European School of Instrumentation in Particle & Astroparticle Physics

European Scientific Institute

Interaction of particles with matter (lecture 1)

A brief review of a few typical situations is going to greatly simplify the subject.

Mean free path of a particle, i.e. average distance travelled between two consecutive collisions in matter :

$$\lambda = \frac{1}{\sigma n}$$

where :

σ total interaction cross-section of the particle

n number of scattering centers per unit volume

example : $n = \frac{\rho N_A}{M}$ for a monoatomic element of molar mass M and specific mass ρ .

N_A Avogadro number

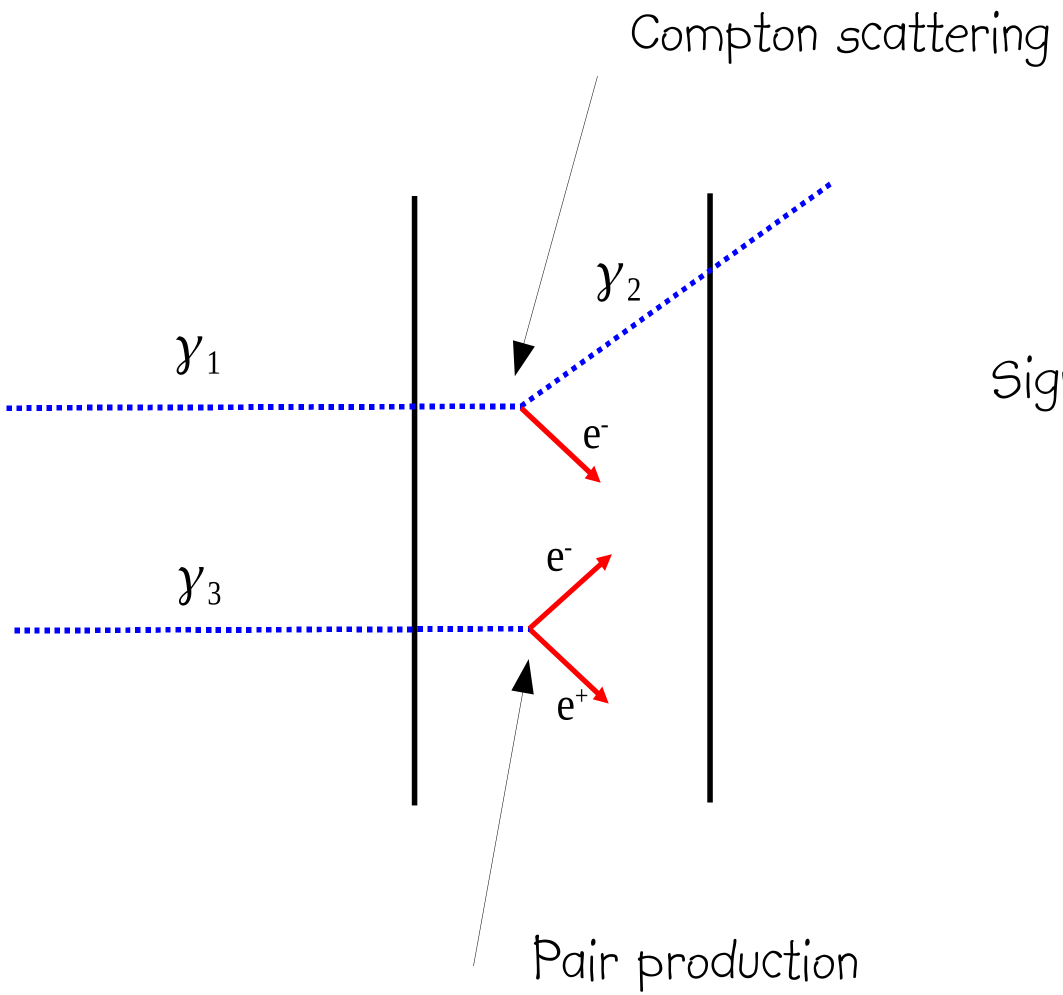
Electromagnetic interaction : $\lambda \leqslant 1 \mu\mathrm{m}$ (charged particles)

Strong interaction : $\lambda \geqslant 1\,\mathrm{cm}$ (neutrons ....)

Weak interaction : $\lambda \geqslant 10^{15}\,\mathrm{m} \simeq 0{,}1$ light year (neutrinos)

A practical signal (>100 interactions or hits) can only come from electromagnetic interaction

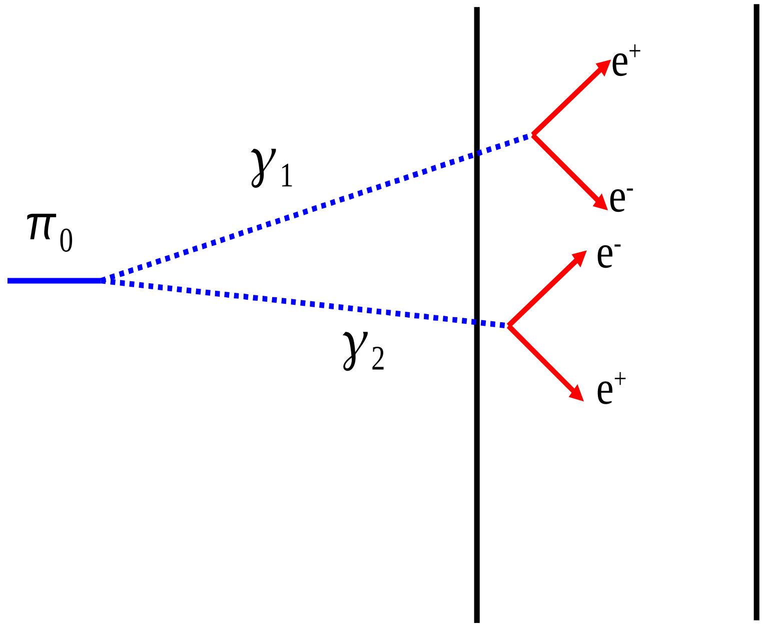
Particle detection proceeds in two steps : 1) primary interaction 2) charged particle interaction producing the signals typical examples : photon detection



