The Magnetic Radius of the Proton

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[Zachary Epstein GP, Joydeep Roy, to appear]
Outline

- The proton electric radius problem
- The proton magnetic radius problem
- Model independent extraction of the proton magnetic radius from electron scattering
- Conclusions and outlook
The proton electric radius problem

[Richard J. Hill, GP PRD 82 113005 (2010)]
Form Factors

Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f) | \sum_q e_q \bar{q} \gamma^\mu q | N(p_i) \rangle = \bar{u}(p_f) \left[ \gamma^\mu F_1(q^2) + \frac{i \sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu \right] u(p_i)$$

Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \quad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \quad G_M^p(0) = \mu_p \approx 2.793$$

The slope of $G_E^p$

$$\langle r^2 \rangle_E^p = 6 \frac{dG_E^p}{dq^2} \bigg|_{q^2=0}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2 \rangle_E^p}$
Lamb shift in muonic hydrogen [Pohl et al. Nature 466, 213 (2010)]
\[ r_E^p = 0.84184(67) \text{ fm} \]
more recently \( r_E^p = 0.84087(39) \text{ fm} \) [Antognini et al. Science 339, 417 (2013)]

CODATA value [Mohr et al. RMP 80, 633 (2008)]
\[ r_E^p = 0.87680(690) \text{ fm} \]
more recently \( r_E^p = 0.87750(510) \text{ fm} \) [Mohr et al. RMP 84, 1527 (2012)]
extracted mainly from (electronic) hydrogen

(more than) 5\( \sigma \) discrepancy!
How to resolve the puzzle?

- Almost 4 years after first measurement puzzle is still not resolved

(Cover story of February 2014 Scientific American)

- Is it new physics?

- Is it a problem with the theoretical prediction?
  [Richard. J. Hill, GP PRL 107 160402 (2011), and in progress]
Proton radii from scattering data

- Apart from regular and muonic hydrogen, the proton radius can be extracted from electron-proton scattering

- Problem: we don’t know the functional form of form factors

- Solution: use analytic properties for a model-independent extraction

  \[ z\text{-expansion: } \text{[Hill, GP PRD 82 113005 (2010)]} \]
  \[ r_E^p = 0.87100(940) \text{ fm} \]

- More consistent with
  - Regular hydrogen \( r_E^p = 0.87680(690) \text{ fm} \) (\( r_E^p = 0.87750(510) \text{ fm} \))
  - Muonic hydrogen \( r_E^p = 0.84184(67) \text{ fm} \) (\( r_E^p = 0.84087(39) \text{ fm} \))
**z expansion**

- \( G^P_E(t) \) is analytic outside a cut \( q^2 = t \in [4m_{\pi}^2, \infty] \)
- \( e - p \) scattering data is in \( t < 0 \) region
- We can map the domain of analyticity onto the unit circle

\[
z(t, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}} - t_0}}
\]

where \( t_{\text{cut}} = 4m_{\pi}^2 \), \( z(t_0, t_{\text{cut}}, t_0) = 0 \)

- Expand \( G^P_E \) in a Taylor series in \( z \):

\[
G^P_E(q^2) = \sum_{k=0}^{\infty} a_k z(q^2)^k
\]

Need to bound \( a_k \) for \( r_E^P \) independent of \( k \):
- Use \( |a_k| \leq 5 \) and \( |a_k| \leq 10 \)
Proton Electric Radius Results

- Proton data: $Q^2 < 0.5 \text{GeV}^2$
  
  $$r_p^E = 0.870 \pm 0.023 \pm 0.012 \text{ fm}$$

- Proton and neutron data
  
  $$r_p^E = 0.880^{+0.017}_{-0.020} \pm 0.007 \text{ fm}$$

- Proton, neutron and $\pi\pi$ data
  
  $$r_p^E = 0.871 \pm 0.009 \pm 0.002 \pm 0.002 \text{ fm}$$
The proton magnetic radius problem
The proton magnetic radius problem

- The proton magnetic radius

\[ \langle r^2 \rangle_p^M = \frac{6}{G_p^M(0)} \frac{dG_p^M(q^2)}{dq^2} \bigg|_{q^2=0} \]

- PDG 2012:
  - Recent high precision data from A1 experiment at Mainz
    \[ r_p^M = 0.777 \pm 0.017 \text{ fm} \] [Bernauer et al. PRL 105, 242001 (2010)]
  - Older data sets
    - \[ r_p^M = 0.876 \pm 0.019 \text{ fm} \] [Borisyuk et al. 2010]
    - \[ r_p^M = 0.854 \pm 0.005 \text{ fm} \] [Belushkin et al. 2007]

  Are we facing a magnetic radius puzzle too?

- We need a model independent extraction of \( r_p^M \)!
Model independent extraction of the proton magnetic radius from electron scattering

[Zachary Epstein GP, Joydeep Roy, to appear]
Bound on $|a_k|$ 

- Analyzing $p$ and $n$ data separate $G_M^p$ and $G_M^n$ to isospin channels
  \[ G_M^{(0)} = G_M^p + G_M^n \]
  \[ G_M^{(0)}(0) = \mu_p + \mu_n \approx 0.88 \]
  \[ \Rightarrow I = 0, \quad a_0 = 0.88 \]

- Vector dominance ansatz:
  - $I = 0$ ($\omega$ exchange) $|a_k| \leq 1.1$
  - $I = 1$ ($\rho$ exchange) $|a_k| \leq 5.1$

- Between $t = 4m_\pi^2$ and $t = 16m_\pi^2$ only $\pi\pi$ contributes
  \[ I = 1: \quad |a_k| \leq 7.2 \]

