Predictive models of Dirac Neutrinos

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In collaboration with
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  - Scans and Results
- Conclusions

KNOW

- νs have masses and $m_2 > m_1$
- Oscillations between 3 flavor states
- θ_{12}, θ_{23}, θ_{13} in mixing matrix
- $\Delta m_{ij}^2 = m_i^2 - m_j^2$ and $m_1 + m_2 + m_3$

DON'T KNOW

- Dirac or Majorana in nature
- the sign of $\Delta m_{32}^2 \to$ normal or inverted hierarchy
- CP phase $\delta \to$ CP violation in the leptonic sector

- These talk present models which predict these unknown quantities using the known data based on some reasonable assumptions

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- What we get following these assumptions:
- (I)Predict the absolute values of the masses of 3 ν's
  (II)Value of CP violating phase δ

The Model and formalism

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- With keV vev for ϕ, the corresponding Yukawa coupling $\sim 10^{-4}$

The Neutrino mass matrix

With 3 assumptions of lepton number conservation, Hermiticity of the neutrino mass matrix, and $\nu_\mu - \nu_\tau$ exchange symmetry, M_ν can be written as,

$$M_\nu = \begin{pmatrix} a & b & b \\ b^* & c & d \\ b^* & d & c \end{pmatrix}$$

- The parameters a, c and d are real, b complex $\rightarrow$ five real parameters

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- The parameters a, c and d are real, b complex $\rightarrow$ five real parameters
- We choose a basis in which the Yukawa couplings for the charged leptons are diagonal $\rightarrow$ PMNS matrix in our model is U_ν

Since the neutrino mass matrix is hermitian, it can then be obtained from

$$M_\nu = U_\nu \mathbf{M}_\nu^{diag} U_\nu^\dagger$$

where,

$$\mathbf{M}_\nu^{diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

and

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where, $c_{ij} = \mathrm{Cos}\theta_{ij}$ and $s_{ij} = \mathrm{Sin}\theta_{ij}$

Experimental data

Parameter	best-fit ($\pm\sigma$)	3σ
$\Delta m^2_{21}[10^{-5}eV^2]$	$7.53^{+0.26}_{-0.22}$	6.99-8.18
$\Delta m^2[10^{-3}eV^2]$	$2.43^{+0.06}_{-0.10}$ $(2.42^{+0.07}_{-0.11})$	2.19(2.17)-2.62(2.61)
$\sin^2\theta_{12}$	$0.307^{+0.018}_{-0.016}$	0.259 - 0.359
$\sin^2\theta_{23}$	$0.386^{+0.024}_{-0.021}(0.392^{+0.039}_{-0.022})$	0.331(0.335)-0.637(0.663)
$\sin^2\theta_{13}$	$0.0241 \pm 0.0025(0.0244^{+0.0023}_{-0.0025})$	0.0169(0.0171)-0.0313(0.032)

Table: The definition of Δm^2 used is $\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$. Thus $\Delta m^2 = \Delta m^2_{31} - m^2_{21}/2$ if $m_1 < m_2 < m_3$ and $\Delta m^2 = \Delta m^2_{32} + m^2_{21}/2$ for $m_3 < m_1 < m_2$.

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- Taking these experimental values in the best-fit($\pm\sigma$) region from Table, for the PMNS mixing matrix, we get

$$U_\nu = \begin{pmatrix} 0.822 & 0.547 & 0.156\exp(-i\delta) \\ -0.432 - 0.081\exp(i\delta) & 0.649 - 0.054\exp(i\delta) & 0.618 \\ 0.347 - 0.101\exp(i\delta) & -0.521 - 0.067\exp(i\delta) & 0.771 \end{pmatrix}$$

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- At first demanding elements $A_{22}(m_{\mu_L\mu_R}) = A_{33}(m_{\tau_L\tau_R})$, we obtain the following 2^{nd} order equation for $\cos\delta$

$$(-123.27m_3^4 - 0.15m_3^2 + 0.0026)\cos^2\delta + (6.66m_3^4 - 6.7m_3^2 - 0.006)\cos\delta \\ +29.654m_3^4 - 3.19m_3^2 + 0.0031 = 0$$

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- It is intriguing that a solution exists, and gives the values of $m_3 = 7.8\times 10^{-2}$ eV and $\delta = 109$ degree
- Don't get a solution for Normal Hierarchy!

Results

- Prediction for the three neutrino masses and the CP violating phase in our model :
  $m_1 = \sqrt{m_3^2 + 0.002315} = 9.16 \times 10^{-2} eV$,
  $m_2 = \sqrt{m_3^2 + 0.00239} = 9.21 \times 10^{-2} eV$,
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- The resulting numerical neutrino mass matrix we obtain

$$M_\nu = \begin{pmatrix} 0.091 & 0.00048 + 0.001i & 0.00044 + 0.0015i \\ 0.00048 - 0.001i & 0.086 & -0.0066 \\ 0.00044 - 0.0015i & -0.0066 & 0.084 \end{pmatrix}$$

  which is of the form

$$M_\nu = \begin{pmatrix} a & b & b^* \\ b^* & c & d \\ b^* & d & c \end{pmatrix}.$$

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- LBNE10 (baseline 1300 Km, E_ν between $1-6$ GeV) would be able to unambiguously shed light both on the mass hierarchy and the CP phase through CP asymmetry $\mathcal{A}_{CP} = 0.17$ + matter effects

Model with lesser parameters: Assumptions:

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- As a result we can fit all experimental data only with four parameters in the model
- Fitting the data, the model also predicts the absolute values of the 3 ν masses in inverted mass hierarchical pattern and a CP violating phase δ