Signal is induced by electrons

neutral pion detection :

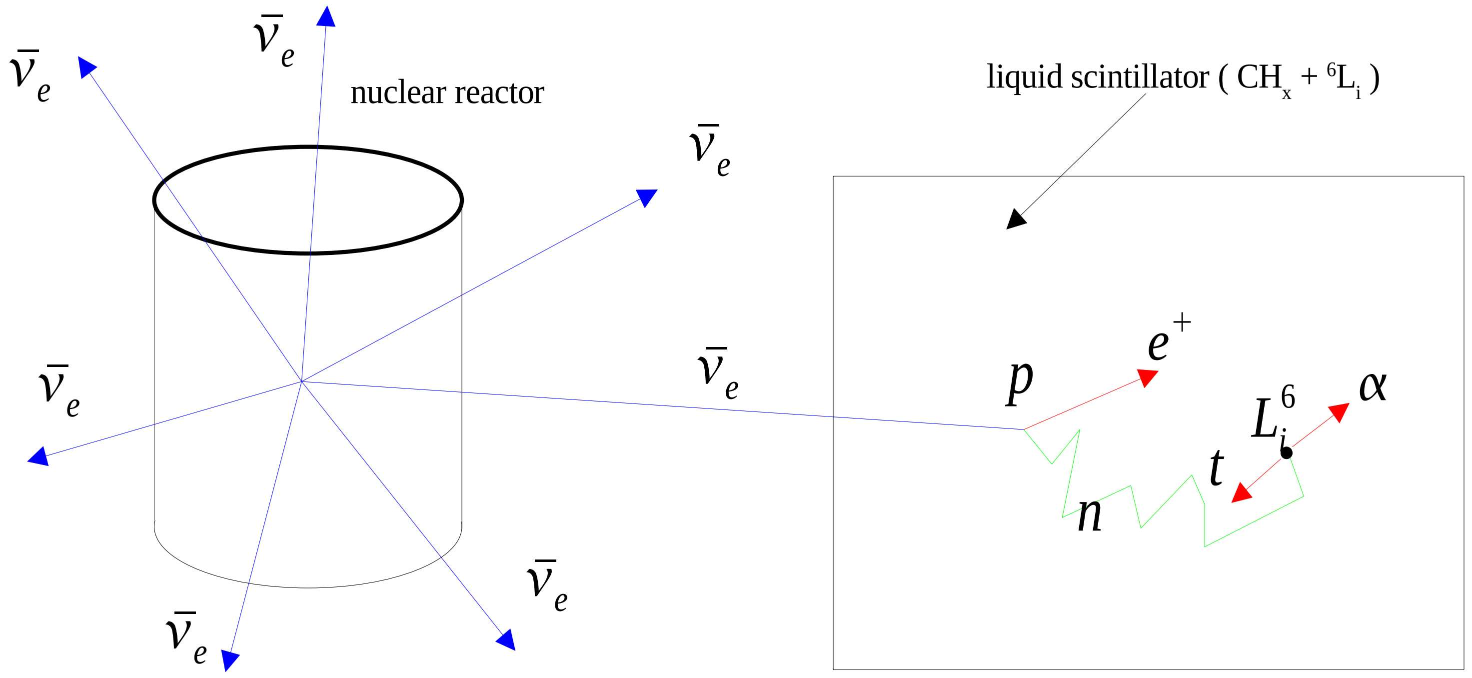
A π^0 decays into two photons with a mean lifetime of 8.5 10^{-17} s.



neutrino detection :

A 2800 MW nuclear power station produces 130 MW of neutrinos !

A detector of 1 m^3 located 20 m away from the reactor core can detect 100 neutrinos/h.



$\epsilon_\nu \sim 0$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

$$n_{th} + L_i^6 \rightarrow \alpha + t + 4{,}8\ MeV$$

Charged particles produce light
in the target scintillator

Interaction of charged particles with matter

For heavy particles ionization and excitation are the dominant processes producing energy loss.

Particle P of Z charge state

Excitation : $P^{(Z)} + \text{atom} \rightarrow \text{atom}^{*} + P^{(Z)}$ followed by : $\text{atom}^{*} \rightarrow \text{atom} + \gamma$

Ionization : $P^{(Z)} + \text{atom} \rightarrow \text{atom} + e^{-} + P^{(Z)}$

Ionization + excitation : $P^{(Z)} + \text{atom} \rightarrow \text{atom}^{*} + e^{-} + P^{(Z)}$

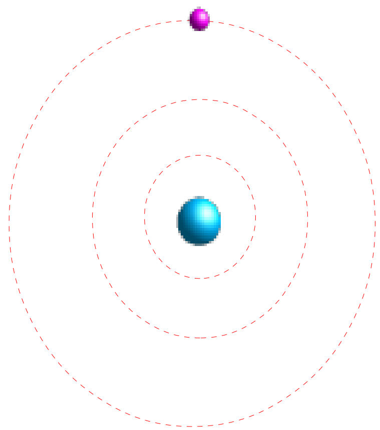
Maximal kinetic energy transferred to an ionized electron :

m_0 $\vec{v}$

$\vec{p} = \gamma m_0 \vec{v}$

E

m_e



atome
Z

$\beta = \frac{v}{c}$

$\gamma = (1-\beta^2)^{-1/2}$

$\beta = (1-\gamma^{-2})^{1/2}$

Hypothesis : $V > \langle v_e \rangle = Z\,\alpha\,c$, speed of deepest atomic orbit electrons where α is the fine structure constant : $\alpha = 1/137$.

