A New Method For the Spin Determination of Dark Matter

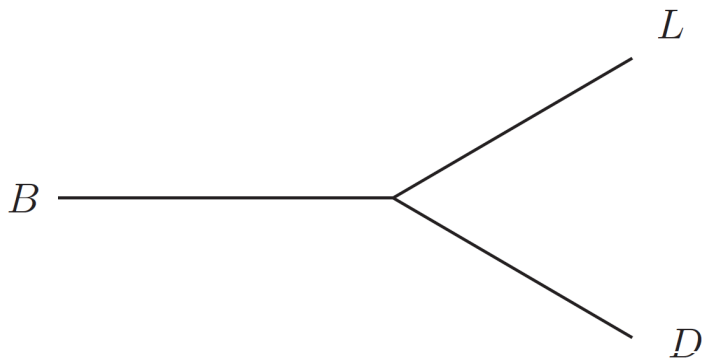
Daniel Salmon - University of Pittsburgh

In Collaboration with Dr. Neil Christensen

arXiv:1311.6465

Pheno 2014

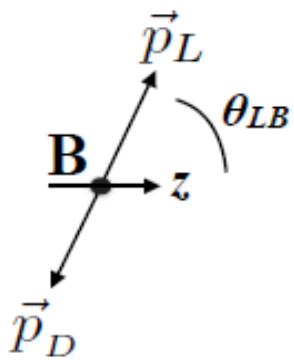
Determining Spin



- To determine the spin of *D* we would like to boost into the CM frame of *B* and make a historgram of the angle of *D*'s decay with respect to the boost direction
- If *B*'s width is narrow, this CM distribution will correspond to a linear combination of squares of the Wigner-*d* functions.

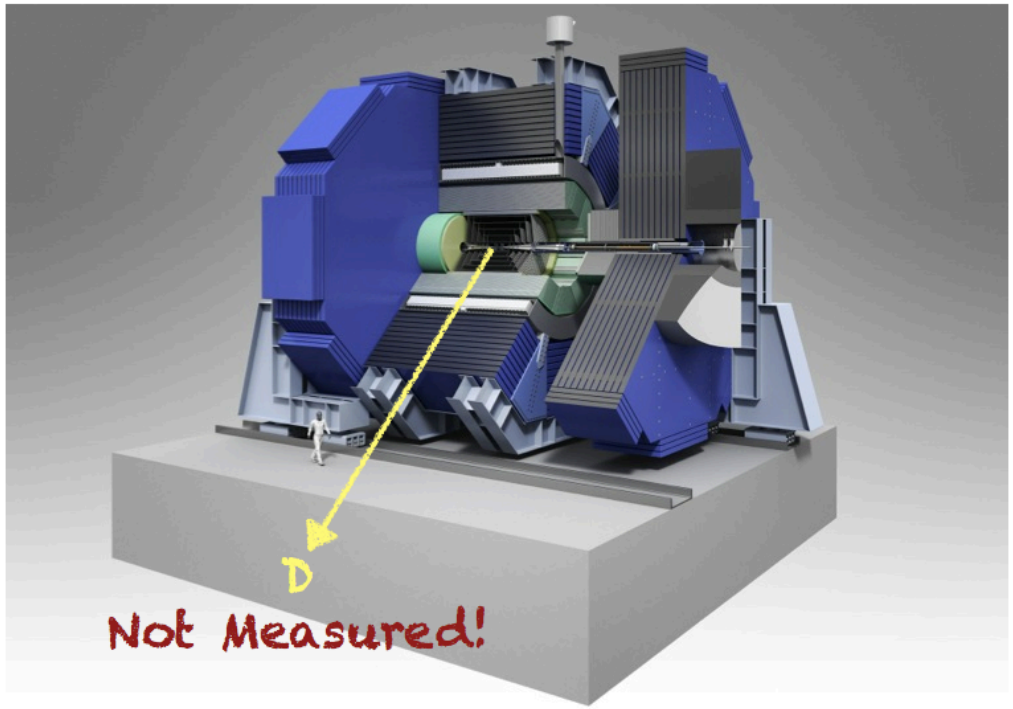
A Simple Matter Right?

- All we must do, then, is
  - Boost into *B*'s CM frame
  - Measure θ_{LB}
  - Compare with the Wigner *d* functions

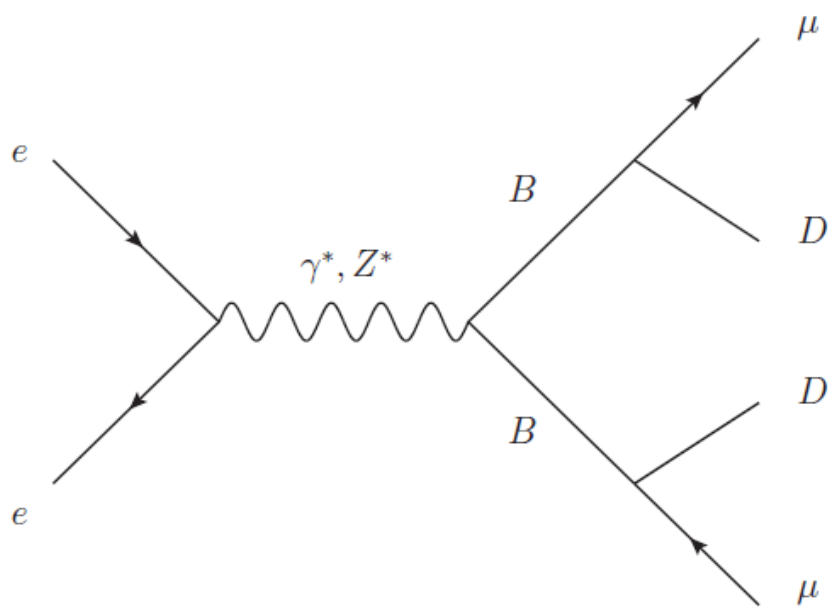


Dark Matter

- Dark matter particles don't interact with detectors!
  - We don't know *D*'s momentum
  - So we don't know *B*'s momentum
  - So we cannot reconstruct *B*'s CM system



Dark Matter Pair Production at the ILC



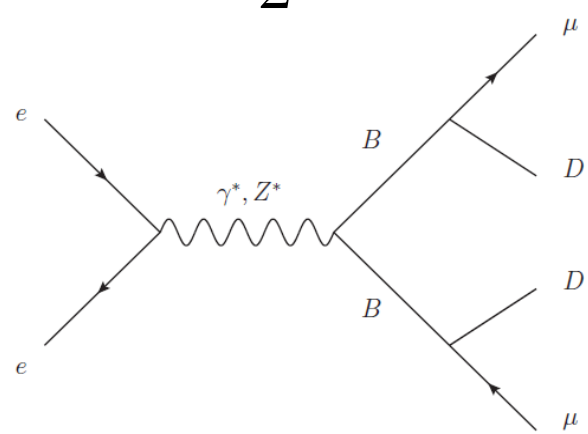
- All possible spin combinations
- General couplings

Equations for Reconstruction

$$|\cos\theta_{\mu B}| = \sqrt{1 - \frac{4M_B^2 p_{\widetilde{T}}^2}{(M_B^2 - M_D^2)^2}}$$

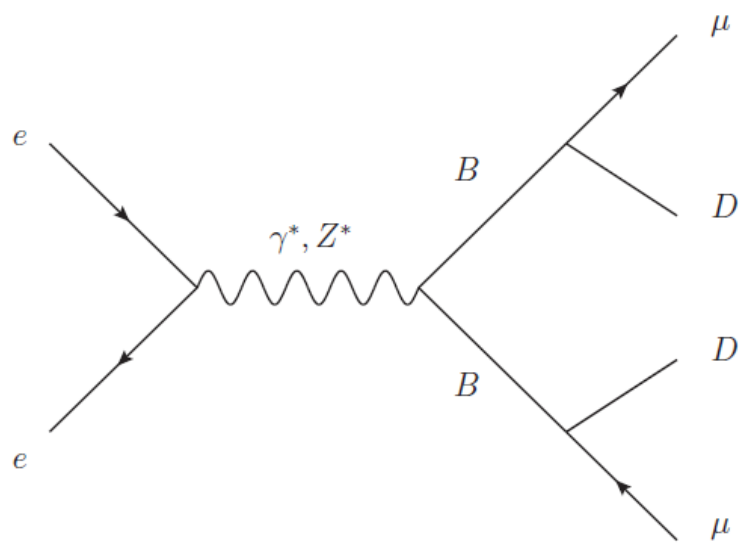
$$p_{\widetilde{T}}^2 = \frac{E_\mu^2(E_D^2 - M_D^2) - \frac{1}{4}(M_D^2 - M_B^2 + 2E_\mu E_D)^2}{(E_\mu + E_D)^2 - M_B^2}$$

