Modern lattice QCD:
progress and prospects

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Phenomenology 2014 Symposium
May 6, 2014

Beyond-the-Standard-Model search strategies

- ✦ The experimental high-energy physics community is presently searching for new physics with two complimentary approaches

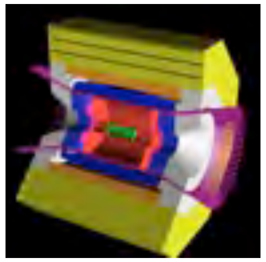
(1) Direct production of new particles at high-energy colliders

- ❖ *E.g.*, the LHC has already discovered a ~125 GeV particle that may be the SM Higgs

(2) Precise measurements of Standard-Model parameters and processes

- ❖ *E.g.*, the quark-flavor factories dramatically improved determinations of CKM matrix elements & CP-violating phase, and measured decay rates for rare processes
- ❖ Upcoming "ultrasensitive" experiments will improve existing measurements and observe some rare processes for the first time
- ❖ Compare measurements to Standard Model predictions and look for inconsistencies

BaBar



KLOE

Belle II

LHCb

Beyond-the-Standard-Model search strategies

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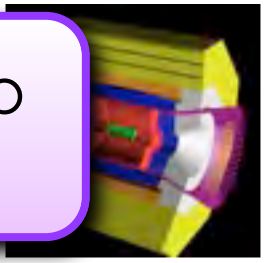
❖ Upcoming "u

existing measurements and observe some rare processes for the first time

Lattice-QCD calculations are needed to interpret many of their results . . .



❖ Compare measurements to Standard Model predictions and look for inconsistencies

Scope of "ultrasensitive" experiments

- Current and planned experiments cover a broad range of topics in particle and nuclear physics

lepton flavor violation

Mu2e

neutrino physics

NOvA

LBNE

Higgs physics

ilc

Muon Accelerator Program

FCC TLEP

B & D physics

Belle II

LHCb

BES-III

muon g-2

μ

g−2

kaon physics

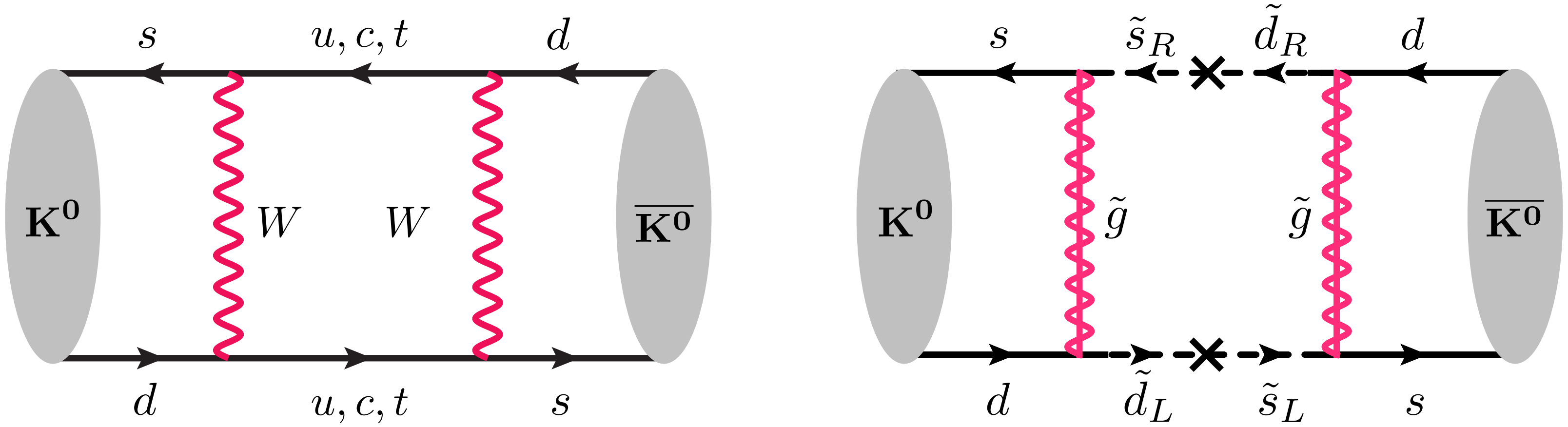
KOTO

NA62

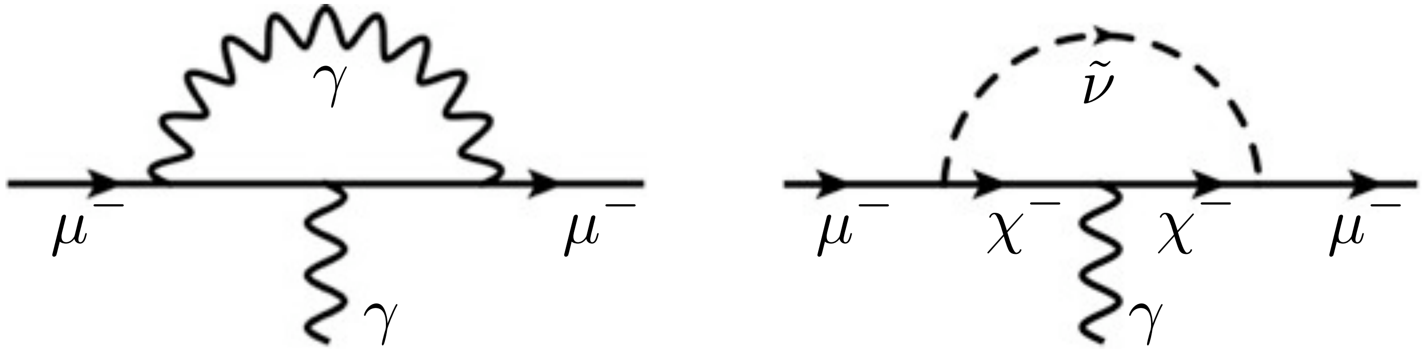
Importance of precision measurements

- Quantum-mechanical loop effects sensitive to physics at higher energy scales than those probed at LHC, in some cases O(1,000 - 10,000 TeV) [Isidori, Nir, Perez, Ann.Rev.Nucl.Part.Sci. 60 (2010) 355]

neutral kaon mixing



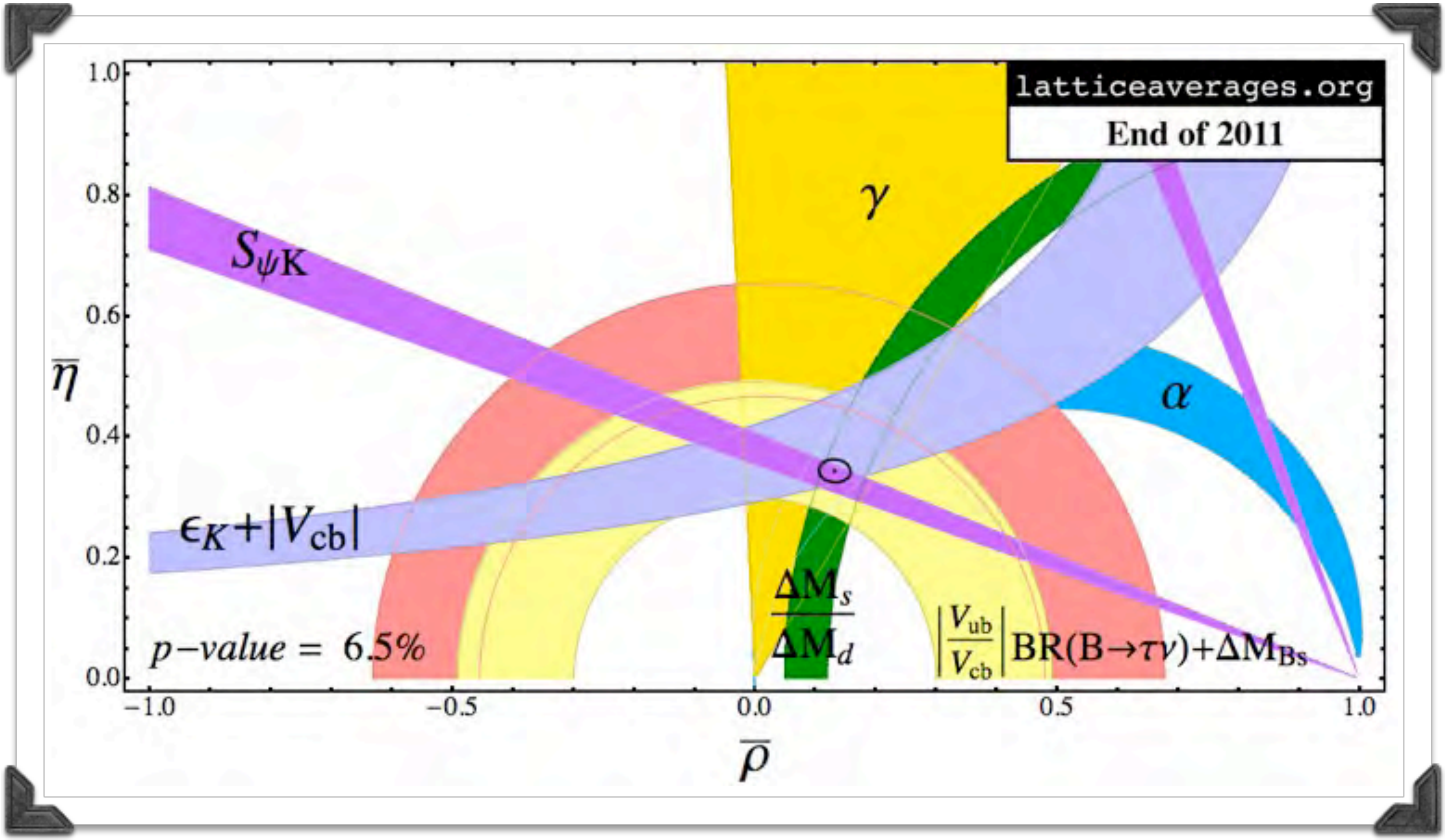
muon g-2



- If new particles are discovered at ATLAS & CMS, precision measurements still needed to **extract the flavor & CP-violating couplings and determine underlying structure of theory**

Role of lattice QCD

- **Comparison between measurements and Standard-Model predictions still limited in most cases by theoretical uncertainties**, often from hadronic matrix elements



