions such that the **final state is macroscopically similar** to the initial state.

First observed in nicotine/water

- multicomponent fluid systems
- gels
- ferroelectrics
- liquid crystals
- binary gases

T. Narayanan and A. Kumar
Physics Reports 249 (1994) 135



C. Hudson
Z. Phys. Chem. 47 (1904) 113.

Single-Rotation Black Holes

$$ds^2 = -\frac{\Delta}{\rho^2}\left[dt - \frac{a\sin^2\theta d\varphi}{\Xi}\right]^2 + \frac{\Sigma\sin^2\theta}{\rho^2}\left[a dt - \frac{\left(r^2+a^2\right)d\varphi}{\Xi}\right]^2 + \frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\Sigma}d\theta^2 + r^2\cos^2\theta d\Omega_{D-2}$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\Sigma = 1 - \frac{a^2}{l^2}\cos^2\theta \quad \Xi = 1 - \frac{a^2}{l^2}$$

$$\Delta = (r^2+a^2)\left(1+\frac{r^2}{l^2}\right) - 2mr^{5-D}$$

$$M = \frac{\omega_{D-2}}{4\pi}\frac{m}{\Xi^2}\left(1+\frac{(D-4)\Xi}{2}\right) \qquad J = \frac{\omega_{D-2}}{4\pi}\frac{ma}{\Xi^2} \qquad \Omega_H = \frac{a}{l^2}\frac{r_+^2+l^2}{r_+^2+a^2},$$

$$T = \frac{1}{2\pi}[r_+(\frac{r_+^2}{l^2}+1)\left(\frac{1}{a^2+r_+^2}+\frac{D-3}{2r_+^2}\right) - \frac{1}{r_+}] \qquad S = \frac{\omega_{D-2}}{4}\frac{(a^2+r_+^2)r_+^{d-4}}{\Xi} = \frac{A}{4}$$