$$E_D = \frac{\sqrt{s}}{2} - E_\mu$$



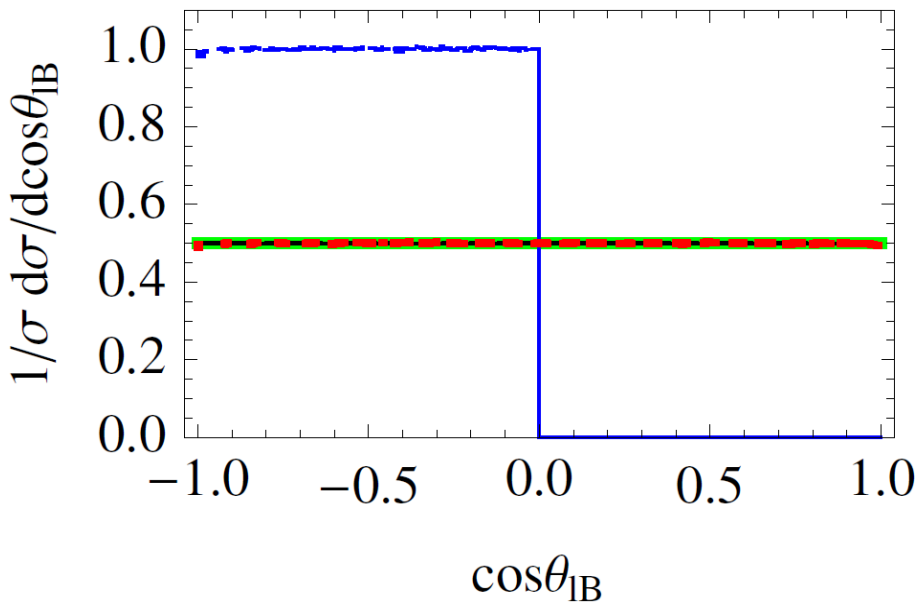
True CM Angular Distribution

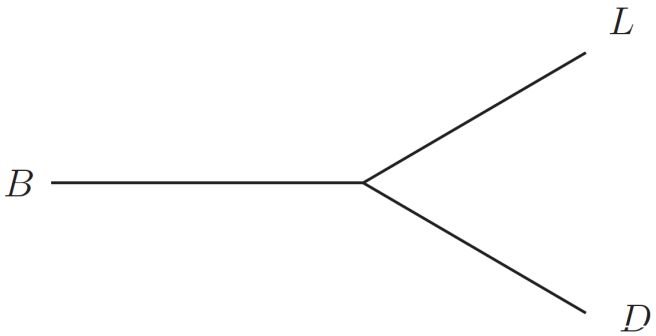
- Always symmetric
  - Independent of chiral couplings
  - Independent of beam polarization
- Z boson makes no difference



Distribution Classes

- Flat: $s_B = 0$ and $s_B = 1/2$ with any s_D

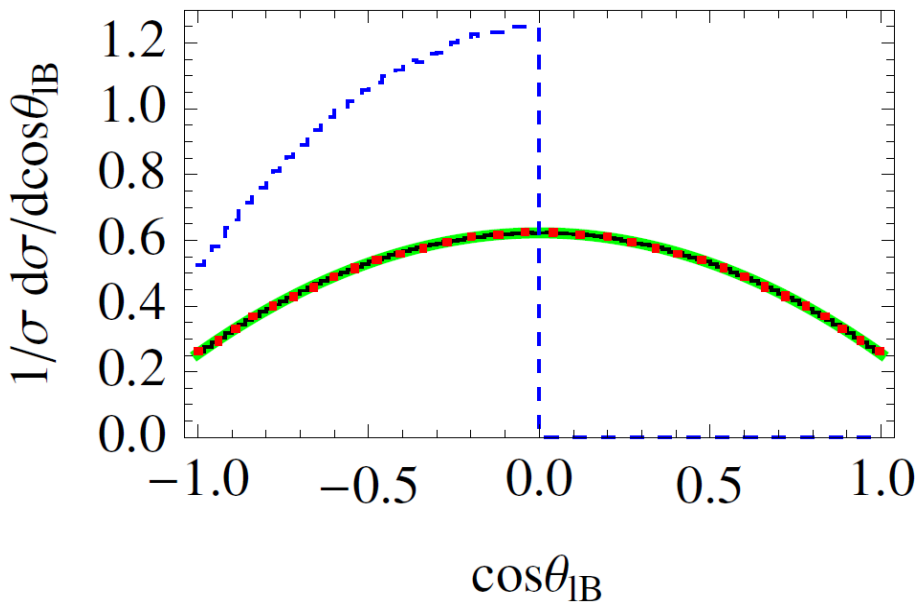


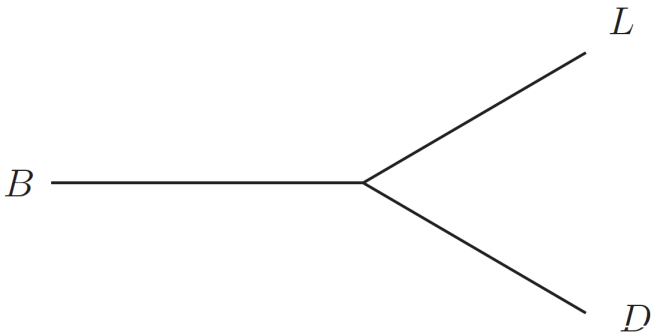


- s_B – Spin of B
- s_D – Spin of D
- ------- Our reconstruction
- ------- Our reconstruction halved and reflected
- ——— True distribution
- ——— Analytic solution

Distribution Classes

- Concave negative parabola: $s_B = 1$, $s_D = 1/2$

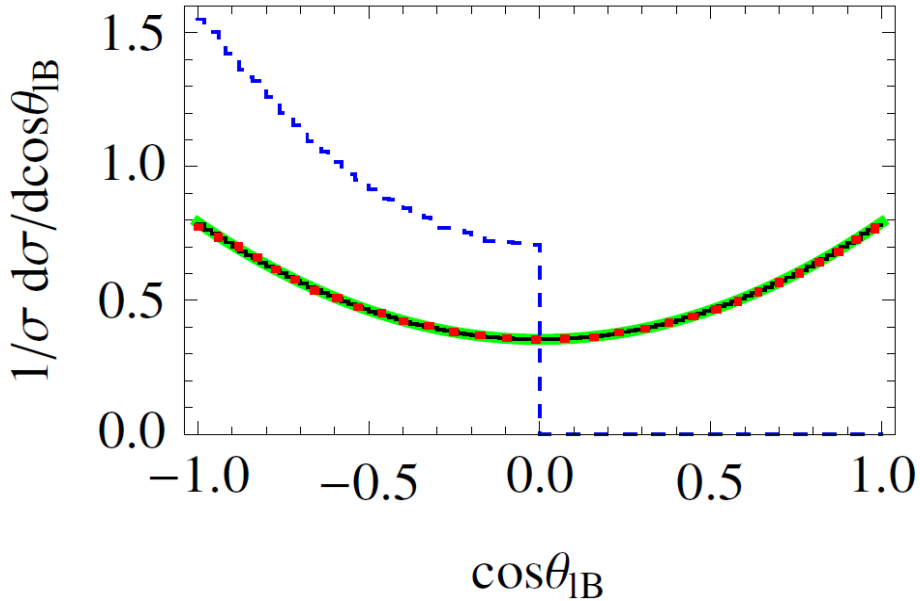


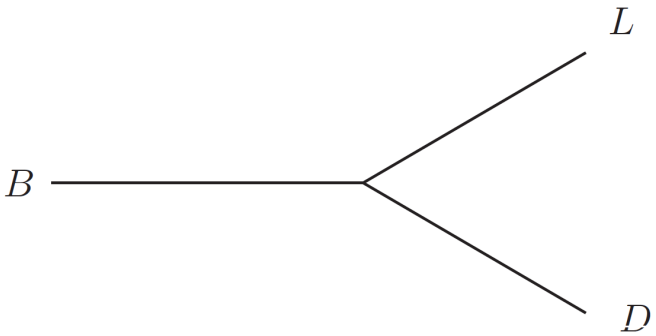


- s_B – Spin of B
- s_D – Spin of D
- ------- Our reconstruction
- ------- Our reconstruction halved and reflected
- ——— True distribution
- ——— Analytic solution

Distribution Classes

- Concave positive parabola: $s_B = 1$, $s_D = 3/2$ and $s_B = 3/2$ with any s_D

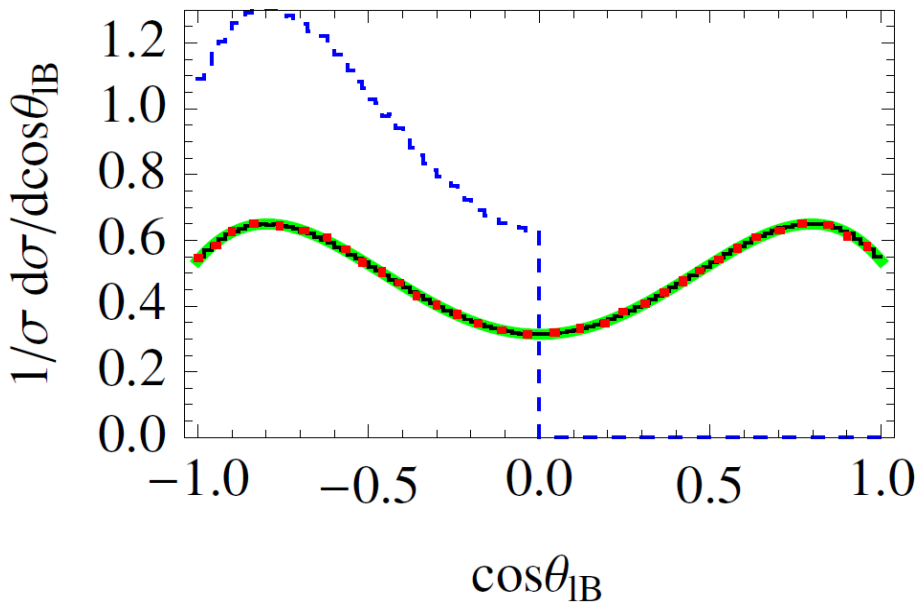


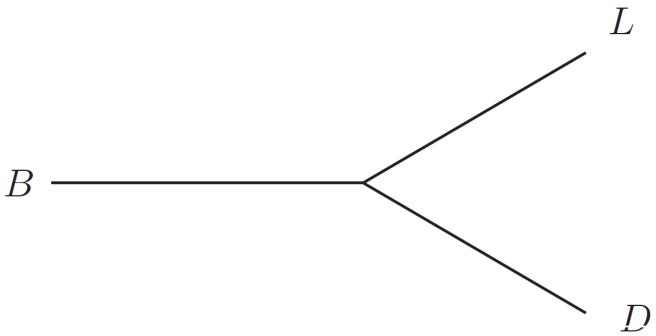


- s_B – Spin of B
- s_D – Spin of D
- ------- Our reconstruction
- ------- Our reconstruction halved and reflected
- ——— True distribution
- ——— Analytic solution

