PROBLEM 2.1

$$\frac{\delta p}{p} = bp \qquad \frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \qquad \begin{cases} b = 5\cdot 10^{-4}\ \text{GeV}^{-1} \\ a = 5\cdot 10^{-2}\ \sqrt{\text{GeV}} \end{cases}$$

1) $E \simeq p$? $\quad m_e \simeq 0.5$ MeV

$$E^2 = p^2 + m^2$$

$$E = \sqrt{p^2+m^2} = p\sqrt{1+\frac{m^2}{p^2}} \simeq p\left(1+\frac{1}{2}\left(\frac{m}{p}\right)^2\right)$$

$$p = \sqrt{E^2-m^2} = E\sqrt{1-\frac{m^2}{E^2}} \simeq E\left(1-\frac{1}{2}\left(\frac{m}{E}\right)^2\right)$$

$$E_e = 1\,\text{GeV} \longrightarrow \frac{m_e}{E_e} = \frac{0.5\cdot 10^{6}\,\text{eV}}{1\cdot 10^{9}\,\text{eV}} = 0.5\cdot 10^{-3}$$

$$\Delta p \simeq -\frac{1}{2}\left(\frac{m_e}{E_e}\right)^2 = -\frac{1}{2}\,0.25\cdot \underline{10^{-6}}$$

↑ bias

$$\left(\frac{m_e}{E_e}\right)^2 = \frac{1}{\gamma^2} \ !!!$$

E_e [GeV]	$\frac{\delta p}{p}$	$\frac{\sigma_E}{E}$
1	$5\cdot 10^{-4}$	$5\cdot 10^{-2}$
10	$5\cdot 10^{-3}$	$1.6\cdot 10^{-2}$
100	$5\cdot 10^{-2}$	$5\cdot 10^{-3}$

2) $$\frac{\delta p}{p} \simeq \frac{a}{\sqrt{p}} \qquad bp \simeq \frac{a}{\sqrt{p}}$$

$$b^2p^2 = \frac{a^2}{p}$$

$$p^3 = \frac{a^2}{b^2} \longrightarrow p = \left(\frac{a}{b}\right)^{2/3} = \left(\frac{\cancel{5}\cdot 10^{-2}\sqrt{\text{GeV}}}{\cancel{5}\cdot 10^{-4}\,\text{GeV}^{-1}}\right)^{2/3}$$

$$= (10^2)^{2/3}\,(\text{GeV}^{3/2})^{2/3}$$

$$\frac{\delta p}{p} < \frac{\sigma_E}{E} \text{ for } p < 21.54\,\text{GeV} \qquad \simeq 21.54\ \text{GeV}$$

4) $E_e = 15$ GeV

I made a check with 15 GeV to see how the impact would be – For 20 GeV see next page...

$$\delta p = bp^2 = 0.5\cdot 10^{-4}\,\text{GeV}^{-1}\,(15)^2\,\text{GeV}^2$$
$$= 0.5\cdot 10^{-4}\,(225)\ \text{GeV} \simeq 0.11\ \text{GeV}$$

$$\frac{\delta p}{p} \simeq 0.7\,\%$$

$$\sigma_E = a\sqrt{E} = 0.5\cdot 10^{-2}\sqrt{15}\ \text{GeV} \simeq 1.9\ \text{GeV}$$

$$\frac{\sigma_E}{E} \simeq 13\,\%$$

$$\langle p\rangle = 15\ \text{GeV}$$

$$\sigma_p = \frac{1}{\frac{1}{\delta p^2}+\frac{1}{\sigma_E^2}} = \sqrt{\frac{1}{\frac{1}{(0.7)^2}+\frac{1}{(13)^2}}} \simeq 0.7\,\% \longleftarrow \text{not useful yet !!!}$$

4) momentum - energy combination

$E_e = 20$ GeV

$$\delta p = bp^2 = 0.5 \cdot 10^{-4} (20)^2 \text{ GeV} = 2\%$$
$$\sigma_E = a\sqrt{E} = 0.5 \cdot 10^2 \sqrt{20} = 2.24\%$$

combination $\rightarrow$ (use weighted mean)

$$\langle p \rangle = 20 \text{ GeV}$$
$$\sigma_p = \left(\frac{1}{\left(\frac{1}{\delta p}\right)^2 + \left(\frac{1}{\sigma_E}\right)^2} \right)^{1/2} =$$
$$= \left(\frac{1}{\left(\frac{1}{0.02}\right)^2 + \left(\frac{1}{0.0224}\right)^2} \right)^{1/2} \cong 1.5\%$$

interesting !!!

