Predicting the Mass of the Dark Matter Particles

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OUTLINE:

I. The basic ideas

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II. An illustrative SU(5) x SU(2) model

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The ratio of n_{DM}/n_B is calculable thermodynamically, and therefore there is a prediction for the mass of the DM particle.

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PHENO 2014

I. THE BASIC IDEAS

A. Cogeneration of dark matter and ordinary matter

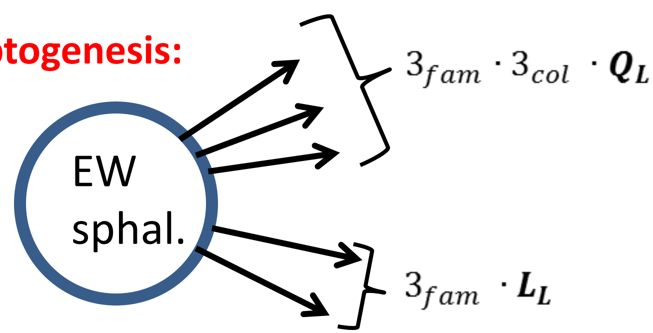
$\Omega_{DM} \sim 5\,\Omega_B \Rightarrow$ **Common origin?**

SMB, R.S.Chivukula, E. Farhi, *Phys. Lett.* **B241**, 387 (1990)
SMB, *Phys. Rev.* **D44**, 3062 (1991)
D.B. Kaplan, *Phys. Rev. Lett.* 68, 741 (1992)

The idea of these early papers was that a **primordial asymmetry in B, L** gets partially converted by **electroweak sphalerons** into an **asymmetry in X**, some other almost-conserved global quantum number. The lightest $X \neq 0$ particles would be stable = DM.
After DM + $\overline{\text{DM}}$ annihilation, one is left with "asymmetric dark matter".
[With S. Nussinov, *Phys. Lett.* **B165**, 55 (1985), these papers were also the earliest to discuss asymmetric DM.]

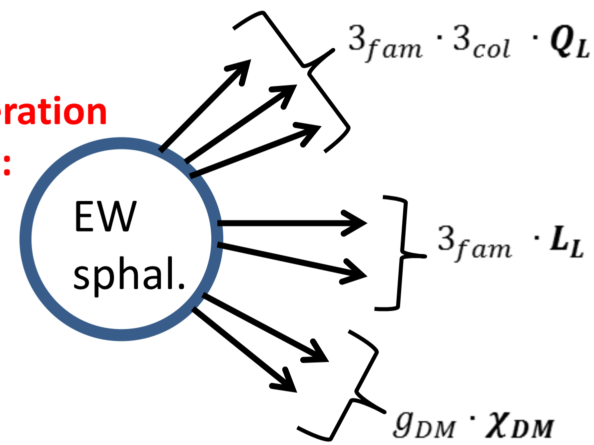
This is similar to the idea of leptogenesis, where a primordial asymmetry in L is partially converted into a baryon asymmetry by EW sphalerons.

Leptogenesis:



$$\Delta B \neq 0 \quad \Delta L \neq 0$$
$$\Delta(B - L) = 0$$

early Cogeneration models:



$$\Delta B \neq 0 \quad \Delta L \neq 0 \quad \Delta X \neq 0$$

I. THE BASIC IDEAS

B. Unification of dark matter and ordinary matter

Unification based on large groups (that is, > SU(5), SO(10)) can have three features that can lead to dark matter:

(1) Standard-Model-singlet fermions, which could be DM
(2) Accidental U(1) global symmetries (like B-L in min. SU(5)), which could be "X"
(3) Extra non-abelian gauge groups whose sphalerons could give co-generation