Distribution Classes

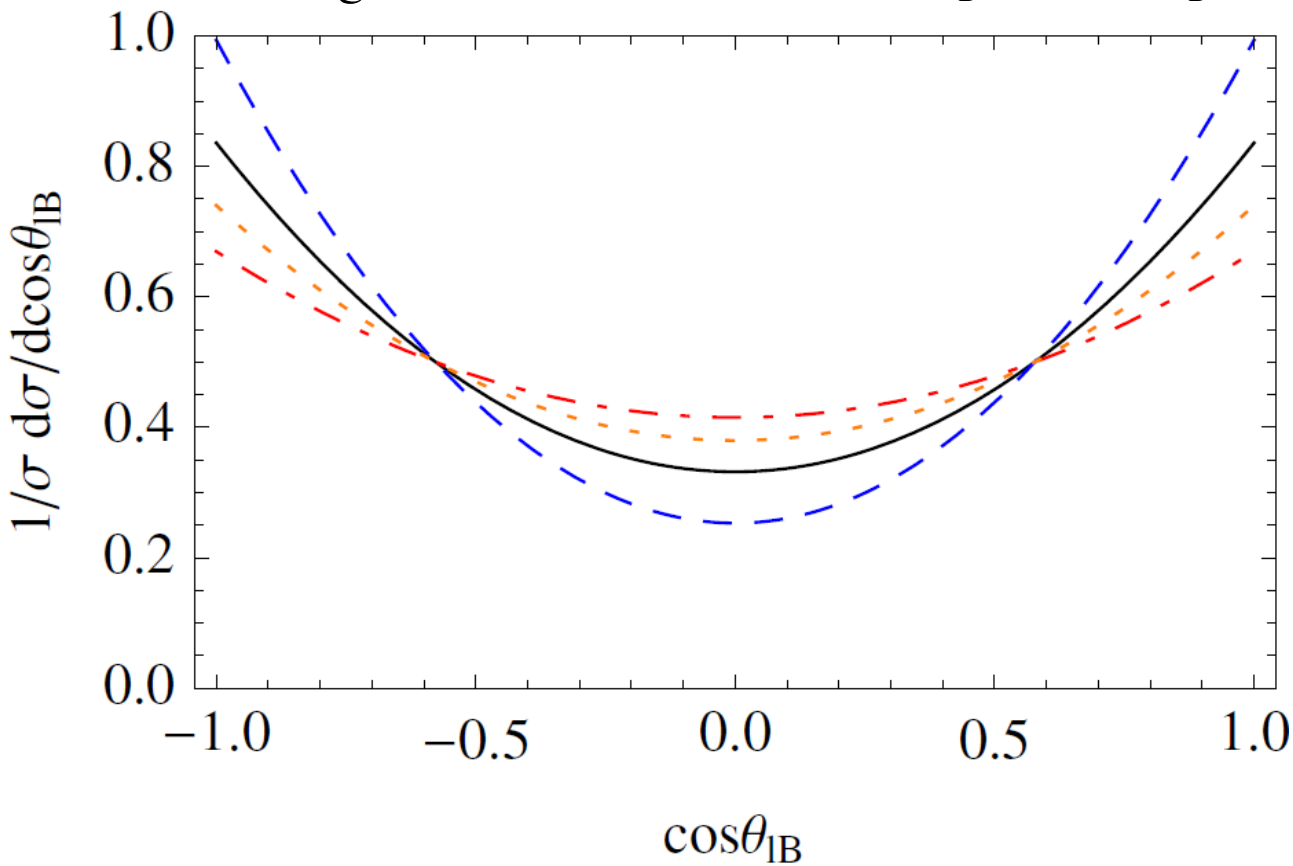
- "M" shaped: $s_B = 2$ with any s_D





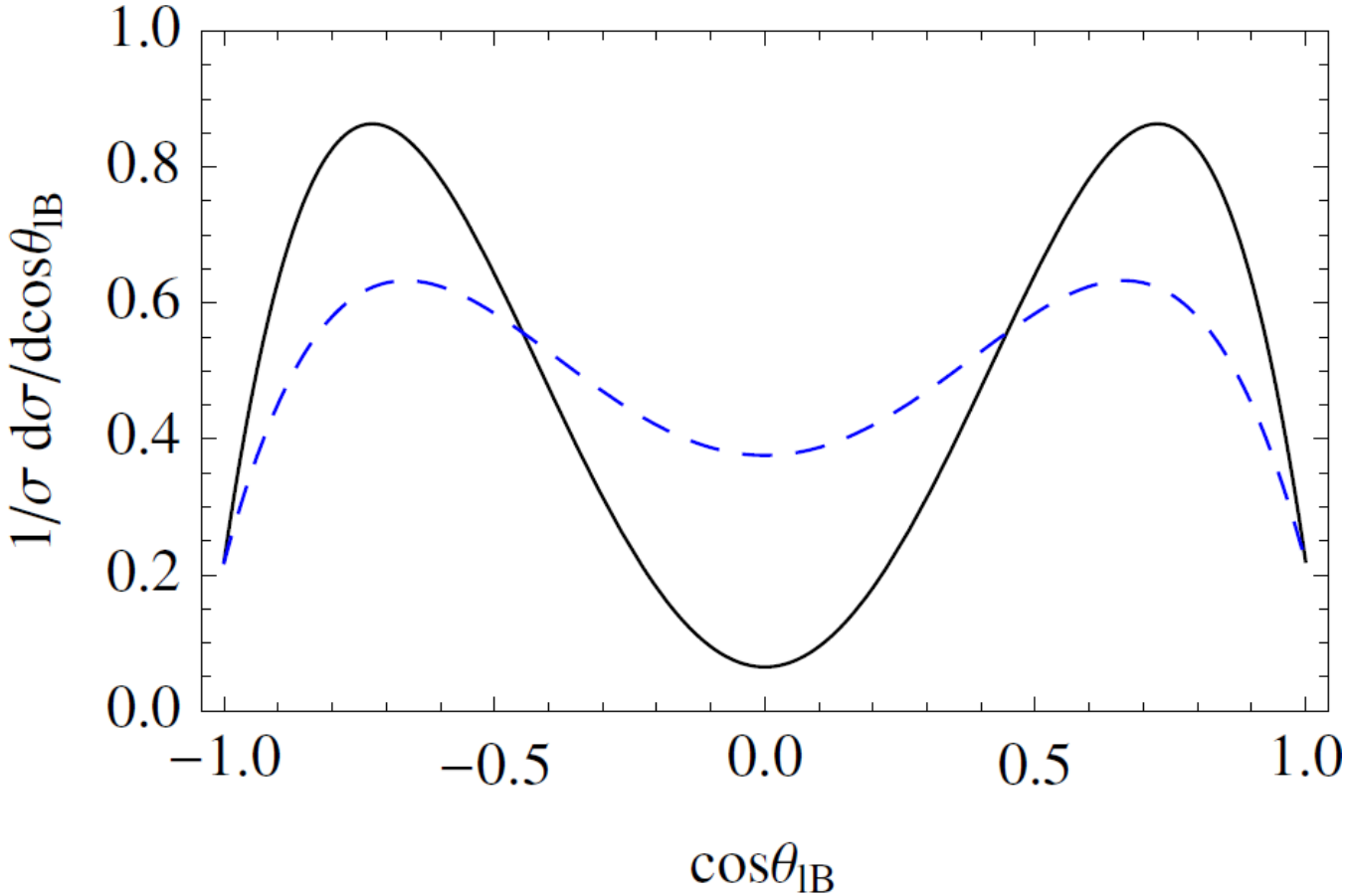
- s_B – Spin of B
- s_D – Spin of D
- ------- Our reconstruction
- ------- Our reconstruction halved and reflected
- ——— True distribution
- ——— Analytic solution

Differentiating between subclasses with $s_B = 1$ and $s_B = 3/2$



- s_B – spin of B
- s_D – spin of D
- ——— $s_B = 1$ and $s_D = 3/2$
- - - - - - $s_B = 3/2$ and $s_D = 0$
- - - - - - $s_B = 3/2$ and $s_D = 1$
- - - - - - $s_B = 3/2$ and $s_D = 2$

