Mode Invisibility and Single Photon Detection

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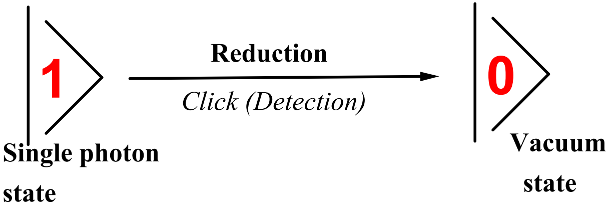
Quantum Measurement Problem

- Photodetection Process



- Photon is destroyed upon detection

Quantum Measurement Problem

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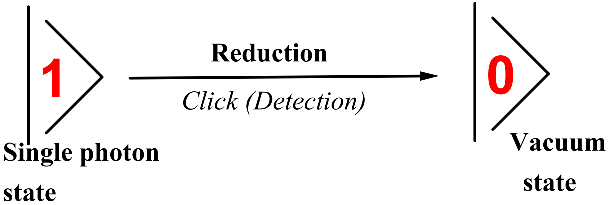

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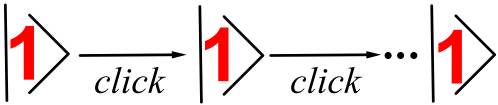
Quantum Non-Demolition (QND) Measurement



- Measurement does not change the state of the measured system
- Successive measurements yield the same result

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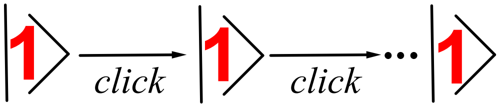

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Seeing a single photon without destroying it

Haroche et al, Nature 400 (1999)

How to measure single photon states in optical cavities

- Use highly off-resonant atomic transition
- Off-resonant $\Rightarrow$ weak coupling between atom-field mode
- Non-negligible global phase: Ramsey interferometery

How can we improve on the measurement precision?

- Use a **resonant** atomic transition
- But wouldn't this make the measurement destructive again?

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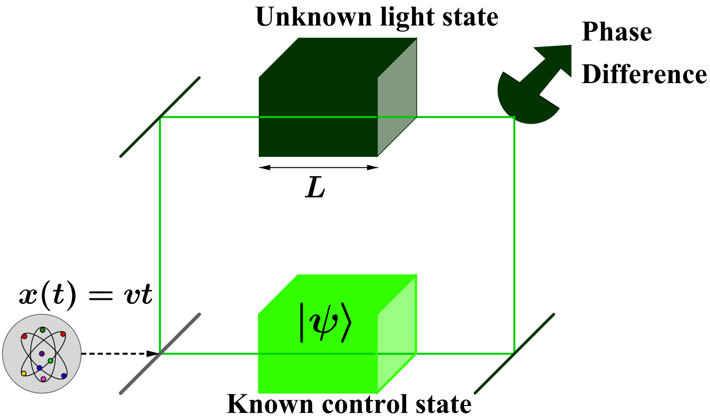
Mode invisibility

Enhancing the optical QND scheme

Onuma-Kalu et al, Phys. Rev. A 88 (2013) 063824



Aim

- Can the relative phase difference non-destructively provide information about the unknown state of light?

Weak adiabatic approximation

- The probability that the whole system remains in the same state is approximately unity, i.e,

$$\left|\langle\psi(0)|U(0,T)|\psi(0)\rangle\right|^2 \approx 1. \tag{1}$$

- Light state stays the same but for a global dynamical phase

$$|\psi(T)\rangle = U(0,T)|\psi(0)\rangle \approx e^{i\gamma}|\psi(0)\rangle, \tag{2}$$

where γ is the phase factor to be determined

Single photon detection

Method

- Setup a resonant interaction between single atom and target cavity modes using the Unruh DeWitt model

$$H_I = \sum_{\kappa} \frac{\lambda}{\sqrt{k_\kappa L}} \left(\sigma^+ e^{i\Omega t} + \sigma^- e^{-i\Omega t}\right)\left(a_\kappa^\dagger e^{i\omega_\kappa t} + a_\kappa e^{-i\omega_\kappa t}\right) \sin(k_\kappa x(t))$$