One may show (exercise) that : $T_e^{max} = E_e^{max} - m_e = \frac{2\,m_e \beta^2 \gamma^2}{(E_{CM}/m_0)^2}$ (In natural units , $c=\hbar=1$)

where : $E_{CM} = (m_0^2 + m_e^2 + 2\,m_e E)^{1/2}$ total energy in center-of-mass frame

Two cases :

$m_0 \gg m_e$, i.e. the incoming particle is not an electron and if its energy is not too big

$$(E_{CM}/m_0)^2 = (\frac{m_0^2}{m_0^2}+\frac{m_e^2}{m_0^2}+\frac{2m_e E}{m_0^2}) \simeq 1 \qquad \text{with} \qquad E=\gamma m_0$$

$\frac{2\gamma m_e}{m_0} \ll 1$ proton $E_p < 50$ GeV , muon $E_\mu < 500$ MeV (medium energy range)

then : $$T_e^{max} = E_e^{max} - m_e = 2 m_e \beta^2 \gamma^2$$

$m_0 = m_e$ the incoming particle is an electron

$$T_e^{max} = (E - m_e)$$

due to undistinguishibility of electrons, max transferable energy = T^{max} / 2

If the incoming particle is not an electron then in practice $m_0 \gg m_e$.

Stopping power of heavy particles by excitation and ionization in matter.

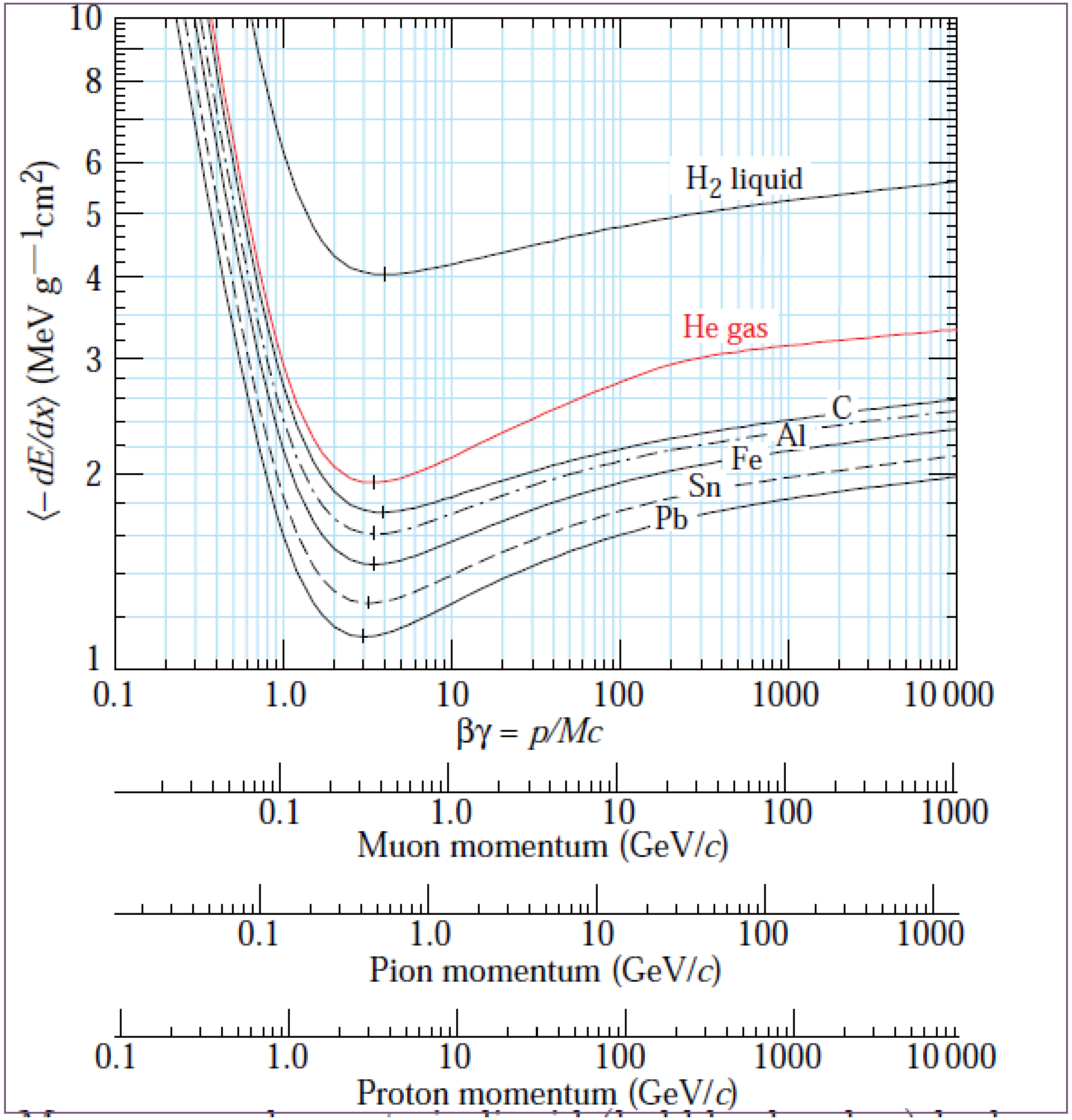
Average energy loss by a charged particle (other than an electron) in matter.

Bethe and Bloch formula

(see Nuclei and particles, Émilio Segré, W.A. Benjamin ; Principles of Radiation Interaction in Matter and Detection, C. Leroy and P.G. Rancoita, World Scientific ; Introduction to experimental particle physics, R. Fernow)

