The Magnetic Radius of the Proton

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[Zachary Epstein GP, Joydeep Roy, to appear]

Outline

- The proton electric radius problem
- The proton magnetic radius problem
- Model independent extraction of the proton magnetic radius from electron scattering
- Conclusions and outlook

The proton electric radius problem

[Richard J. Hill, GP PRD **82** 113005 (2010)]

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)|\sum_q e_q\,\bar{q}\gamma^\mu q|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2(q^2)q^\nu\right]u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\langle r^2\rangle_E^p = 6\frac{dG_E^p}{dq^2}\bigg|_{q^2=0}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

Charge radius from atomic physics

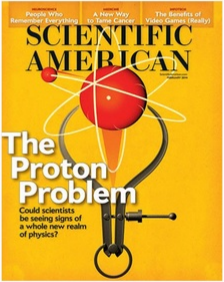
- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]

- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm
  more recently $r_E^p = 0.87750(510)$ fm [Mohr et al. RMP **84**, 1527 (2012)]
  extracted mainly from (electronic) hydrogen

- (more than) **5σ discrepancy!**

How to resolve the puzzle?

- Almost 4 years after first measurement puzzle is still not resolved



(Cover story of February 2014 Scientific American)

- Is it new physics?

- Is it a problem with the theoretical prediction?
  [Richard. J. Hill, GP PRL **107** 160402 (2011), and in progress]

Proton radii from scattering data

- Apart from regular and muonic hydrogen, the proton radius can be extracted from electron-proton scattering

- Problem: we don't know the functional form of form factors

- Solution: use analytic properties for a model-independent extraction
  z-expansion: [Hill, GP PRD **82** 113005 (2010)]
  $r_E^p = 0.87100(940)$ fm

- More consistent with

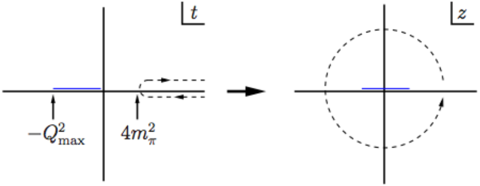
- Regular hydrogen $r_E^p = 0.87680(690)$ fm $\left(r_E^p = 0.87750(510) \text{ fm}\right)$
  Than
- Muonic hydrogen $r_E^p = 0.84184(67)$ fm $\left(r_E^p = 0.84087(39) \text{ fm}\right)$

z expansion

- $G_E^p(t)$ is analytic outside a cut $q^2 = t \in [4m_\pi^2, \infty]$
  $e - p$ scattering data is in $t < 0$ region
- We can map the domain of analyticity onto the unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$



- Expand G_E^p in a Taylor series in z: $G_E^p(q^2) = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$

  Need to bound a_k for r_E^p independent of k:
  Use $|a_k| \leq 5$ and $|a_k| \leq 10$

Proton Electric Radius Results

- Proton data: $Q^2 < 0.5\,\mathrm{GeV}^2$

$$r_E^p = 0.870 \pm 0.023 \pm 0.012 \text{ fm}$$

- Proton and neutron data

$$r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007 \text{ fm}$$

- Proton, neutron and $\pi\,\pi$ data

$$r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002 \text{ fm}$$

The proton magnetic radius problem

The proton magnetic radius problem

- The proton *magnetic* radius

$$\langle r^2 \rangle_M^p = \frac{6}{G_M^p(0)} \frac{dG_M^p(q^2)}{dq^2}\bigg|_{q^2=0}$$

- PDG 2012:

- Recent high precision data from A1 experiment at Mainz
  $r_M^p = 0.777 \pm 0.017$ fm [Bernauer et al. PRL **105**, 242001 (2010)]
  Older data sets
- $r_M^p = 0.876 \pm 0.019$ fm [Borisyuk et al. 2010]
- $r_M^p = 0.854 \pm 0.005$ fm [Belushkin et al. 2007]
  Are we facing a magnetic radius puzzle too?

- We need a model independent extraction of r_M^p!

Model independent extraction of the proton magnetic radius from electron scattering

[Zachary Epstein GP, Joydeep Roy, to appear]

Bound on $|a_k|$

- Analyzing p and n data separate G_M^p and G_M^n to isospin channels

$$G_M^{(0)} = G_M^p + G_M^n \qquad G_M^{(1)} = G_M^p - G_M^n$$
$$G_M^{(0)}(0) = \mu_p + \mu_n \approx 0.88 \qquad G_M^{(1)}(0) = \mu_p - \mu_n \approx 4.7$$
$$\Rightarrow I = 0, \quad a_0 = 0.88 \qquad \Rightarrow I = 1, \quad a_0 = 4.7$$

- Vector dominance ansatz:
  - $I = 0$ (ω exchange) $|a_k| \leq 1.1$
  - $I = 1$ (ρ exchange) $|a_k| \leq 5.1$

- Between $t = 4m_\pi^2$ and $t = 16m_\pi^2$ only $\pi\pi$ contributes
  $I = 1$: $|a_k| \leq 7.2$

- Above $t = 4m_N^2$ use $e^+e^- \to N\bar{N}$: negligible contribution to a_k

- Conclusion:
  $|a_k| \leq 5$ not conservative enough, use $|a_k| \leq 10$ and $|a_k| \leq 15$

r_M^p from proton data (*Preliminary*)