Observation

- Strong atom-field interaction evident from the excitation transition probability

$$P_{|e\rangle}(T) = \lambda^2 \left[|I_{-,\kappa}|^2 n + |I_{+,\kappa}|^2 (n+1) + \sum_{\beta \neq \kappa} |I_{+,\beta}|^2 \right]$$

$$I_{\pm,\beta} = \frac{1}{\sqrt{k_\beta L}} \int_0^T dt e^{i(\omega_\beta \pm \Omega)t} \sin[k_\beta x(t)]$$ violates QND technique

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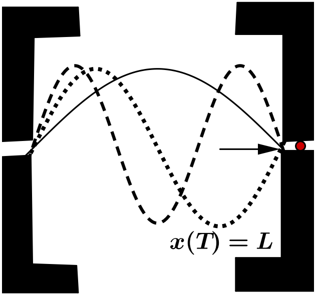
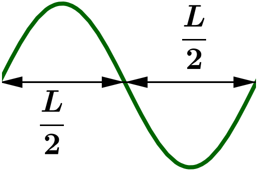
Mode invisibility technique





- Idea– Cancel the largest contribution to transition probability
- Effect of interaction through length $x \in [0, \frac{L}{2}]$ is canceled in $x \in [\frac{L}{2}, L]$ before interaction is turned off

Mode invisibility continued

Excitation transition probability

$$P_{|e\rangle}(T) = \lambda^2 \left[|I_{-,\alpha}|^2 n + |I_{+,\alpha}|^2 (n+1) + \sum_{\beta \neq \alpha} |I_{+,\beta}|^2 \right]$$

$$I_{-,\beta} = \frac{[(-1)^\beta - 1]L}{(\beta\pi)^{3/2} v} = 0, \qquad \beta = 2, 4, \cdots$$

$$P_{|e\rangle}(T) = \lambda^2 \left[|I_{+,\alpha}|^2 (n+1) + \sum_{\beta \neq \alpha} |I_{+,\beta}|^2 \right] \approx 10^{-20}$$

Phase shift on atomic state

- Leading order correction to phase factor

$$|\psi_T^{(2)}\rangle = -\lambda^2\left[n\frac{C_{-,\alpha}}{k_\alpha L}+\sum_{\beta\neq\alpha}\frac{C^*_{+,\beta}}{k_\beta L}+(n+1)\frac{C^*_{+,\alpha}}{k_\alpha L}\right]|\psi(0)\rangle+|\psi(T)\rangle_\perp$$

$$\eta = -i\ln\left(1-\lambda^2\left[n\frac{C_{-,\alpha}}{k_\alpha L}+\sum_{\beta\neq\alpha}\frac{C^*_{+,\beta}}{k_\beta L}+(n+1)\frac{C^*_{+,\alpha}}{k_\alpha L}\right]\right),$$

$$C_{\pm,\beta} = \int_0^T \mathrm{dt}\int_0^t \mathrm{dt}'\; e^{i(\omega_\beta\pm\Omega)(t-t')}\sin[k_\beta x(t)]\sin[k_\beta x(t')].$$

- Then phase on atomic state is

$$\gamma = \mathrm{Re}(\eta)$$

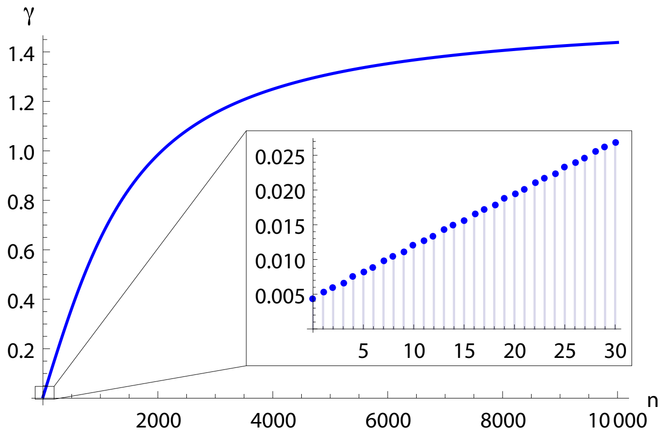


Figure : Phase plot as a function of photon number

- Interest is to measure difference between phases for different states containing n and $n+m$ photons.