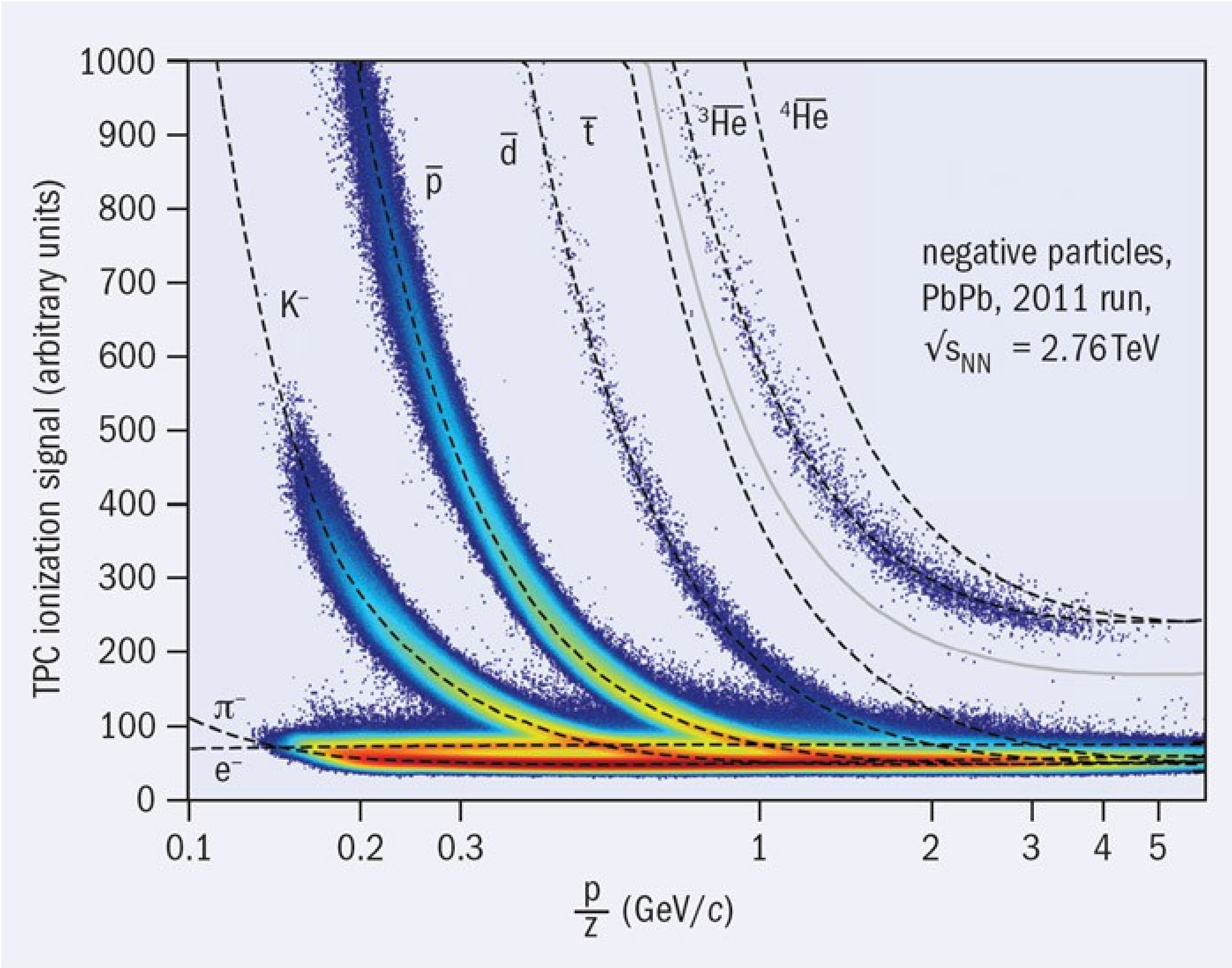
$$-\left(\frac{dE}{dx}\right)\left[\frac{\mathrm{MeV}}{\mathrm{g/cm^2}}\right] = \frac{0.3071}{A\,(\mathrm{g\ mol^{-1}})}\,\frac{z^2\ Z}{\beta^2}\,\left(\frac{1}{2}\ln\left(\frac{2\,m_e\,\beta^2\,\gamma^2\,T_e^{max}}{I^2}\right)-\beta^2-\frac{\delta(\gamma\beta)}{2}-\frac{C_e}{Z}\right)$$

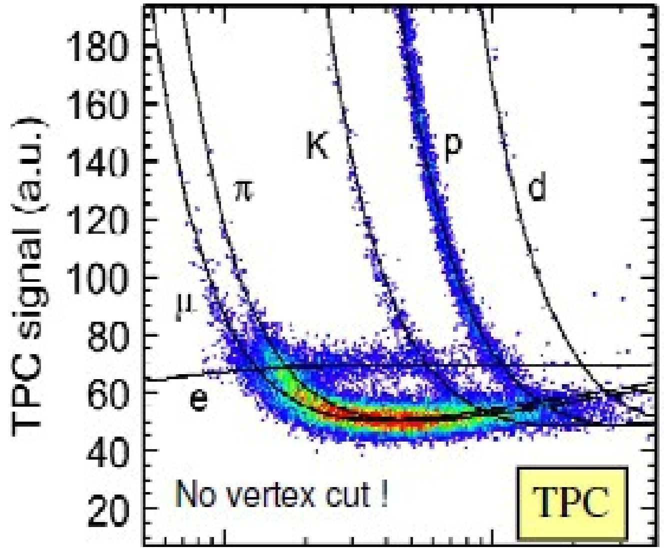
- $\frac{dE}{dx}$: Stopping power or mean specific energy loss
- z: charge of incoming particle
- Z: Z of medium
- $\delta(\gamma\beta)$: density effect correction at high energy
- dx: Surface mass density of medium dx = ρ dl (or mass thickness of medium)
- A: Atomic mass of medium
- I: mean excitation energy
- C_e: Atomic shell correction at low energy (not covered in this lecture, see Leroy & Rancoita)