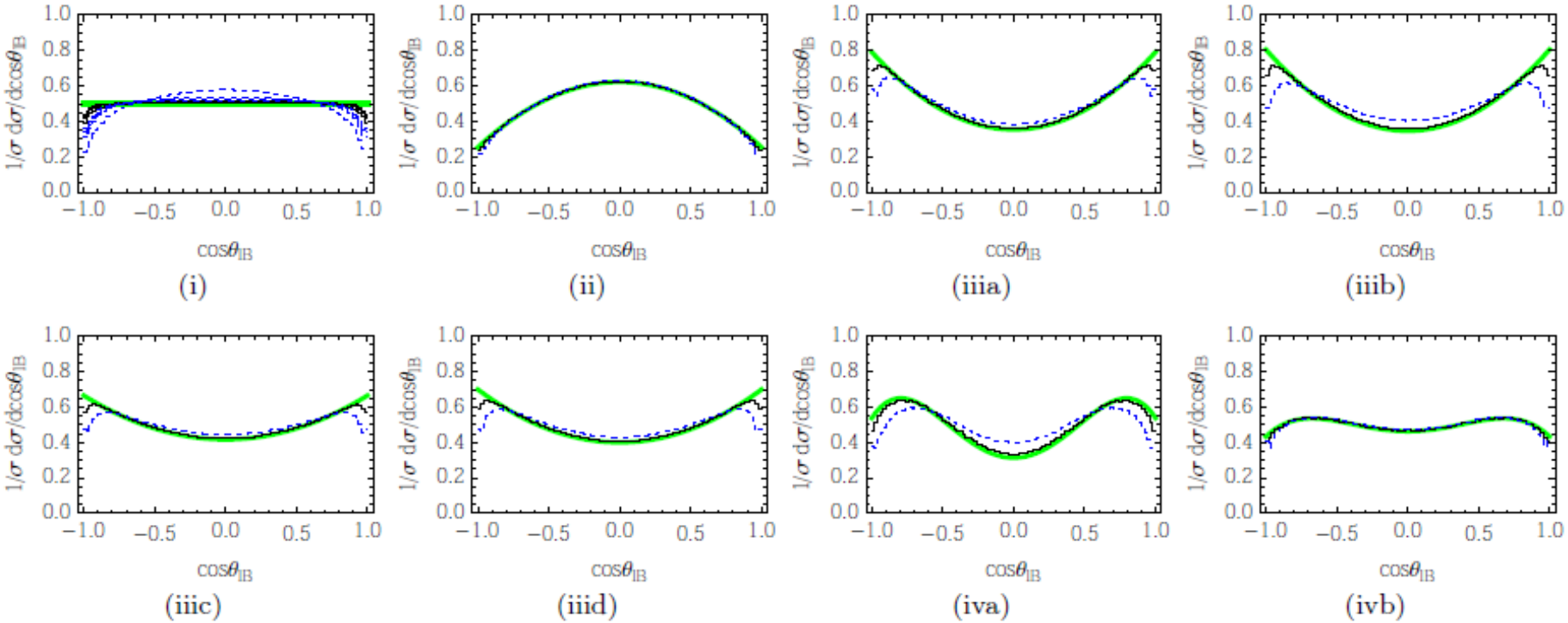
Differentiating between subclasses with $s_B = 2$



- s_B – spin of B
- s_D – spin of D
- ——— $s_B = 2$ and $s_D = 1/2$
- - - - - - $s_B = 2$ and $s_D = 3/2$

Finite Widths

$s_B = 0$ or $1/2$ with any s_D | $s_B = 1$ and $s_D = 1/2$ | $s_B = 1$ and $s_D = 3/2$ | $s_B = 3/2$ and $s_D = 0$

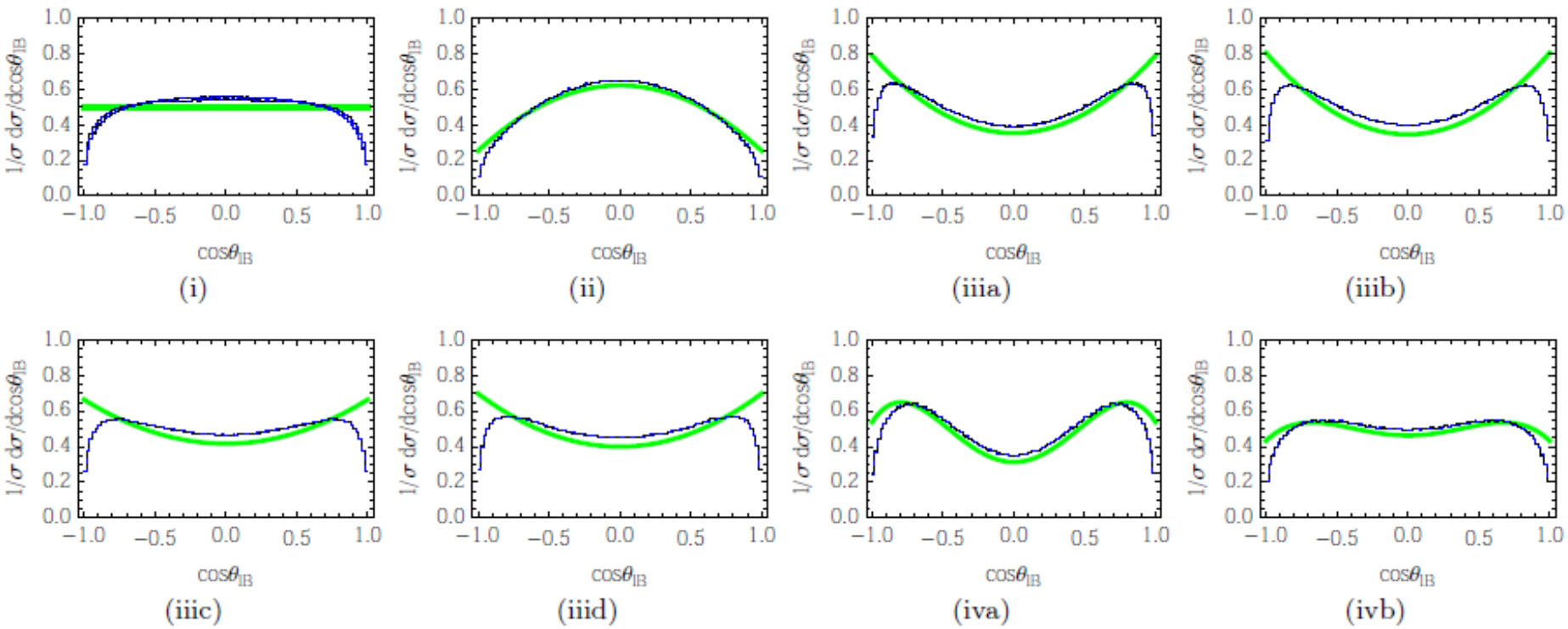


$s_B = 3/2$ and $s_D = 1$ | $s_B = 3/2$ and $s_D = 2$ | $s_B = 2$ and $s_D = 1/2$ | $s_B = 2$ and $s_D = 3/2$

- Analytic solution
- Width of B: 1%
- Width of B: 5%

ISR, Beamstrahlung and Cuts

$s_B = 0$ or $1/2$ with any s_D | $s_B = 1$ and $s_D = 1/2$ | $s_B = 1$ and $s_D = 3/2$ | $s_B = 3/2$ and $s_D = 0$

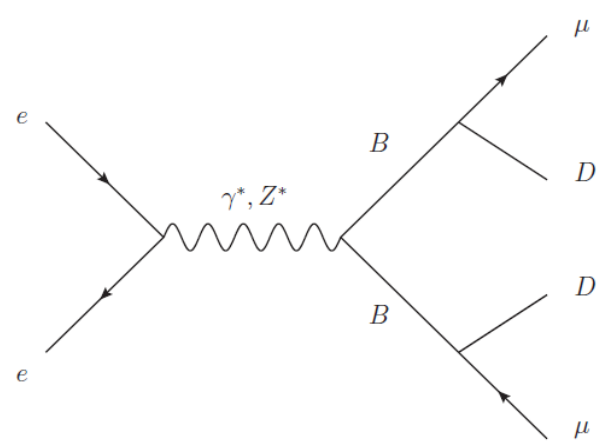


$s_B = 3/2$ and $s_D = 1$ | $s_B = 3/2$ and $s_D = 2$ | $s_B = 2$ and $s_D = 1/2$ | $s_B = 2$ and $s_D = 3/2$