$$\frac{D-3}{D-2}M = TS + \Omega_H J - \frac{2VP}{D-2}$$

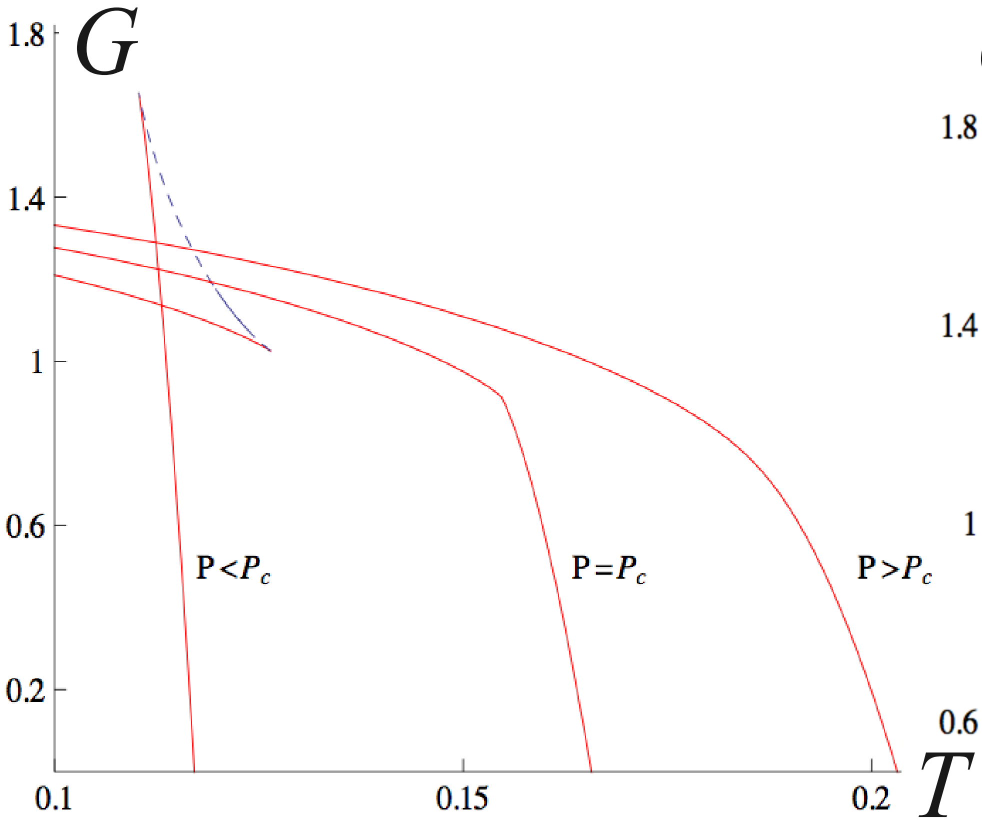
Smarr Relation

$$dM = TdS + VdP + \Omega_H dJ$$

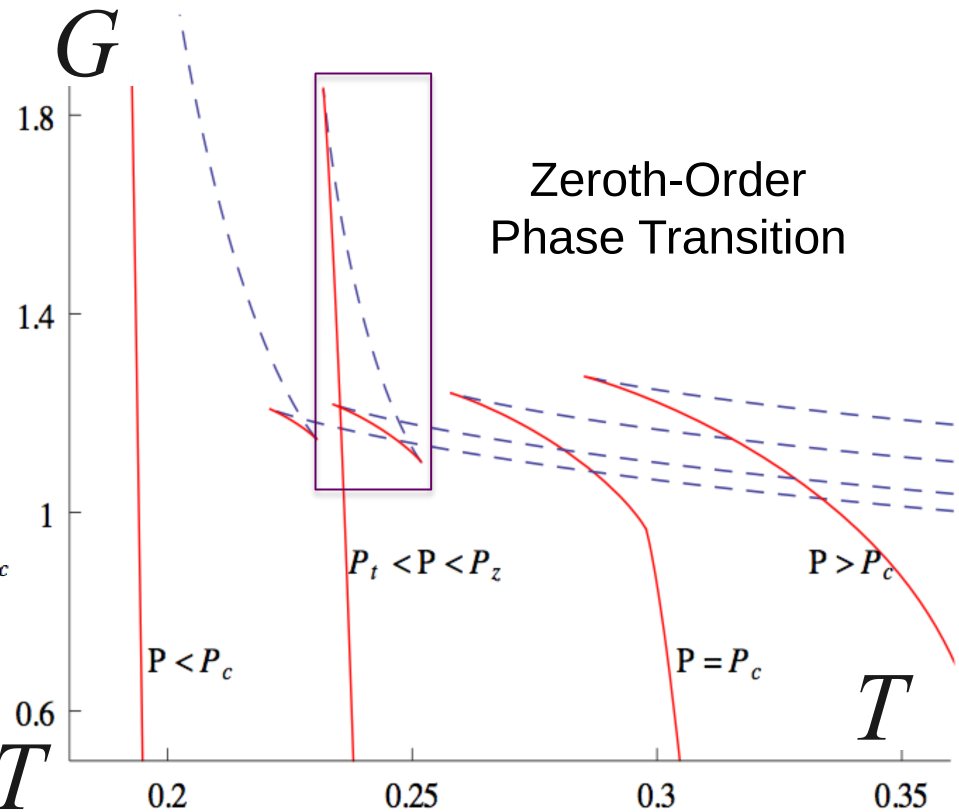
First Law