(1) Standard-Model-singlet fermions: Consider example of SU(6). The simplest anomaly-free set of fermions that gives a family is

$$
\begin{aligned}
& \mathbf{15} + \overline{\mathbf{6}} + \overline{\mathbf{6}} \\
&= \psi^{AB} + \psi_A + \psi_A \qquad A,B = 1,\dots,6 \quad \alpha,\beta = 1,\dots,5 \\
&= \left(\psi^{\alpha\beta} + \psi^{\alpha 6}\right) + \left(\psi_\alpha + \psi_6\right) + \left(\psi_\alpha + \psi_6\right) \\
&= (10 + 5) + \left(\overline{5} + 1\right) + \left(\overline{5} + 1\right) \\
&= \underbrace{10 + \overline{5}}_{family} + \underbrace{5 + \overline{5}}_{vectorlike} + \underbrace{1 + 1}_{singlets}
\end{aligned}
$$

(2) Accidental U(1) global symmetries

Example: minimal SU(5) $\quad (10\;10)\;5_H \;+\; (10\;\bar{5})\;5_H^{\,*}$

$$T = \;1\;\;1\;\;-2 \qquad 1\;\;-3\;\;2$$

Higgs VEV breaks both T and Y/2 leaving unbroken $\quad B - L = \frac{1}{5}\,T \;+\; \frac{4}{5}\left(\frac{Y}{2}\right)$

(3) Extra non-abelian gauge groups whose sphalerons could give co-generation

Example based on $\quad SU(7) \longrightarrow SU(5)\times SU(2)_* \longrightarrow G_{SM}\times SU(2)_*$

$$\begin{aligned}
& 21 \;+\; \bar{7} \;+\; \bar{7} \;+\; \bar{7} \\
&= \psi^{AB} + \psi_A + \psi_A + \psi_A \qquad A,B = 1,\dots,7 \quad \alpha,\beta = 1,\dots,5 \\
&= \left(\psi^{\alpha\beta} + \psi^{\alpha I} + \psi^{IJ}\right) \;+\; 3\times\left(\psi_\alpha + \psi_I\right) \qquad I,J = 1,2 \\
&= \left((10,1) + (5,2) + (1,1)\right) \;+\; 3\times\left((\bar{5},1) + (1,2)\right) \quad \text{under } SU(5)\times SU(2)_* \\
&= \underbrace{10 + \bar{5}}_{family} \;+\; \underbrace{2\times(5 + \bar{5})}_{vectorlike} \;+\; \underbrace{7\times 1}_{singlets} \quad \text{under SU(5)}
\end{aligned}$$

Note that the following transform non-trivially under $SU(2)_*$

(5,2) = heavy "extra" quarks and anti-leptons

(1,2) = dark matter

So the sphalerons of $SU(2)_*$ will violate B, L, and X as desired (if the DM has $X \neq 0$).

The model I shall outline is based on a different group, but the consequences are very similar

$$E_6 \supset SU(6)\times SU(2)_* \supset SU(5)\times SU(2)_* \supset G_{SM}\times SU(2)_*$$

$$\mathbf{27} \longrightarrow (\mathbf{15},\mathbf{1}) + (\overline{\mathbf{6}},\mathbf{2}) \longrightarrow (\mathbf{10},\mathbf{1}) + (\mathbf{5},\mathbf{1}) + \left(\overline{\mathbf{5}},\mathbf{2}\right) + (\mathbf{1},\mathbf{2}) \text{ under } SU(5)\times SU(2)_*$$
$$\longrightarrow \left[\mathbf{10} + \overline{\mathbf{5}}\right] + [\mathbf{5} + \overline{\mathbf{5}}] + \mathbf{2}\times\mathbf{1} \text{ under SU(5)}$$

Comment: For the purposes of co-generation, one does not need unification, only $G_{SM} \times SU(2)_*$ with some quarks, leptons and dark matter particles being non-singlets under $SU(2)_*$. However, we see from these two examples that GUTs are a natural context for such a structure.

Second comment: Why do we want the sphalerons to be of a new SU(2) rather than of $SU(2)_{EW}$ as in the 1990-2 models of SMB, Chivukula, Farhi; SMB; and Kaplan?

