ASTROPARTICLES
Tutorials

ESIPAP - 2014

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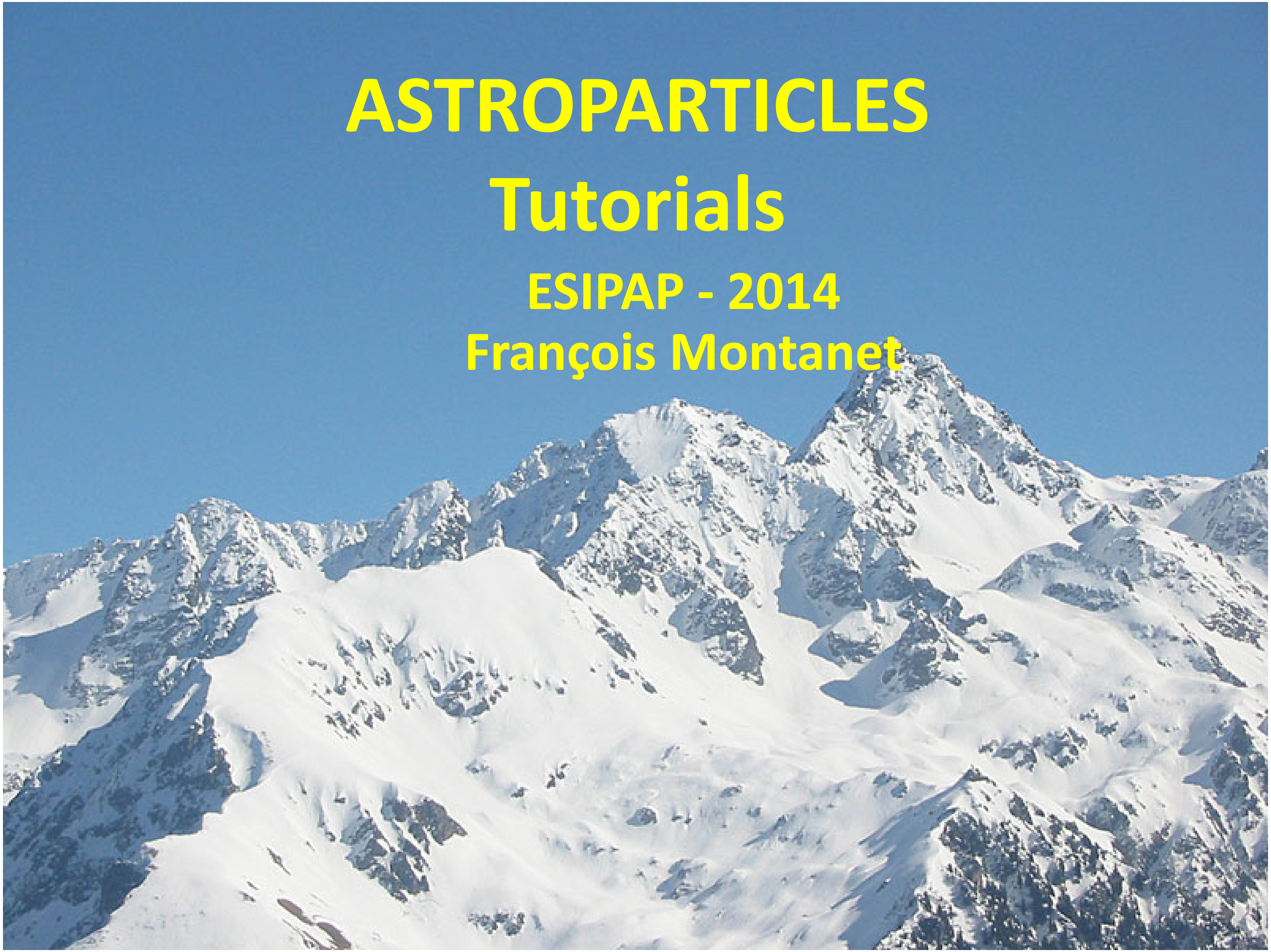
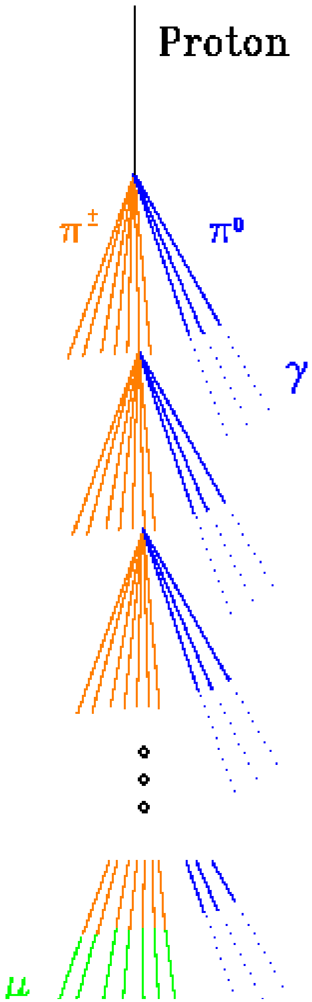