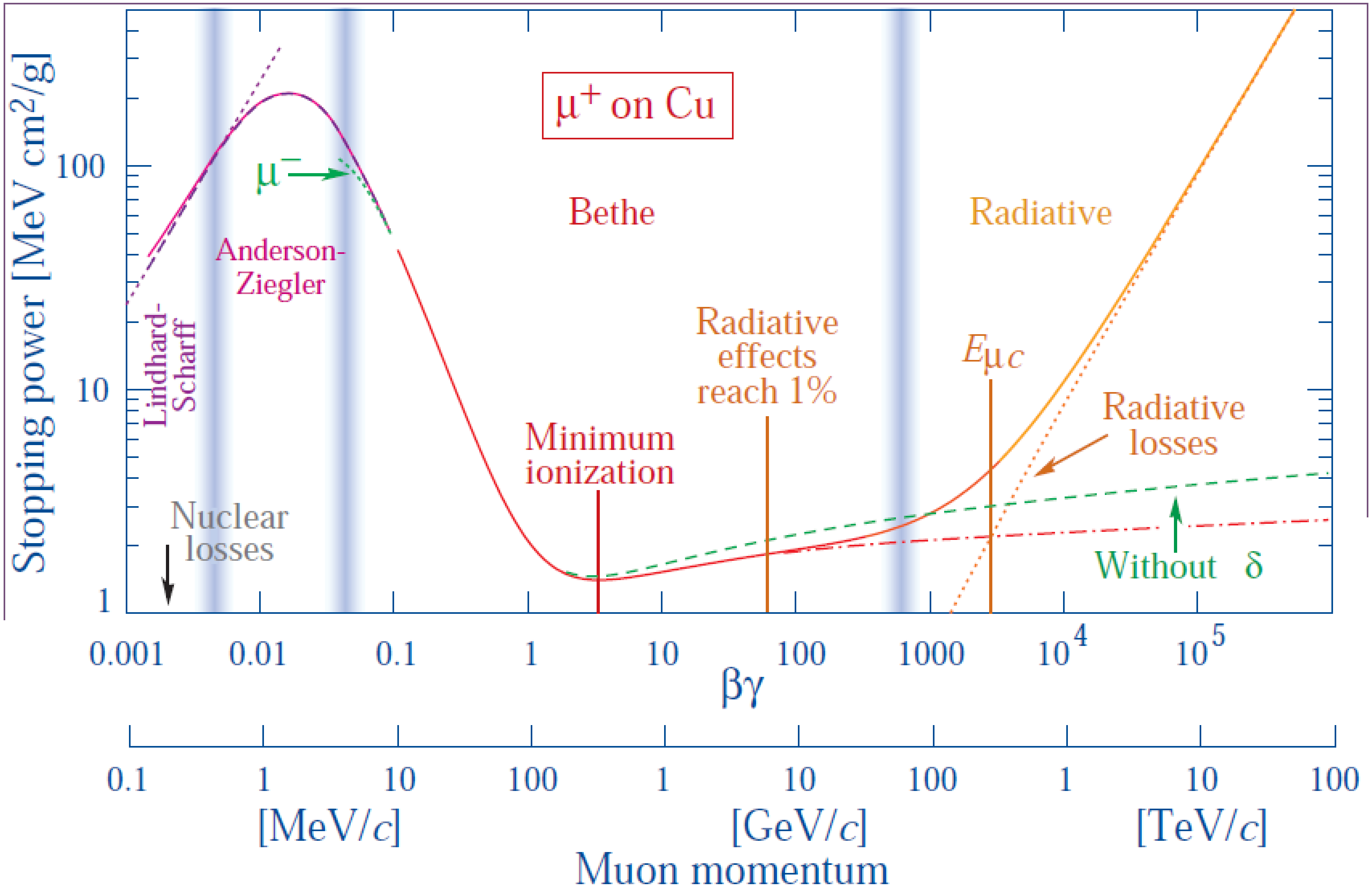
show that :

$$\beta^2 = \frac{(\beta\gamma)^2}{1+(\beta\gamma)^2}$$





Particle identification in Alice TPC


μ+ on Cu
Stopping power [MeV cm²/g]
100
10
1
Lindhard-Scharff
Anderson-Ziegler
μ−
Bethe
Radiative
Radiative effects reach 1%
Eμc
Minimum ionization
Radiative losses
Nuclear losses
Without δ
0.001
0.01
0.1
1
10
100
1000
10^4
10^5
βγ
0.1
1
10
100
1
10
100
1
10
100
[MeV/c]
[GeV/c]
[TeV/c]
Muon momentum

few remarks :

- for $\beta\gamma < 1$: $-\frac{dE}{dx} \sim \beta^{-5/3}$ non relativistic particles

- for $\beta\gamma \sim 3\text{-}4$: $-\frac{dE}{dx}$ is minimal over a large energy plateau . A particle in this state is called a minimum ionizing particle (MIP)

In media composed of light elements : $-\frac{dE}{dx}^{MIP} \simeq 2 \frac{\text{MeV}}{\text{g cm}^{-2}}$

- for $\beta\gamma > 4$: relativistic increase of $-\frac{dE}{dx}$ as $\ln(\gamma)$ wich is tempered by $-\delta/2$ correction.

- I : mean excitation and ionization energy , I = 15 eV for atomic H and 19.2 eV for H_2

I = 41.8 for He

I = 15 $Z^{0.9}$ eV for Z > 2

At medium energy : $\frac{2\gamma m_e}{m_0} \ll 1$ $T_e^{max} = 2 m_e \beta^2 \gamma^2$

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right] = \frac{0.3071}{A(g)} \cdot \frac{z^2 Z}{\beta^2} \left[\ln\left(\frac{2 m_e \beta^2 \gamma^2}{I}\right) - \beta^2 - \frac{\delta}{2} - \frac{C_e}{Z}\right]$$

When energy increases, stopping power decreases to a minimum ($1/\beta^2$ dependance) and then starts rising again due to logarithmic term. In fact, the max. transverse electric field increases as γ but its influence is screened by nearby atoms beyond a distance of 70 $(A(g)/\rho(g)Z)^{1/2}$ Å (shown by Bohr). This density effect tempers the relativistic rise.

Studies have be carried-out by Sernheimer, Peierls, Berger & Seltzer (see Leroy & Rancoita).

