HIGH-PRECISION
HALF-LIFE AND
BRANCHING-RATIO
MEASUREMENTS FOR
THE SUPERALLOWED
$\beta^+$ EMITTER $^{26}\text{Al}^m$

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Overview

- Introduction
  - Superallowed Fermi $\beta$ decay – Why $^{26}\text{Al}_m$?
- Experiment
  - Half-life of $^{26}\text{Al}_m$
  - Branching Ratios for $^{26}\text{Al}_m$ Decay
- Results and Discussion
  - The $^{26}\text{Al}_m ft$ and $Ft$ values
  - Impact
- Future Work
Superallowed Fermi $\beta$ Decay: Corrections

$$\mathcal{F}t = ft \left( 1 + \delta'_R \right) \left( 1 + \delta_{\text{NS}} - \delta_C \right) = \frac{K}{2G_V^2 \left( 1 + \Delta^V_R \right)} = \text{constant}$$

- $\Delta^V_R$ = nucleus independent inner radiative correction: 2.361(38)%
- $\delta'_R$ = nucleus dependent radiative correction to order $Z^2\alpha^3$: ~1.4%
  - depends on electron’s energy and $Z$ of nucleus
- $\delta_{\text{NS}}$ = nuclear structure dependent radiative correction: -0.3% – 0.03%
- $\delta_C$ = nucleus dependent isospin-symmetry-breaking correction: 0.2% – 1.5%
  - strong nuclear structure dependence

$$\delta_C = \delta_{C1} + \delta_{C2} \quad \text{(isospin mixing plus radial overlap)}$$
Several approaches to ISB corrections

→ Nuclear Shell Model

→ Relativistic Hartree-Fock

→ Random Phase Approximation

→ Energy Density Functional theory
Two approaches to $\delta_{C2}$

Use radial wave functions derived from a Woods-Saxon (WS) potential, or use Hartree-Fock (HF) eigenfunctions.

Isospin-Symmetry-Breaking Corrections

Two approaches to \( \delta_{C2} \)

Use radial wave functions derived from a Woods-Saxon (WS) potential, or use Hartree-Fock (HF) eigenfunctions.

\[
\langle \bar{F}t \rangle = 3071.81 \pm 0.79_{\text{stat}} \pm 0.27_{\text{syst}} \text{ s}
\]

I.S. Towner and J.C. Hardy, Physical Review C 77, 055501 (2008)

J.C. Hardy and I.S. Towner, Physical Review C 79, 055502 (2009)
\[ ft = 3036.9(5)f(9)_{T_{1/2}} \text{s} \]

\[ Ft = 3072.4(6)\delta_R(8)\delta_{C-NS}(9)ft\text{ s} \]

\[ \delta_{C-NS} = 0.305(27)\% \]

J.C. Hardy and I.S. Towner, Physical Review C 79, 055502 (2009)
TRIUMF: Canada’s National Laboratory for Nuclear and Particle Physics Research
Measuring Superallowed Half Lives

- **Beam**
  - **Cycle**
    - Implant 6-14 s
    - Allow $^{26}\text{Na}$ to decay 26-34 s
    - Move tape to detector and count $^{26}\text{Al}^m$ decays for ~20, 25, 30 half-lives, then repeat.
    - Change detector voltage, discriminator setting, and swap fixed, nonextendable dead times between two MCS units to investigate systematic effects.