A difficult decision

You are an eminent member of an important scientific council that has to decide to found or not a expensive project that aims at studying the very high energy gamma-rays from extragalactic sources such as AGNs.

- The gamma-ray detection threshold of the propose detector is 1PeV and the collection surface is quite large (>10km^2).
- You decide:
- a) to found the project because it opens a new unexplored domain
- b) to ask further questions about the source candidates and the expected fluxes? Where do you think could there be a real problem with this proposal ?

A simplified development model



Energy per pion	$N_{\pi^{\pm}}$
$E_0/10$	$\frac{2}{3}10$
$E_0/100$	$(\frac{2}{3}10)^2$
$E_0/1000$	$(\frac{2}{3}10)^3$
⋮	⋮
$E_c = E_0/10^{n_c}$	$(\frac{2}{3}10)^{n_c}$

A simplified development model

Using a simplified model la Heitler, we want to model the EM and the muonic componant of shower induced by a proton with an energy $E_0 = 10^{18}$eV. One gives the critical energy to be (fixed) $\varepsilon_\pi = 100$ GeV and $\lambda_\pi = 100$ g/cm^2 is the pion interaction length. We will do the following hypotheses:

1. At each fixed interaction length λ, all the non decayed hadrons with energy $> \varepsilon_\pi$, interact with a nucleus of the atmosphere.
2. Each interaction produces secondary hadrons (only pions) with multiplicity m, $2/3$ of them are $\pi^\pm$ and $1/3$ are π^0. One assume that the multiplicity is fixed and its value is $m = 10$.
3. Each of the m pions produced takes away a fraction of the parent energy $1/m$.
4. The π^0 decay immediatly in two γ that will feed the EM componant.
5. When charged pions reach the critical energy ε_π, they all decay and produce each one muon that propagates to the ground. We will assume that the muons takes away $1/2$ of the critical energy.

A simplified development model

1. Give the number of interaction length before reaching the critical energy and give an approximate value using the given parameters.

2. Compute what is the fraction of the total energy transfered to the EM componant.

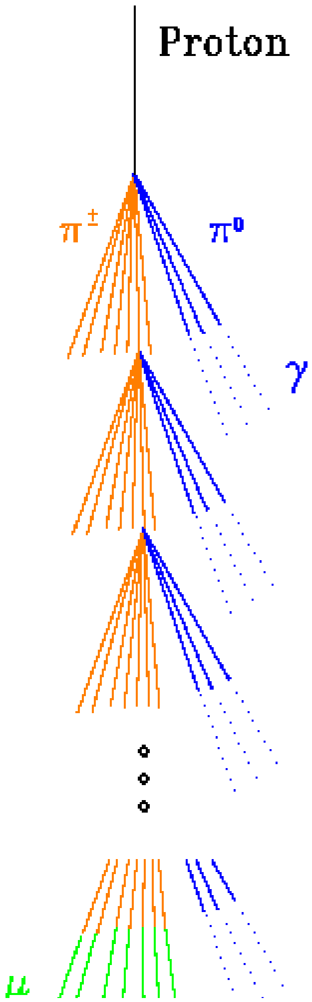
3. Compute the number of muons produced at the end of the development. What fraction of energy do they take away.