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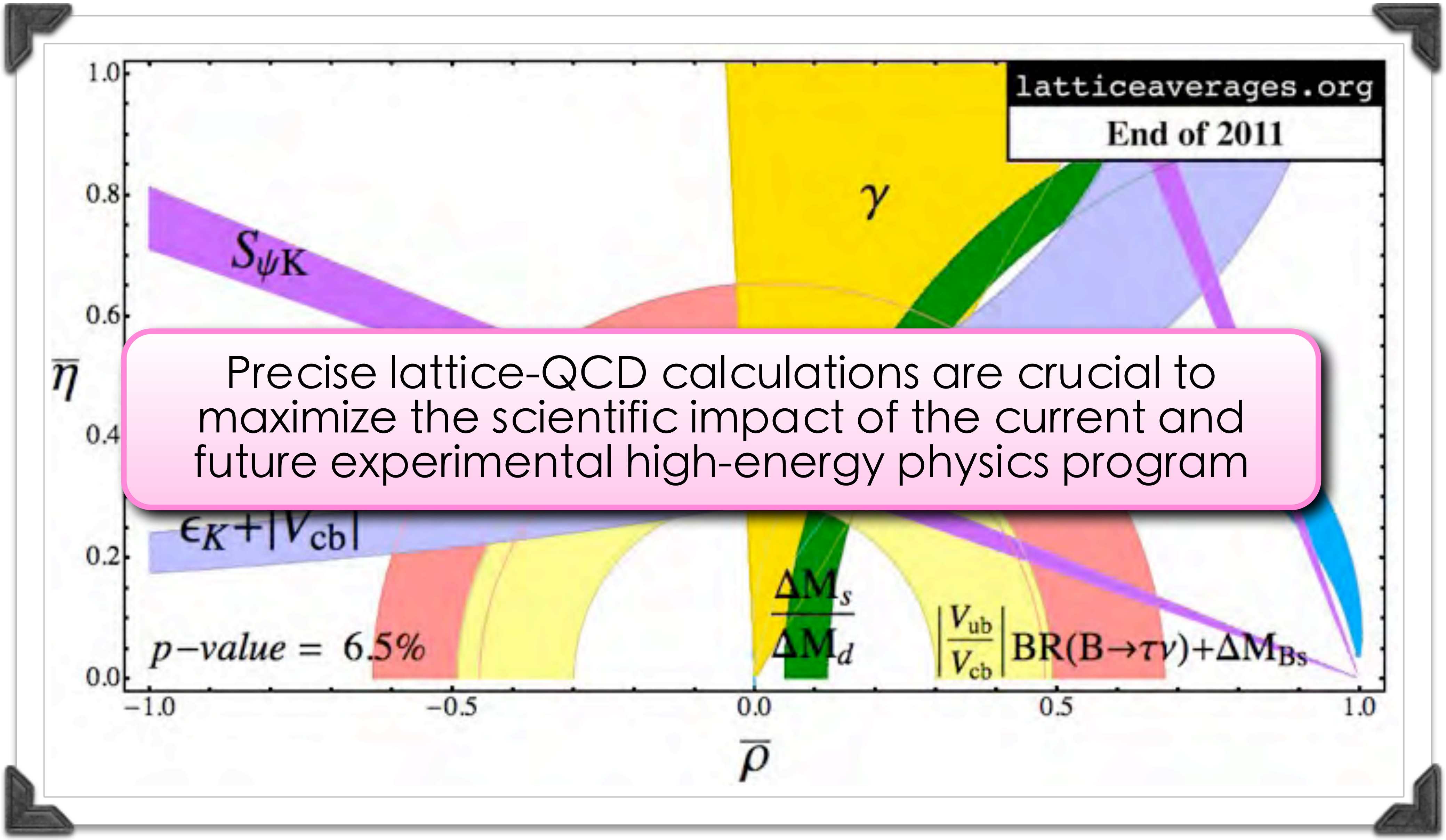
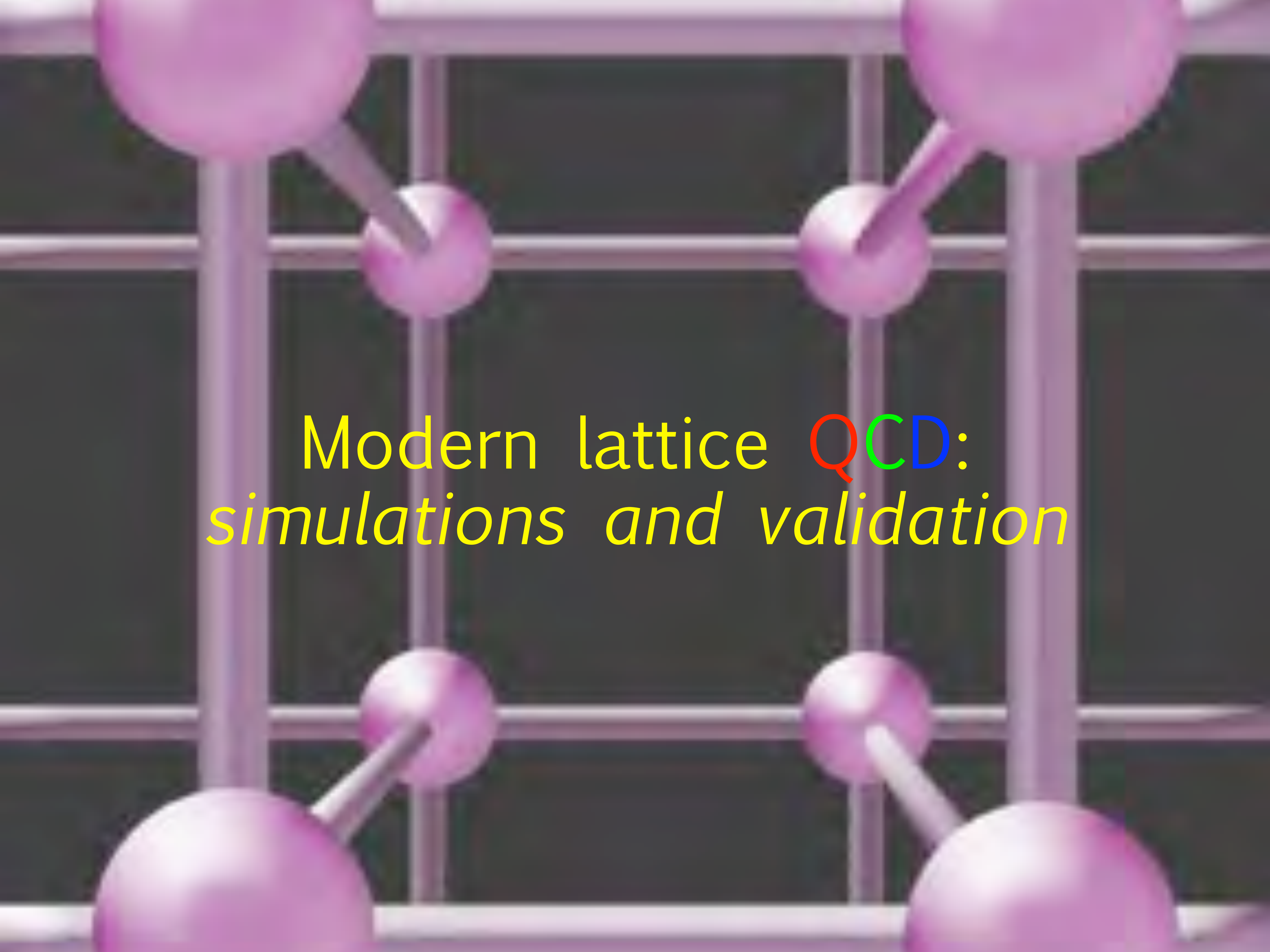


Precise lattice-QCD calculations are crucial to maximize the scientific impact of the current and future experimental high-energy physics program



Modern lattice QCD:
simulations and validation

Quantum Chromodynamics

- QCD Lagrangian contains 1 + n_f + 1 parameters:

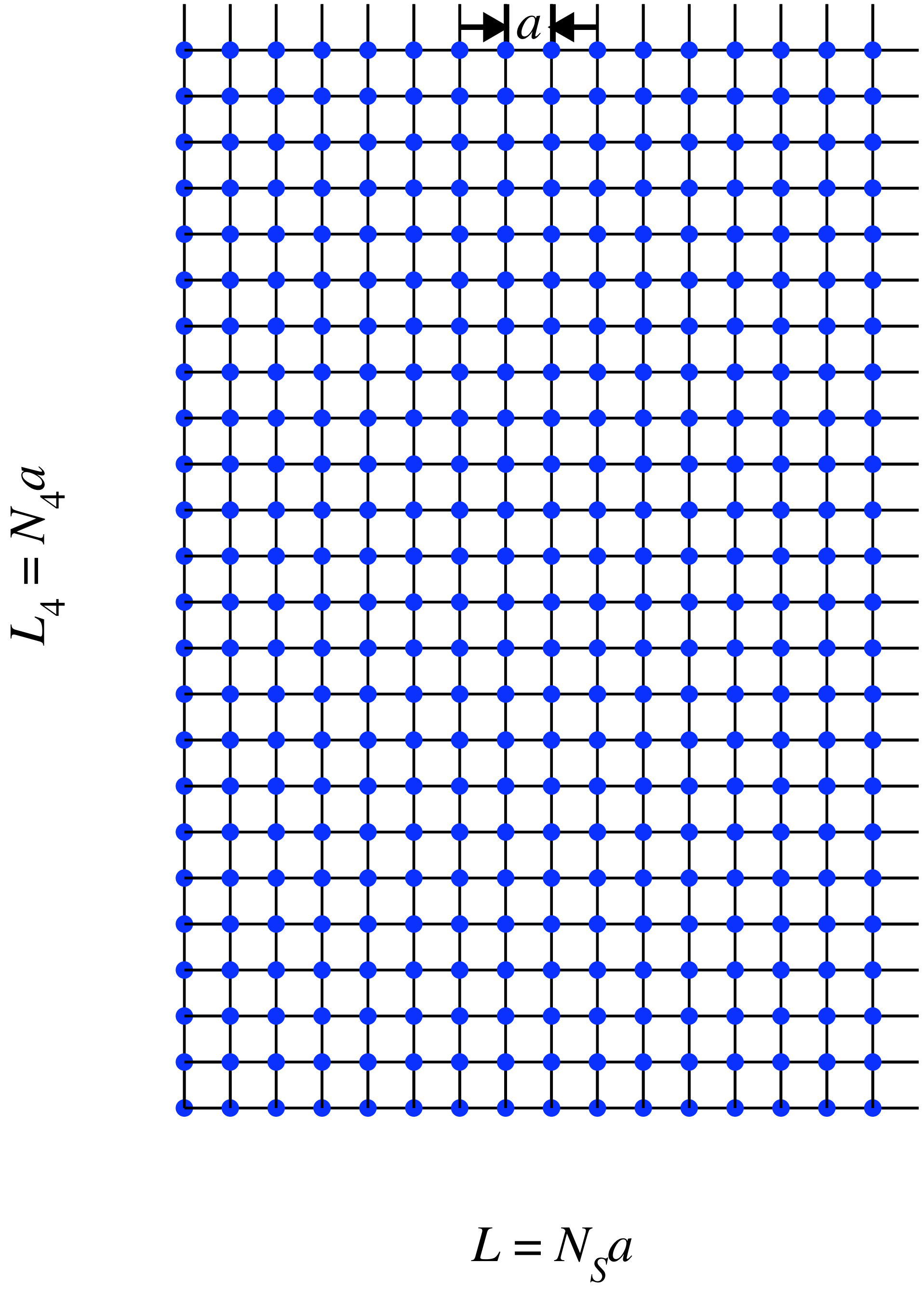
$$\mathcal{L}_{\rm QCD} = \frac{1}{2g^2}\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right] - \sum_{f=1}^{n_f}\bar{\psi}_f\left(\not{D} + m_f\right)\psi_f + \underbrace{\frac{i\bar{\theta}}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left[F_{\mu\nu}F_{\rho\sigma}\right]}_{\text{violates } CP}$$

❖ Gauge coupling g^2	r_1, m_Ω, Υ(2S-1S), or f_π
❖ n_f quark masses m_f	m_π, m_K, $m_{J/\psi}$, m_Υ, …
❖ Experimental bound on $\lvert\theta\rvert<10^{-10}$ from neutron EDM	$\theta = 0$

- Once the parameters of the QCD Lagrangian are fixed, everything else is a prediction of the theory

Lattice Quantum Chromodynamics

- Systematic method for calculating hadronic parameters from QCD first principles



- Define QCD on a (Euclidean) spacetime lattice
- Replace derivatives by discrete differences and integrals by sums, e.g.:

$$\partial\psi(x) \longrightarrow \frac{\psi(x+a) - \psi(x-a)}{2a}$$

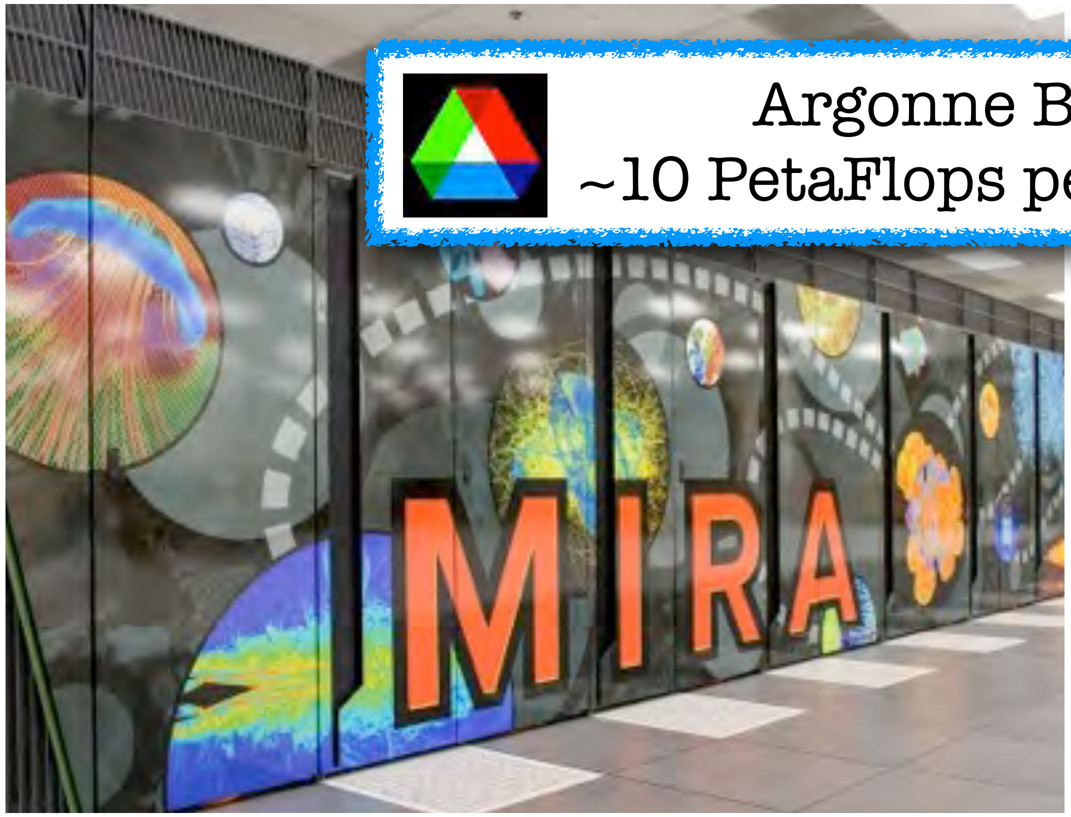
$$\psi(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x}\tilde{\psi}(k) \longrightarrow \sum_k e^{-ik\cdot x}\tilde{\psi}(k)$$

- Many choices for how to discretize QCD action
  - Different lattice fermion formulations are optimal for different physical quantities
- All recover continuum QCD when lattice spacing a→0 and box size L→∞

Lattice computing

- Simulate numerically using **Monte Carlo methods and importance sampling**
    - Sample from all possible field configurations using a distribution given by exp(-S_{QFT})
- Run codes upon a variety of high-performance computing hardware
    - Depending upon the problem size, use **large supercomputers at DOE and NSF leadership-class facilities** and **dedicated PC and GPU-accelerated clusters at national laboratories**