- $^{26}\text{Al}^m$: $T_{1/2} = 6.3465$ s
- $^{26}\text{Na}$: $T_{1/2} = 1.072$ s
- $^{26}\text{Al}^g$: $T_{1/2} = 7.4 \times 10^5$ yrs
Counts per 500 ms

\[ \chi^2 / \nu = 0.81 \]

\[ T_{1/2}(^{26}\text{Al}^m) = 6.344(4) \text{ s} \]

41 cycles

Fractional uncertainty = 0.077\%
$T_{1/2}^{^{26}\text{Al}} = 6.34649 \pm 0.00046 \text{ s}$

$\chi^2/\nu = 1.020$

Stat. uncertainty = 0.007%
Assigning a systematic uncertainty

\[ \sigma = \sqrt{2.67 \sigma_{\text{stat.}}} = 0.75 \text{ ms} \]

\[ \sigma_{\text{stat.}} = 0.46 \text{ ms} \]

\[ \sigma_{\text{syst.}} = 0.59 \text{ ms} \]

\[ \sigma_{\text{dead}} = 0.045 \text{ ms} \]

Two independent analyses

1) 6346.59 ms
2) 6346.49 ms

\[ \sigma_{\text{ind}} = 0.05 \text{ ms} \]

\[ T_{1/2} = 6346.54(76) \text{ ms} \]
Comparison with previous results

New world average: $6.34643(68)$ s

This work: $6.34654(76)$ s

$\chi^2/\nu = 0.45$

The $8\pi$ Spectrometer and SCEPTAR at ISAC-I

- 20 Compton-Suppressed HPGe detectors
- 20 plastic scintillators
- 8$\pi$ Spectrometer
- Mass 26 beam from ISAC
- Tape transport

SCEPTAR
Branching Ratios for $^{26}\text{Al}^m$ Decay

$^{26}\text{Na}$

$Q_{\beta^-} = 9312$

$Q_{EC}(3^+) = 291.1$ keV

$Q_{EC}(0^+)_1 = 644.1$ keV

$Q_{EC}(2^+_2) = 1294.3$ keV

$Q_{EC}(2^+_1) = 2424.0$ keV

$Q_{EC}(0^+) = 4232.66(12)$ keV

$^{26}\text{Mg}$

$1.072$ s

$3^+$

$0^+$

$2^+$

$2133$ keV [38%]

$1003$ keV [62%]

$1780$ keV

$1130$ keV [90%]

$2938$ keV [10%]

$1809$ keV

$0^+$

$5^+$

$1228$ keV

$0$

$7.17 \times 10^5$ yrs

$Q = 3.4643$ s

$^{26}\text{Al}$

$228.3$
Cycle Structure

First a 2s tape move

4s background counting

Implant for 21s

Start of photopeak time structure analysis window

Start of late times analysis window

End of analysis time windows

β counts / 0.2 s

Time (s)
Determining $^{26}\text{Al}^m$ non-analog intensity

$^{2561(140)}\text{Al}^m$ counts

$^{1809\text{keV}}\gamma$-ray activity

Fit

$^{26}\text{Na}$ activity

$^{26}\text{Al}^m$ activity

Background activity
Determining $^{26}\text{Al}^m$ non-analog intensity

$^{26}\text{Al}^m$ counts

$17163(202)$

$-1.0 +/- 10\text{ppm}$

$^{26}\text{Al}^m$ 1809 BR
Fit 1809 keV peak area vs. time with $^{26}\text{Na}$ and $^{26}\text{Al}^m$ components.
Fit 1809 keV peak area vs. time with $^{26}$Na and $^{26}$Al$^m$ components.
Fit 1809 keV peak area vs. time with $^{26}\text{Na}$ and $^{26}\text{Al}^m$ components.
Fit 1809 keV peak area vs. time with $^{26}\text{Na}$ and $^{26}\text{Al}_m$ components.
Figure: Fit 1809 keV peak area vs. time with $^{26}$Na and $^{26}$Al$^m$ components.
158(186) $^{26}\text{Al}^m$ counts

$\chi^2/\nu = 1.03$

5.5+/-6.5ppm $^{26}\text{Al}^m$ BR
**26Al** Non-Analog Branching Ratios

All measured BR consistent with zero

Total non-analogue decay:

- 5.5+/−6.5 ppm (peak area vs. time)
  - ≤ 10 ppm @ 67% CL
  - ≤ 15 ppm @ 90% CL

Direct feeding of 1809 keV:

- -0.9+/−5.7 ppm (late time analysis)
  - ≤ 5 ppm @ 67% CL
  - ≤ 12 ppm @ 90% CL

\[ ft = \frac{ft_{\frac{1}{2}} (1 + P_{EC})}{SBR} \]

\[ P_{EC} = 0.082 \% \]

\[ ft = 478.237(80) \]

\[ t_{\frac{1}{2}} = 6346.43(68) \text{ ms} \]

\[ SBR = 100.0000_{-0.0015}^{+0} \% \]

\[ ft = 3037.58(51) f(32)_{T_{\frac{1}{2}}} (5)_{BR} \text{ s} \]

Most precise \( ft \) for any superallowed emitter
$\delta'_R = 1.478(20) \%$

$\delta_{NS} = 0.005(20) \%$

$\delta_C = 0.310(18) \%$

$\mathcal{F}t = \mathcal{F}t(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)$

$= 3073.1(6)\delta'_R, (8)\delta_C - \delta_{NS}(6)\mathcal{F}t$ s

$\mathcal{F}t_{26Al^{m}} = 3073.1(12)$ s

$\mathcal{F}t_{\text{other}} = 3072.0(10)$ s

Based on J.C. Hardy and I.S. Towner, Physical Review C 79, 055502 (2009)
$\delta'_{R} = 1.478(20) \%$

$\delta_{NS} = 0.005(20) \%$

$\delta_{C} = 0.440(51) \%$

$Ft = ft(1 + \delta'_{R})(1 + \delta_{NS} - \delta_{C})$

$= 3069.1(6)\delta'_{R}(17)\delta_{C} - \delta_{NS}(6)ft \ s$

$Ft_{26Al^m} = 3069.1(19) \ s$

$Ft_{other} = 3072.3(10) \ s$

Based on J.C. Hardy and I.S. Towner, Physical Review C 79, 055502 (2009)
“Experimental” $\delta_C$

$$\delta_C = 1 + \delta_{NS} - \frac{\mathcal{F} t \,(^{26}\text{Al}^m)}{ft \,(1 + \delta'_R)}$$

-Woods-Saxon $\delta_C$ continue to form an impressively consistent set

-Hartree-Fock $\delta_C$ do not exhibit the same degree of conformity
$\bar{Ft}$ ($^{26}\text{Al}^m$)
WS: 3073.0(12) s

$\bar{Ft}$ (no $^{26}\text{Al}^m$)
WS: 3072.0(10) s
\( \mathcal{F}_t \left( ^{26}\text{Al}^m \right) \)

WS: 3073.0(12) s

HF: 3069.0(19) s

\( \overline{\mathcal{F}_t} \ (\text{no} \ ^{26}\text{Al}^m) \)

WS: 3072.0(10) s

HF: 3072.3(10) s
Hartree-Fock vs. Woods-Saxon and World-Averaged $\mathcal{F}t$

$\mathcal{F}t$ ($^{26}\text{Al}^m$)
WS: 3073.0(12) s  
HF: 3069.0(19) s

$\overline{\mathcal{F}t}$ (no $^{26}\text{Al}^m$)
WS: 3072.0(10) s  
HF: 3072.3(10) s

$\overline{\mathcal{F}t}$ (with $^{26}\text{Al}^m$)
WS: 3072.38(75) s  
HF: 3071.59(87) s

Systematic uncertainty  
$= 0.79$ s
Further tests of ISB corrections??????

- Need to test superallowed corrections but independent of superallowed data!