- Above $t = 4m_N^2$ use $e^+e^- \to N\bar{N}$: negligible contribution to $a_k$

- Conclusion:
  \[ |a_k| \leq 5 \text{ not conservative enough, use } |a_k| \leq 10 \text{ and } |a_k| \leq 15 \]
$r^p_M$ from proton data (Preliminary)

- $G^p_M(q^2)$ values from $e - p$ scattering data
  [Arrington et al. PRC 76, 035205 (2007)]

- Extracted values don’t depend on number of parameters
  (results shown for $k_{max} = 8$)

- $Q^2 \leq 0.5$ GeV$^2$
  - $|a_k| \leq 10$: $r^p_M = 0.91^{+0.03}_{-0.06}$ fm
  - $|a_k| \leq 15$: $r^p_M = 0.92^{+0.04}_{-0.07}$ fm

- $Q^2 \leq 1.0$ GeV
  - $|a_k| \leq 10$: $r^p_M = 0.90^{+0.03}_{-0.07}$ fm
  - $|a_k| \leq 15$: $r^p_M = 0.91^{+0.04}_{-0.07}$ fm
$r^p_M$ from proton and neutron data (*Preliminary*)

- $G^p_M(q^2)$ from [Arrington et al. PRC 76, 035205 (2007)]
- $G^n_M(q^2)$ from [Lachniet et al. PRL 102, 192001 (2009); Anderson et al. PRC75, 034003 (2007); Kubon et al. PLB 524, 26 (2002); Xu et al. PRL 85, 2900 (2000); Anklin et al. PLB 428, 248 (1998); Anklin et al. PLB 336, 313 (1994); Gao et al. PRC 50, 546 (1994); Lung et al. PRL 70, 718 (1993)]

- Fit both $G^{(0)}_M$ and $G^{(1)}_M$

- $Q^2 \leq 0.5$ GeV$^2$
  - $|a_k| \leq 10$: $r^p_M = 0.87^{+0.04}_{-0.05}$ fm
  - $|a_k| \leq 15$: $r^p_M = 0.87^{+0.05}_{-0.05}$ fm

- $Q^2 \leq 1.0$ GeV
  - $|a_k| \leq 10$: $r^p_M = 0.88^{+0.02}_{-0.05}$ fm
  - $|a_k| \leq 15$: $r^p_M = 0.88^{+0.04}_{-0.05}$ fm
$r_M^p$ from proton and neutron and $\pi\pi$ data \textit{(Preliminary)}

- $G_M^p(q^2)$ from [Arrington et al. PRC 76, 035205 (2007)]
- $G_M^n(q^2)$ from [Lachniet et al. PRL 102 192001 (2009); Anderson et al. PRC 75, 034003 (2007); Kubon et al. PLB 524, 26 (2002); Xu et al. PRL 85, 2900 (2000); Anklin et al. PLB 428, 248 (1998); Anklin et al. PLB 336, 313 (1994); Gao et al. PRC 50, 546 (1994); Lung et al. PRL 70, 718 (1993)]
- $\text{Im } G_M^{(1)}$ between $t = 4m_{\pi}^2$ and $t = 16m_{\pi}^2$ from $\pi\pi$ data [Höhler, Landolt-Börnstein database Vol. 9b1 (1983); Amendolia et al. PLB 138, 454 (1984); Achasov et al. JETP 101, 1053 (2005)]

- $Q^2 \leq 0.5$ GeV$^2$
  - $|a_k| \leq 10$: $r_M^p = 0.87^{+0.01}_{-0.02}$ fm
  - $|a_k| \leq 15$: $r_M^p = 0.87^{+0.01}_{-0.02}$ fm

- $Q^2 \leq 1.0$ GeV
  - $|a_k| \leq 10$: $r_M^p = 0.87^{+0.01}_{-0.01}$ fm
  - $|a_k| \leq 15$: $r_M^p = 0.88^{+0.01}_{-0.02}$ fm
Conclusions and outlook
Conclusions

- Proton electric radius problem not resolved yet

- Are we facing a magnetic radius puzzle too?

- Model independent extraction of magnetic radius
  
  *Preliminary results:*
  - Proton data
    \[ r^p_M = 0.91^{+0.03}_{-0.06} \pm 0.02 \text{ fm} \]
  - Proton and neutron data
    \[ r^p_M = 0.87^{+0.04}_{-0.05} \pm 0.01 \text{ fm} \]
  - Proton, neutron and $\pi \pi$ data
    \[ r^p_M = 0.87^{+0.01}_{-0.02} \text{ fm} \]

- Consistent results, independent of $k_{\text{max}}$ and cut on $Q^2$
Outlook

- Model independent extraction of magnetic radius
  
  Preliminary results:
  
  - Proton data: \( r_M^p = 0.91^{+0.03}_{-0.06} \pm 0.02 \) fm
  
  - Proton and neutron data: \( r_M^p = 0.87^{+0.04}_{-0.05} \pm 0.01 \) fm
  
  - Proton, neutron and \( \pi \pi \) data: \( r_M^p = 0.87^{+0.01}_{-0.02} \) fm
  
  PDG 2012:
  
  \( r_M^p = 0.777 \pm 0.017 \) fm [Bernauer et al. PRL 105, 242001 (2010)]
  
  - \( r_M^p = 0.876 \pm 0.019 \) fm [Borisyuk et al. 2010]
  
  - \( r_M^p = 0.854 \pm 0.005 \) fm [Belushkin et al. 2007]
  
  Future direction:
  
  analyze other data sets, e.g. high precision data from [Bernauer et al. PRL 105, 242001 (2010)]