Critical Phenomena in Higher Dimensional Gravity Using Adaptive Mesh Refinement

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Initial Data:

\[ \psi(R, t = 0) = AR^\delta \exp \left[ -\left( \frac{R - R_0}{B} \right)^2 \right] \]

- For \( A > A^* \) black holes form
- For \( A < A^* \) matter disperses
- Near criticality geometrical quantities scale as\(^1\):
  \[ \ln(M) = \gamma \ln(A - A^*) + f(A - A^*) \]
  \( f \) is periodic
- Both \( \gamma \) and period depend on \( n \)
- Echoing:
  \[ \psi(Re^\Delta, te^\Delta) = \psi(R, t) \]

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\(^1\) Choptuik, Phys. Rev. Lett. 70, 9 (1993)
Introduction

Gravitational collapse in GR and other theories
Higher dimensions interesting for several reasons:
- Asymptotic limit of critical exponent
- AdS/CFT correspondence
- Other higher dimensional theories

Problems in higher dimensions
- Stability near $R = 0$
- Horizon radii decrease
- Time to formation increases
\[
\gamma = 0.373 \pm 0.001, \quad \Delta = 3.45 \pm 0.03 - \text{Cusps for } \Delta \\
\text{Agree with accepted values}\textsuperscript{2}
\]

A Much Closer Look at Cusps

Figure: Zoom in of cusp on a scaling plot in 4 dimensions. Similar behaviour in higher dimensions.
Nature of Cusps

Coordinate Time, $t$

Areal Radius, $R$

$t = \text{constant slice}$

Use generalized flat-slice (PG) coordinates

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Higher Dimensional Critical Phenomena

(6/12)
Results - 5D

Radius Plot for GR in 5 Dimensions

\[ 0.417 \pm 0.001 \]
\[ \Delta = 3.27 \]

0.408 \pm 0.008 \quad \text{Sorkin & Oren, Phys. Rev. D 71, 124005 (2005)}

0.412 \pm 0.004 \quad \text{Bland et al., Classical Quantum Gravity 22, 5355 (2005)}

0.413 \pm 0.002 \quad \text{Taves & Kunstatter(2011). PhysRevD.84.044034}

0.416 \pm 0.002 \quad \text{Taves & Kunstatter(2011). PhysRevD.84.044034}
Results - 6D

Radius Plot for $\tilde{\alpha}=0e+00$ in 6 Dimensions

$0.428 \pm 0.003$

$\Delta = 2.99 \pm 0.04$

0.422 ± 0.008  Sorkin & Oren, Phys. Rev. D 71, 124005 (2005)
0.430 ± 0.003  Bland et al., Classical Quantum Gravity 22, 5355 (2005)
0.424           Garfinkle, Cutler, & Duncan, Phys. Rev. D 60, 104007 (1999)
0.429 ± 0.003  Taves & Kunstatter(2011). PhysRevD.84.044034
0.428 ± 0.002  Taves & Kunstatter(2011). PhysRevD.84.044034
Preliminary Results - 7D

Radius Plot for $\alpha=0e+00$ in 7 Dimensions

$0.440 \pm 0.003$

$0.440 \pm 0.003$

$\Delta = 2.7 \pm 0.1$

0.429 ± 0.008  Sorkin & Oren, Phys. Rev. D 71, 124005 (2005)
0.441 ± 0.007  Bland et al., Classical Quantum Gravity 22, 5355 (2005)
0.440 ± 0.005  Taves & Kunstatter (2011). PhysRevD.84.044034
0.440 ± 0.006  Taves & Kunstatter (2011). PhysRevD.84.044034
Numerical Techniques

- Adaptive mesh refinement
- 6th order in space and time - $E_{ADM}$
- Dissipation applied as filter (near $R = 0$)
- l’Hôpital’s trick for stability:

$$\frac{f}{R} = f_{,R} - R(f/R)_{,R}$$

- Variable time step size
- Echoing period decreases in higher $n$...
  Smaller time steps: $\rho_t > \rho_s$ sufficient?
  Decrease time step early? Horizon function?
Investigating critical phenomena in higher $D$ poses challenges:

- Stable equations - l’Hôpital’s Trick & variable time stepping
- Sufficient resolution - Adaptive Mesh Refinement
- High order for energy conservation
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