Fermilab lqcd clusters
~245 TFlops peak

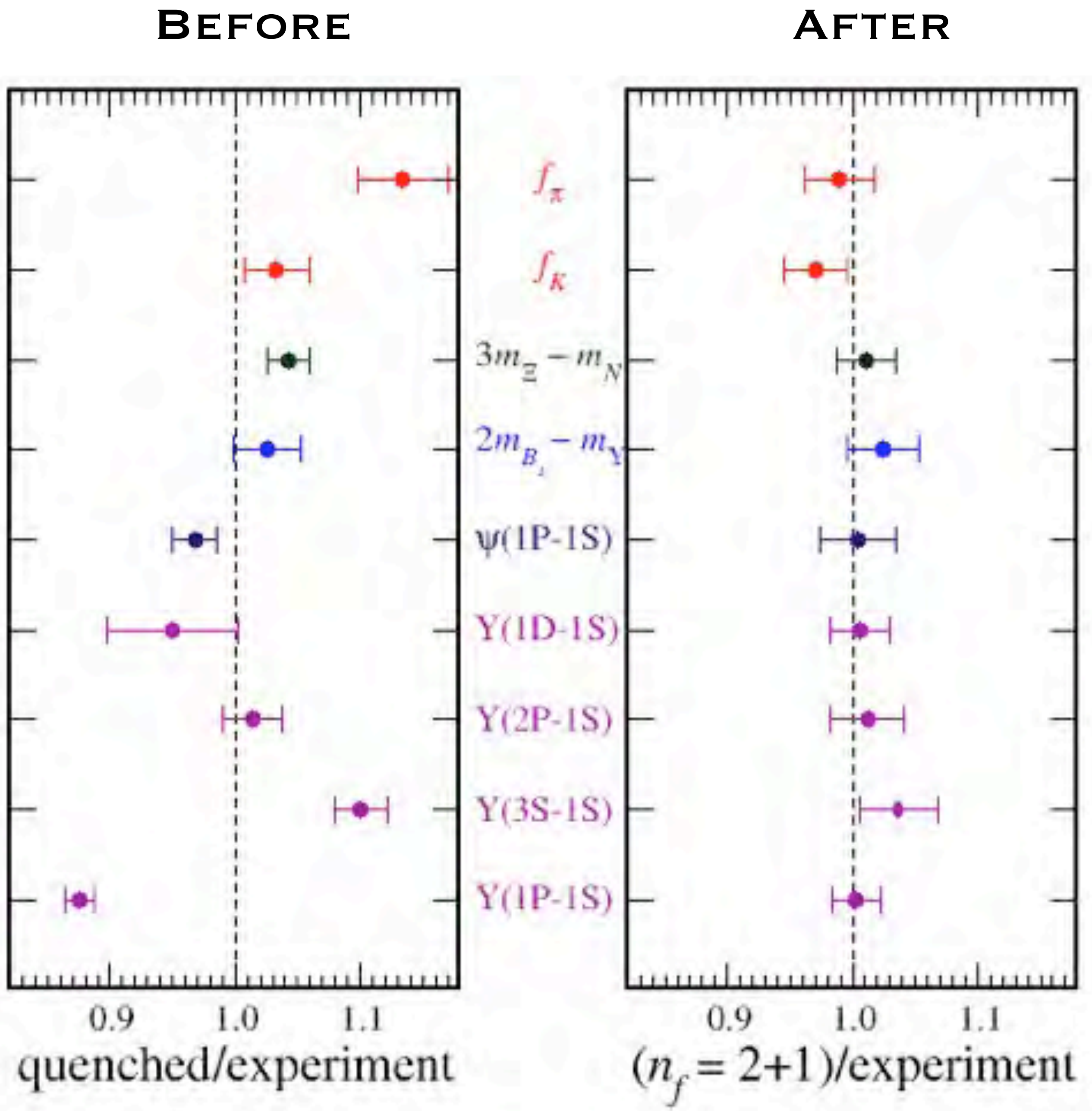


Argonne BG/Q
~10 PetaFlops peak

Lattice QCD in the 21st century

- ✦ **For the past decade, it has been possible to simulate realistic QCD** including the effects of the dynamical u, d, & s quarks in the vacuum
- ✦ Over this time, lattice methods have been used to calculate many simple quantities with **controlled uncertainties and complete error budgets**
- ✦ Most precise results are for matrix elements with only hadron in initial state and at most one hadron in final state, where all hadrons are stable under QCD

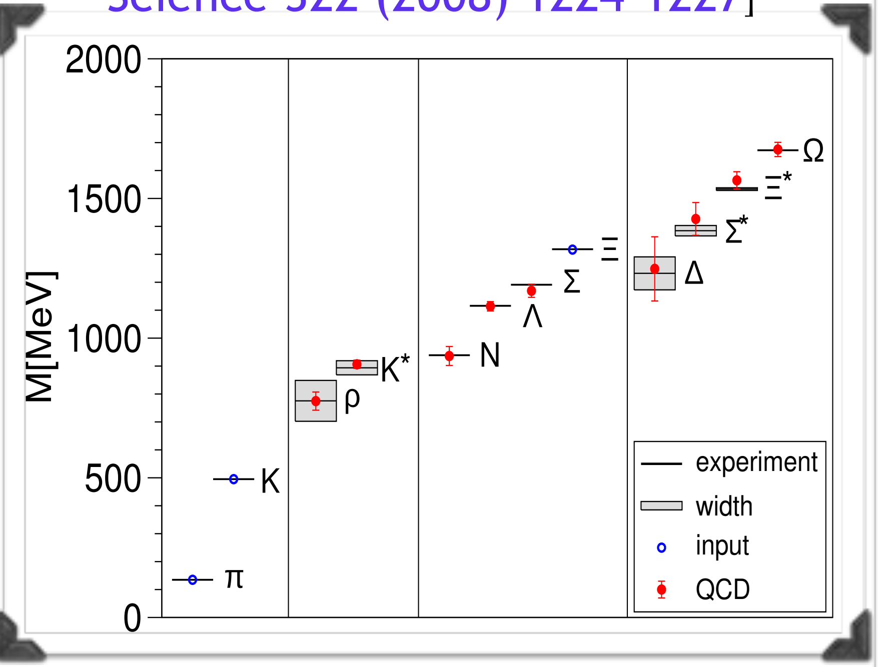
Lattice methods tested and errors verified by (i) comparison with experiment, and (ii) comparison of independent lattice calculations sensitive to different systematics



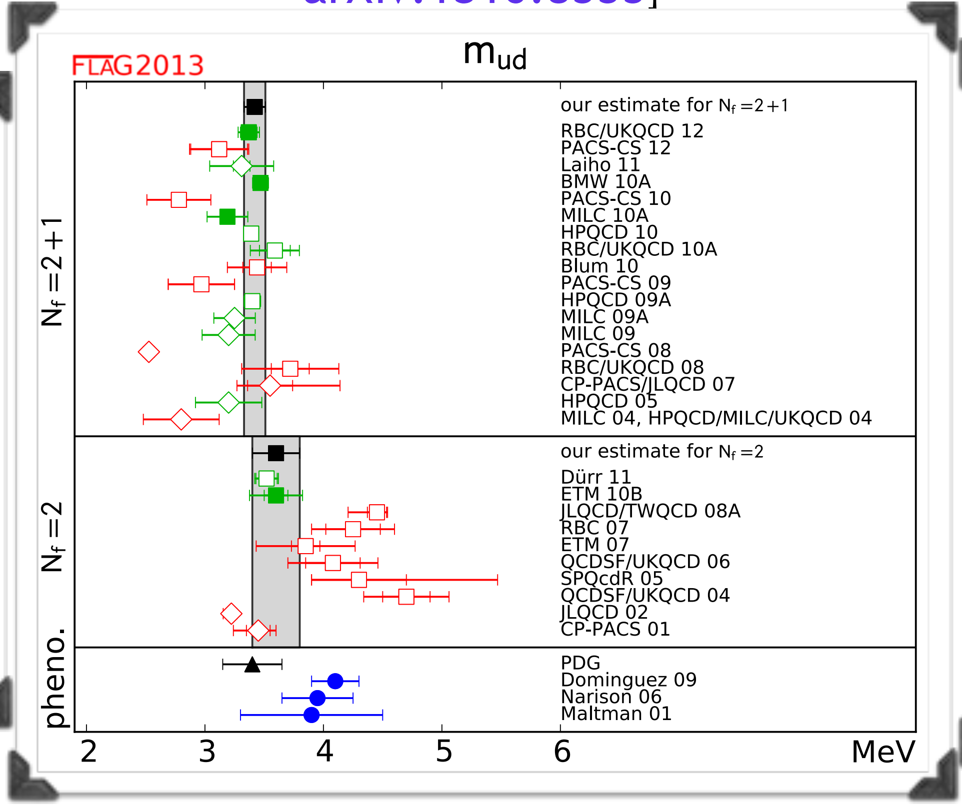
[HPQCD, MILC, & Fermilab Lattice Collaborations Phys.Rev.Lett.92:022001,2004]

Successes of lattice QCD

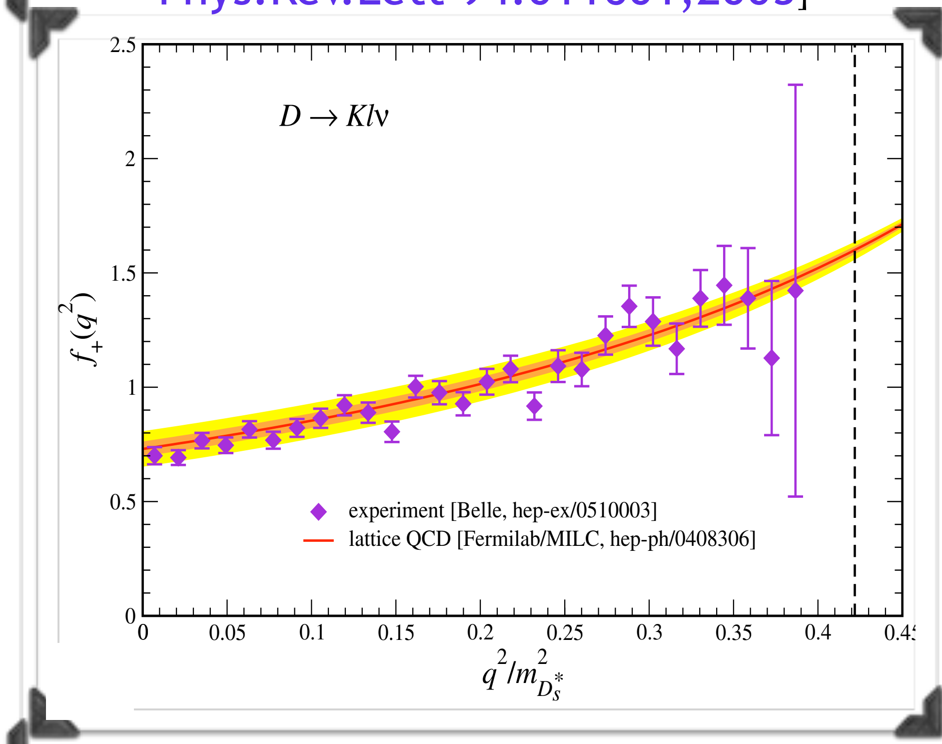
[BMW Collaboration,
Science 322 (2008) 1224-1227]



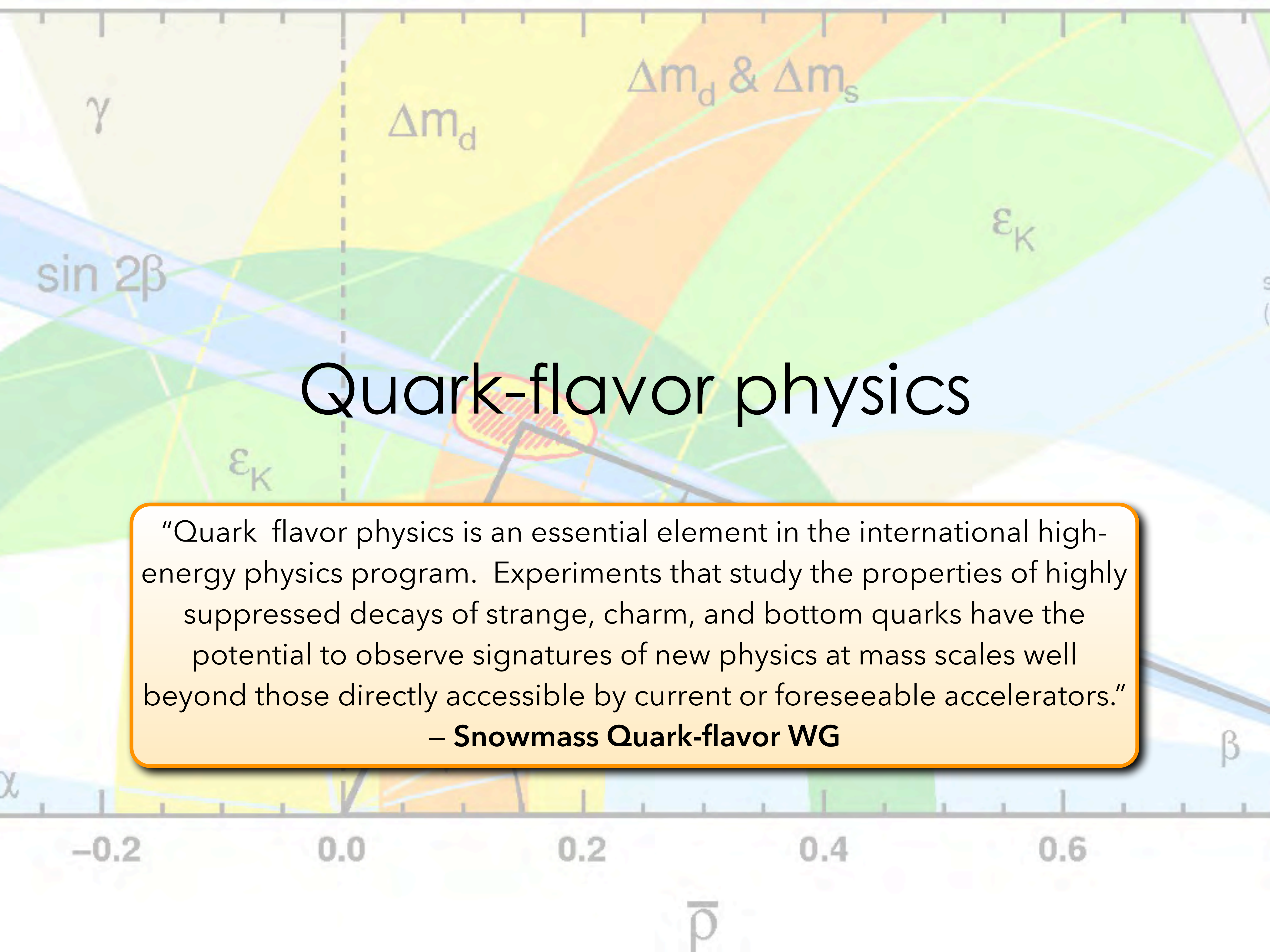
[Flavor Lattice Averaging Group,
arXiv:1310.8555]