If $SU(2)_{EW}$ sphalerons do the co-generation, one has a model-building dilemma:

Either (A), the DM fields (which have $X \neq 0$) are chiral under $SU(2)_{EW}$, in which case their masses would come from the breaking of the EW group (like the SM fermions). The danger is that the DM particles will be light enough to have been seen, or will contribute too much to the S,T,U parameters, or to H $\rightarrow$gg, H $\rightarrow \gamma\gamma$, etc.

Or (B), the DM fields are vectorlike under $SU(2)_{EW}$, in which case both DM and anti-DM doublets (which have opposite values of X), would be produced by the EW sphalerons, so that these sphalerons would not violate X and no x asymmetry would be generated.

III. The Model

Based on the group and matter multiplets shown before:

$$SU(5) \times SU(2)_* \supset G_{SM} \times SU(2)_*$$

$$(10,1) \quad + \quad (5,1) \quad + \quad (\overline{5},2) \quad + \quad (1,2)$$

$$\begin{pmatrix} \ell^c & \begin{matrix} u \\ d \end{matrix} & u^c \end{pmatrix} + \begin{pmatrix} \overline{\ell} \\ \overline{\nu} \end{pmatrix} \overline{d^c} \quad + \begin{pmatrix} \ell_I \\ \nu_I \end{pmatrix} {d^c}_I \quad + \quad \chi_I \qquad\qquad I = 1,2$$

$SU(2)_*$ breaking is done by a $(1,2)$ scalar field: $\langle \Omega_I \rangle \sim M_*$

So the "extra" vectorlike $5 + \overline{5}$ fermions get mass from

$$(\overline{5},2)\,(5,1)\,\langle (1,2)_h \rangle = {d^c}_I\, \overline{d^c}\, \langle \Omega_I \rangle + (\ell_I\, \nu_I) \begin{pmatrix} \overline{\ell} \\ \overline{\nu} \end{pmatrix} \langle \Omega_I \rangle$$

We want the DM particles in (1,2) to have non-zero X, so they must get Dirac masses.
Thus we assume that there are six gauge-singlet fermions that we denote χ^c.
The DM masses also come from $SU(2)_*$ breaking:

$$\sum_{a=1}^{6} Y_a\, \chi_I\, {\chi^c}_a \langle \Omega_I \rangle$$

In addition to these six gauge-singlet fermions, there need to be at least one other, which will be massless, so that DM particle and anti-particles can annihilate into it, and that dark energy will be red-shifted away". We will call that fermion S.

The quarks and leptons get mass when the EW symmetry breaks, from two Higgs multiplets: $(5,1)_h \equiv h$ and $(\overline{5},2)_h \equiv h'_I$.

up q masses: $(\mathbf{10},\mathbf{1})\,(\mathbf{10},\mathbf{1})\,\langle(\mathbf{5},\mathbf{1})_h\rangle = u\,u^c\,\langle h\rangle$

down q & ch. lepton masses $(\mathbf{10},\mathbf{1})\,(\overline{\mathbf{5}},\mathbf{2})\,\left\langle(\overline{\mathbf{5}},\mathbf{2})_h\right\rangle = d\,d^c{}_I\langle h'_I\rangle + \ell^c\,\ell_I\,\langle h'_I\rangle$

neutrinos can get see-saw masses:

$$(\overline{\mathbf{5}},\mathbf{2})\,(\overline{\mathbf{5}},\mathbf{2})\,\left\langle(\overline{\mathbf{5}},\mathbf{2})_h{}^*\right\rangle\left\langle(\overline{\mathbf{5}},\mathbf{2})_h{}^*\right\rangle \quad \nu_I\ \nu_J\,\langle h'^*{}_I\rangle\langle h'^*{}_J\rangle$$

What is the scale M_* of $SU(2)_*$ breaking? Of course, it must be large enough that the extra vectorlike quarks and leptons have not been seen. But the DM particles will turn out to have masses of order 1 GeV. If one of the Yukawa constants Y_a is of order $10^{-4} - 10^{-5}$, then M_* can be of order 10$-\,10^2$ TeV.