The density correction effect term, δ is given by :

for $\beta\gamma < 10^{S_0}$: $\delta = \delta_0 \left(\frac{\beta\gamma}{10^{S_0}}\right)^2$

for $10^{S_0} < \beta\gamma < 10^{S_1}$: $\delta = 2\ln(\beta\gamma) + C + a\left[\frac{1}{\ln(10)}\ln\left(\frac{10^{S_1}}{\beta\gamma}\right)\right]^{md}$

for $\beta\gamma > 10^{S_1}$: $\delta = 2\ln(\beta\gamma) + C$

where : $C = -2\ln\left(\frac{I}{h\nu_p}\right) - 1$

with $\nu_p = \sqrt{\frac{n r_e c^2}{\pi}}$ in which n is the density of electrons and r_e the classical radius of e^- : r_e = 2.82 fm

$$h\nu_p \simeq 28.7\sqrt{\frac{\rho(g/cm^3)}{A(g)}Z}\ eV$$

show that at very high energy : $\beta\gamma > 10^{S_1}$

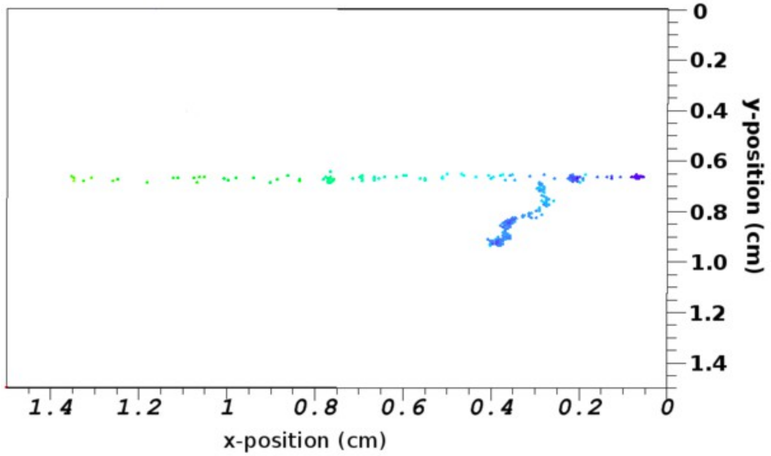
$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right] = 0.3071\frac{z^2 Z}{2.A(g)}\left[\ln\left(\frac{2m_e T_e^{max}}{(h\nu_p)^2}\right) - 1\right]$$

Table 2.1 Values of Z, Z/A, I, ρ in units of g/cm^3, $h\nu_p$ and density-effect parameters S_0, S_1, a, md, and δ_0 for elemental substances.

El.	Z	Z/A	I eV	ρ	$h\nu_p$ eV	S_0	S_1	a	md	δ_0
He	2	0.500	41.8	1.66 10^{-4}	0.26	2.202	3.612	0.134	5.835	0.00
Li	3	0.432	40.0	0.53	13.84	0.130	1.640	0.951	2.500	0.14
O	8	0.500	95.0	1.33 10^{-3}	0.74	1.754	4.321	0.118	3.291	0.00
Ne	10	0.496	137.0	8.36 10^{-4}	0.59	2.074	4.642	0.081	3.577	0.00
Al	13	0.482	166.0	2.70	32.86	0.171	3.013	0.080	3.635	0.12
Si	14	0.498	173.0	2.33	31.06	0.201	2.872	0.149	3.255	0.14
Ar	18	0.451	188.0	1.66 10^{-3}	0.79	1.764	4.486	0.197	2.962	0.00
Fe	26	0.466	286.0	7.87	55.17	-0.001	3.153	0.147	2.963	0.12
Cu	29	0.456	322.0	8.96	58.27	-0.025	3.279	0.143	2.904	0.08
Ge	32	0.441	350.0	5.32	44.14	0.338	3.610	0.072	3.331	0.14
Kr	36	0.430	352.0	3.48 10^{-3}	1.11	1.716	5.075	0.074	3.405	0.00
Ag	47	0.436	470.0	10.50	61.64	0.066	3.107	0.246	2.690	0.14
Xe	54	0.411	482.0	5.49 10^{-3}	1.37	1.563	4.737	0.233	2.741	0.0
Ta	73	0.403	718.0	16.65	74.69	0.212	3.481	0.178	2.762	0.14
W	74	0.403	727.0	19.30	80.32	0.217	3.496	0.155	2.845	0.14
Au	79	0.401	790.0	19.32	80.22	0.202	3.698	0.098	3.110	0.14
Pb	82	0.396	823.0	11.35	61.07	0.378	3.807	0.094	3.161	0.14
U	92	0.387	890.0	18.95	77.99	0.226	3.372	0.197	2.817	0.14

Data are from [Sternheimer, Berger and Seltzer (1984)]

Restricted energy loss



Knock-on electron (delta ray) generated by a 180 GeV muon as observed by the experiment GridPix at CERN SPS.

High energy transfers generate delta rays that may espace the detector if it is too thin. So average energy deposits are very often much smaler thant predicted by B&B.

If T_0 is the average maximal delta ray energy that can be absorbed in the detecting medium, a better estimate of the average deposited energy is given by :

$$-\left(\frac{dE}{dx}\right)\left[\frac{\text{MeV}}{\text{g/cm}^2}\right] = \frac{0.3071}{A\,(\text{g mol}^{-1})}\frac{z^2\,Z}{\beta^2}\left(\frac{1}{2}\ln\left(\frac{2m_e\beta^2\gamma^2T_0}{I^2}\right)-\frac{\beta^2}{2}\left(1+\frac{T_0}{2m_e\beta^2\gamma^2}\right)-\frac{\delta(\gamma\beta)}{2}-\frac{C_e}{Z}\right)$$

At extremely high energies, when $\beta\gamma>10^{S_1}$, stopping power reaches a constant called Fermi plateau.