Dimensional Dependence of G

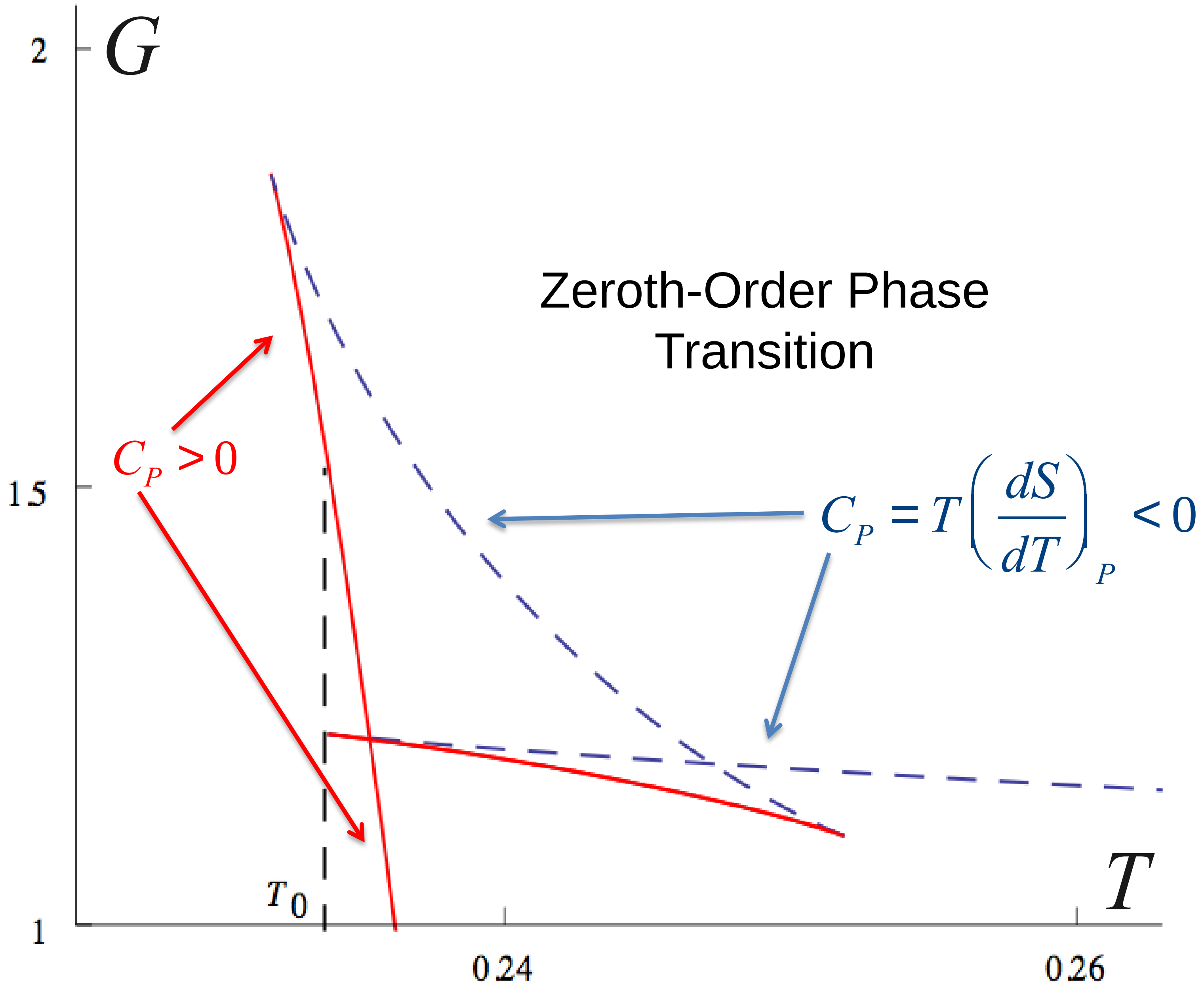
$D = 5$



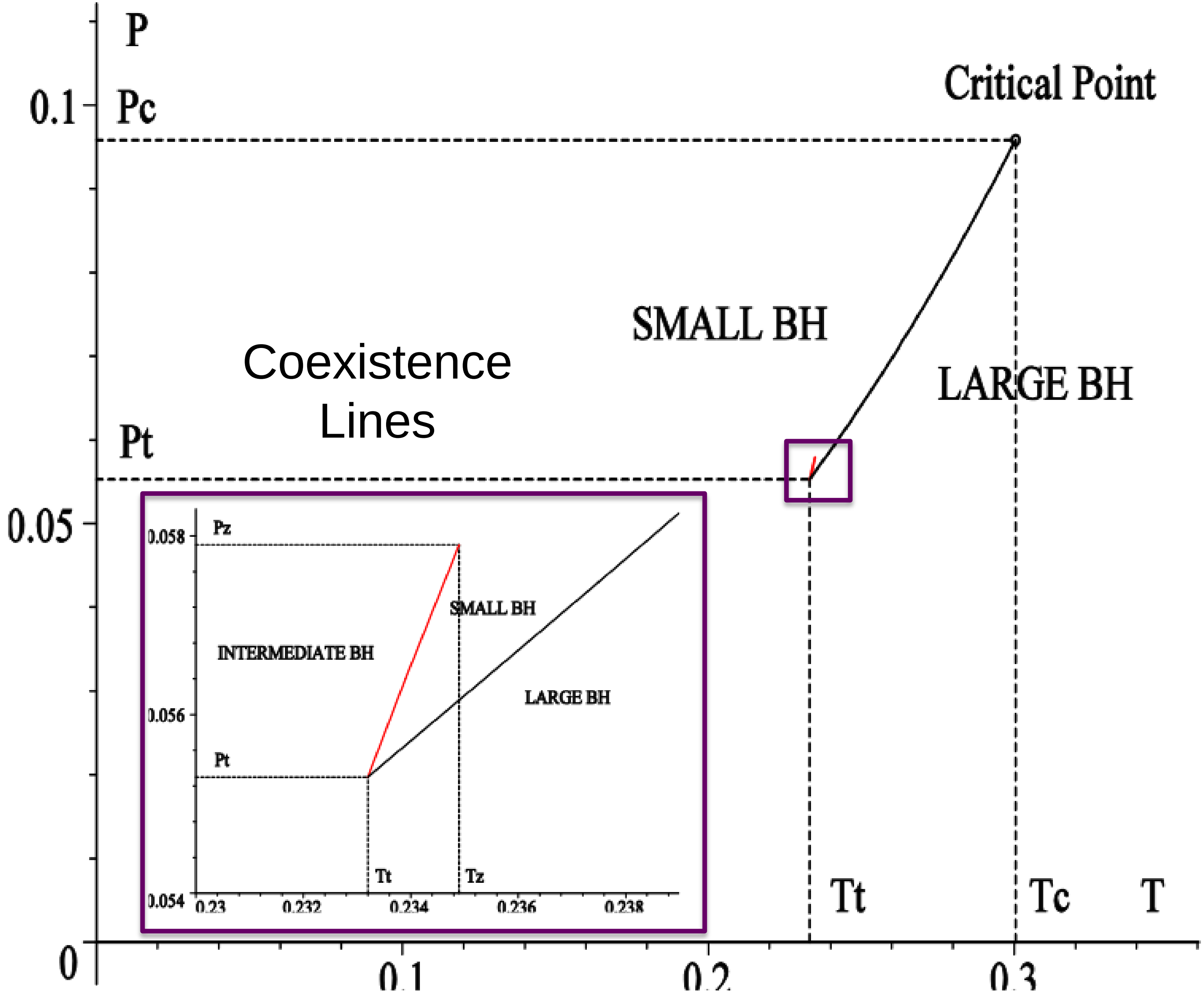
$D > 5$



Reentrant Phase Transitions in D>5



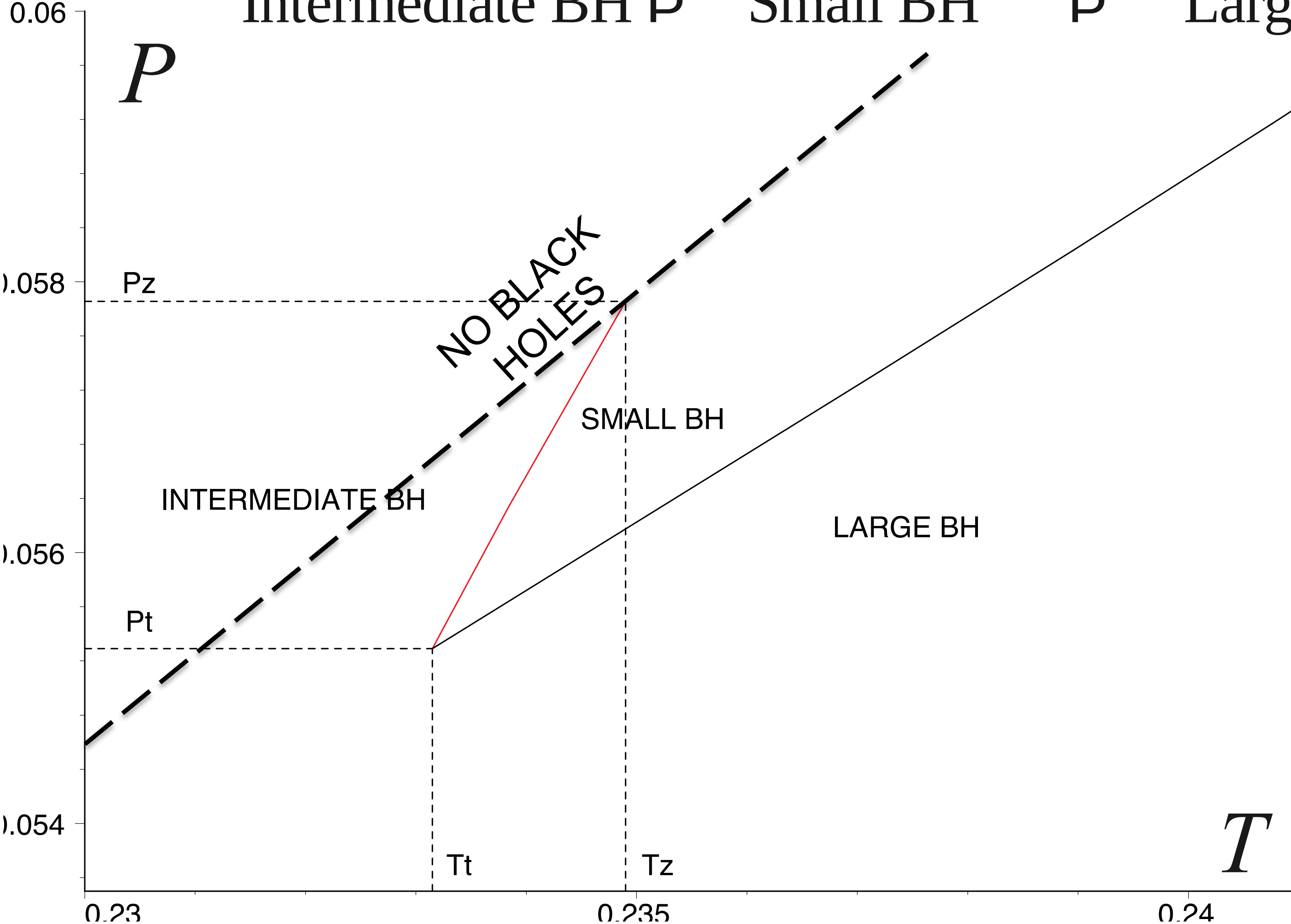