- Want to avoid assuming CVC
**T=1/2 Mirror Nuclei**

- Dominated by experimental uncertainties.
- Also require $\delta_C$ corrections.
- Different nuclear structure than superallowed

\[ A_{SM} = \frac{-\lambda_{JJ} \rho^2}{1 + \rho^2} = 2Ft_{\text{mirror}}^0 \left( 1 + \frac{f_A}{f_V} \rho^2 \right) = 2Ft_{0^+\rightarrow0^+} = \frac{K}{G_F^2 V_{ud}^2 (1 + \Delta_R^V)} \]

Mirror $ft$ values at TRIUMF

Approved Experiments:

S1192 – Half-life and BR, $^{19}$Ne
S1385 – Half-life for $^{21}$Na
S1517 – Half-life and Q-value for $^{35}$Ar

Q-value measurement @ TITAN

$Q_{EC} = m_m - m_d$

- Precision advantage
  - PRL 107, 272501 (2011)
  - $\frac{\Delta m}{m} = \frac{1}{2\pi \nu}
  - @ BTN

- Threshold charge breeding for isotopic separation

He-like charge state

$^{35}$Ar$: (SCI from ISAC)$

$^{20}$O$: (SCI isotopically pure from OLI)$

Description of $^{35}$Ar half-life measurement

$T_{1/2}(^{35}$Ar$) = 1.7754 \pm \varepsilon$

- $^{37}$K: $T_{1/2} = 64.761 \pm 0.004$ (2003)
- $^{39}$Na: $T_{1/2} = 1.0546 \pm 0.004$ (2005)
- $^{41}$Ca: $T_{1/2} = 116.121 \pm 0.003$ (2008)
- $^{39}$K: $T_{1/2} = 924.46 \pm 0.46$ (2010)
- $^{41}$Ar: $T_{1/2} = 6.34654 \pm 0.00004$ (2011)
- $^{41}$Ca: $T_{1/2} = 1.672 \pm 0.001$ (in preparation)
- $^{40}$Ca: $T_{1/2} = 19.29 \pm 0.01$ (in preparation)

Cycle Structure

- Implant ~1 s.
- Move tape (2 s) to detector and count $^{35}$Ar decays for between 15 and 25 half-lives, then repeat.
- Change detector voltage, discriminator setting, dwell times, and swap fixed, nonextendable dead times between two MCA units to investigate systematic effects. → Many cycles.
Mirror $f_t$ values at TRIUMF
High-precision half-life and branching-ratio measurements for $^{26}\text{Al}^m$, carried out at TRIUMF, have resulted in the most precise superallowed $ft$ and $Ft$ values for any superallowed emitter to date.

This unrivaled precision for the $^{26}\text{Al}^m ft$ and $Ft$ values yields one of the most demanding consistency tests of leading isospin-symmetry-breaking corrections for these decays, required in order to extract $V_{ud}$, and currently a leading source of uncertainty.

Going forward, $ft$-value measurements for the isospin $T=1/2$ mirror nuclei offer an excellent opportunity to test and refine these calculations, with the goal of improving the uncertainty in $V_{ud}$ and further constraining physics beyond the Standard Model.
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The Cabibbo-Kobayashi-Maskawa (CKM) matrix

The CKM matrix plays a central role in the Standard Model
describes the mixing of different quark generations:
weak interaction eigenstates ≠ quark mass eigenstates

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
d' \\
s' \\
b'
\end{pmatrix}
= 
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}
\]

\[|V_{ud}| = \frac{G_V}{G_F}\]

In the Standard Model the CKM matrix
describes a unitary transformation.

\[V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1\]

The first row of the CKM matrix provides the most
demanding experimental test of this unitarity condition.
To first order, $\beta$ decay $ft$ values can be expressed as:

$$ft = \frac{K}{|M_{fi}|^2 g^2}$$

phase space (Q-value) \hspace{2cm} \text{constants} \hspace{2cm} \text{Weak coupling strength}

half-life, branching ratio \hspace{2cm} \text{matrix element}

For the special case of $0^+ \rightarrow 0^+$ (pure Fermi) $\beta$ decays between isobaric analogue states (superallowed) the matrix element is that of an isospin ladder operator:

$$|M_{fi}|^2 = (T - T_Z)(T + T_Z + 1) = 2 \hspace{0.5cm} \text{(for } T=1)$$

$$G_V^2 = \frac{K}{2 ft} \hspace{2cm} |V_{ud}| = \frac{G_V}{G_F}$$

Vector coupling constant \hspace{2cm} Fermi coupling constant
Superallowed $ft$-values

\[ ft \approx \frac{K}{2 G_v^2} = \text{constant (CVC)} \]