If M_* > 100 TeV, the $SU(2)_*$ interactions turn out to be too slow to keep the DM in equilibrium with the ordinary matter, and energy that is trapped in the DM sector will cause problems for primordial nucleosynthesis.

So we assume that M_* is of order 10$-\,10^2$ TeV.

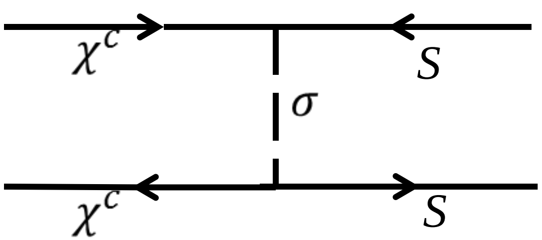
Global Symmetries:

There is a global charge X, which is +1 for χ_I
-1 for χ^c
+1 for a singlet scalar σ
and 0 for everything else. X is conserved by all perturbative low-energy couplings, but violated by $SU(2)_*$ sphalerons.

There is a global charge W, which is +1 for the massless singlet fermions S, and -1 for the singlet scalar σ. W is conserved by everything. It forbids the fermion S from getting mass. It allows the coupling

$$(\chi^c\ S)\ \sigma$$

The scalar σ, which is assumed to be very light (~ 10 GeV) allows the DM particles and antiparticles to annihilate:



IV. The genesis of the DM asymmetry

There are four eras that we need to think about:

(1) T $\gg T_*$, where T_* is the temperature when $SU(2)_*$ sphalerons freeze out.
In this era a primordial asymmetry in some combination of B, L, X is produced.
Which asymmetry is not important, as we shall see.

(2) $T \geq T_*$
The sphaleron* processes of both $SU(2)_{EW}$ and $SU(2)_*$ are taking place, and violate B, L, X, and reshuffle the asymmetries in these quantum numbers. The equilibrium conditions for the two kinds of sphalerons determine the ratios of B,L, and X.

(3) $T_{EW} < T < T_*$
$SU(2)_*$ sphalerons have frozen out, so B-L and X are henceforth fixed to their values at the end of era (2). But EW sphaleron processes continue and reshuffle B and L. The equilibrium condition for these EW sphaleron processes determine the ratio B/L, which then remains at the value it has at the end of era (3).

(4) $T < M_{DM}$
When T falls below the mass of the DM particles (about 1 GeV), the DM antiparticles annihilate with the DM particles, leaving just the asymmetric component. The same happens to the baryons and antibaryons. Since the ratios of B, L, X are known we know n_{DM}/n_B and thus m_{DM}/m_p.

From thermodynamics, we can calculate the ratios of B,L,X.
[J.A. Harvey and M.S. Turner, *Phys. Rev.* **D42**, 3344 (1990)]

Era (2), $T > T_*$

We will assume that above T_* the $SU(2)_*$ gauge symmetry is unbroken. That would mean that

(a) all particles within an irrep of $G_{SM} \times SU(2)_*$ have the same chemical potential μ
(b) the chemical potentials of the $G_{SM} \times SU(2)_*$ gauge bosons vanish.

Second, we use the fact that the scattering processes involving the Yukawa interactions and interactions among the scalars are in equilibrium and give relations among the chemical potentials that allow all of them to be expressed in terms of just five
$\mu_Q, \mu_L, \mu_\chi, \mu_h, \mu_\sigma$

$\mu_\Omega = \mu_{\Omega^*} \implies \mu_\Omega = 0.$ The existence of a $h\, h'_I\, \Omega_I$ term $\implies \mu_{h'} = -\mu_h$.
The Yukaws of the SM quarks and leptons $\implies \mu_{u^c} = -\mu_Q - \mu_h, \;\; \mu_{d^c} = -\mu_Q + \mu_h$,
$\mu_{\ell^c} = -\mu_L + \mu_h$
The extra vectorlike quarks and leptons have the couplings $L_I\, \bar{L}\, \Omega_I \; + \; {d^c}_I\, \overline{d^c}\; \Omega_I$
$\implies \mu_{\bar{L}} = -\mu_L, \;\; \mu_{\overline{d}^c} = -\mu_{d^c}$

And the mass terms of the singlet fermions and singlet scalar imply that
$\implies \mu_{\chi^c} = -\mu_\chi \quad , \qquad \mu_S = \mu_\chi - \mu_\sigma$

The third step is to use the fact that Q=0 and W=0 .