Reentrant Phase Transitions in D>5



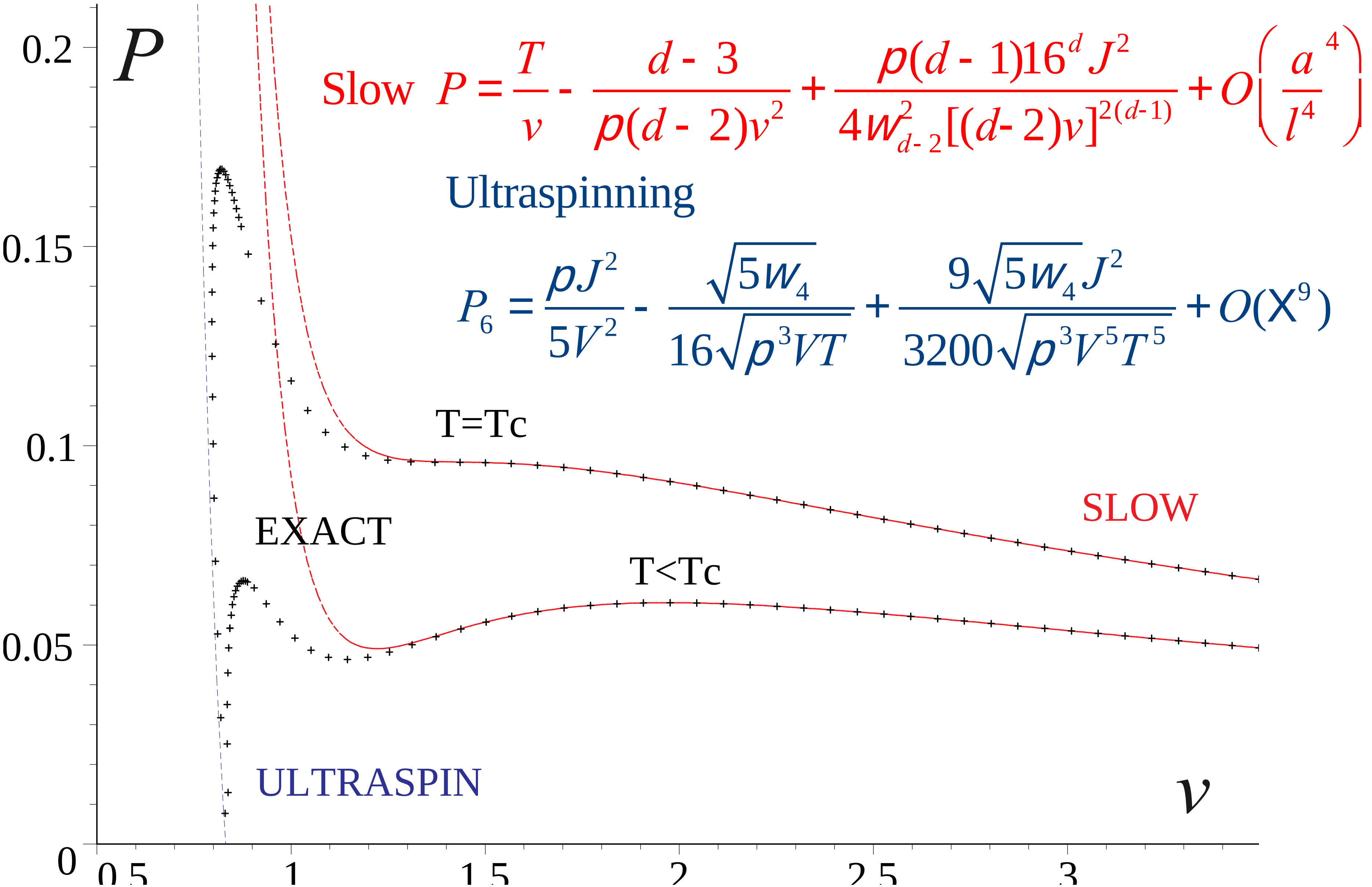
Low T Medium T High T

mixed Þ water/nicotine Þ mixed

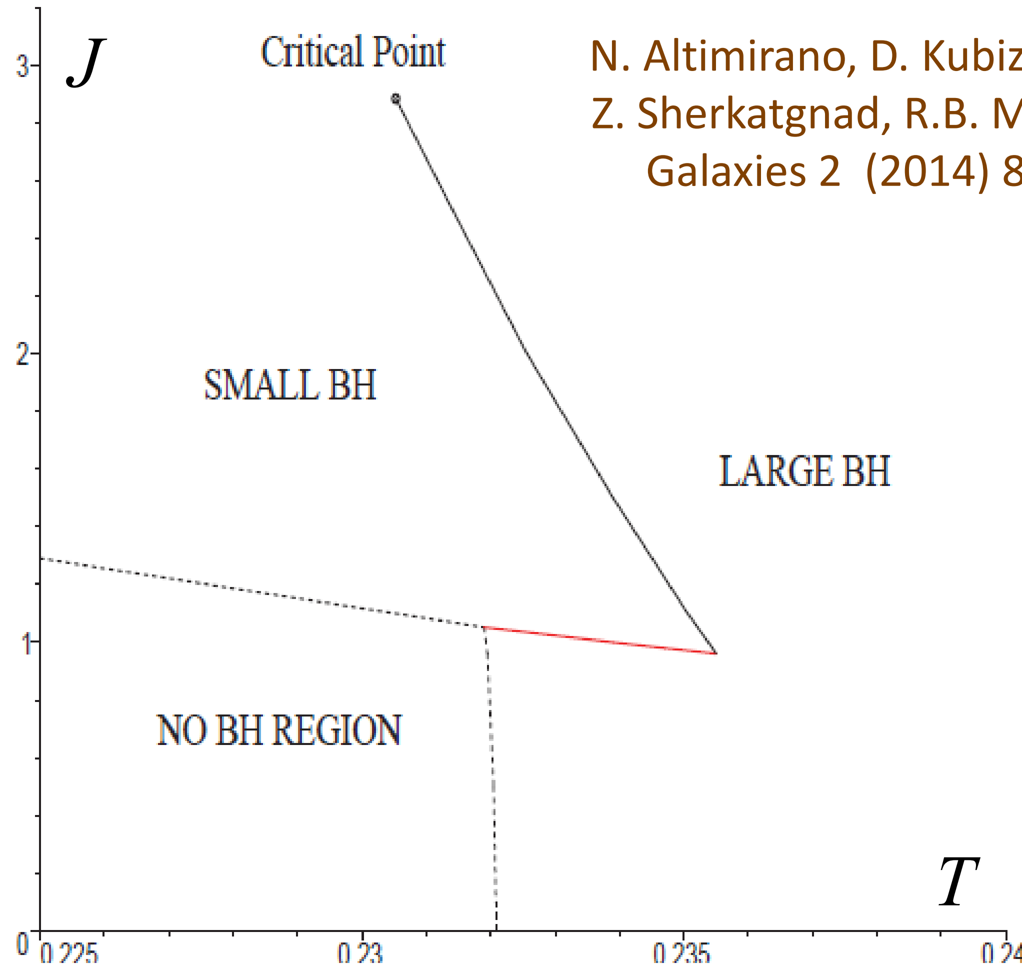
Intermediate BH Þ Small BH Þ Large BH



Slow and Ultraspinning Limits



J/T Phase Diagram



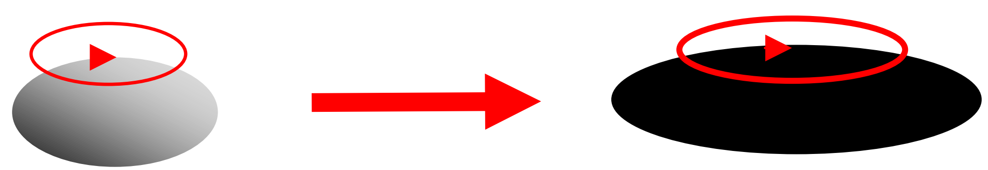
N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann Galaxies 2 (2014) 89