Combining the measurements becomes interesting when the relative resolution are similar (if they are <u>uncorrelated</u>)

3) The momentum resolution improves linearly with the magnetic field

$$\frac{\delta p}{p} \propto \frac{1}{B} \qquad \frac{\left(\frac{\delta p}{p}\right)_1}{\left(\frac{\delta p}{p}\right)_2} = \frac{B_2}{B_1} =$$

$$\left(\frac{\delta p}{p}\right)_2 = \left(\frac{\delta p}{p}\right)_1 \cdot \frac{B_1}{B_2} \cong 0.67 \frac{\delta p}{p}$$

$\sim 30\%$ improvement

PROBLEM 2.2

$m_{\pi^\pm} = 139.6$ MeV $\qquad E = 20$ GeV

$m_{K^\pm} = 493.7$ MeV

$$\begin{cases} p = m\gamma\beta \\ E = m\gamma \end{cases} \qquad \gamma_\pi = \frac{20 \cdot 10^9}{139.6 \cdot 10^6} = 1.43 \cdot 10^2 \qquad \gamma_k = \frac{20 \cdot 10^9}{493.7 \cdot 10^6} = 0.40 \cdot 10^2$$

1) $\beta = \frac{p}{m\gamma}$

$$\beta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \longrightarrow \beta_\pi = \sqrt{\frac{(1.43 \cdot 10^2)^2 - 1}{(1.43 \cdot 10^2)^2}} = 0.999975$$

$$\beta_k = \sqrt{\frac{(0.40 \cdot 10^2)^2 - 1}{(0.40 \cdot 10^2)^2}} = 0.99969$$

2) $\beta_{th} = \frac{1}{n} \rightarrow n = \frac{1}{\beta_{th}} = \frac{1}{\beta_k} = 1.000312$

3) $\cos\theta_c(\pi) = \frac{1}{n\beta_\pi} = 0.9997 \longrightarrow \theta_c(\pi) = 1.37°$

4) 50% of Kaons would be above threshold $E_k > 20$ GeV !

lowest energy for pions would be $E_\pi^{min} = 19$ GeV

$$\gamma_{min\,\pi} = \frac{19 \cdot 10^9}{139.6 \cdot 10^6} = 1.36 \cdot 10^2$$

$$\beta_\pi^{min} = \left(\frac{(1.36 \cdot 10^2)^2 - 1}{(1.36 \cdot 10^2)^2}\right)^{1/2} = 0.999946 > \beta_k \;!!!$$

even lowest energy pions would still emit Cherenkov light

5) This can be tricky to solve properly - Let's do it the easy (and not-exactly-correct) way!

$\overline{N}_\gamma(\pi) = 80\% \cdot 5 = 4$ $\qquad$ on average, we would collect 4 γ from a π passing our radiator

Such a small number is distributed according to Poisson - The probability not to see any photons is: and to mistake a K for a π

$$P(0, \underbrace{\lambda = 4}_{k}) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{4^0 e^{-4}}{0!} \simeq 2\%$$

PROBLEM 2.3

1) To produce the $\Upsilon(2S)$ at rest you need symmetric beam energies, so that the momentum of the CM frame would be null in the LAB frame

$e^+ \; E_1 \; p_1 \longrightarrow \longleftarrow E_2 \; p_2 \; e^-$

$$m_\Upsilon = 10.58 \text{ GeV}$$

$$E_1 = E_2 = \frac{m_\Upsilon}{2} \simeq 5.29 \text{ GeV}$$

$$|\vec{p}_1| = |-\vec{p}_2| \simeq E_1 \longrightarrow$$ given the energy, the e^+ and e^- are ultrarelativistic: $E \simeq p$ and you can ignore m_e