[Fermilab Lattice & MILC,
Phys.Rev.Lett 94:011601,2005]



- Lattice-QCD calculations now reproduce experimental results for a wide variety of hadron properties and provide the only *ab initio* QCD calculation of others, *e.g.*:
  - Low-lying light- and heavy-hadron spectrum
  - Quark masses & most accurate determination of strong coupling constant
  - **Predictions** of B_c meson mass, decay constants f_D & f_{Ds}, and D→Klν form factor
- Demonstrate that **lattice calculations are reliable with controlled systematic errors**



Quark-flavor physics

> "Quark flavor physics is an essential element in the international high-energy physics program. Experiments that study the properties of highly suppressed decays of strange, charm, and bottom quarks have the potential to observe signatures of new physics at mass scales well beyond those directly accessible by current or foreseeable accelerators."
>
> – **Snowmass Quark-flavor WG**

Lattice QCD and the CKM matrix

- CKM quark-mixing matrix elements & phase are fundamental SM parameters that cannot be calculated from first principles
- Parametric inputs to SM predictions for many flavor-changing processes such as neutral kaon mixing and K → πνν decays
- Simple lattice processes with 0 or 1 hadron in initial & final states enable determinations of all CKM elements except $|V_{tb}|$

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\ & K \to \pi\ell\nu & B \to \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

(Experiment) = (known) x (CKM factors) × (Hadronic Matrix Element)

$\Delta m_{(d,s)}$,

$$\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2}, \frac{d\Gamma(B \to D^{(*)}\ell\nu)}{dw}, \dots$$

Parameterize nonperturbative QCD effects as decay constants, form factors, and bag-parameters that must be computed numerically with **LATTICE QCD**

Lattice QCD and the CKM matrix

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- ✦ Simple lattice processes with 0 or 1 hadron in initial & final states enable determinations of all CKM elements except $|V_{tb}|$

$$\begin{pmatrix} \begin{array}{c}\mathbf{V_{ud}}\\ \pi\to\ell\nu\\ \\ \end{array} & \begin{array}{c}\mathbf{V_{us}}\\ K\to\ell\nu\\ K\to\pi\ell\nu\end{array} & \begin{array}{c}\mathbf{V_{ub}}\\ B\to\ell\nu\\ B\to\pi\ell\nu\end{array} \\ \begin{array}{c}\mathbf{V_{cd}}\\ D\to\ell\nu\\ D\to\pi\ell\nu\end{array} & \begin{array}{c}\mathbf{V_{cs}}\\ D_s\to\ell\nu\\ D\to K\ell\nu\end{array} & \begin{array}{c}\mathbf{V_{cb}}\\ B\to D\ell\nu\\ B\to D^*\ell\nu\end{array} \\ \begin{array}{c}\mathbf{V_{td}}\\ \langle B_d|\bar{B}_d\rangle\end{array} & \begin{array}{c}\mathbf{V_{ts}}\\ \langle B_s|\bar{B}_s\rangle\end{array} & \begin{array}{c}\mathbf{V_{tb}}\\ \\ \end{array} \end{pmatrix}$$

(Experiment) = (known) x (CKM factors) × (Hadronic Matrix Element)

Lattice community has well-established and successful program to calculate weak matrix elements needed to obtain elements and phase of CKM matrix

(see Flavor Lattice Averaging Group (FLAG) review, 1310.8555)

Example:

$|V_{cb}|$ from B→D*$\ell\nu$ at zero recoil

- B → D*$\ell\nu$ semileptonic form factor allows determination of $|V_{cb}|$ via:

$$\frac{d\Gamma(B \to D^* l\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |\mathbf{V_{cb}}|^{\mathbf{2}} |\mathcal{F}_{\mathbf{B\to D^*}}(\mathbf{w})|^{\mathbf{2}} \quad \Big\} \; w \equiv v_B \cdot v_D$$

- Only need one normalization point, so choose zero recoil (w=1) because it can be computed most precisely

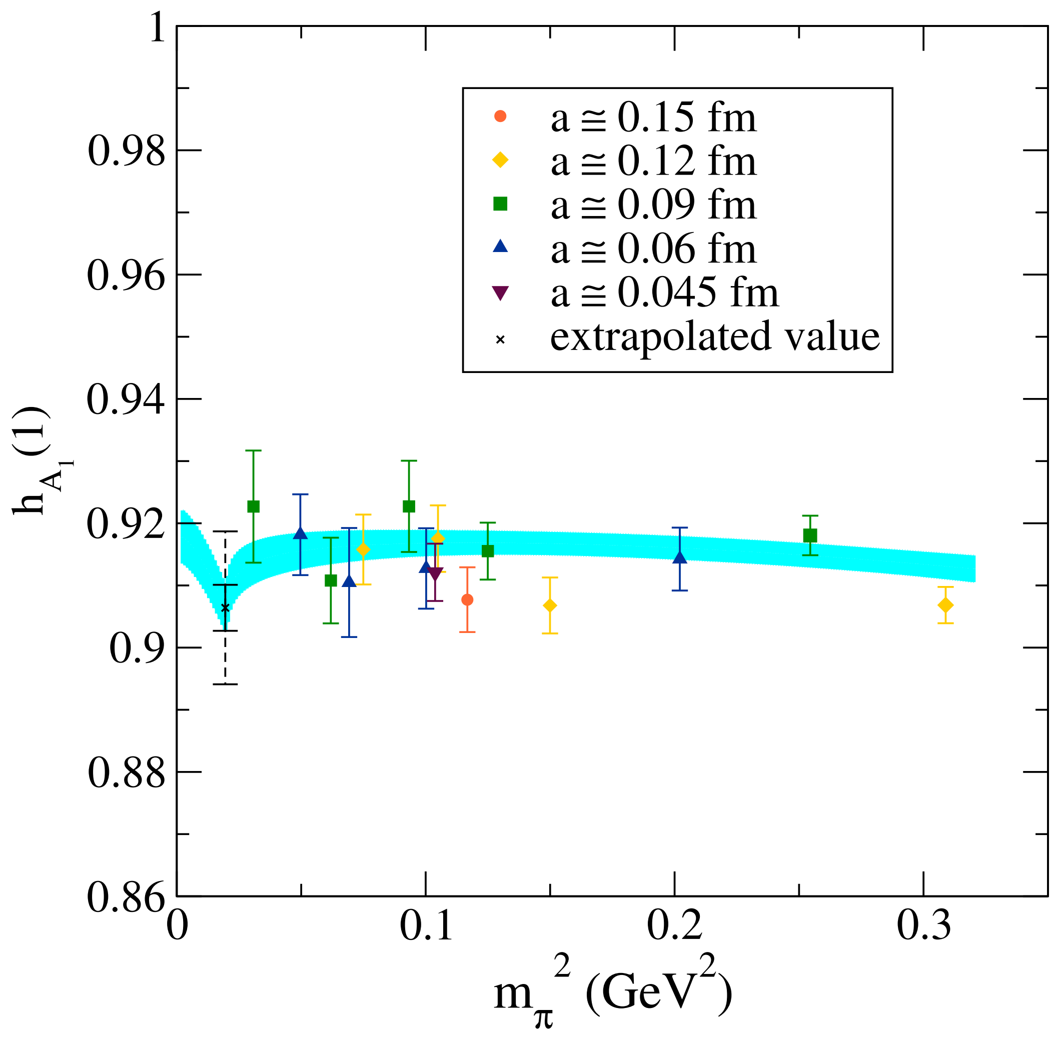
- Fermilab Lattice & MILC Collaborations recently **updated F(1) with increased statistics, lighter quark masses, & finer lattice spacings**, obtaining $|V_{cb}|$ to 1.9% precision

- **QCD error in $|V_{cb}|$ now commensurate with the experimental error**

$$F(1) = 0.906(4)_{\rm stat}(12)_{\rm sys}$$

$$|V_{cb}| = [39.04(49)_{\rm expt}(53)_{\rm LQCD} \pm (19)_{\rm QED}] \times 10^{-3}$$

[Bailey *et al.*, arXiv:1403.0635]



Recent highlight:

K→πℓν form factor at the physical pion mass

- K→πℓν form factor can be combined with experimentally-measured branching fraction to obtain $|V_{us}|$ in the Standard Model via:

$$\Gamma(K \to \pi \ell \nu) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)^2$$

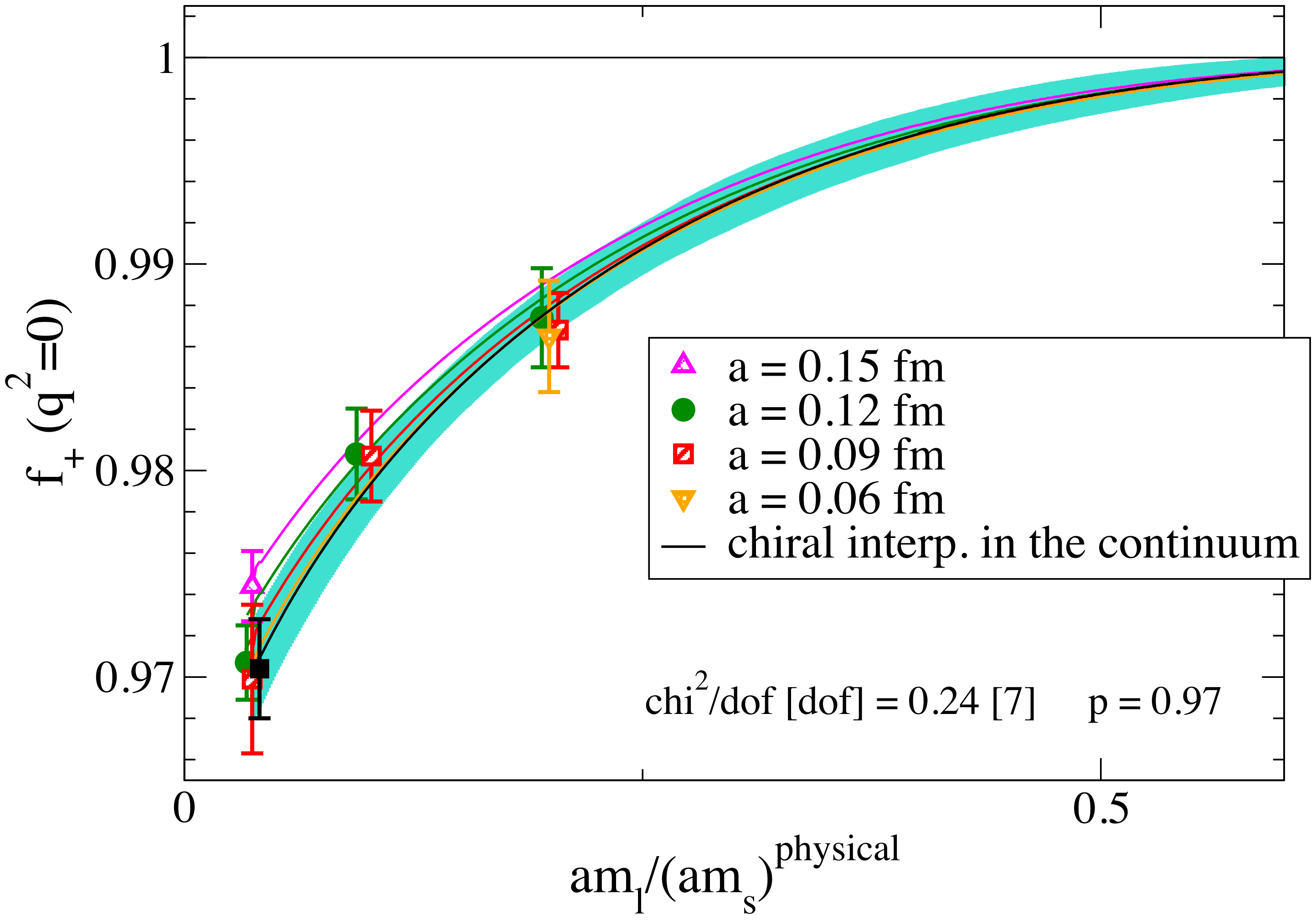
- Fermilab Lattice and MILC recently obtained the **first result for the $f_+^{K\pi}(q^2=0)$ at the physical pion mass, removing previously dominant uncertainty from chiral extrapolation**