- $G_M^p(q^2)$ values from $e-p$ scattering data
  [Arrington et al. PRC **76**, 035205 (2007)]

- Extracted values don't depend on number of parameters
  (results shown for for $k_{\max}=8$)

- $Q^2 \leq 0.5$ GeV^2
  - $|a_k| \leq 10$: $r_M^p = 0.91^{+0.03}_{-0.06}$ fm
  - $|a_k| \leq 15$: $r_M^p = 0.92^{+0.04}_{-0.07}$ fm

- $Q^2 \leq 1.0$ GeV
  - $|a_k| \leq 10$: $r_M^p = 0.90^{+0.03}_{-0.07}$ fm
  - $|a_k| \leq 15$: $r_M^p = 0.91^{+0.04}_{-0.07}$ fm

r_M^p from proton and neutron data (*Preliminary*)

- $G_M^p(q^2)$ from [Arrington et al. PRC **76**, 035205 (2007)]
- $G_M^n(q^2)$ from [Lachniet et al. PRL **102** 192001 (2009); Anderson et al. PRC**75**, 034003 (2007); Kubon et al. PLB **524**, 26 (2002); Xu et al. PRL **85**, 2900 (2000); Anklin et al. PLB **428**, 248 (1998); Anklin et al. PLB **336**, 313 (1994); Gao et al. PRC **50**, 546 (1994); Lung et al. PRL **70**, 718 (1993)]

- Fit both $G_M^{(0)}$ and $G_M^{(1)}$

- $Q^2 \leq 0.5$ GeV^2
  - $|a_k| \leq 10$: $r_M^p = 0.87^{+0.04}_{-0.05}$ fm
  - $|a_k| \leq 15$: $r_M^p = 0.87^{+0.05}_{-0.05}$ fm

- $Q^2 \leq 1.0$ GeV
  - $|a_k| \leq 10$: $r_M^p = 0.88^{+0.02}_{-0.05}$ fm
  - $|a_k| \leq 15$: $r_M^p = 0.88^{+0.04}_{-0.05}$ fm

r_M^p from proton and neutron and $\pi\pi$ data (*Preliminary*)

- $G_M^p(q^2)$ from [Arrington et al. PRC **76**, 035205 (2007)]
- $G_M^n(q^2)$ from [Lachniet et al. PRL **102** 192001 (2009); Anderson et al. PRC**75**, 034003 (2007); Kubon et al. PLB **524**, 26 (2002); Xu et al. PRL **85**, 2900 (2000); Anklin et al. PLB **428**, 248 (1998); Anklin et al. PLB **336**, 313 (1994); Gao et al. PRC **50**, 546 (1994); Lung et al. PRL **70**, 718 (1993)]
- $\mathrm{Im}\, G_M^{(1)}$ between $t = 4m_\pi^2$ and $t = 16m_\pi^2$ from $\pi\pi$ data [Höhler, Landolt-Börnstein database Vol. 9b1 (1983); Amendolia et al. PLB **138**, 454 (1984); Achasov et al. JETP **101**, 1053 (2005)]

- $Q^2 \leq 0.5$ GeV^2
  - $|a_k| \leq 10$: $r_M^p = 0.87^{+0.01}_{-0.02}$ fm
  - $|a_k| \leq 15$: $r_M^p = 0.87^{+0.01}_{-0.02}$ fm

- $Q^2 \leq 1.0$ GeV
  - $|a_k| \leq 10$: $r_M^p = 0.87^{+0.01}_{-0.01}$ fm
  - $|a_k| \leq 15$: $r_M^p = 0.88^{+0.01}_{-0.02}$ fm

Conclusions and outlook

Conclusions

- Proton electric radius problem not resolved yet

- Are we facing a magnetic radius puzzle too?

- Model independent extraction of magnetic radius
  Preliminary results:

- Proton data

$$r_M^p = 0.91^{+0.03}_{-0.06} \pm 0.02 \text{ fm}$$

- Proton and neutron data

$$r_M^p = 0.87^{+0.04}_{-0.05} \pm 0.01 \text{ fm}$$

- Proton, neutron and $\pi\pi$ data

$$r_M^p = 0.87^{+0.01}_{-0.02} \text{ fm}$$

- Consistent results, independent of $k_{\rm max}$ and cut on Q^2

Outlook

- Model independent extraction of magnetic radius

  Preliminary results:

- Proton data :$r_M^p = 0.91^{+0.03}_{-0.06} \pm 0.02$ fm
- Proton and neutron data: $r_M^p = 0.87^{+0.04}_{-0.05} \pm 0.01$ fm
- Proton, neutron and $\pi\,\pi$ data: $r_M^p = 0.87^{+0.01}_{-0.02}$ fm

- PDG 2012:

  $r_M^p = 0.777 \pm 0.017$ fm [Bernauer et al. PRL **105**, 242001 (2010)]

- $r_M^p = 0.876 \pm 0.019$ fm [Borisyuk et al. 2010]
- $r_M^p = 0.854 \pm 0.005$ fm [Belushkin et al. 2007]

- Future direction:

  analyze other data sets, e.g. high precision data from [Bernauer et al. PRL **105**, 242001 (2010)]