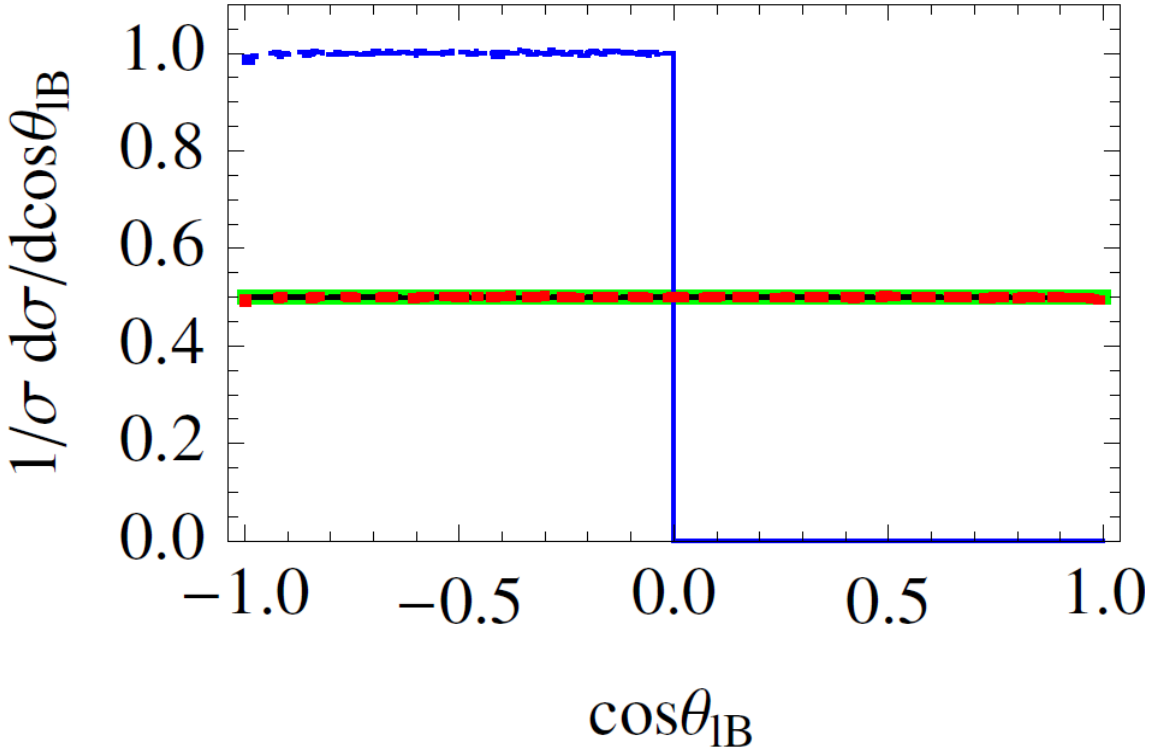
- Analytic solution
- ISR and Beamstrahlung
- ISR, Beamstrahlung and Cuts

Summary

- We are able to reconstruct the CM angular distribution, even in cases in which the CM system is unknown
- Our method gives exact agreement in the narrow width limit in the absence of ISR and beamstrahlung
- Finite width effects are negligible and subdominant
- We should still be able to determine spin even with ISR and beamstrahlung



Extra Slides Follow



s_B=0 and s_B= ½ with any s_D

- Parity symmetry requires equal contributions from $d^j_{m,+m'}$ and $d^j_{m,-m'}$
- For spin-1/2

$$d^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}}(\theta_{\mu B}) = \cos(\theta_{\mu B})$$

$$d^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_{\mu B}) = -\sin(\theta_{\mu B})$$

$$\frac{1}{\sigma_i}\frac{d\sigma_i}{d\cos\theta_{\ell B}} = \frac{1}{2}\left[\left(d^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}}(\theta_{\ell B})\right)^2 + \left(d^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_{\ell B})\right)^2\right] = \frac{1}{2}$$

$s_B = 1,\ s_D = 1/2$

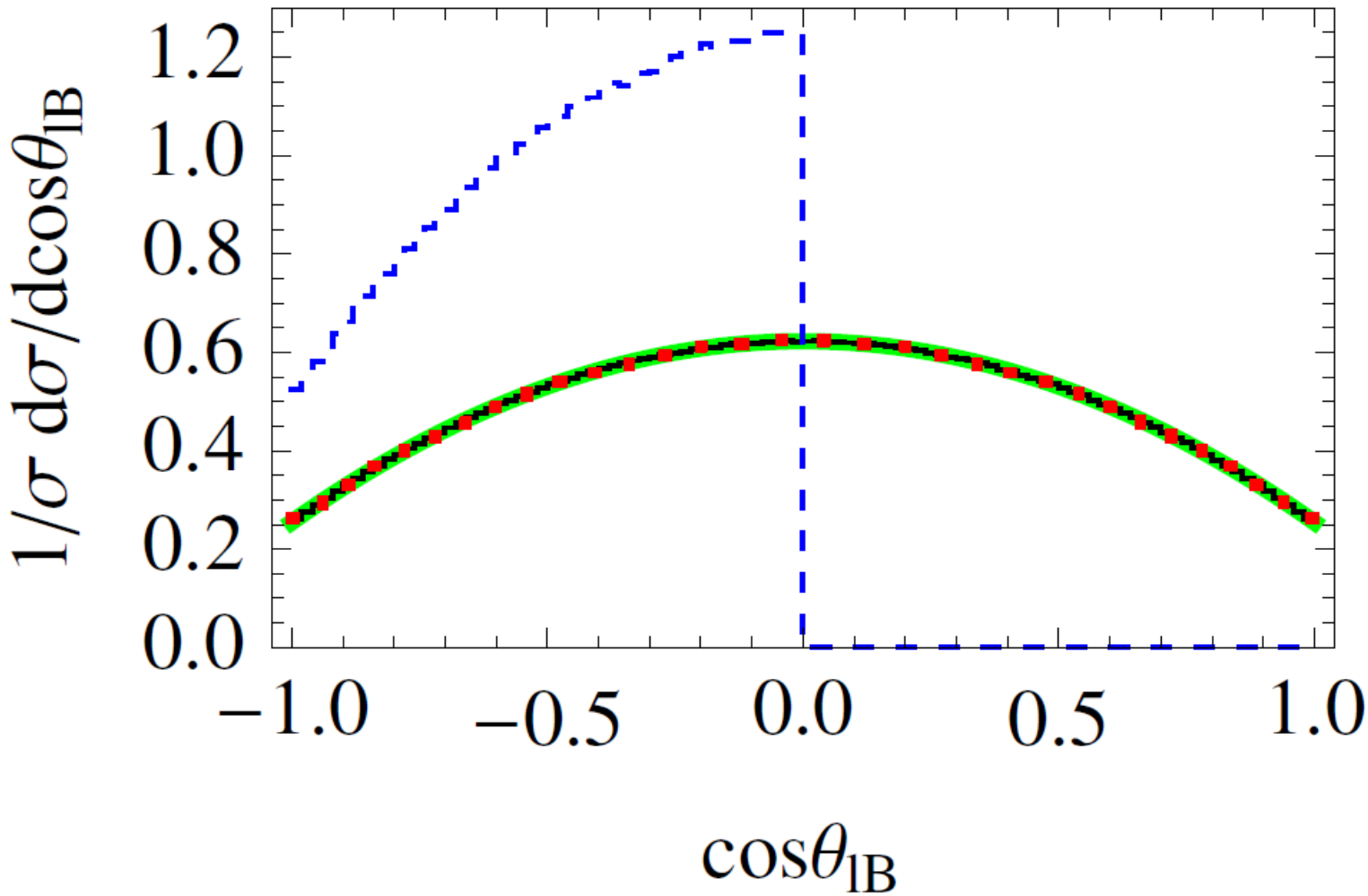
$$\frac{1}{\mathcal{N}_{ii}}\left(\mathcal{A}_{ii} - \mathcal{B}_{ii}\cos^2\theta_{\mu B}\right)$$

$$\mathcal{A}_{ii} = 3M_B^2\left(s^2 - 4M_B^2 s + 4M_B^2 M_D^2 + 8M_B^4\right)$$

$$\mathcal{B}_{ii} = 3s\left(M_B^2 - M_D^2\right)\left(s - 4M_B^2\right)$$

$$\mathcal{N}_{ii} = 2\left(2M_B^2 + M_D^2\right)\left(s^2 - 4M_B^2 s + 12M_B^4\right)$$

