Heavy-light diquark masses from QCD sum rules and constituent diquark models of XYZ states

Robin Kleiv
2014 CAP Congress
June 18, 2014
Outline

1. Heavy quarkonium-like states and exotic hadrons
2. QCD Laplace sum rules (QSR)
3. Heavy quarkonium-like four-quark states
4. QSR and diquarks
5. Results and Implications
6. Summary and Acknowledgements
Heavy quarkonium-like states

- Heavy quarkonia:
  - Charmonium ($c\bar{c}$)
  - Bottomonium ($b\bar{b}$)

- Charmonium spectrum
  - predicted $c\bar{c}$ states
  - unexpected states!
  - Some in Bottomonium spectrum, too.

- Heavy quarkonium-like states ("XYZ’s")
  - Difficult to interpret as $c\bar{c}$ or $b\bar{b}$.

(S. Godfrey, hep-ph/0910.3409)
Exotic hadrons

• All $c\bar{c}$ states below 3.7 GeV have been discovered
  ○ Potential models reproduce masses of these very accurately.
  ○ The XYZ’s were not predicted by potential models.
  ○ These only consider $c\bar{c}$ states.

• Quark model (assumed by potential models): hadronic spectrum just $q\bar{q}$ mesons, $qqq$ baryons
  ○ QCD suggests hadrons other than $q\bar{q}$ and $qqq$

• Exotic hadrons: hadrons outside the constituent quark model
  ○ What if the XYZ’s are not $c\bar{c}$ states? Could they be exotics?
  ○ Use QCD sum rules (QSR) to predict properties of exotics containing heavy quarks.
QCD Laplace sum rules (QSR)

**Duality**

Hadrons can be described via QCD or as resonances.

\[ \Pi(Q^2) \quad \leftrightarrow \quad \rho(t) \]

QCD correlation function

\[ \rho(t) \]

hadronic spectral function

Measure or model

Problem: unsuitable for studying hadrons in this form!

- Large contributions from excited states
- Unknown subtraction constants
- Field-theoretic divergences
QCD Laplace sum rules (QSR)

- $\Pi(Q^2)$ satisfies Schwarz reflection: $\Pi(Q^2) = \overline{\Pi(\overline{Q}^2)}$.
  → It satisfies a dispersion relation, such as

\[
\Pi(Q^2) = \Pi(0) + Q^2 \Pi'(0) + \frac{Q^4}{\pi} \int_{t_0}^{\infty} dt \frac{\text{Im}\Pi(t)}{t^2 (t + Q^2)}
\]

- $\text{Im}\Pi(t)$ is related to hadronic spectral function $\rho^{\text{had}}(t)$
  → something that can be measured/modelled
QCD Laplace sum rules (QSR)

- Parametrize $\rho_{\text{had}}(t)$ in terms of resonance and continuum parts:

  $$\rho_{\text{had}}(t) = \rho_{\text{res}}(t) + \theta(t - s_0) \text{Im}\Pi(t)$$

- Taking the Borel (inverse Laplace) transform of the dispersion relation yields

  $$\mathcal{R}_{k}^{\text{QCD}}(\tau, s_0) = \frac{1}{\pi} \int_{t_0}^{\infty} t^k e^{-t\tau} \rho_{\text{res}}(t) dt$$

  $$\mathcal{R}_{k}^{\text{QCD}}(\tau, s_0) = \hat{B} \left[ (-Q^2)^k \Pi(Q^2) \right] - \frac{1}{\tau} \int_{s_0}^{\infty} t^k e^{-t\tau} \text{Im}\Pi(te^{-i\pi}) dt$$

- Excited states suppressed, subtraction constants removed
QCD Laplace sum rules (QSR)

• $\Pi(q)$ is calculated within the **Operator Product Expansion**

$$\Pi(q) = \sum_n C_n(q) \langle O_n \rangle \rightarrow C_n(q) \text{ expanded in } \alpha$$

$$\langle O_n \rangle \in \{ I , m_q\langle \bar{q}q \rangle , \langle \alpha G^2 \rangle , \langle \bar{q}\sigma Gq \rangle , \alpha\langle \bar{q}q \rangle^2 , \ldots \}$$

• Resonance model in terms of **hadron mass** $M$

$$\rho_{\text{res}}(t) = \pi f^2 \delta(t - M^2) \rightarrow M = \sqrt{R_1^{\text{QCD}}(\tau, s_0) / R_0^{\text{QCD}}(\tau, s_0)}$$

• **Stability of sum rule**: $M$ has minimal $\tau$ dependence
Heavy quarkonium-like four-quark states

- Many XYZ’s have been interpreted as four-quark states.