RPTs do not require variable cosmological constant

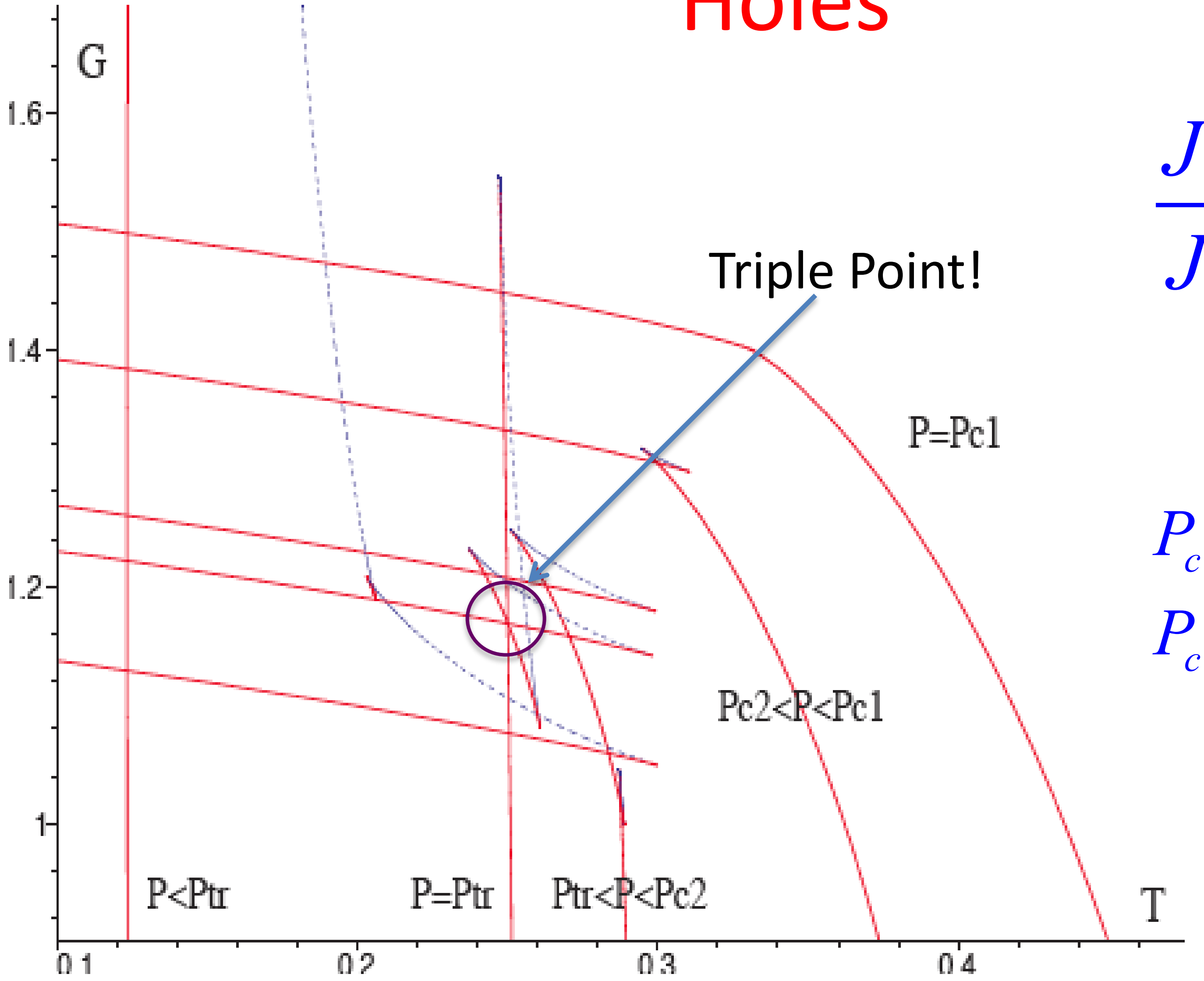
Occurs in any *d* ³ 6

“two components”

BH vs.Blackbrane?