For low number densities, one has the following expression for the number density of particle of type i:

$$n^i(T) \cong c_i(m_i, T)\, \frac{\mu_i}{T}\ , \quad \text{where } c_i(m_i, T) = g_i \begin{Bmatrix} f(m_i/T) \\ b(m_i/T) \end{Bmatrix} T^3$$

and g_i is the statistical weight of particle type i.

$$f(x) \equiv \frac{1}{4\pi^2} \int_0^\infty \frac{y^2 dy}{cosh^2(\frac{1}{2}\sqrt{y^2+x^2})} \text{ for fermions,} \quad f(x) \cong \begin{cases} 1/6, & for\ x \ll 1 \\ 2\left(\frac{x}{2\pi}\right)^{3/2} e^{-x}, & for\ x \gg 1 \end{cases}$$

$$b(x) \equiv \frac{1}{4\pi^2} \int_0^\infty \frac{y^2 dy}{sinh^2(\frac{1}{2}\sqrt{y^2+x^2})} \text{ for bosons,} \quad b(x) \cong \begin{cases} 1/3, & for\ x \ll 1 \\ 2\left(\frac{x}{2\pi}\right)^{3/2} e^{-x}, & for\ x \gg 1 \end{cases}$$

So that for T large compared to the particle mass $\quad n^i(T) \propto g_i\, \mu_i \begin{Bmatrix} 1/6 \\ 1/3 \end{Bmatrix}$

So the condition that $Q = 0$ gives:

$$0 = Q$$

$$\propto 6\,(-1)\mu_L \;+\; 3\,(+1)\mu_{\ell^c} \;+\; 3\,(+1)\,\mu_{\bar{L}}$$
$$+\,9\left(+\tfrac{2}{3}\right)\mu_Q \;+\; 9\left(-\tfrac{2}{3}\right)\mu_{u^c} + 9\left(-\tfrac{1}{3}\right)\mu_Q + 18\left(+\tfrac{1}{3}\right)\mu_{d^c} + 9\left(-\tfrac{1}{3}\right)\mu_{\bar{d}^c}$$
$$+\,\frac{b(0)}{f(0)}\left[(+1)\,\mu_h + 2\,(-1)\mu_{h'}\right]$$

$$\Longrightarrow \quad 0 = \; -12\,\mu_L + 24\,\mu_h \quad \Longrightarrow \quad \mu_h = \frac{1}{2}\,\mu_L$$

Similarly, the condition that $W = 0$ gives:

$$0 = W$$
$$\propto (+1)\mu_S + \frac{b(0)}{f(0)}\,(-1)\,\mu_\sigma$$
$$\Longrightarrow \; \mu_S = 2\,\mu_\sigma \;\Longrightarrow\; \mu_\chi - \mu_\sigma = 2\,\mu_\sigma \Longrightarrow \; \mu_\sigma = \frac{1}{3}\,\mu_\chi$$

So these two relations allow one to express all chemical potentials in terms of three, namely

$$\mu_Q\,,\;\; \mu_L\,,\;\; \mu_\chi$$

Finally, we have the two equilibrium conditions for the EW and $SU(2)_*$ sphaleron processes:

EW sphalerons: $\mathbf{0 = 3_{fam} 3_{col}\, \mu_Q + 3_{fam} 2_{SU(2)*}\, \mu_L + 3_{fam} \mu_{\bar{L}}}$

$SU(2)_*$ sphalerons: $\mathbf{0 = 3_{fam} 2_{EW}\, \mu_L + 3_{fam} 3_{col} \mu_{d^c} + 3_{fam} \mu_\chi}$

These two relations allow one to solve for the chemical potentials of the quarks and Dark Matter particles in terms of that of the leptons:

After some simple algebra one ends up with $\mu_Q = -\frac{1}{3}\mu_L$ and $\mu_\chi = -\frac{9}{2}\mu_L$

Now we can calculate the number densities of B, L, and X:

$$
\begin{aligned}
n_B &\propto \frac{1}{3}\left[18\,\mu_Q - 9\,\mu_{u^c} - 18\mu_{d^c} + 9\,\mu_{\bar{d}^c}\right] &&= 18\,\mu_Q - 6\,\mu_h &&= -9\,\mu_L \\
n_L &\propto 12\,\mu_L - 6\,\mu_{\bar{L}} - 3\,\mu_{\ell^c} &&= 21\,\mu_L - 3\,\mu_h &&= \frac{39}{2}\,\mu_L \\
n_\chi &\propto 6\,\mu_\chi - 6\,\mu_{\chi^c} + \frac{b(0)}{f(0)}\,\mu_\sigma &&= 12\,\mu_\chi + 2\,\mu_\sigma &&= -57\,\mu_L
\end{aligned}
$$

Consequently: $\frac{X}{B-L} = \frac{n_X}{n_B - n_L} = 2,$ which remains fixed after era (2)

Now we turn to era (3), where $T_{EW} < T < T_*$. In this era, the $SU(2)_*$ sphalerons have frozen out, but the EW sphalerons continue to shuffle *B* and *L* (but *B-L* and *X* are unaffected).

Here we assume that $SU(2)_*$ is broken and that $SU(2)_{EW}$ is unbroken. So, $\mu_{W^\pm} = 0$, and particles within a Standard Model multiplet have the same μ

We don't need to consider the quantum number *X*, or any of the $X \neq 0$ particles, as they do not affect the ratio of *B* and *L*.

The important chemical potentials are $\mu_Q,\ \mu_L,\ \mu_\chi$.

We can solve for one of these using the condition $Q = 0$, as before. (The condition that gives on the chemical potentials is different than in era (2), because some of the charged particles (i.e. the heavy vectorlike ones) have disappeared as their masses are much larger than *T* in era (2).

Then one uses the single equilibrium condition for the EW sphalerons to solve for the remaining chemical potential.

I won't go through the algebra, which is similar to that we did for era (2) (but simpler). One obtains

$$\frac{L}{B} = \frac{n_L}{n_B} = -\frac{3}{4}\,\frac{14+3\,c_h}{6+c_h}$$

where, $c_h \equiv \left[b(m_h/T_{EW}) + b\left(m_{h'_1}/T_{EW}\right) + b\left(m_{h'_2}/T_{EW}\right)\right]/b(0)$
So that $0 < c_h < 3.$

If one assumes that the SM Higgs doublet has mass $< T_{EW}$, while the other two scalar doublets are heavier than that, one has that $c_h \cong 1.$

If we combine the equation we just found for L/B, with the result we found before (from era (2)) for X/(B-L), we obtain

$$\frac{X}{B} = \frac{n_X}{n_B} = \frac{66 + 13\,c_h}{2(6 + c_h)}$$

So that

$$5.5 < \frac{n_X}{n_B} < 5.833$$

With $c_h \cong 1$, $n_X/n_B \cong 5.64 \Rightarrow m_{DM} \cong 1$ GeV.

Final Remarks:

(1) In this kind of scheme, the two equilibrium conditions for the EW sphalerons and the $SU(2)_*$ sphalerons allow one to compute X/B and L/B. Thus one knows the number density of DM particles and consequently can predict their mass. The calculation depends primarily on the particle content of the model.

(2) The dark matter that is produced in this kind of scheme is neutral under all the Standard Model interactions. It would thus not be seen in direct searches or produced directly at accelerators.

(3) However, one generally has heavy "$5 + \overline{5}$" fermions in these models, and these can be produced in accelerators. They would then decay either into a SM fermion plus a DM pair (e.g. in the "E6" model) or into another heavy fermion plus a DM pair (e.g. the SU(7) model). It depends on the group theory.