$s_B = 1,\ s_D = 1/2$

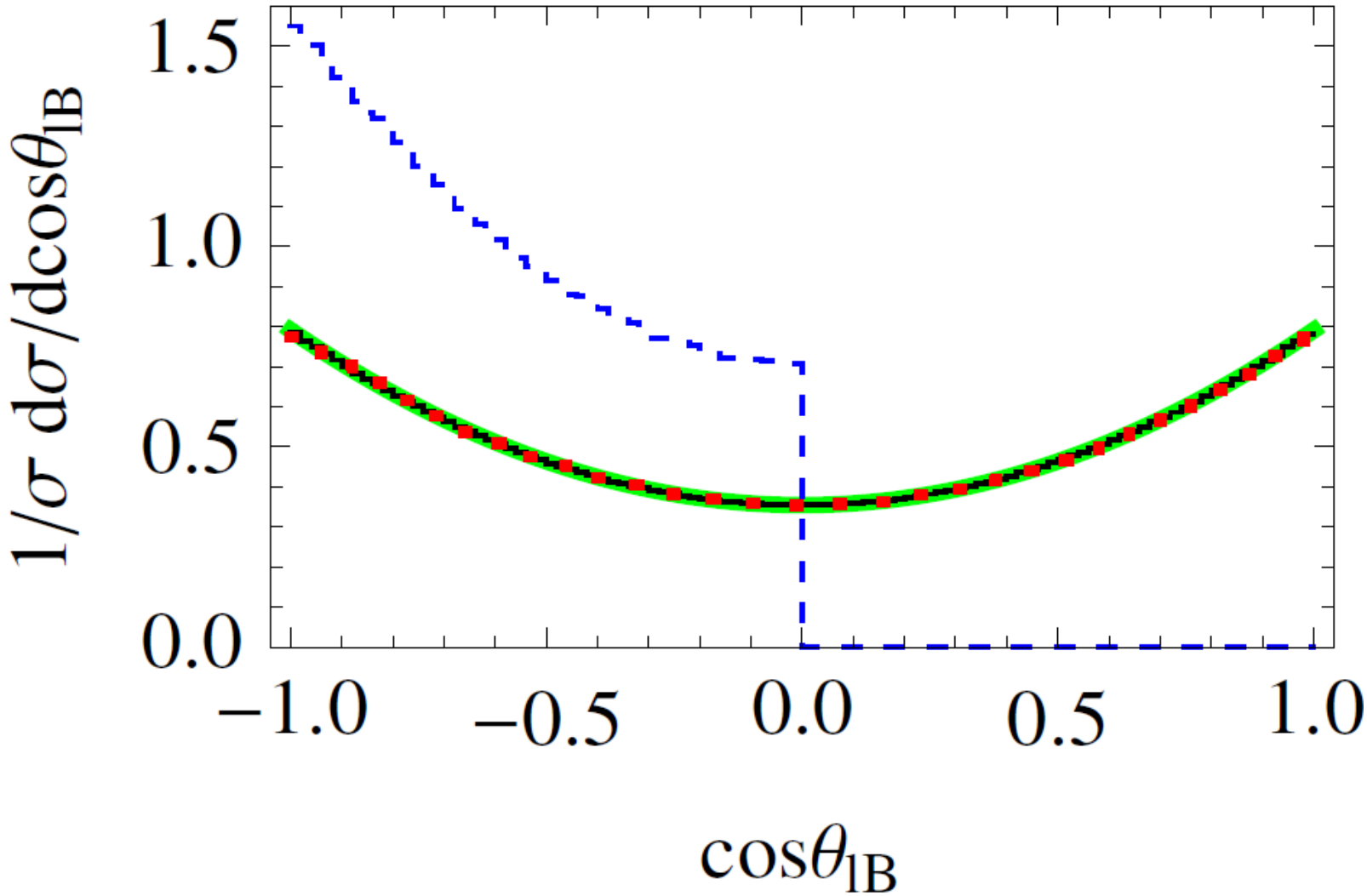

1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0
$1/\sigma\ d\sigma/d\cos\theta_{lB}$
$\cos\theta_{lB}$

$s_B = 1,\ s_D = 3/2$

$$\frac{1}{\mathcal{N}_{iii}}\left(\mathcal{A}_{iii} + \mathcal{B}_{iii}\cos^2\theta_{\mu B}\right)$$

$$\mathcal{A}_{iiia} = 12M_B^2\left(M_D^2 s^2 - 4M_B^2 M_D^2 s + M_B^6 + 10M_B^4 M_D^2 + M_B^2 M_D^4\right.$$
$$\mathcal{B}_{iiia} = 3s\left(M_B^2 - M_D^2\right)^2\left(s - 4M_B^2\right)$$
$$\mathcal{N}_{iiia} = 2\left(M_B^4 + 10M_B^2 M_D^2 + M_D^4\right)\left(s^2 - 4M_B^2 s + 12M_B^4\right)$$

$s_B = 1,\ s_D = 3/2$

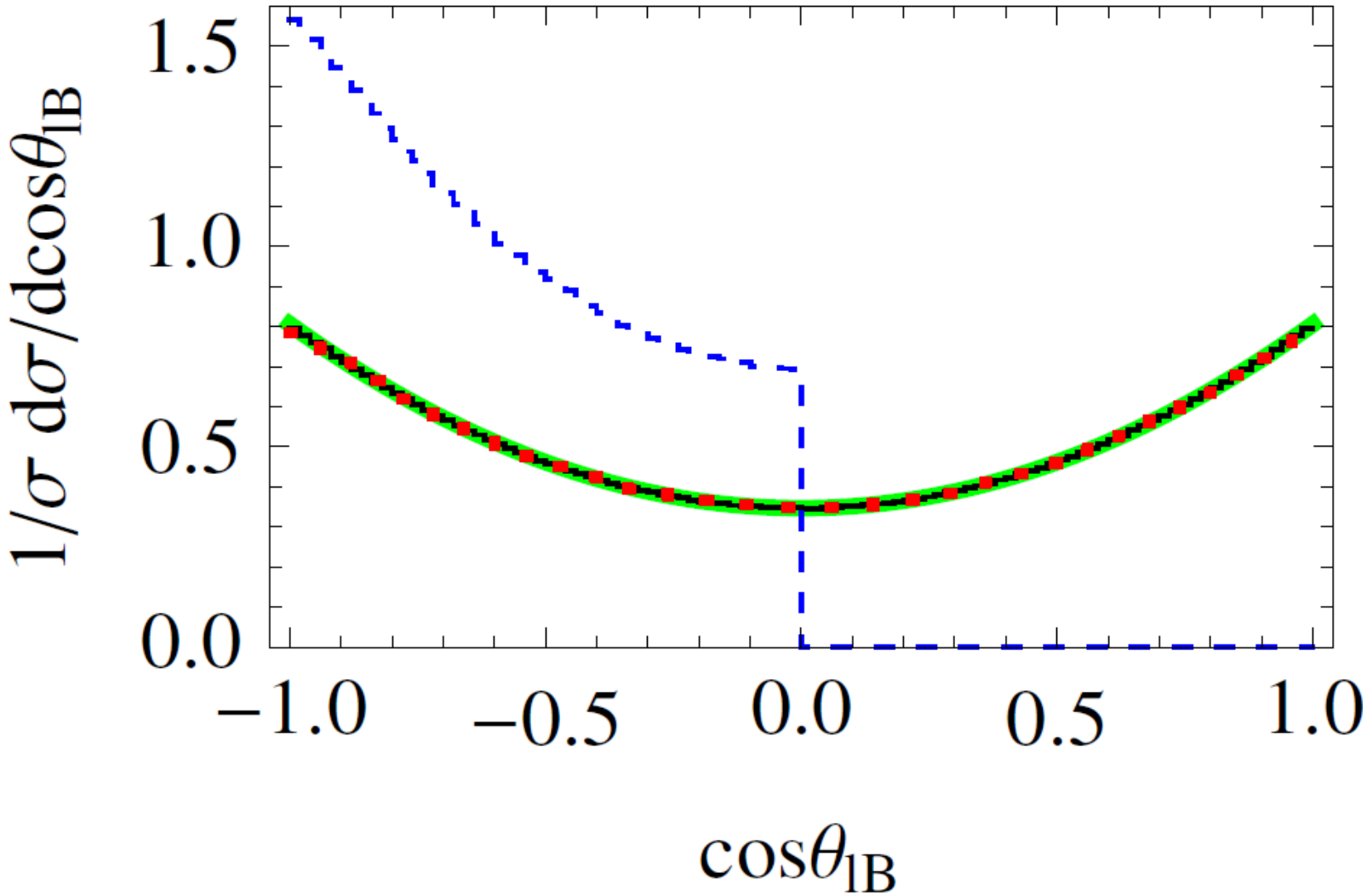

1.5
1.0
0.5
0.0
−1.0
−0.5
0.0
0.5
1.0
$1/\sigma\ d\sigma/d\cos\theta_{lB}$
$\cos\theta_{lB}$

$s_B = 3/2,\ s_D = 0$

$$\frac{1}{\mathcal{N}_{iii}}\left(\mathcal{A}_{iii} + \mathcal{B}_{iii}\cos^2\theta_{\mu B}\right)$$

$$\mathcal{A}_{iiib} = s^3 - 2M_B^2 s^2 + 4M_B^4 s + 72M_B^6$$
$$\mathcal{B}_{iiib} = 3s\left(s - 4M_B^2\right)\left(s + 2M_B^2\right)$$
$$\mathcal{N}_{iiib} = 4\left(s^3 - 2M_B^2 s^2 - 2M_B^4 s + 36M_B^6\right)$$