- Single most precise result for $f_+(0)$ enables 0.3% determination of $|V_{us}|$

$f_+^{K\pi}(0) = 0.9704(24)_{\rm stat}(22)_{\rm sys}$

$|V_{us}| = 0.22290(74)_{\rm theo}(52)_{\rm exp}$

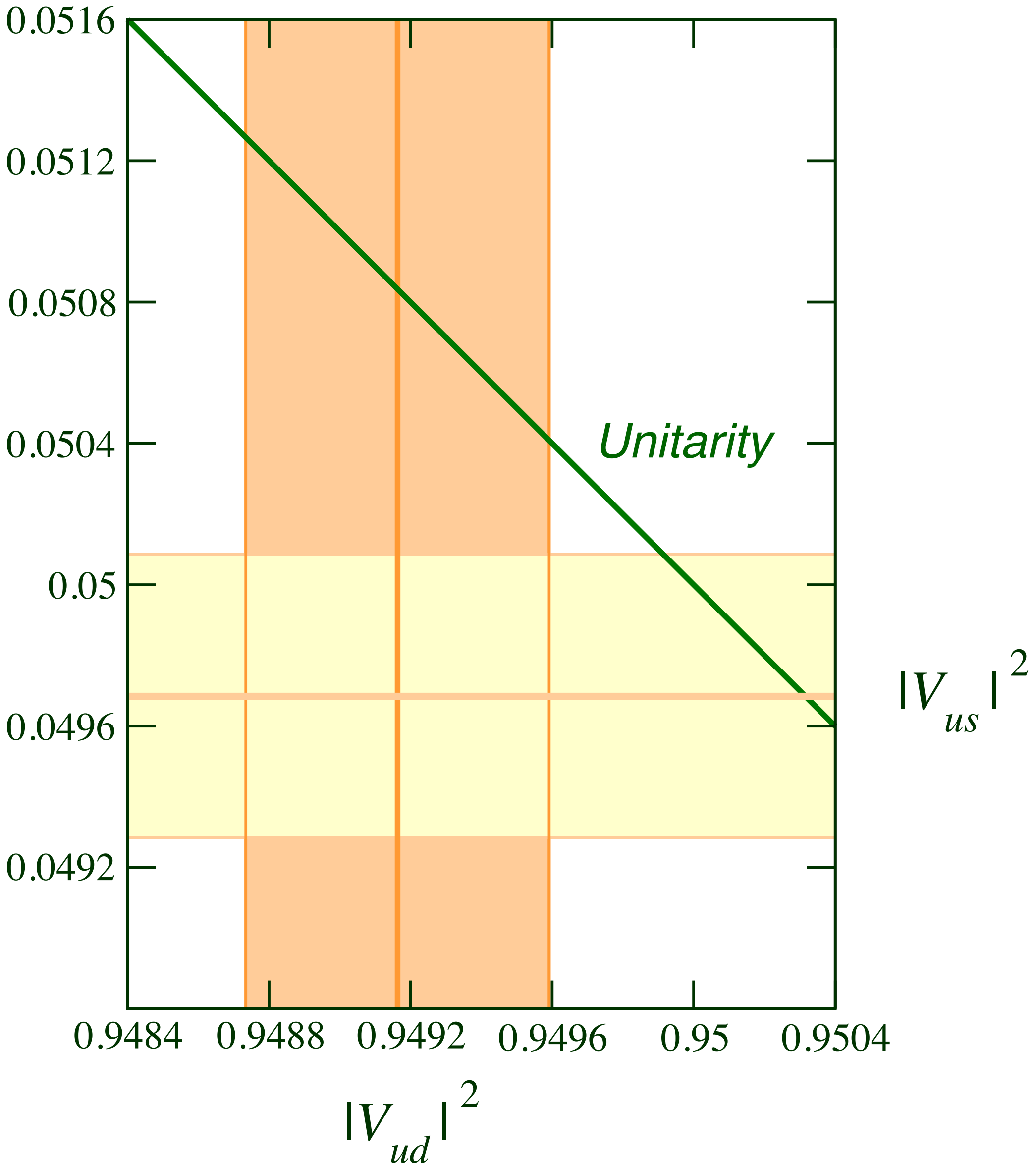
[Bazavov *et al.* PRL 112 (2014) 112001]



Recent highlight:
$K \to \pi \ell \nu$ form factor at the physical pion mass

In test of first-row unitarity, error from $|V_{us}|$ now smaller than that from $|V_{ud}|$:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00115(40)_{V_{us}}(43)_{V_{ud}}$$



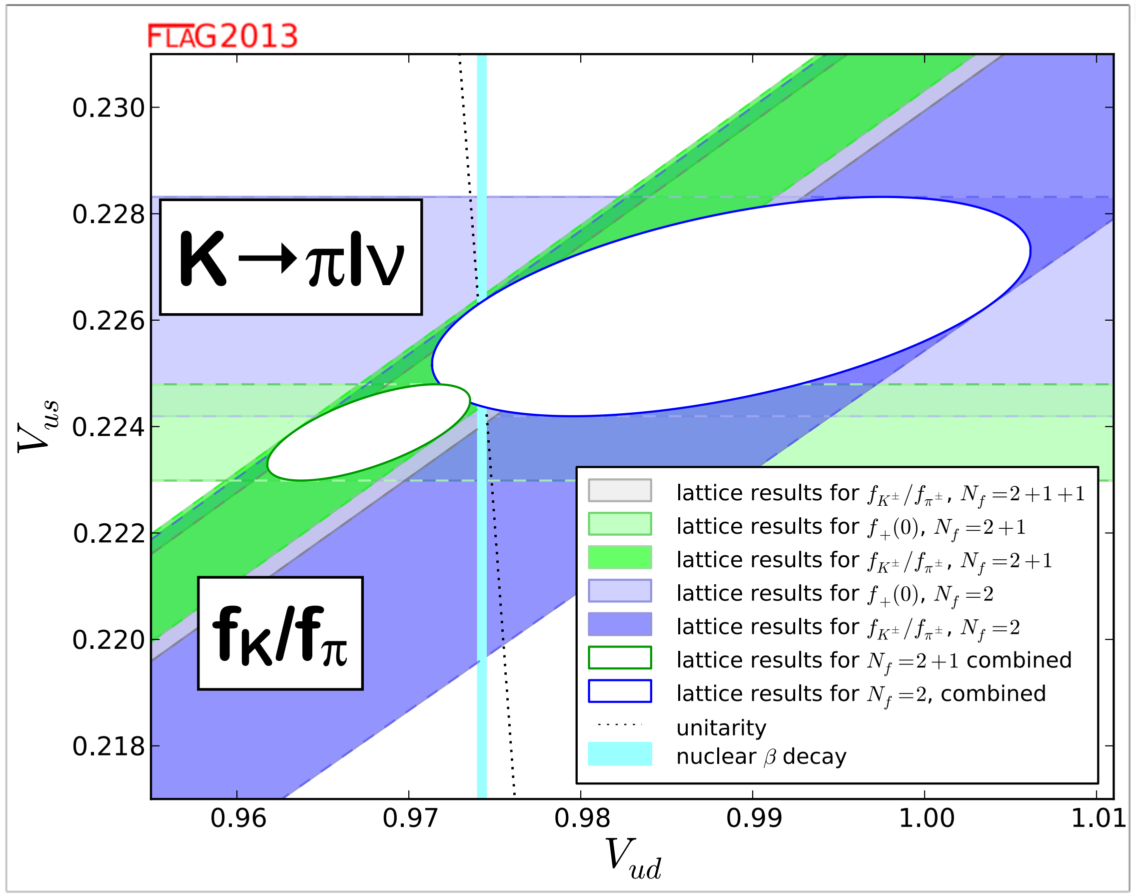
➜ Worth revisiting error on $|V_{ud}|$ from nuclear β decays?

Room for improvement

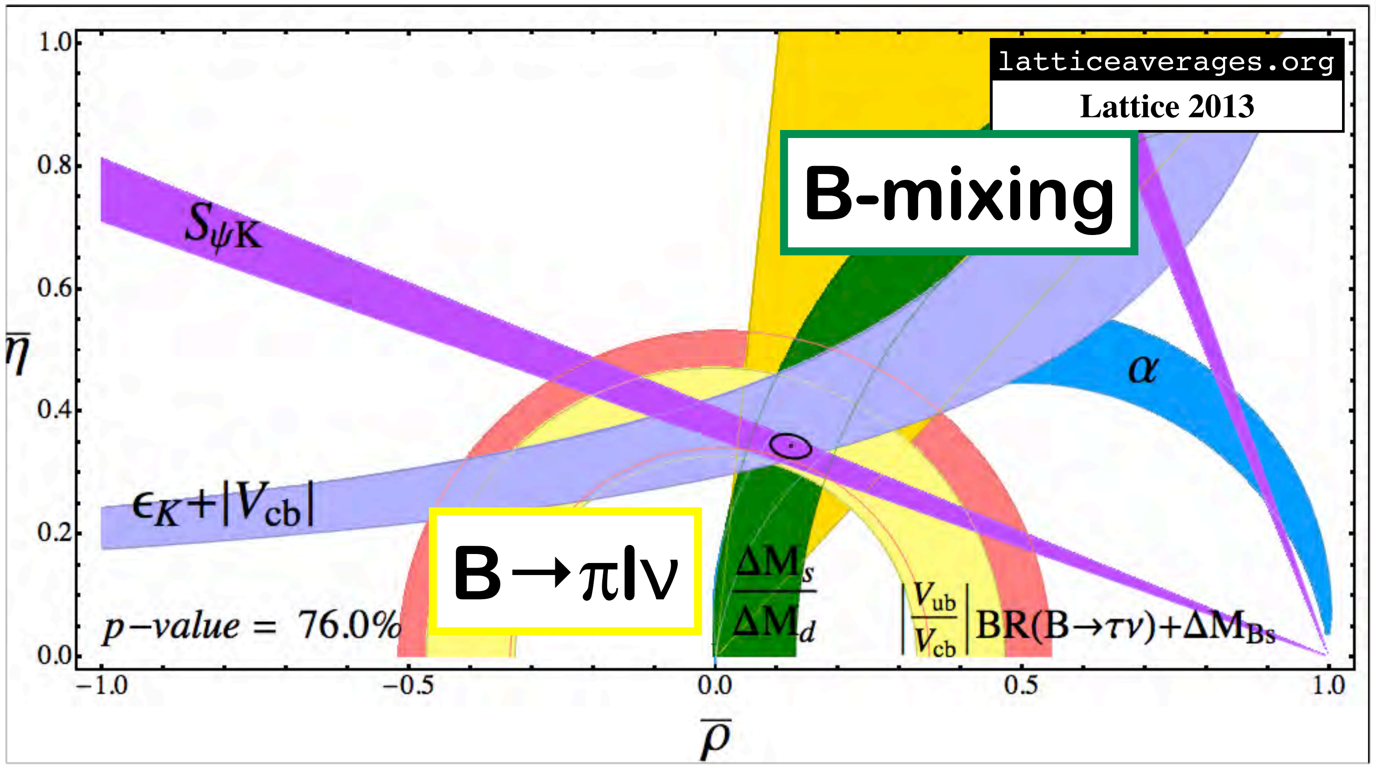
- For many quark-flavor changing processes, lattice errors still larger than those from experiment (in particular semileptonic form factors and neutral meson mixing parameters)

- Must continue to improve precision on “standard” lattice matrix elements to squeeze the vise on the Standard-Model CKM framework with existing quark-flavor data