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right]=0.3071\frac{z^2 Z}{2.A(g)}\ln\left(\frac{2m_eT_0}{(h\nu_p)^2}\right)$$

Fermi plateau measured in silicon

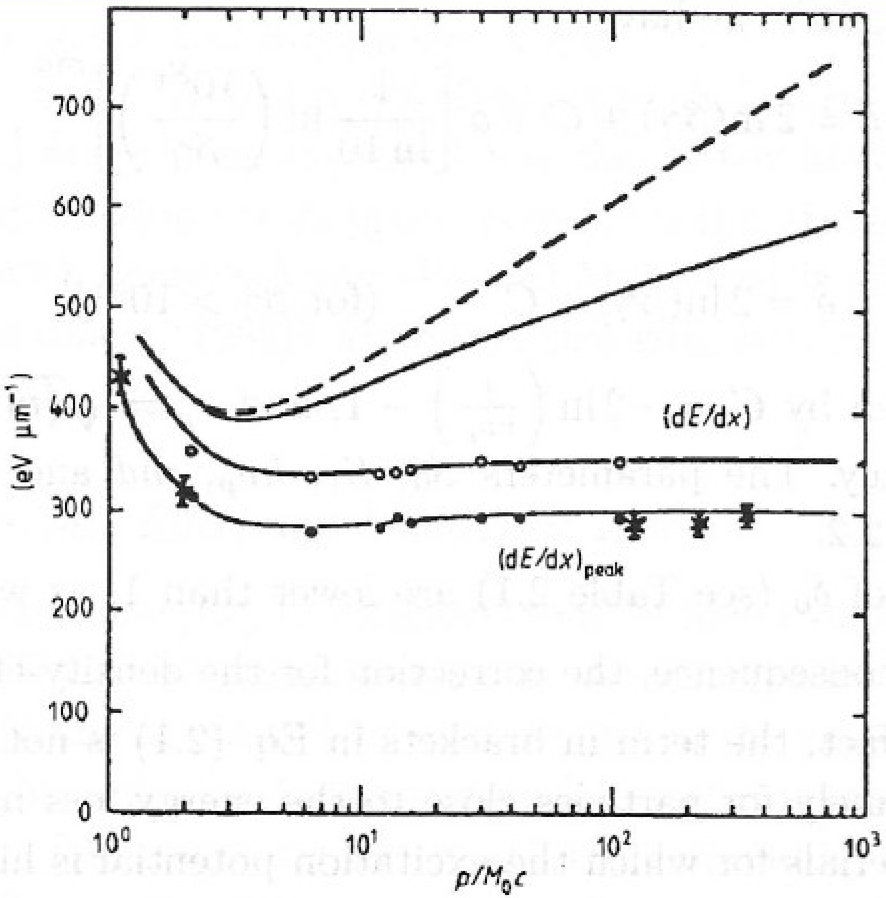


Fig. 2.5 Energy loss in silicon (in units of eV/μm) versus $\beta\gamma$ ($= p/M_0c$, where M_0 is the rest mass of the incoming particle) from [Rancoita (1984)]. From the top the first two curves are: the $-dE/dx$ without (broken curve) and with (full curve) the density-effect correction. The following second two curves are compared to experimental data for detector thicknesses of 300 ($\times$ from [Hancock, James, Movchet, Rancoita and Van Rossum (1983)]) and 900 μm ($\circ$ and $\bullet$ from [Esbensen et al. (1978)]): the restricted energy loss with the density-effect taken into account and the prediction of the most probable energy loss.

Delta rays (secondary electrons)

The differential probability to generate a delta ray of kinetic energy T is given by :

$$\frac{dw(T,E)}{dTdx}=0.3071\frac{z^2 Z}{2.A(g)\beta^2}\frac{F(T)}{T^2}\ MeV^{-1}cm^2 g^{-1}$$

F(T) is a spin-dependent factor.

$$F(T)=F_0(T)=(1-\beta^2\frac{T}{T_{max}})$$ for spin-0 particles

$$F(T)=F_{1/2}(T)=F_0(T)+\frac{1}{2}(\frac{T}{E})^2$$ for spin-1/2 particles

$$F(T)=F_1(T)=F_0(T)(1+\frac{1}{3}\frac{T m_e}{m_0^2})+\frac{1}{3}(\frac{T}{E})^2(1+\frac{1}{2}\frac{T m_e}{m_0^2})$$ for spin-1 particles

Delta rays (secondary electrons)

For $T \ll T_{max}$ and $T \ll m_0^2 / m_e$,

$$\frac{dw(T,E)}{dTdx} = 0.3071 \frac{z^2 Z}{2.A(g)\beta^2} \frac{1}{T^2}\ MeV^{-1} cm^2 g^{-1}$$

This allows to compute an approximate probability to generate a delta ray of kinetic energy greater than T_s in a thin absorber of mass thickness x :

$$w(Ts, E, x) \simeq 0.3071\, x \frac{z^2 Z}{2.A(g)\beta^2} \frac{1}{T_s}$$

show this expression

So far, only the average energy loss has been considered. But energy loss is subjected to large fluctuations that in thin absorbers results in asymmetric distributions. The subject is quite complex and has no general exact solutions, but a few approximate formulas help to estimate it.

For thin absorbers in which $\epsilon / T_{max} \ll 1$, where : $\epsilon = 0.3071\, x \frac{z^2 Z}{2.A(g)\beta^2}\ MeV$

The problem was first studied by Landau and then Vavilov. Their distribution functions are not analitic. A useful approximation of the Landau distribution is :

$$L(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(\lambda + e^{-\lambda})\right)$$

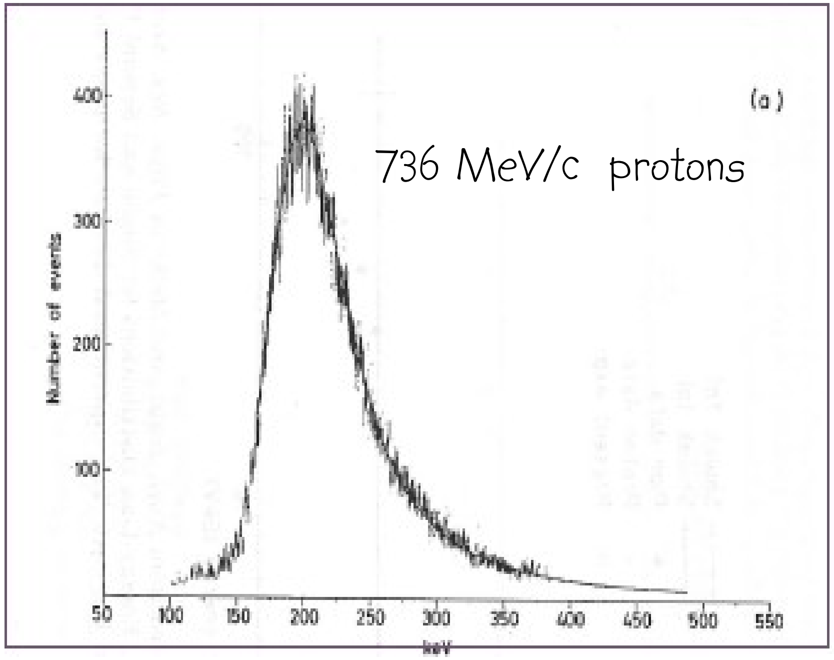
where : $\lambda = \frac{\Delta E - \Delta E_{MP}}{\epsilon}$ and ΔE is the energy loss ΔE_{MP} is the most probable energy loss.