Triple Points in Multiply Rotating Black Holes



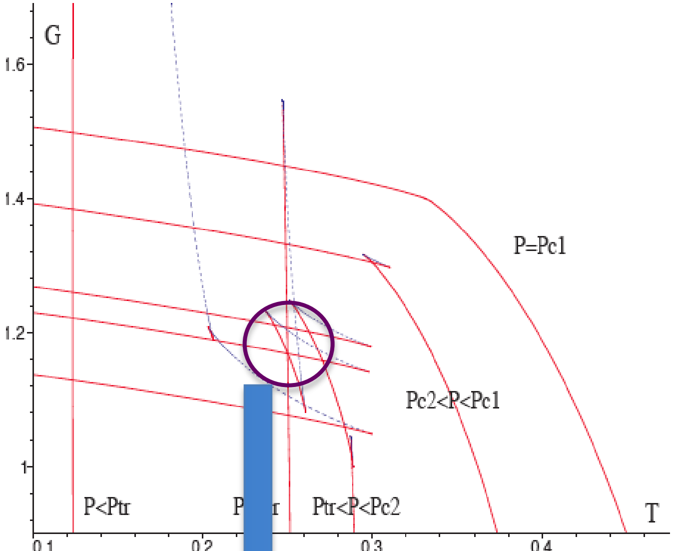
$$\frac{J_2}{J_1} = 0.05$$

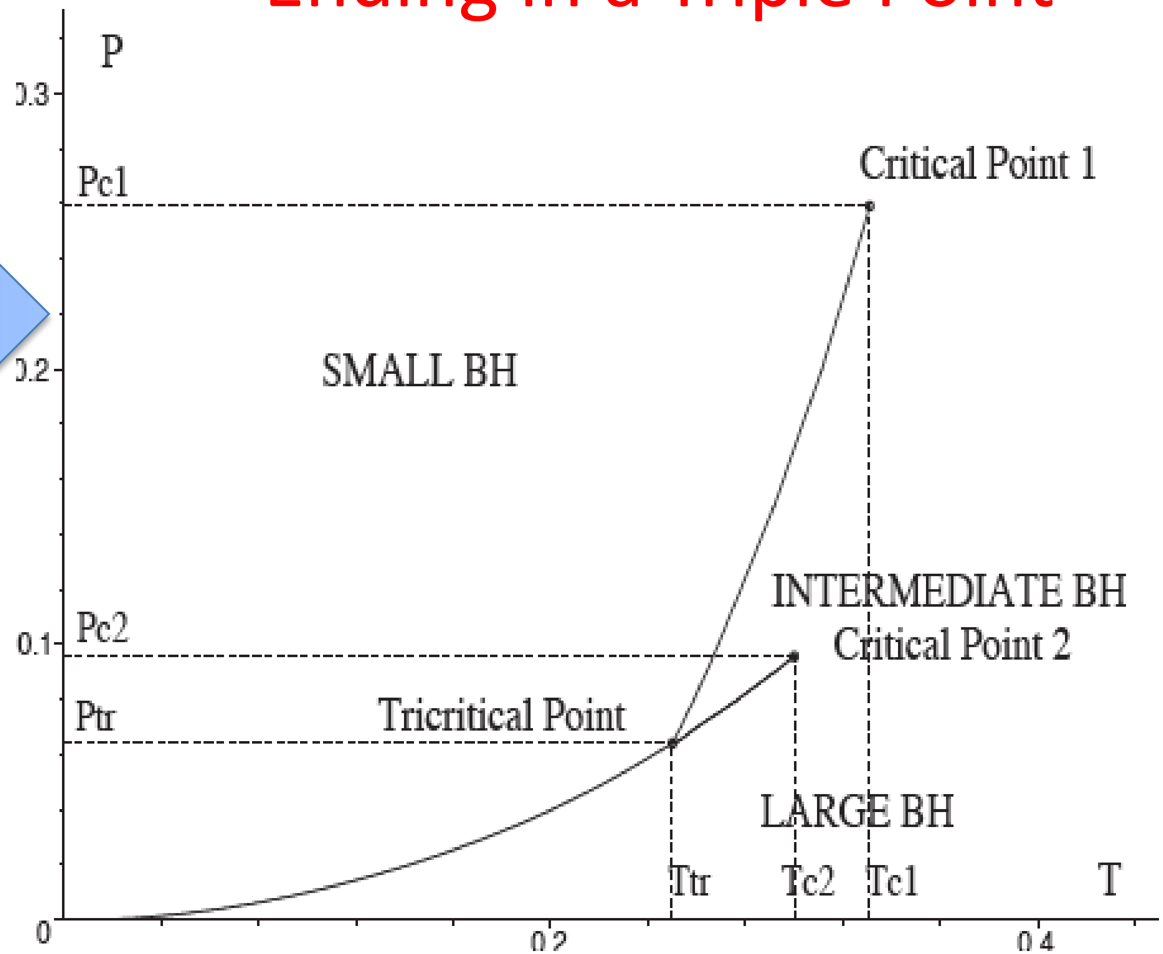
$$P_{c1} = 0.259$$

$$P_{c1} = 0.0956$$

N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann
CQG 31 (2104) 042001 arXiV:1308.2672

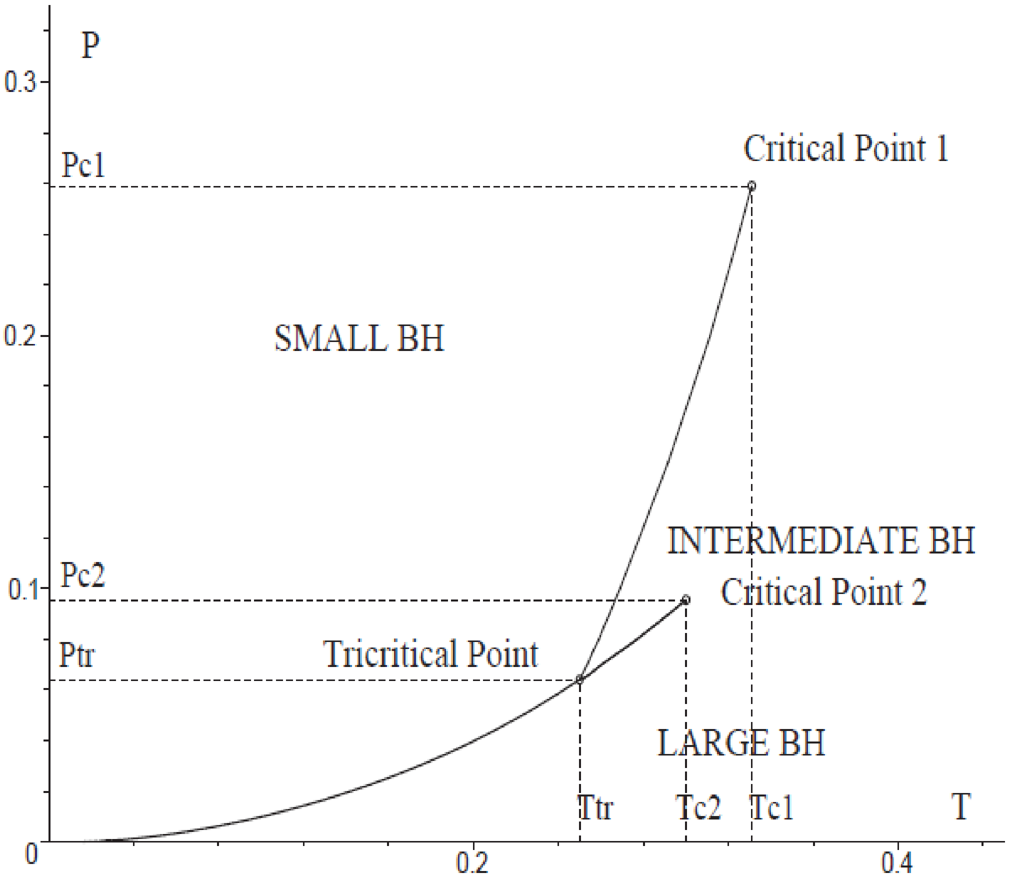
Reentrant Phase transition Ending in a Triple Point