First-row unitarity

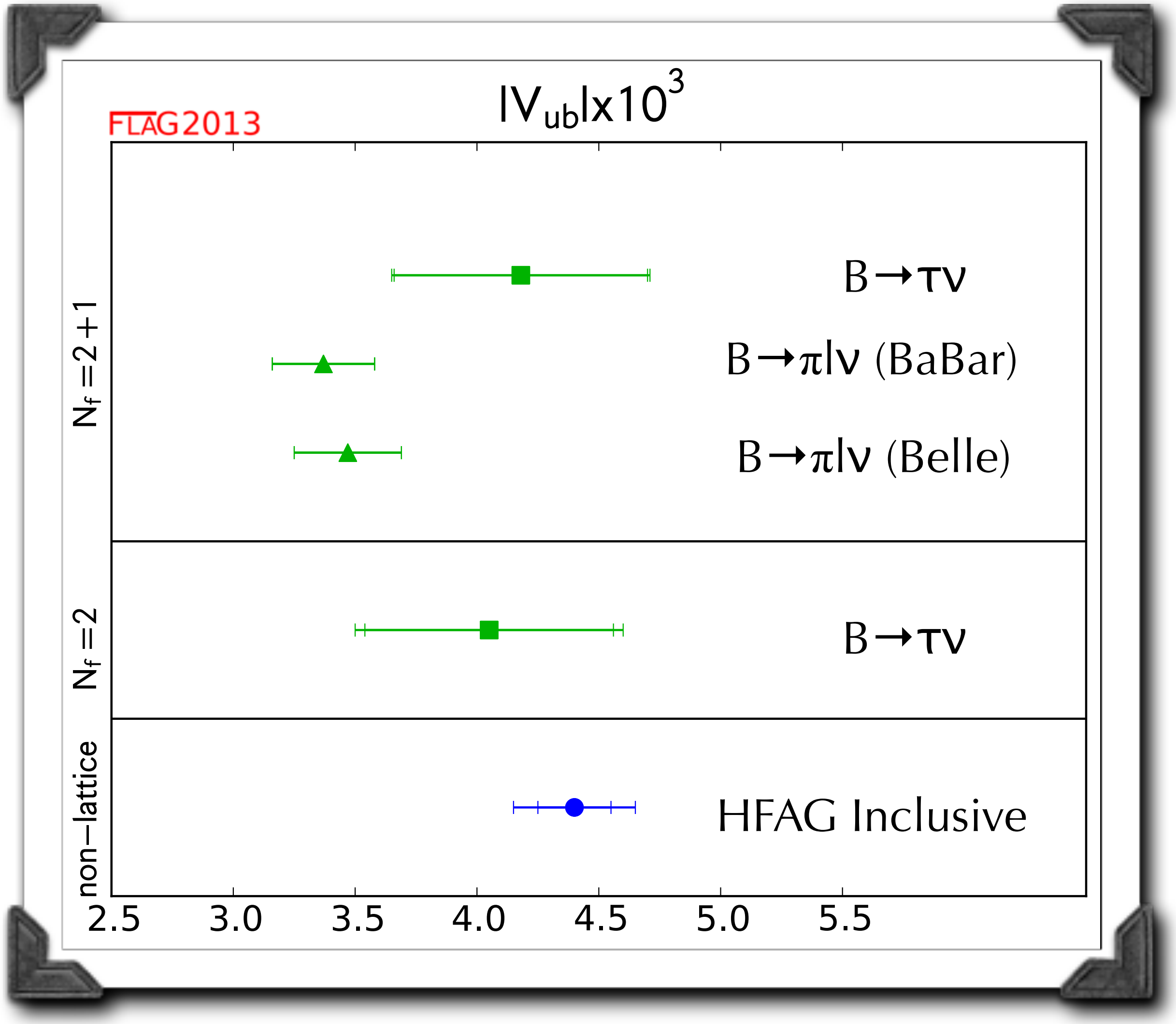


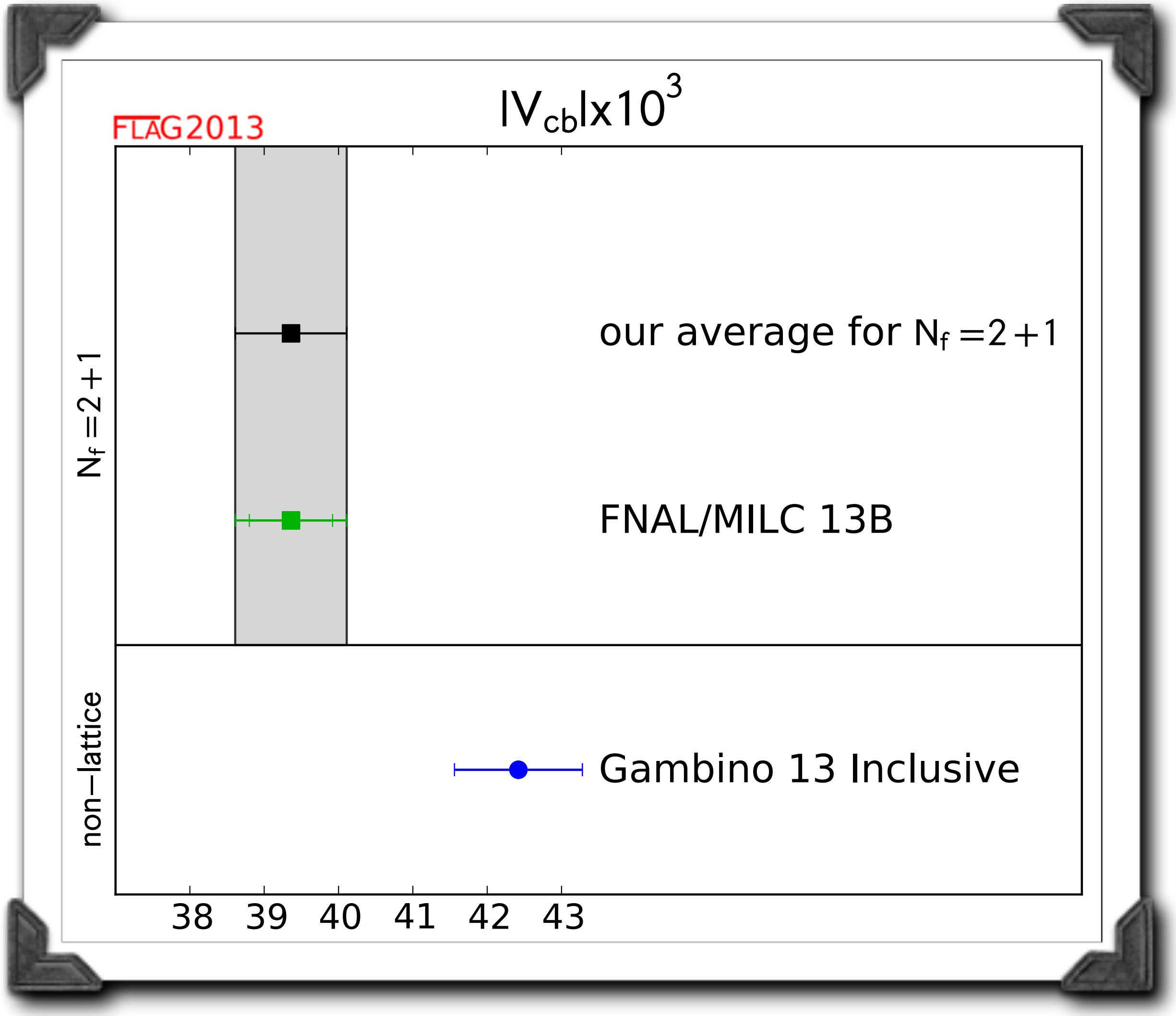
Global CKM UT Fit



Outstanding puzzles

- Long-standing ~3σ tension between determinations of $|V_{ub}|$ and $|V_{cb}|$ from inclusive and exclusive semileptonic B-decays still needs resolution



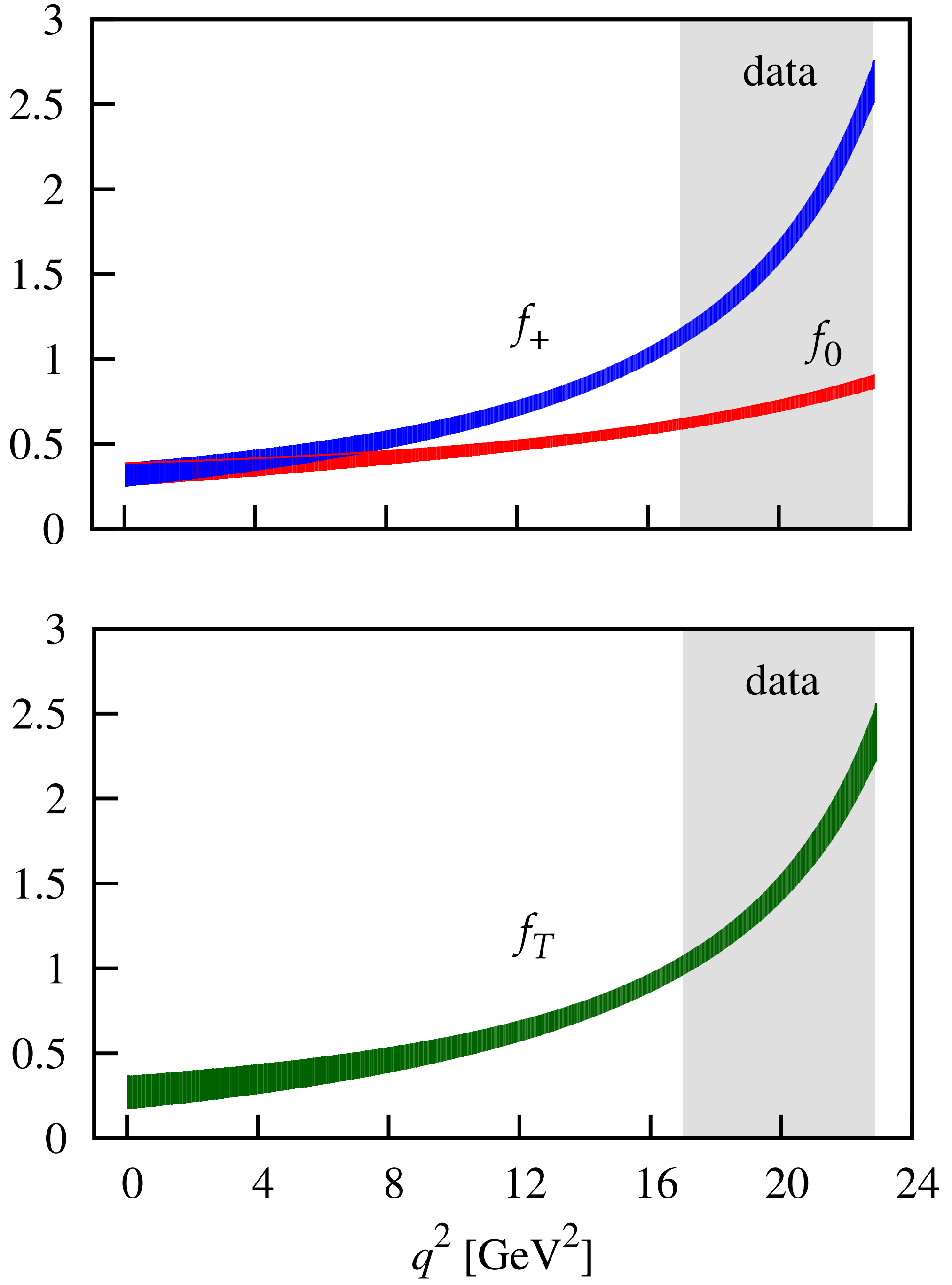


- **Determinations from other exclusive decays will provide important checks:**
  - Lattice-QCD calculations underway of form factors for **B_s→Kμν to obtain $|V_{ub}|$** (will be measured @ LHCb) and **B→Dlν to obtain $|V_{cb}|$** (N_f=2+1 result coming soon)

New-physics searches with rare decays

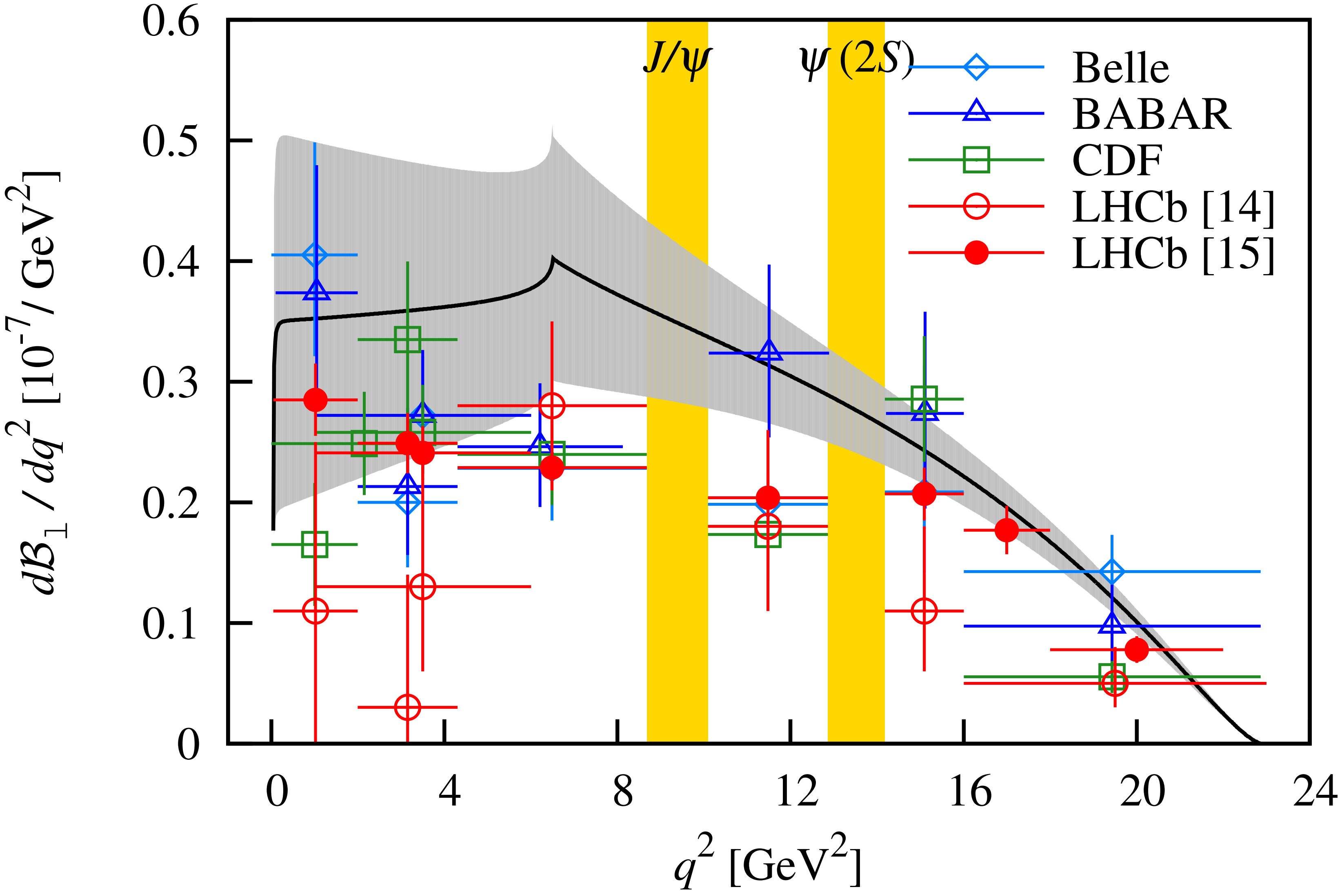
- Rare decay B→K$\ell^+\ell$ can proceed only through loop diagrams in the Standard Model, making it a particularly sensitive probe for new physics
- Accurate Standard-Model predictions are important and timely as experimental measurements becoming more precise, and require parameterization of hadronic form factors over full q^2 range
- HPQCD Collaboration recently obtained the **first (2+1)-flavor result for the three form factors $f_+(q^2)$, $f_0(q^2)$, and $f_T(q^2)$, which are sufficient to parameterize B→Kl^+l both in the Standard Model and in all possible beyond-the-SM theories**

[Bouchard *et al.* PRL111 (2013) 162002, PRD88 (2013) 054509]



New-physics searches with rare decays

For $q^2 > 10$ GeV^2, results more precise than previous Standard-Model predictions, and for all q^2, results consistent with previous calculations and experiment.