- $X(3872)$, for example:
  - D-meson molecule (cf. Deuteron), or
  - Tetraquark (diquark-antidiquark bound state).

- **Diquarks**
  - Two-quark ($qq$) clusters inside hadrons
  - Sometimes useful to think of as hadronic constituents
  - Spin-0 (scalar) or Spin-1 (vector)
  - Scalar and vector heavy-light ($Qq$) diquark masses should be degenerate

(M. Nielsen et al., PR 497 (2010) 41)
Constituent diquark models of tetraquarks

- Tetraquarks can be studied using constituent diquark models:
  - $M_{Qq}$ from fits to $X(3872), Y_b(10890)$
  - $0^+$ and $1^+$ diquarks assumed degenerate
  - $M_{cq} = 1.93$ GeV (Maiani et al., PRD71 (2005) 014028).

- Charged partners:
  - Predicted by Maiani et al. on basis on constituent diquark model
  - Other charged heavy quarkonium-like states found recently.

- QSR studies of heavy-light diquarks: calculate heavy-light constituent diquark mass.
  - QCD-based test of constituent diquark models.
QSR studies of four-quark states

• QSR studies of four-quark states among XYZ’s.
  ○ Most use four quark currents and are leading-order calculations.

• Molecular and tetraquark currents mix under Fierz transformations.
  ○ QSR calculations that use four quark currents cannot distinguish between the molecular and tetraquark scenarios.

• Fierz transformation ambiguity can be addressed by using diquark currents (Zhang, Huang, Steele, PRD76 (2007) 036004).
  ○ QSR determination of constituent diquark mass.
  ○ Diquarks are relevant for tetraquarks, not molecules.
  ○ Study pure tetraquark states via QSR.
NLO QSR studies of four-quark states

• Composite operator renormalization needed for NLO QSR calculations.
  ◦ Composite operators can mix with lower dimensional operators with the same quantum numbers.
  ◦ (Ex) Four-quark currents mix with other operators.
  ◦ Operator mixing is a significant technical barrier to NLO QSR calculations.

• The diquark current is protected from operator mixing:
  ◦ No lower dimensional operators with the same quantum numbers.
  ◦ Renormalizes multiplicatively.

• Four-quark current vs. diquark current approach to studying four-quark states in QSR:
  ◦ Diquark operator does not mix with other composite operators,
  ◦ QSR analyses of diquarks can be extended to higher orders more easily.
Diquark current renormalization

• We have calculated the scalar \((J^P = 0^+)\) diquark renormalization factor to two-loop order:
  ○ Kleiv, Steele, JPG38 (2011) 025001.

• Exploited the relationship between diquark and meson currents.
  ○ Easy to extend to \(J^P = 0^{-} , 1^\pm\) diquarks at one-loop level.

• The \(J^P = 0^{\pm} , 1^{\pm}\) diquark operator renormalization factors are needed in NLO QSR studies of diquarks.
  ○ Used in NLO calculation of heavy-light diquark mass.
QSR studies of diquarks

• Original QSR studies of light \((qq)\) diquarks:

• Updated by Zhang, Huang, Steele, PRD76 (2007) 036004
  ○ Constituent light diquark mass \(\rightarrow\) light tetraquarks

• Heavy-light \((Qq)\) diquarks :
  ○ Wang, EPJC71 (2011) 1524
    \(\rightarrow\) LO calculation of constituent diquark mass
  ○ **Kleiv, Steele, Zhang, Blokland, PRD 87 (2013) 125018**
    \(\rightarrow\) NLO calculation of constituent diquark mass, including negative parity diquarks
Diquark Correlation Function

\[ \Pi (q) = \int d^4 x \, e^{i q \cdot x} \langle 0 | T \left[ J_\alpha (x) \, S_{\alpha \omega} [x, 0] \, J^\dagger_\omega (0) \right] | 0 \rangle \]

Current and \( J^P \)

<table>
<thead>
<tr>
<th>( \mathcal{O} )</th>
<th>( J^P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_5 )</td>
<td>0^+</td>
</tr>
<tr>
<td>( I )</td>
<td>0^-</td>
</tr>
<tr>
<td>( \gamma_\mu )</td>
<td>1^+, \ldots</td>
</tr>
<tr>
<td>( \gamma_\mu \gamma_5 )</td>
<td>1^-, \ldots</td>
</tr>
</tbody>
</table>
Diquark Correlation Function

\[ \Pi(q) = C^I(q) + C^{qq}(q)\langle \bar{q}q \rangle + C^{GG}(q)\langle \alpha G^2 \rangle + C^{qGq}(q)\langle \bar{q}\sigma Gq \rangle + C^{qqqa}(q)\langle \bar{q}q \rangle^2 + \ldots \]
Diquark Correlation Function