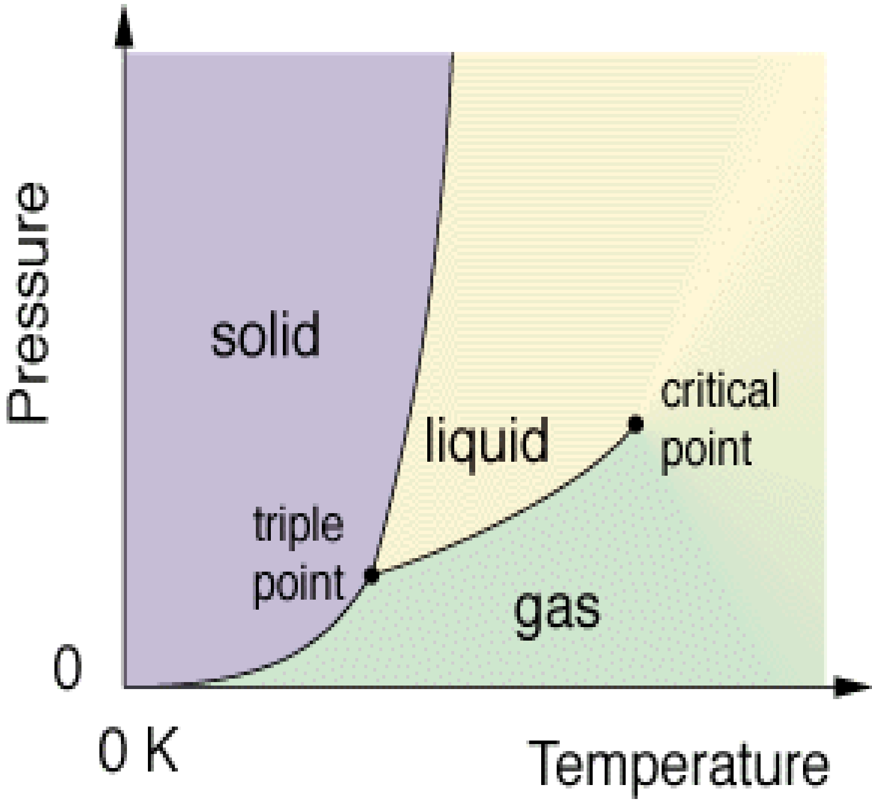




$$\frac{J_2}{J_1} = 0.05$$

The Black Hole Triple Point





N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann CQG 31 (2014) 042001

Variable L and AdS/CFT?

$$4pN = \left(\frac{\ell}{l_P}\right)^4 \qquad \left(\frac{\ell}{l_s}\right)^4 = g_{YM}^2 N$$

$$\text{Varying L} \Leftrightarrow \text{varying } N$$

$$\text{CFT} \Rightarrow g_{YM} \text{ does not run}$$

Do we get RG flow?

- Continuous variation of N_is probably OK.

- Classical gravity corresponds to $N \to \infty$
  (similar to TD limit…can vary number of moles continuously)
  Quantized N…quantum gravity effects?

- "Grand-canonical ensemble of stringy vacua" with conjugate quantity playing role of "chemical potential"?

Summary

- Cosmological Pressure can be understood as a Thermodynamic Quantity
  - First Law is modified to include a pressure-volume term
- Smarr Law needs to be modified to respect scaling
- Mass becomes Enthalpy
- "Everyday Chemical Thermodynamics" is manifest
  - Van der Waals Transitions
    - Small/large ←→ liquid/gas phase
    - Critical Phenomena analogous
  - Reentrant Phase Transitions
  - Triple Points

These phenomena do not require a variable cosmological constant!

Open Questions

- Meaning of Conjugate Volume?
- Compressibility and Stability?
- Gauge/Gravity Duality interpretation?
- More exotic black holes?
  - Lovelock Black Holes?
  - Lifshitz Black Holes?
- 1+1 Pressure-Volume?
- Meaning of de Sitter Thermodynamics?
- Other "everyday analogues"?

Mo/Li/Liu; Wei/Liu; Ballik/Lake
Dutta/Jain/Soni; Zhou/Zhang/Wang
Breton/Vergliaffa; Cai/Cao/Zhang;
Allahverdizadeh/Lemos/Sheykhi;
Lu/Pang/Pope/Vasquez-Portiz

D. Kubiznak, N. Altimirano, W. Brenna, M. Park, F. Simovic,
Z. Sherkatgnad, J. Mureika, S. Solodukhin