4. Compare the EM, muonic and initla energy. Do you observed missing energy? What is taking this missing energy away?

5. Suppose now that the incident particle is an iron nucleus with the same total energy than the proton. Assuming superposition principle holds, what is now the muons energy fraction? Conclude if this muons energy fraction can be used to deduce the nature of the incident particle.

A few (may be) usefull numerical values :
$(2/3)^7 \approx 5.85 \times 10^{-2}$
$\log_{10}(2/3) \approx -0,18$
$\log_{10}(56) \approx 1,75$
$(2/3)^{-\log_{10}(56)} \approx 2,03$

A simplified development model



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$$n_c = \log_{10}\left(\frac{E_0}{E_c}\right)$$

$$\begin{aligned} N_\mu &\approx N_{\pi^\pm} = \left(\frac{2}{3}\times 10\right)^{n_c} \\ \log_{10} N_\mu &\approx \left(1 + \log_{10}\left(\frac{2}{3}\right)\right) n_c \\ &= 0.82 \log_{10}(E_0/E_c) \end{aligned}$$

$$\Rightarrow N_\mu \approx \left(\frac{E_0}{E_c}\right)^{0.82}$$

A simplified development model

Superposition principle: a nucleus ${}^{A}N$ is equivalent to A protons.

Given: $N_\mu \approx \left(\frac{E_0}{E_c}\right)^{0.82}$

at equivalent total energy, the superposition principle says:

$$N_\mu^A(E) \propto A \left(\frac{E_0}{A}\right)^{0.82}$$

Thus

$$\begin{aligned} N_\mu^A(E) &\approx A^{(1-0.82)} \times N_\mu^p(E) \approx A^{0.18} \times N_\mu^p(E) \\ N_\mu^{Fe} &\approx 2 \times N_\mu^p(E) \end{aligned}$$

In fact one observes 80% more muons
for a ${}^{56}Fe$ primary compared to a proton with the same totoal energy.

A simplified development model

The size (number of électrons at max) is proportional to the primary energy:

$$N_e^{max} \approx S_0 E_0/\epsilon_0 = E_0/(1.7\text{GeV})$$

The depth of max is proportional to the log of the energy:

$$X_{max} \approx X_0 \log(E_0/\epsilon_0) \Rightarrow 80\text{g/cm}^2 \text{ par décade}$$

Showers from heavier nuclei produce more muons than lighter ones.

$$N_\mu^{Fe} \approx 2 \times N_\mu^p(E)$$

Shower from heavier nuclei start higher up and reach max higher up too.

$$X_{max}^{Fe} < X_{max}^p$$

Power laws and stochastic processes

- The power laws observed in differential energy spectra follow naturally from cyclic acceleration mechanisms with constant energy gain and constant escape probabilities:
  - Initial energy: E_0
  - Energy gain at each cycle: $\Delta E = \varepsilon E$
  - Particle energy after n iterations: $E_n = E_0(1+\varepsilon)^n$
  - Escape probability from the acceleration zone: P_{esc}
  - Probability to remain in the acceleration zone: $(1-P_{esc})^n$

Power laws and stochastic processes

- Particle energy after n iterations: $E_n = E_0(1+\varepsilon)^n$
- Probability to remain in the acceleration zone: $(1-P_{esc})^n$

Number of iterations to reach an energy E:

$$n = \frac{\ln(E/E_0)}{\ln(1+\varepsilon)}$$

Proportion of particles accelerated up to an energy equal or greater than E :

$$N(\geq E) = N_0 \sum_{m=n}^{\infty} (1-P_{esc})^m = N_0 \frac{(1-P_{esc})^n}{P_{esc}}$$

thus :

$$\frac{\ln(P_{esc}N/N_0)}{\ln(1-P_{esc})} = n = \frac{\ln(E/E_0)}{\ln(1+\varepsilon)}$$

eliminating n:

$$N(\geq E) \propto \left(\frac{E}{E_0}\right)^{-\gamma}$$

Power laws are natural !

with $\gamma = \frac{-\ln(1-P_{esc})}{\ln(1+\varepsilon)} \approx \frac{P_{esc}}{\varepsilon} = \frac{1}{\varepsilon}\frac{T_{cycle}}{T_{esc}}$