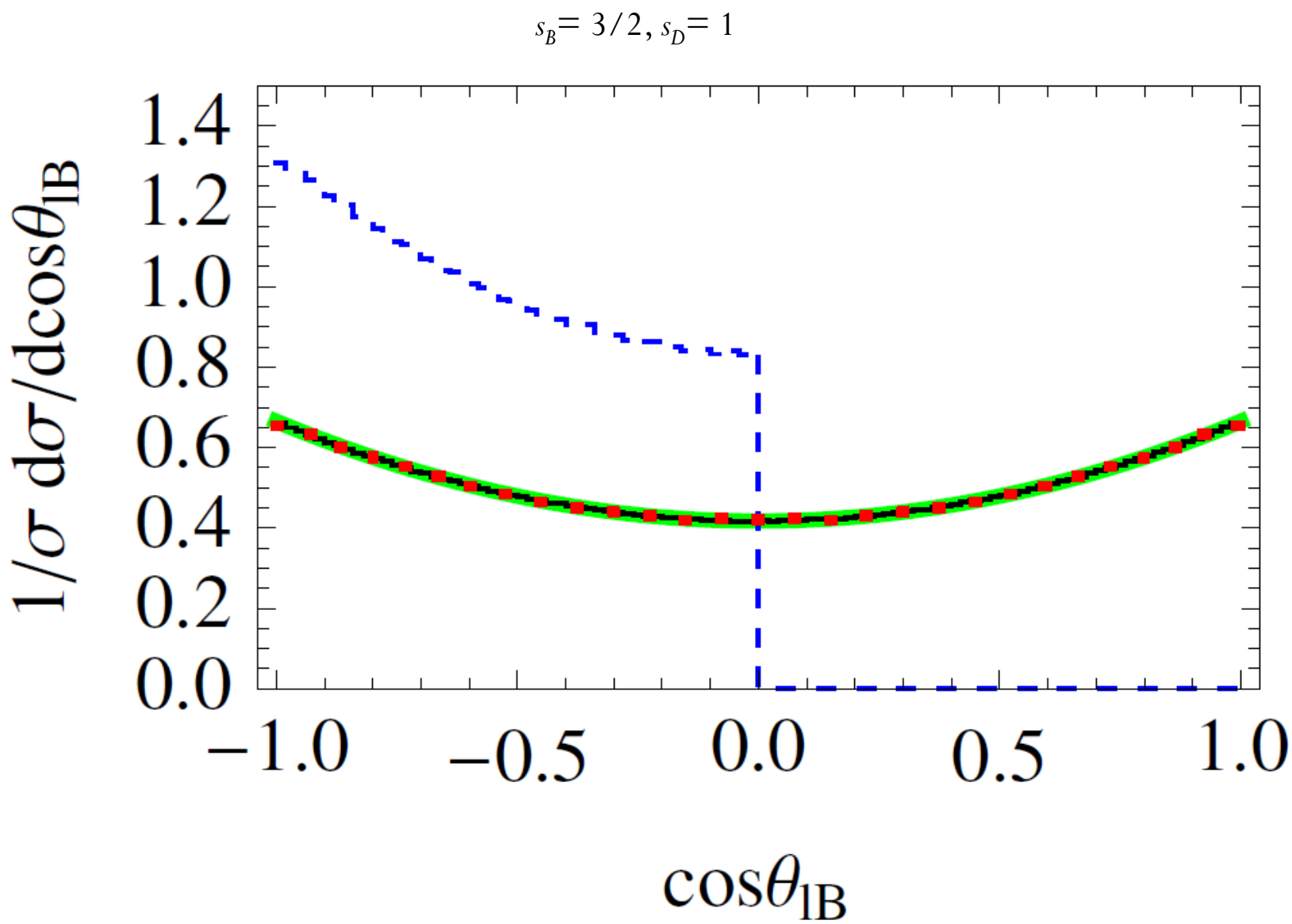
$s_B = 3/2,\ s_D = 0$


1.5
1.0
0.5
0.0
−1.0
−0.5
0.0
0.5
1.0
$1/\sigma\ d\sigma/d\cos\theta_{lB}$
$\cos\theta_{lB}$

$s_B = 3/2$, $s_D = 1$

$$\frac{1}{\mathcal{N}_{iii}}\left(\mathcal{A}_{iii} + \mathcal{B}_{iii}\cos^2\theta_{\mu B}\right)$$

$$\mathcal{A}_{iiic} = \left(s^3 - 2M_B^2 s^2 + 4M_B^4 s + 72M_B^6\right)\left(M_B^4 + 10M_B^2 M_D^2 + M_D^4\right) + 12M_B^2 M_D^2 s\left(s + 2M_B^2\right)\left(s - 4M_B^2\right)$$
$$\mathcal{B}_{iiic} = 3s\left(M_B^2 - M_D^2\right)^2\left(s - 4M_B^2\right)\left(s + 2M_B^2\right)$$
$$\mathcal{N}_{iiic} = 4\left(M_B^4 + 10M_B^2 M_D^2 + M_D^4\right)\left(s^3 - 2M_B^2 s^2 - 2M_B^4 s + 36M_B^6\right)$$

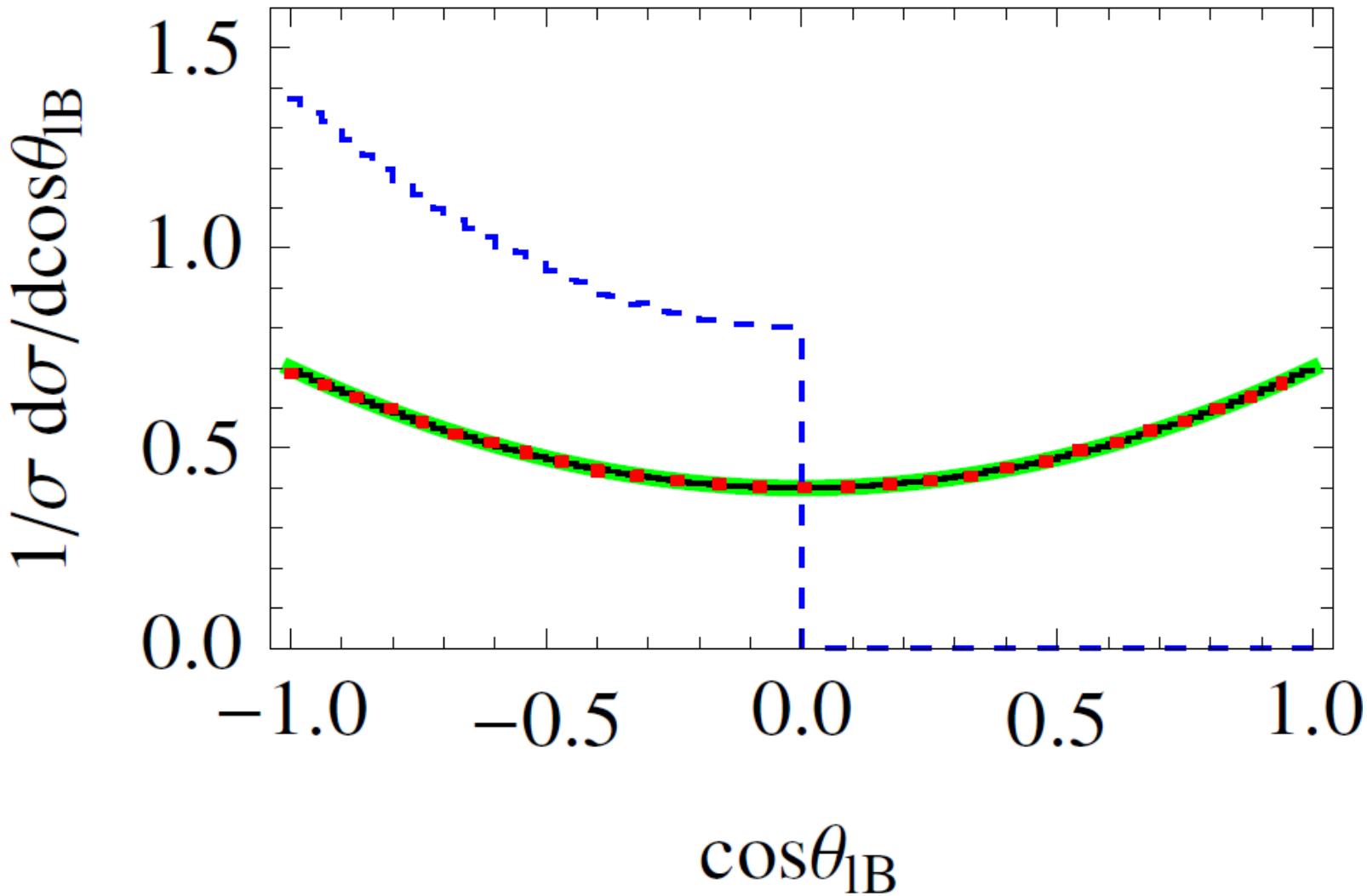

$s_B = 3/2,\ s_D = 1$
$1/\sigma\ d\sigma/d\cos\theta_{lB}$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0
$\cos\theta_{lB}$

$s_B = 3/2,\ s_D = 2$

$$\frac{1}{\mathcal{N}_{iii}}\left(\mathcal{A}_{iii} + \mathcal{B}_{iii}\cos^2\theta_{\mu B}\right)$$

$$\begin{aligned}
\mathcal{A}_{iiid} &= \left(2M_B^6 + 47M_B^4M_D^2 + 128M_B^2M_D^4 + 3M_D^6\right)\left(s^3 - 2M_B^2s^2 - 2M_B^4s + 36M_B^6\right) \\
&\quad + 6M_B^4\left(2M_B^6 + 11M_B^4M_D^2 - 16M_B^2M_D^4 + 3M_D^6\right)\left(s + 6M_B^2\right) \\
\mathcal{B}_{iiid} &= 3s\left(2M_B^6 + 11M_B^4M_D^2 - 16M_B^2M_D^4 + 3M_D^6\right)\left(s - 4M_B^2\right)\left(s + 2M_B^2\right) \\
\mathcal{N}_{iiid} &= 4\left(2M_B^6 + 29M_B^4M_D^2 + 56M_B^2M_D^4 + 3M_D^6\right)\left(s^3 - 2M_B^2s^2 - 2M_B^4s + 36M_B^6\right)
\end{aligned}$$