- **Gauge invariance**
  - **Schwinger string** exactly cancels gauge dependence of perturbative contributions
  - Condensates calculated in background gauge: $x^\mu A_\mu^a = 0$

- **Renormalization**
  - Use known diquark current renormalization factor
  - Renormalization-induced contributions from LO perturbative contribution
Heavy-Light diquark mass predictions

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$M_{cq}$ (GeV)</th>
<th>$J^P$</th>
<th>$M_{bq}$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0$^+$</td>
<td>1.86 ± 0.05</td>
<td>0$^+$</td>
<td>5.08 ± 0.04</td>
</tr>
<tr>
<td>1$^+$</td>
<td>1.87 ± 0.10</td>
<td>1$^+$</td>
<td>5.08 ± 0.04</td>
</tr>
</tbody>
</table>

- Scalar/vector masses are degenerate within uncertainty
- Negative parity sum rules are unstable → unable to make mass predictions (in agreement with QSR results for light diquarks)
Implications for the XYZ states

1. QSR determined heavy-light diquark masses are consistent with constituent models.
   → QCD support for constituent diquark models of tetraquarks.

2. Strengthens tetraquark interpretation of $X(3872)$, $Y_b(10890)$.

3. Support for tetraquark interpretation of charged heavy quarkonium-like states
Summary

• QSR predictions for heavy-light diquarks
  ○ QCD-based test of constituent diquark models
  ○ QSR support for tetraquark interpretation of $X(3872)$, $Y_b(10890)$ and charged heavy quarkonium-like states
  ○ Will help to determine the identities of the XYZ states

• Exciting time for hadron spectroscopy:
  ○ Many XYZ states discovered, perhaps more to come?
  ○ New experiments: Belle-II, BES-III, Glue-X, LHCb, PANDA.
  ○ *Lots of data to come!*

[belle2.kek.jp]  [bes3.ihep.ac.cn]  [halld.org]  [cern.ch]  [panda.gsi.de]
Acknowledgements

- Collaborators
  - Tom Steele (Saskatchewan), Ailin Zhang (Shanghai), Ian Blokland (Alberta)

- Financial support
  - National Science and Engineering Research Council of Canada
  - University of the Fraser Valley and St. Peter’s College

- CAP 2014 Conference Organizers

- DTP and DNP divisions

- Preprints and links to published papers are available at
  - http://arxiv.org/a/kleiv_r_1

Thanks!
Backup Slides
**$J^{PC}$ quantum numbers**

- Hadrons are classified by their $J^{PC}$:
  - $J = L + S$ is the angular momentum
  - $P$ is parity ($P = +$ or $-$)
  - $C$ is charge conjugation ($C = +$ or $-$)

- Conventional quarkonia have $P = (-)^{L+1}$ and $C = (-)^{L+S}$
  - **Possible (non-exotic) $J^{PC}$**: $0^{--}$, $1^{--}$, $1^{+-}$, $0^{++}$, $1^{++}$, $2^{++}$ ...
  - **Impossible (exotic) $J^{PC}$**: $0^{++}$, $0^{+-}$, $1^{--}$, $2^{++}$ ...

- The observation of a state with exotic $J^{PC}$ is a “smoking gun” for the existence of exotic hadrons.
Note: Here Y(3940) is listed as X(3915).
Sum rule window

- We need to balance perturbative and non-perturbative effects.
  - If perturbative effects are too large, continuum contributions dominate.
  - If non-perturbative effects are too large, we can have large errors in our mass prediction.
- An acceptable range of $\tau$ values can be determined using

\[
\begin{align*}
  f_{\text{cont}}(\tau, s_0) &= \frac{\mathcal{L}_{1}^{QCD}(\tau, s_0) / \mathcal{L}_{0}^{QCD}(\tau, s_0)}{\mathcal{L}_{1}^{QCD}(\tau, \infty) / \mathcal{L}_{0}^{QCD}(\tau, \infty)} \geq 0.7 \\
  f_{\text{pow}}(\tau, s_0) &= \left| \frac{\mathcal{L}_{1}^{QCD}(\tau, s_0) / \mathcal{L}_{0}^{QCD}(\tau, s_0)}{\mathcal{L}_{1}^{\text{pert}}(\tau, s_0) / \mathcal{L}_{0}^{\text{pert}}(\tau, s_0)} - 1 \right| \leq 0.15
\end{align*}
\]

- The mass function $M^2 = \mathcal{L}_{1}^{QCD}(\tau, s_0) / \mathcal{L}_{0}^{QCD}(\tau, s_0)$ must stabilize (have a critical point) within the sum-rule window.
- Optimal $s_0$ found though best fit of $M^2$ to a constant.