Examining Dynamics in AdS Spacetime in Einstein-Gauss-Bonnet Gravity

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Motivation

- AdS-CFT correspondence
- To examine gravitational collapse in higher dimensional AdS spacetime
- Wish to extend to Einstein-Gauss-Bonnet
- Is AdS spacetime stable against the formation of black holes?
Motivation

- To repeat previous work done by Buchel et al., Bizoń et al., and Garfinkle but in different coordinates
- Motivated by results of the Gauss-Bonnet work done by Deppe et al.
- To be able to simulate many bounces off of infinity before finally collapsing to form a black hole
Background: Conformal Diagrams

$\mathbb{S}^{d-1}$

$\text{AdS}_{d+1}$

$r=0$

$r=\infty$

$\text{time}$
Background: Stability

Horizon radius vs amplitude for initial data

Figure: Bizón, Rostworowski, arXiv:1104.3702
Metric:

\[ ds^2 = -N^2 dt^2 + \Lambda^2 (dx + N_r dt)^2 + R^2 d\Omega^{(n-2)} \]

Gauge fixing conditions:

\[ R = f(x) = \frac{Lx}{L - x} \]  \hspace{1cm} (1)

\[ \Lambda^{-2} = \frac{1}{f_x^2} \left( 1 - \frac{2\lambda f^2(x)}{(n-1)(n-2)} \right) \]  \hspace{1cm} (2)
Equations of motion:

\[
\psi_t = \frac{\delta H}{\delta P_\psi} = N \left[ \frac{\tilde{P}_\psi}{\Lambda} + \left( \frac{N_r}{N} \right) \psi_x \right]
\]

\[
\tilde{P}(\psi)_t = \frac{1}{f(x)^{n-2}} \frac{\delta H}{\delta \psi} = \frac{1}{f(x)^{n-2}} \left[ \frac{N}{\Lambda} f(x)^{n-2} \psi_x + \left( \frac{N_r}{N} \right) \tilde{P}_\psi \right], x
\]

Constraint Equation:

\[
M_x = f(x)^{n-2} \frac{f(x),x}{\Lambda} \left[ \frac{1}{\Lambda} \left( \frac{\tilde{P}_\psi^2 + \psi^2_x}{2} \right) + \left( \frac{N_r}{N} \right) \tilde{P}_\psi \psi_x \right]
\]

Consistency Condition:

\[
\frac{N,x}{N} = - \left[ \frac{\Lambda}{f(x),x} \right] \frac{f(x),x}{\Lambda} + \frac{f(x)^{n-2} \tilde{P}_\psi \psi_x f(x),x}{M,\left( \frac{N_r}{N} \right)} \frac{1}{\Lambda}
\]
Ignoring self-interaction (i.e. \( N_r \to 0 \) and \( N \to 1 \)) we obtain the spherically symmetric wave equation

\[
\partial_t^2 \psi = (n - 2) \frac{\partial_x f(x)}{f(x)} \partial_x \psi + \partial_x^2 \psi
\]

Assume \( \psi(x, t) = e^{i\omega t} \phi(x) \):

\[
0 = \omega^2 \phi + (n - 2) \frac{L}{x(L - x)} \partial_x \phi + \partial_x^2 \phi
\]

Which gives the asymptotic solution:

\[
\phi(x) = \phi(0) + \phi(2)(x)^2 + \phi(4)(x)^4 + \phi(6)(x)^6 + \cdots
\]
Boundary Conditions - At Infinity

- Much more subtle than at $R = 0$
- At infinity the metric must tend to the AdS metric define $z = L - x$ so that as $x \to L$, $z \to 0$ and $x \to 0$, $z \to L$
- We have chosen for our first gauge choice, $f(x) = \frac{Lx}{L-x}$

Will need to solve Bessel’s differential equation about $x = L$

\[
\Phi(t, x \approx L) = (L - x)^{n-2} \left[ \Phi(0) + \Phi(2)(L - x)^2 + \Phi(4)(L - x)^4 + \cdots \right]
\]

\[
\Pi_\psi(t, x \approx L) = (L - x)^{n-1} \left[ (\Pi_\psi)_0 + (\Pi_\psi)_2(L - x)^2 + (\Pi_\psi)_4(L - x)^4 + \cdots \right]
\]
Current State and Future Goals

- Last summer: put our equations into the code to test them → issues
- Removed the gravitational self-interaction (i.e. set $N_r = 0$) → still issues
- Now have properly finite differenced equations and fully functional code
Thanks

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