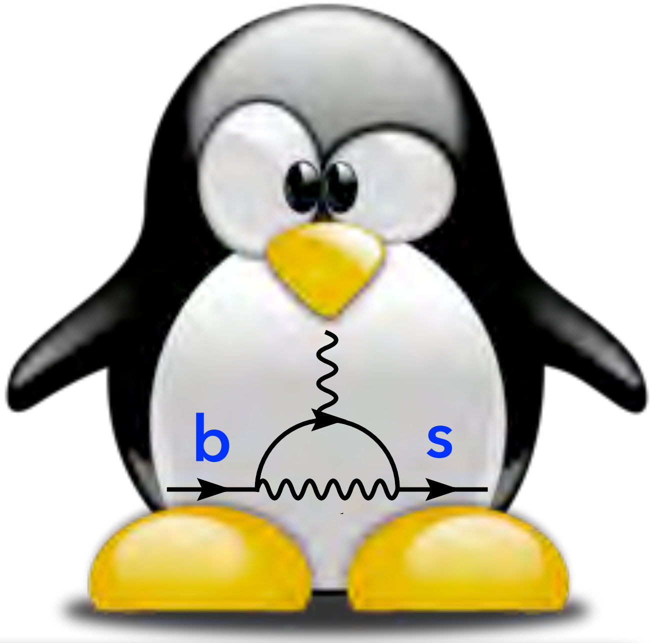


(But both experimental and theory uncertainties will continue to improve…)

Open challenges

$B\to K^*$ AND RELATED FORM FACTORS

- $B\to K^*ll$, $B\to K^*\gamma$, and $B_s\to\varphi\gamma$ have been observed experimentally and rate measurements will continue to improve; comparisons with SM predictions require form factors over full kinematic range
- Lattice calculations "very challenging" because final-state K^* and φ are unstable in QCD; moreover, their widths increase as the light-quark masses approach the physical point
- Initial step recently taken by Prelovsek *et al.*, who completed first lattice study of the $K^{*(}872)$ decay width [Phys.Rev. D88 (2013) 054508]

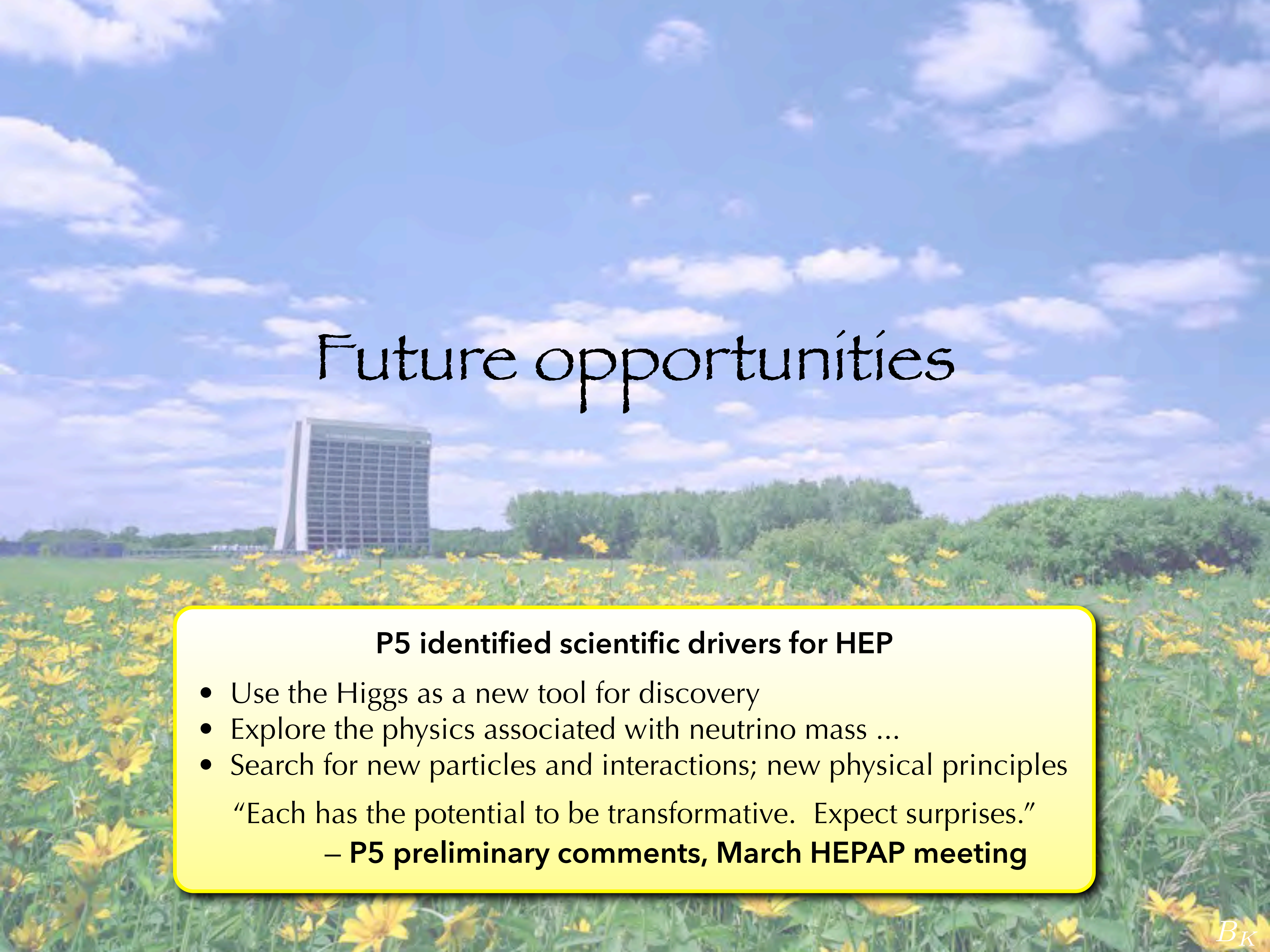


FCNCs mediated by b→s penguins potentially sensitive to new physics

D-MESON MATRIX ELEMENTS

- Important in light of recent experimental evidence for CP-violation in $D\to\pi\pi(KK)$ decays and mixing ➜ *now in the same situation as we've been in for decades with ε'!*
- Particularly difficult aspect is dealing with intermediate 4π, 6π, etc., states in finite box
- Progress with generalization of Lüscher formalism to 3π case [Polejaeva & Rusetsky, Briceno & Davoudi, Hansen & Sharpe], but more ideas and hard work are needed

Future opportunities



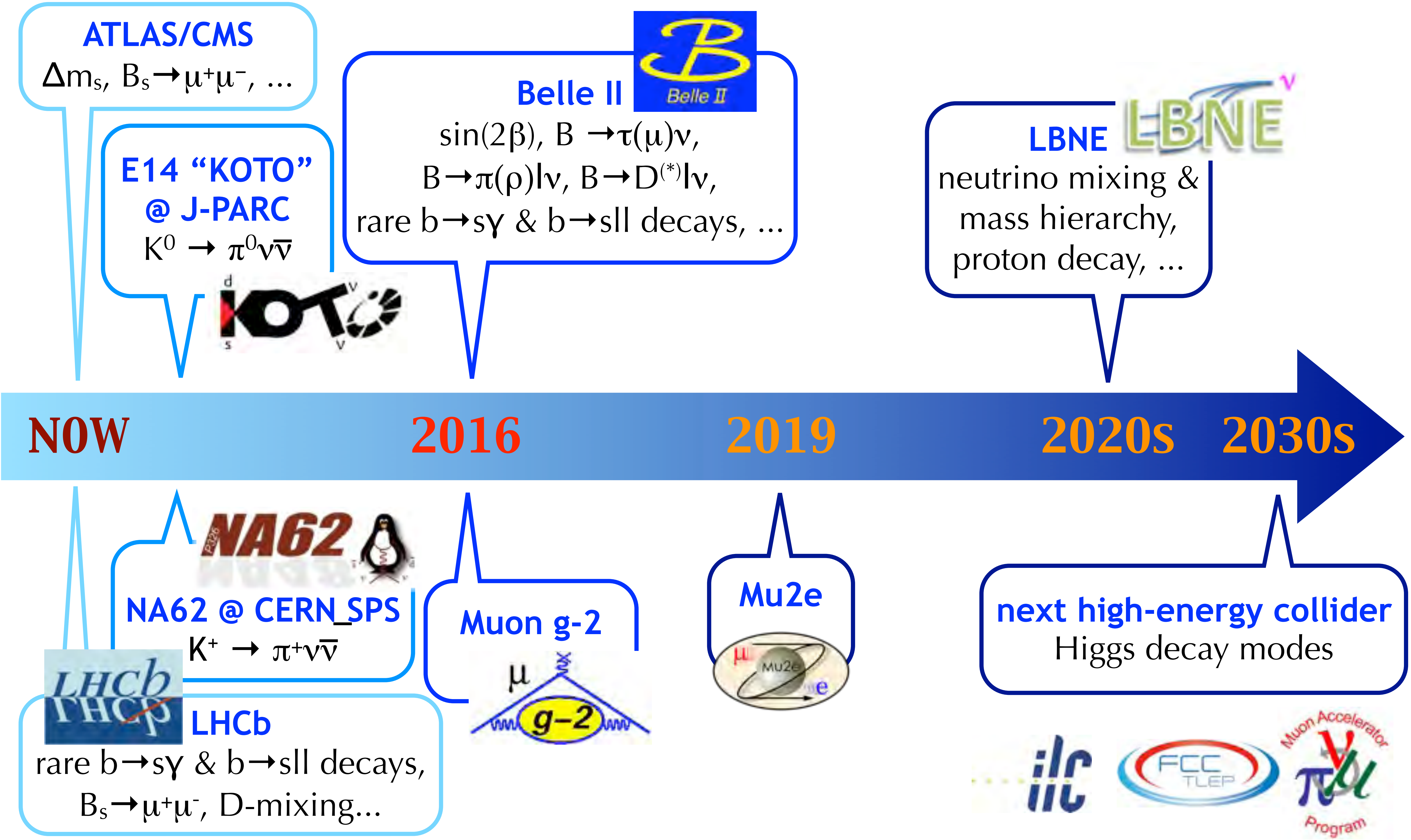
P5 identified scientific drivers for HEP

- Use the Higgs as a new tool for discovery
- Explore the physics associated with neutrino mass ...
- Search for new particles and interactions; new physical principles

> "Each has the potential to be transformative. Expect surprises."
>
> – **P5 preliminary comments, March HEPAP meeting**

Likely experimental horizon

(Not comprehensive;
later experiments more "off shell")



ATLAS/CMS
Δm_s, $B_s \to \mu^+\mu^-$, ...

E14 "KOTO" @ J-PARC
$K^0 \to \pi^0 \nu\bar{\nu}$

Belle II
$\sin(2\beta)$, $B \to \tau(\mu)\nu$,
$B \to \pi(\rho) l\nu$, $B \to D^{(*)} l\nu$,
rare $b \to s\gamma$ & $b \to sll$ decays, ...

LBNE
neutrino mixing & mass hierarchy,
proton decay, ...

NA62 @ CERN SPS
$K^+ \to \pi^+ \nu\bar{\nu}$

LHCb
rare $b \to s\gamma$ & $b \to sll$ decays,
$B_s \to \mu^+\mu^-$, D-mixing...