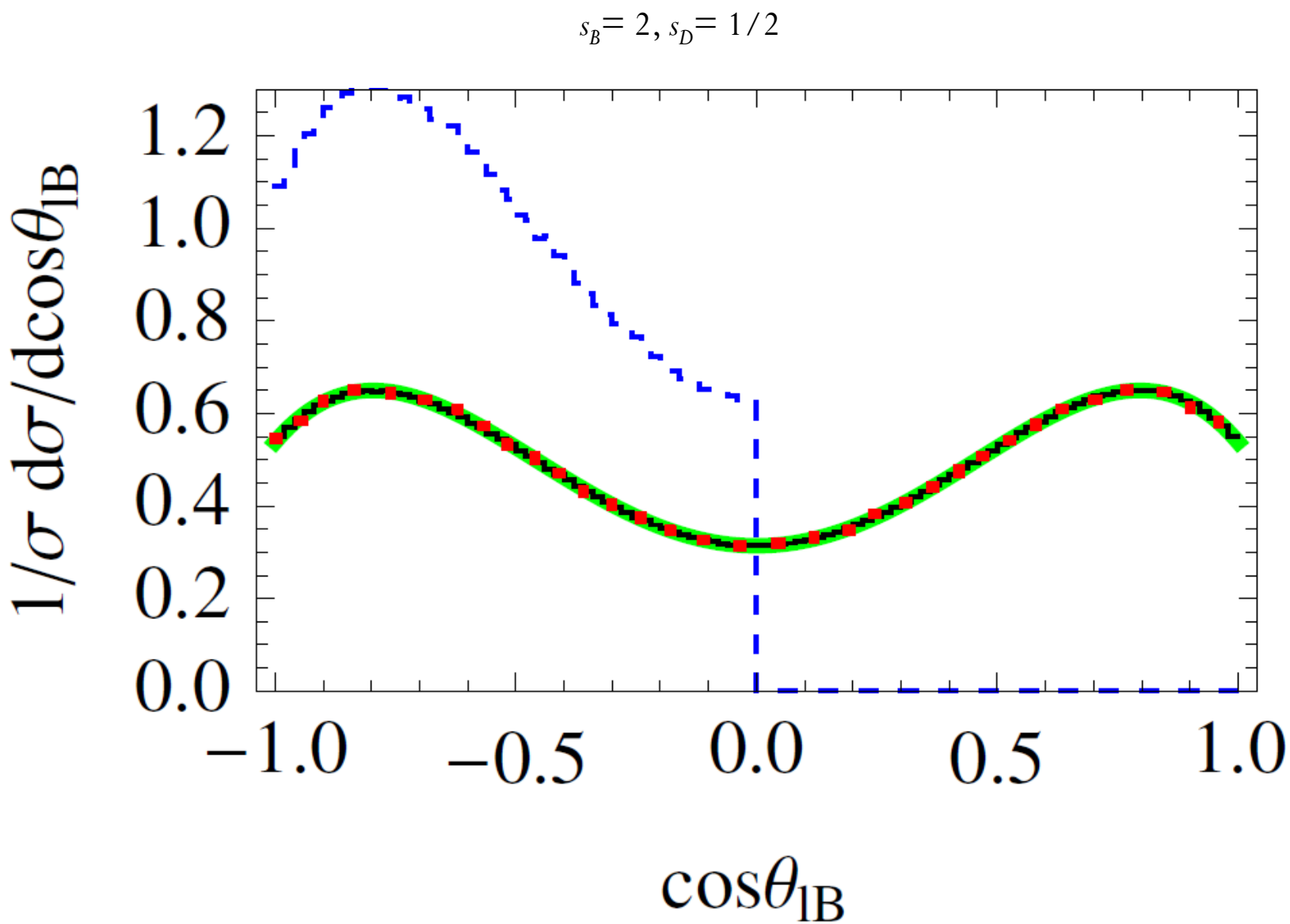
$s_B = 3/2,\ s_D = 2$


1.5
1.0
0.5
0.0
−1.0
−0.5
0.0
0.5
1.0
$1/\sigma\ d\sigma/d\cos\theta_{lB}$
$\cos\theta_{lB}$

$s_B = 2,\ s_D = 1/2$

$$\frac{1}{\mathcal{N}_{iv}}\left(\mathcal{A}_{iv} + \mathcal{B}_{iv}\cos^2\theta_{\mu B} - \mathcal{C}_{iv}\cos^4\theta_{\mu B}\right)$$

$$\mathcal{A}_{iva} = 5\left(M_D^2 s^4 - 8M_B^2 M_D^2 s^3 + 28M_B^4 M_D^2 s^2 + 27M_B^6 s^2 - 48M_B^6 M_D^2 s - 108M_B^8 s + 144M_B^8 M_D^2 + 216M_B^{10}\right)$$
$$\mathcal{B}_{iva} = -15s\left(s - 4M_B^2\right)\left(-3M_B^2 s^2 + 2M_D^2 s^2 - 8M_B^2 M_D^2 s + 12M_B^4 s - 12M_B^4 M_D^2 - 9M_B^6\right)$$
$$\mathcal{C}_{iva} = 45s^2\left(M_B^2 - M_D^2\right)\left(s - 4M_B^2\right)^2$$
$$\mathcal{N}_{iva} = 4\left(3M_B^2 + 2M_D^2\right)\left(s^4 - 8M_B^2 s^3 + 46M_B^4 s^2 - 120M_B^6 s + 180M_B^8\right)$$

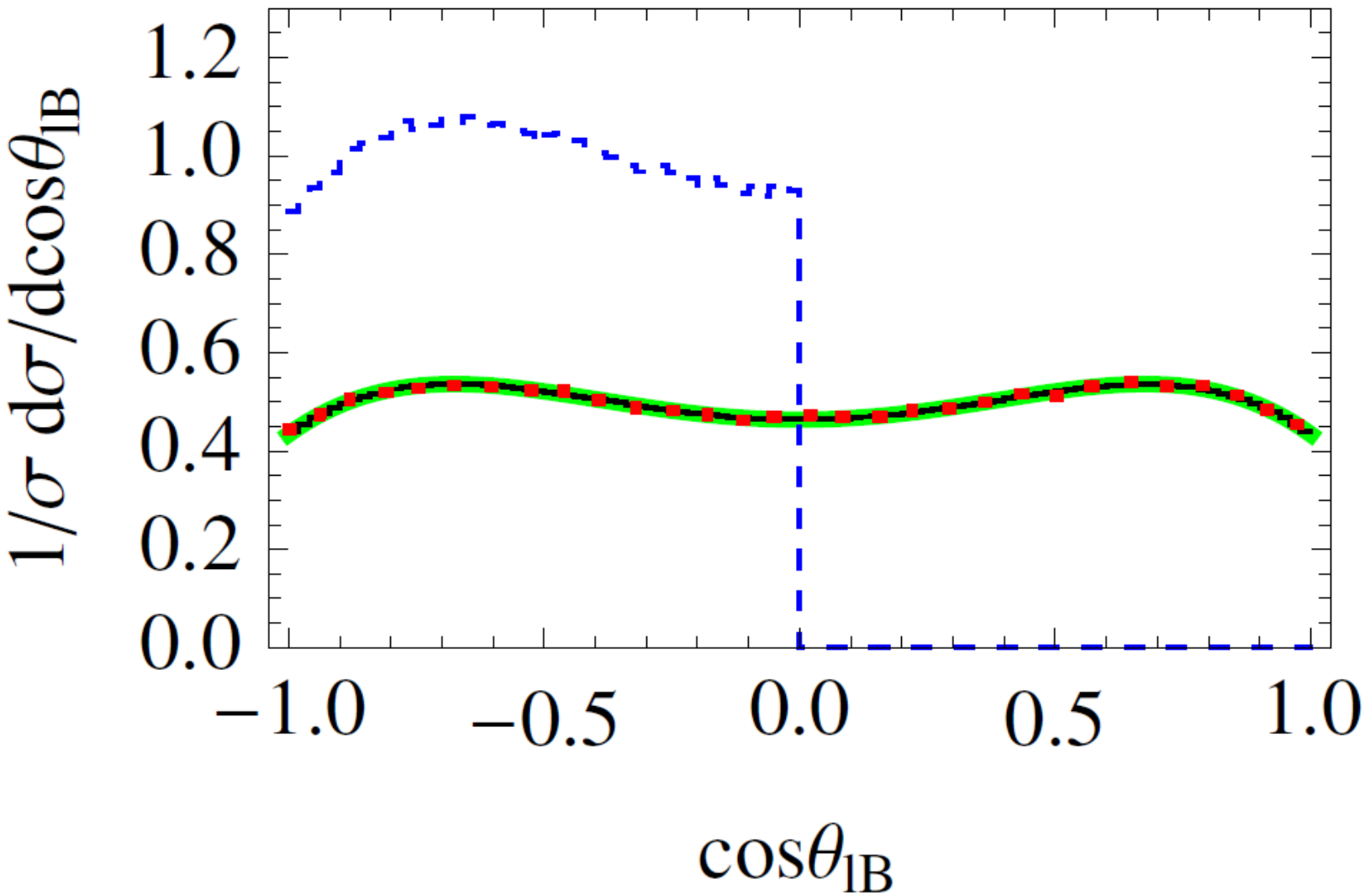

$s_B = 2,\ s_D = 1/2$
$1/\sigma\ d\sigma/d\cos\theta_{lB}$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0
$\cos\theta_{lB}$

$s_B = 2,\ s_D = 3/2$

$$\frac{1}{\mathcal{N}_{iv}}\left(\mathcal{A}_{iv} + \mathcal{B}_{iv}\cos^2\theta_{\mu B} - \mathcal{C}_{iv}\cos^4\theta_{\mu B}\right)$$

$$\mathcal{A}_{ivb} = 889s^4 - 7112M_B^2 s^3 + 49804M_B^4 s^2 - 142320M_B^6 s + 216720M_B^8$$
$$\mathcal{B}_{ivb} = 270s\left(s - 4M_B^2\right)\left(11s^2 - 44M_B^2 s + 6M_B^4\right)$$
$$\mathcal{C}_{ivb} = 3375s^2\left(s - 4M_B^2\right)^2$$
$$\mathcal{N}_{ivb} = 2408\left(s^4 - 8M_B^2 s^3 + 46M_B^4 s^2 - 120M_B^6 s + 180M_B^8\right)$$

$s_B = 2,\ s_D = 3/2$


1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0
$1/\sigma\ d\sigma/d\cos\theta_{lB}$
$\cos\theta_{lB}$