$$\Delta_m \gamma(n) = \gamma(m+n) - \gamma(n)$$

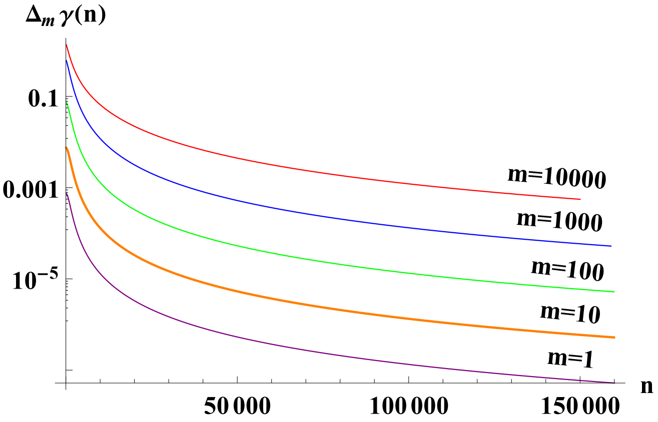


Figure : Phase resolution required to distinguish between n photons and $n+m$ photons.

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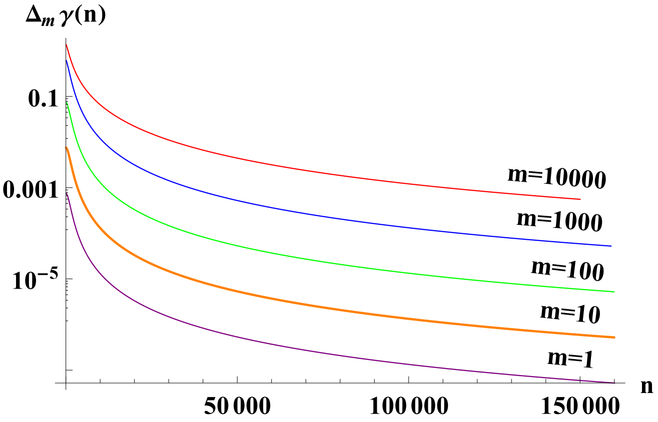


Figure : Phase resolution required to distinguish between n photons and $n+m$ photons.

Visibility factor $= \exp[-|\mathrm{Im}(\eta)|]$

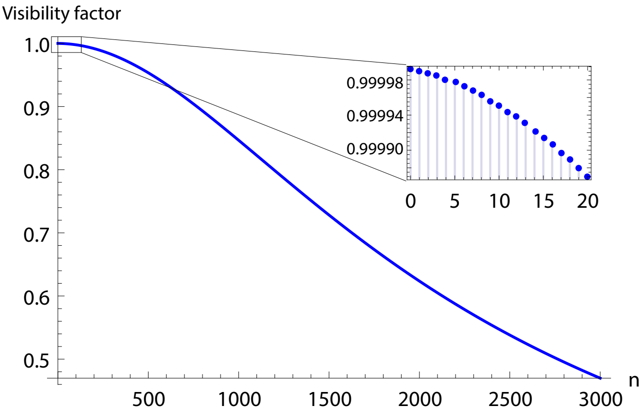


Figure : Visibility factor as a function of photon number *n*

Current work

- QND measurement of general states of light

$$\rho_0 = |g\rangle\langle g| \otimes |\zeta, \alpha\rangle_\kappa \langle \zeta, \alpha|_\kappa$$

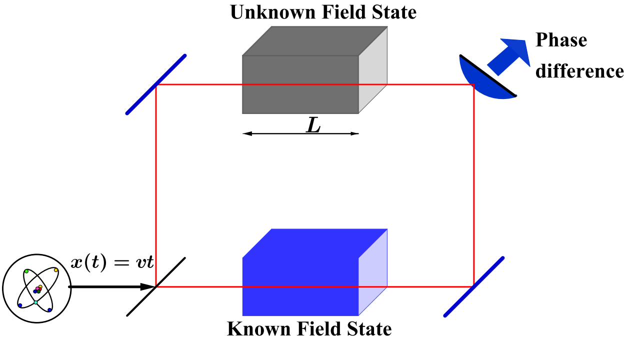


Figure : Measurement setup

Squeezed coherent state

$$
\begin{aligned}
|\alpha, \zeta\rangle =& S(\zeta) D(\alpha)|0\rangle \\
D(\alpha) =& \exp\left(\alpha \hat{a}^{\dagger} - \alpha^{*}\hat{a}\right), \qquad \alpha = |\alpha| e^{i\theta} \\
S(\zeta) =& \exp\left(\frac{1}{2}\zeta^{*}\hat{a}\hat{a} - \frac{1}{2}\zeta\hat{a}^{\dagger}\hat{a}^{\dagger}\right), \qquad \zeta = re^{i\phi}
\end{aligned}
$$