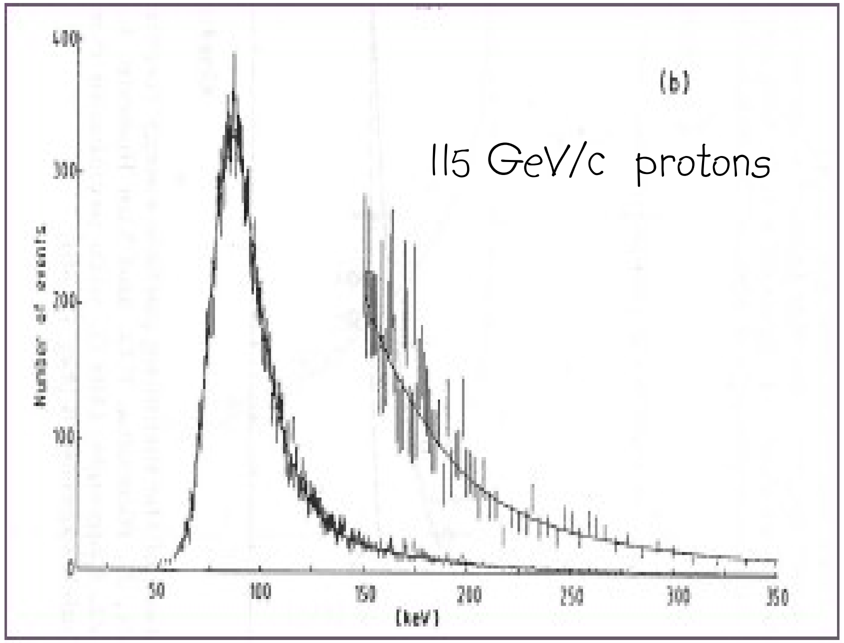
$$\Delta E_{MP} = \Delta E_{Bethe} + \epsilon \left(\beta^2 + \ln\left(\frac{\epsilon}{T_{max}}\right) + 0.194\right)\ MeV$$

Energy straggling

Still for thin absorbers, an improved generalized energy loss distribution that takes into the distant collisions that are neglected in Landau's approach, can be obtained by convoluting a Landau distribution with a Gaussian distribution :

$$f(\Delta E, x)_I = \frac{1}{\sqrt{2\pi\sigma_I^2}} \int_{-\infty}^{+\infty} L(\Delta E - \Delta E', x) \exp\left(\frac{-\Delta E'}{2\sigma_I^2}\right) d(\Delta E')$$





Energy deposited by protons in silicon

Fig. 2.10 Curves (a) and (b) (adapted and republished with permission from Hancock, S., James, F., Movchet, J., Rancoita, P.G. and Van Rossum, L., *Phys. Rev. A* **28**, 615 (1983); Copyright (1983) by the American Physical Society) show the energy loss spectra at 0.736 and 115 GeV/c of incoming particle momentum. Continuous curves are the complete fit to experimental data, i.e., the Landau straggling function folded over the Gaussian distribution taking into account distant collisions.

In thick absorbers in which $\epsilon / T_{max} \gg 1$, both the Landau and the Vavilov distributions tend to a Gaussian :

$$f(\Delta E, x) \simeq \frac{1}{\sqrt{2\pi T_{max}\epsilon(1-\frac{\beta^2}{2})}} \exp\left(-\frac{(\Delta E - \Delta E_{Bethe})^2}{2 T_{max}\epsilon(1-\frac{\beta^2}{2})}\right)$$

with : $\sigma \simeq \sqrt{\epsilon T_{max}(1-\frac{\beta^2}{2})}$

Stopping power of electrons by ionization and excitation in matter.

Incoming and outgoing particles are identical.
Energy transfer is bigger .

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right]=\frac{0.3071}{A(g)}\cdot\frac{Z}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{T\,m_e\beta^2\gamma^2}{2I^2}\right)+\frac{1}{2\gamma^2}\left(1-(2\gamma-1)\ln(2)\right)+\frac{1}{16}\left(\frac{\gamma-1}{\gamma}\right)^2\right]$$

Kinetic energy of incoming electron : $T=(\gamma-1)m_e=E-m_e$

Stopping power of positrons by ionization and excitation in matter.

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right]=\frac{0.3071}{A(g)}\cdot\frac{Z}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{T\,m_e\beta^2\gamma^2}{2I^2}\right)-\frac{\beta^2}{24}\left(23+\frac{14}{\gamma+1}+\frac{10}{(\gamma+1)^2}+\frac{4}{(\gamma+1)^3}\right)\right]$$

When a positron comes to a rest it annihilates : $e^+ + e^- \rightarrow \gamma\ \gamma$ of 511 KeV each

A positron may also undergo an annihilation in flight according to the following cross section :

$$\sigma(Z,E)=\frac{Z\pi r_e^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right)-\frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]$$

Stopping power of a compound medium

$$\frac{dE}{dx} \approx \sum_i f_i \left.\frac{dE}{dx}\right|_i$$

$$f_i = \frac{m_i}{m} \quad , \quad \sum_i m_i = m$$ where f_i is the massic ratio of element i

$\left.\frac{dE}{dx}\right|_i$ is the stopping power of element i