Neutrino mass matrix with 4 parameters

- With assumptions of Hermiticity of the neutrino mass matrix and $\nu_{\mu_R} \rightarrow$ - ν_{τ_R} symmetry along with a CP transformation, M_ν can be written as,

$$M_\nu = \begin{pmatrix} a & b & -b^* \\ b^* & c & -c \\ -b & -c & c \end{pmatrix}$$

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- To fit the model, we use U_ν with $\theta_{23} = 45$ degree($\sin^2\theta_{23} = 0.5$) which is within 3σ in experimental data in Table-I

Scans over parameter space of unknown parameters - I

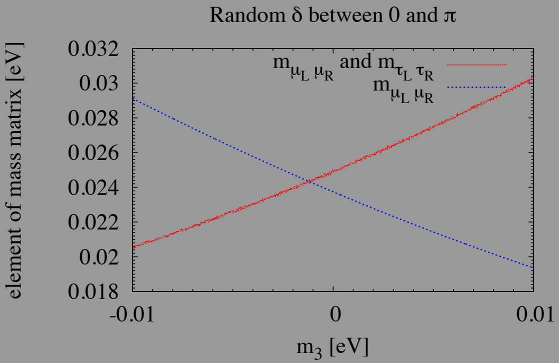

Figure: The M_ν mass matrix elements $A_{22}(m_{\mu_L\mu_R})$, $A_{33}(m_{\tau_L\tau_R})$ and real part of $A_{23}(m_{\mu_L\tau_R})$ of the Dirac neutrino mass matrix as a function of m_3. From this plot $m_3 = -1.198 \times 10^{-3}$eV

Scans over parameter space of unknown parameters - II

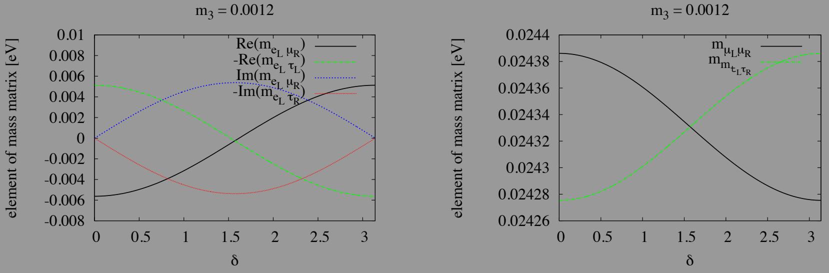

Figure: Left panel: The real and imaginary part of the elements $A_{12}(m_{e_L\mu_R})$ and $A_{13}(m_{e_L\tau_R})$ of M_ν as a function of $\delta \to \delta = \pi/2$ Right panel: The diagonal elements $A_{22}(m_{\mu_L\mu_R})$ and $A_{33}(m_{\tau_L\tau_R})$ of M_ν as a function of $\delta \to \delta = \pi/2$

Results

- Prediction for the three neutrino masses and the CP violating phase in the model :

$$m_1 = \sqrt{m_3^2 + 0.002315} = 4.81 \times 10^{-2} eV$$

$$m_2 = \sqrt{m_3^2 + 0.00239} = 4.89 \times 10^{-2} eV$$

$$m_3 = -1.2 \times 10^{-3} eV$$

$$\delta = 90 \text{ degree}$$

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  $\delta = 90$ degree

- The resulting numerical neutrino mass matrix we obtain

$$M_\nu = \begin{pmatrix} 0.047 & 0.000219 - 0.00536i & -0.000219 - 0.00536i \\ 0.000219 + 0.00536i & 0.0243 & -0.0243 \\ -0.000219 + 0.00536i & -0.0243 & 0.0243 \end{pmatrix}$$

which is of the form

$$M_\nu = \begin{pmatrix} a & b & -b^* \\ b^* & c & -c \\ -b & -c & c \end{pmatrix}.$$

Conclusions:

5 parameter M_ν

- Basic assumptions:
  (i) L number conservation,
  (ii) Hermiticity of M_ν and
  (iii) $\nu_\mu - \nu_\tau$ symmetry
- $$M_\nu = \begin{pmatrix} a & b & b \\ b^* & c & d \\ b^* & d & c \end{pmatrix}$$
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4 parameter M_ν

- Basic assumptions:
  (i) L number conservation,
  (ii) hermiticity of M_ν
  (iii) $\nu_{\mu_R} \rightarrow$ - ν_{τ_R} + CP of L
  and one relaxation: $\sin^2\theta_{23} = 0.5$
- $$M_\nu = \begin{pmatrix} a & b & -b^* \\ b^* & c & -c \\ -b & -c & c \end{pmatrix}.$$
- m_1=4.81×10^{-2} eV,
  m_2= 4.89×10^{-2} eV
  m_3=-1.1×10^{-3} eV and
  $\delta = 90$ degree

- Inverted mass hierarchy (close the degenerate pattern)

Conclusions:

5 parameter M_ν

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  (iii) $\nu_\mu - \nu_\tau$ symmetry
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- Inverted mass hierarchy (close the degenerate pattern)
- No neutrinoless double beta decay
- Definite predictions of three ν masses and CP violating phase delta: testable in the future neutrino experiments!

Thank you!

BACKUP: Experimental status and Sensitivity

Unknowns	Experimental Sensitivity
Absolute ν masses	$m_\beta < 2.3$ eV(Tritium) $\rightarrow$ 0.2eV(KATRIN)
CP violating phase delta	LBNE: delta $<$ 10 degree possible(in 5σ)
$\beta\beta 0\nu$ decay	KamLAND-Zen: $< m_{\beta\beta} > \rightarrow 10^{-2}$eV