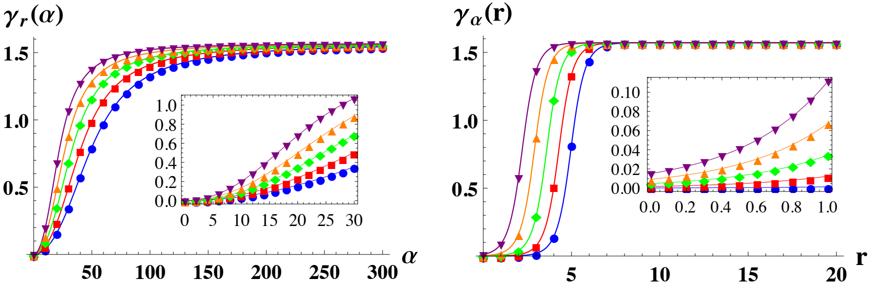
- $|\alpha|$ and θ are the amplitude and phase of the coherent operator
- r and ϕ are the amplitude and phase of the squeezed state

Excitation transition probability

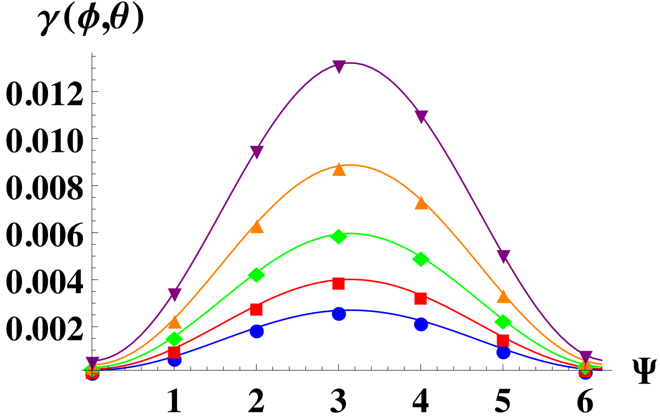
$$
\begin{aligned}
P^{\alpha,r}_{|e\rangle} = \Bigg[& \frac{\lambda^2}{k_\kappa L}(|I_{-,\kappa}|^2 + |I_{+,\kappa}|^2)\big(\cosh^2(r) + \sinh^2(r)\big)|\alpha|^2 \\
& - \frac{2\lambda^2}{k_\kappa L}(|I_{-,\kappa}|^2 + |I_{+,\kappa}|^2)\sinh(r)\cosh(r)\,\mathrm{Re}\left[|\alpha|^2 e^{i(2\theta-\phi)}\right] \\
& + (|I_{-,\kappa}|^2 + |I_{+,\kappa}|^2)\frac{\lambda^2 \sinh^2(r)}{k_\kappa L} + \sum_\gamma \frac{\lambda^2 |I_{+,\gamma}|^2}{k_\gamma L}\Bigg] \approx 10^{-21}
\end{aligned}
$$

Phase shift on atomic state is

$$
\begin{aligned}
\gamma = \mathrm{Re}\Bigg[- i \ln \Bigg[1 - \lambda^2 \Bigg(& \frac{C_{\kappa,\kappa}}{k_\kappa L}\sinh^2(r) + \sum_\gamma \frac{c_{+,\gamma}}{k_\gamma L} \\
& + \frac{C_{\kappa,\kappa}}{k_\kappa L}(\cosh^2(r) + \sinh^2(r))|\alpha|^2 \\
& - \frac{2}{k_\kappa L} C_{\kappa,\kappa}\sinh(r)\cosh(r)\,\mathrm{Re}\left[|\alpha|^2 e^{i(2\theta-\phi)}\right]\Bigg)\Bigg]\Bigg]
\end{aligned}
$$



$\gamma_r(\alpha)$
1.5
1.0
0.5
50 100 150 200 250 300
α
1.0 0.8 0.6 0.4 0.2 0.0
0 5 10 15 20 25 30
γ_α(r)
1.5
1.0
0.5
5 10 15 20
r
0.10 0.08 0.06 0.04 0.02 0.00
0.0 0.2 0.4 0.6 0.8 1.0



$\Psi = 2\theta - \phi$

Conclusion

- The mode invisibility technique allows for on-resonance QND measurements of single photon states.
- Being on resonance allows us to amplify the phase-shift.
- Not limited to single photon states (squeezed light, QND determination of Wigner function,...)

Future work

Weak measurement of the Wigner function of states of light

Thank you