2) To produce the $\Upsilon(2S)$ with $\beta = 0.6$, you'll need asymmetric beam energies (as it is done in the PEP-II collider). You'll need:

- The sum of the beam energies to equal the Υ energy (boosted)
- The CM energy to equal the Υ mass, to guarantee production

$$\beta = 0.6 \longrightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.25$$

$$E_\Upsilon = \gamma m = 1.25 \times 10.58 \text{ GeV} \simeq 13.2 \text{ GeV}$$

center of mass energy:

$$\sqrt{s} = \sqrt{(E_1+E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}$$ ignore m_e: $E_i \sim p_i$

$$= \sqrt{(E_1+E_2)^2 - (E_1 - E_2)^2}$$

$$= \sqrt{4E_1E_2}$$

conditions:

$$\begin{cases} E_1 + E_2 = E_\Upsilon = \gamma m_\Upsilon \\ \sqrt{4E_1E_2} = m_\Upsilon \end{cases} \longrightarrow \text{solve for } E_1 \text{ or } E_2$$

$$E_1 = \gamma m - E_2$$

$$\frac{m^2}{4} = (\gamma m - E_2)E_2 \longrightarrow E_2^2 - \gamma m E_2 + \frac{m^2}{4} = 0$$

$$E_2 = \frac{1}{2}\left[\gamma m \pm \sqrt{(\gamma m)^2 - m^2}\right] =$$

$$= \frac{1}{2}\gamma m \left[1 \pm \underbrace{\sqrt{1 - \frac{1}{\gamma^2}}}_{\beta}\right] =$$

one solution is E_1, the other is E_2 ... $\longrightarrow$

$$= \frac{1}{2}\gamma m \left[1 \pm \beta\right]$$

$$E_1 = \frac{1}{2} E_\Upsilon (1+\beta) \simeq 10.6 \text{ GeV}$$

$$E_2 = \frac{1}{2} E_\Upsilon (1-\beta) \simeq 2.6 \text{ GeV}$$

3) Consider B mesons produced at rest in $\Upsilon(4S)$ frame:

$$m_{B\pm} \simeq \frac{1}{2} m_{\Upsilon(4S)}$$

The only boost the B mesons will have comes from Υ boost ($\beta = 0.6$)

CM: $E^*_B = \frac{m_\Upsilon}{2}$, $p^*_B = 0$

LAB: $E_B = \gamma E^*_B = \gamma \frac{m_\Upsilon}{2} \simeq \gamma m_B$

$p_B = \gamma\beta m_B$

The B meson half life increases because of the γ boost:

$$\tau = \gamma \tau^* \qquad \tau^* = 1.6 \text{ ps} = 1.6 \cdot 10^{-12} \text{ s}$$

The B mesons flights at speed βc, thus in 1 halflife they travel:

$$\begin{aligned} \ell &= \beta c \gamma \tau^* = \\ &= 0.6 \cdot 300\, 10^6 \text{ m/s} \cdot 1.25 \cdot 1.6 \cdot 10^{-12} \text{ s} \\ &= 360 \cdot 10^{-6} \text{ m} = 360\ \mu\text{m} \end{aligned}$$

4) The probability of a particle to survive after a time t is:

$$P(t) = e^{-t/\tau}$$

in order to travel 2 mm, the B meson would need to survive for a time

$$t(2\text{mm}) = \frac{2 \text{ mm}}{\beta c} = \frac{2 \cdot 10^{-3} \text{ m}}{0.6 \cdot 300 \cdot 10^6 \text{ m/s}} \simeq 0.011 \cdot 10^{-9} \text{ s} \simeq 11 \text{ ps} \simeq 5.5\, \underbrace{\gamma\tau^*}_{\tau}$$

the probability for this to happen is

$$P(t(2\text{mm})) \cong e^{-\frac{11\text{ps}}{\tau}} = e^{-5.5} \cong 0.4\% \quad \text{(not very high!)}$$

ANY QUESTION OR DOUBT?
FOUND A MISTAKE?
DO NOT HESITATE TO CONTACT ME:

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