J.C. Hardy and I.S. Towner, Physical Review C 79, 055502 (2009)
\[ Q \text{-value established to 0.003\%} \]  
\[ T_{1/2} \text{ dominant uncertainty in ft.} \]  
\[ \text{Superallowed BR is } \sim 100\% \]

Counting the number of $\beta$ particles

Read out event by event

Energies → Low-energy cut

Times → Gate on good events

Distinguish $\beta$ and $\beta\gamma$ triggers → Scale down $\beta$ singles
SCEPTAR Efficiency

\[
\text{Beta Eff.} = 76.8(16)\% \\
\text{2\% uncertainty}
\]

\begin{align*}
\text{Ground State, } Q_{\beta^+} &= 3210.66 \text{ keV} \\
1809 \text{ keV, } &Q_{\beta^+} = 1402.96 \text{ keV} \\
2938 \text{ keV, } &Q_{\beta^+} = 272.32 \text{ keV}
\end{align*}

\[\beta^+ \text{ efficiency relative to } \beta^- \text{ efficiency for } ^{26}\text{Na} \text{ (1809 keV)}\]

\[\text{Beta Eff.} = 76.8(16)\%\]
$8\pi$ Efficiency

\[ \ln(\epsilon) = \sum_{i=0}^{8} a_i (\ln E)^i \]

\[ \text{Eff.}(1809 \text{ keV}) = 0.750(15)\% \]
Transitions to other excited states dominated by EC, so beta-anti-coincidence gamma-ray data used in this analysis at late times in the cycle.

<table>
<thead>
<tr>
<th>Level (J^π, E)</th>
<th>γ ray (keV)</th>
<th>Peak area (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3_1^+ , 3942 keV</td>
<td>1003</td>
<td>11(12)</td>
</tr>
<tr>
<td>0_1^+ , 3589 keV</td>
<td>2133</td>
<td>-10(6)</td>
</tr>
<tr>
<td>2_2^+ , 2938 keV</td>
<td>1130</td>
<td>3(10)</td>
</tr>
<tr>
<td>5^+ , ^{26}Al^9</td>
<td>228</td>
<td>3(15)</td>
</tr>
</tbody>
</table>
The Radial Overlap Correction: $\delta_{C2}$

J.C. Hardy and I.S. Towner, Physical Review C 79, 055502 (2009)
Z-dependence in Radial Overlap Correction
The Resulting Precision in $G_V$

Prior to this work:  

$$G_V^2 = \frac{K}{2(1 + \Delta^V_R)Ft} ,$$

$$G_V^2 / (\hbar c)^6 = 1.29126(33)_{\text{stat.}} (11)_{\delta C} (48)_{\Delta^V_R} \times 10^{-10} \text{ GeV}^{-4} ,$$

$$G_V / (\hbar c)^3 = 1.13633(15)_{\text{stat.}} (5)_{\delta C} (21)_{\Delta^V_R} \times 10^{-5} \text{ GeV}^{-2} ,$$

$$= 1.13633(26) \times 10^{-5} \text{ GeV}^{-2} .$$

Following this work:

$$G_V^2 / (\hbar c)^6 = 1.29118(32)_{\overline{F}_{\text{stat.}}} (17)_{\overline{F}_{\text{syst.}}} (48)_{\Delta^V_R} \times 10^{-10} \text{ GeV}^{-4}$$

$$G_V / (\hbar c)^3 = 1.13630(14)_{\overline{F}_{\text{stat.}}} (8)_{\overline{F}_{\text{syst.}}} (21)_{\Delta^V_R} \times 10^{-5} \text{ GeV}^{-2} ,$$

$$= 1.13630(27) \times 10^{-5} \text{ GeV}^{-2}$$