Muon g-2

Mu2e

next high-energy collider
Higgs decay modes

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(Not comprehensive;
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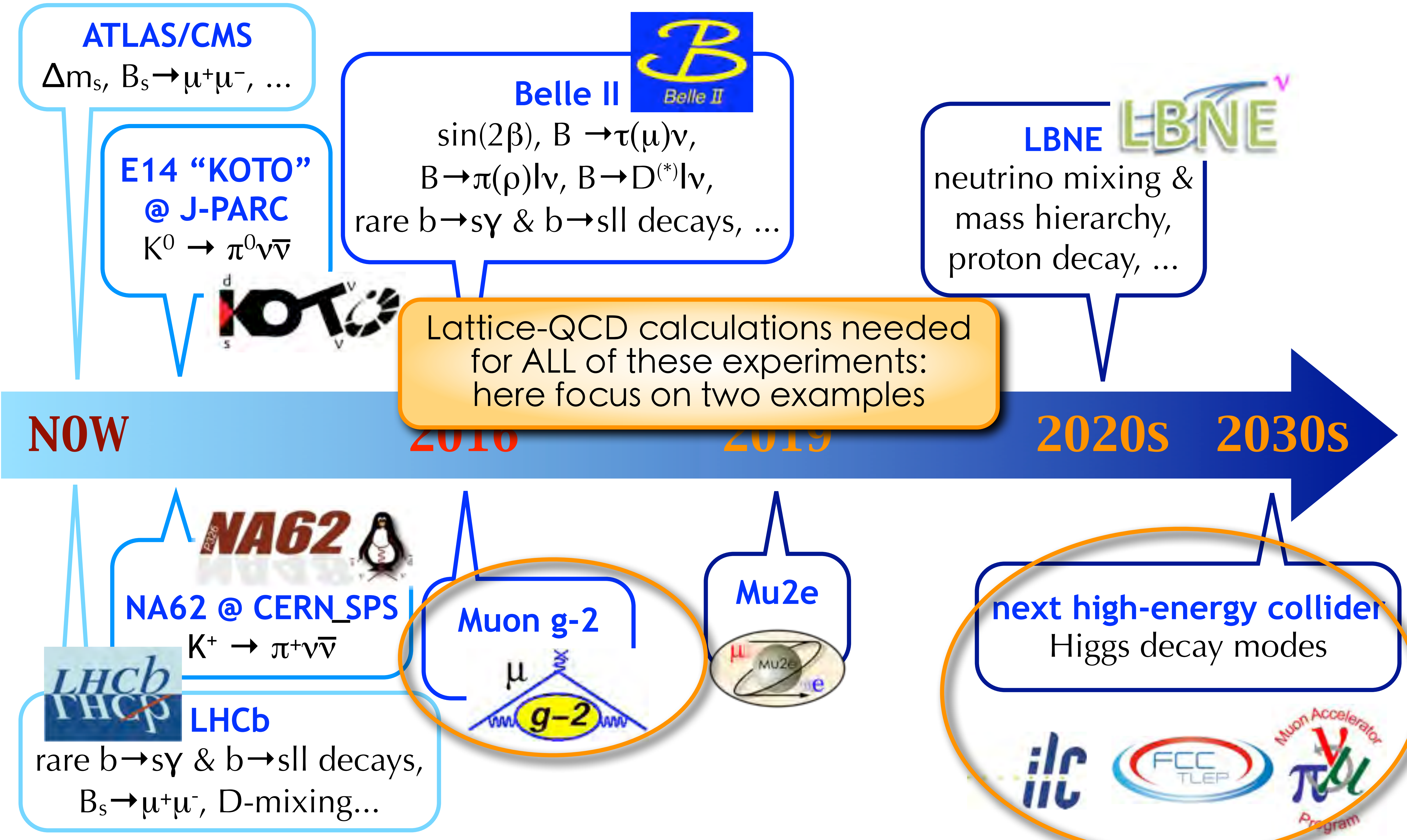

ATLAS/CMS
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$\sin(2\beta)$, $B \to \tau(\mu)\nu$,
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LBNE
neutrino mixing &
mass hierarchy,
proton decay, ...

Lattice-QCD calculations needed
for ALL of these experiments:
here focus on two examples

NA62 @ CERN SPS
$K^+ \to \pi^+ \nu\bar{\nu}$

LHCb
rare $b \to s\gamma$ & $b \to sll$ decays,
$B_s \to \mu^+\mu^-$, D-mixing...

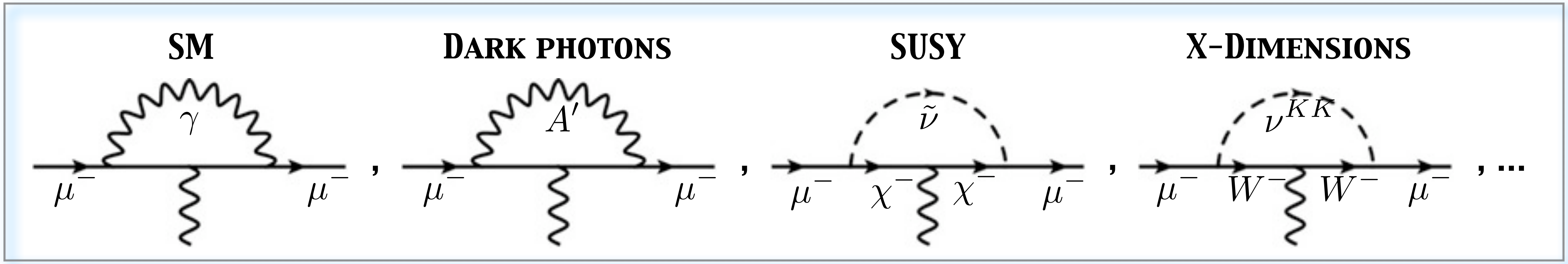
Muon g-2

Mu2e

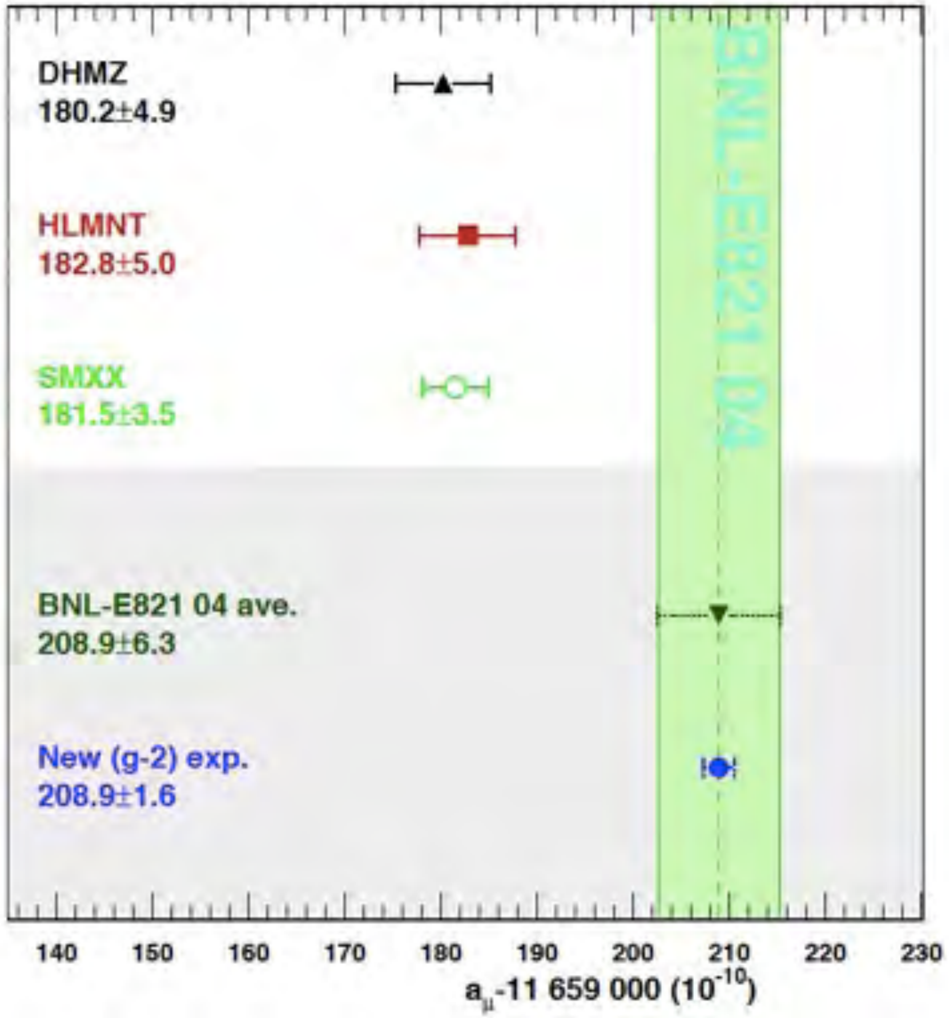
next high-energy collider
Higgs decay modes

Muon anomalous magnetic moment

- Muon g-2 provides precise test of SM and constraints on its extensions

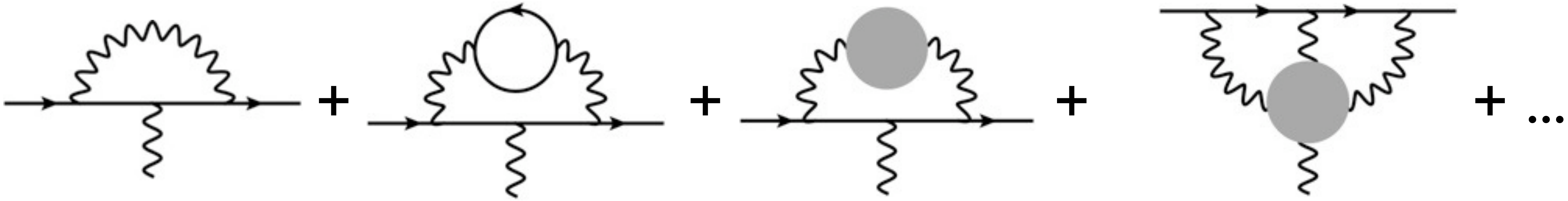


- BNL measurement disagrees with SM by >3σ, and Fermilab Muon g-2 Experiment aims to reduce experimental error by factor of four
- To leverage improved experimental precision, theoretical uncertainty in SM prediction must be shored-up and brought to a comparable precision
- Lattice QCD can provide hadronic contributions to muon g-2 from first principles with controlled uncertainties that are systematically improvable



[Blum *et al.*, arXiv:1311.2198]

Standard-Model contributions to g-2



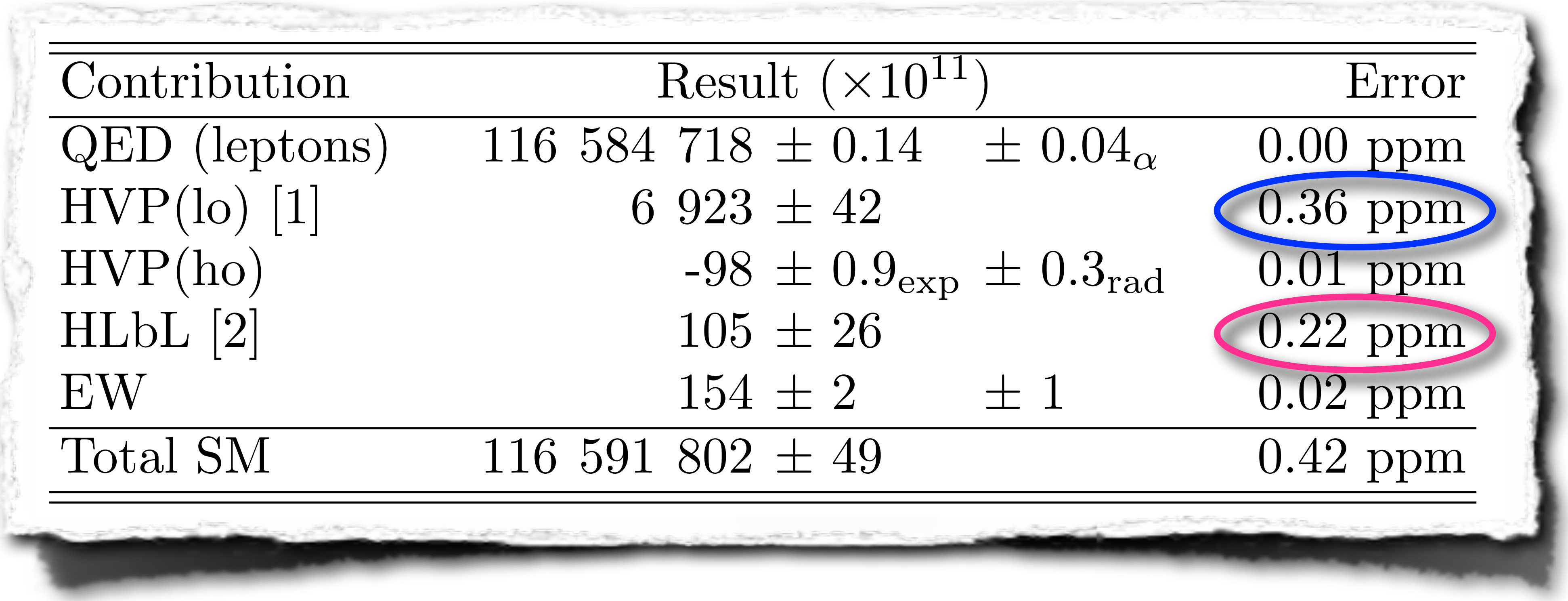
Hadronic vacuum polarization (HVP):

Hadronic light-by-light (HLbL):

QED (4 loops) & EW (2 loops)

from experimental result for $e^+e^- \rightarrow$ hadrons plus dispersion relation

estimated from models such as large N_c, χPT, vector meson dominance, etc...



Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	116 584 718 $\pm$ 0.14	$\pm$ 0.04_α	0.00 ppm
HVP(lo) [1]	6 923 $\pm$ 42		0.36 ppm
HVP(ho)	-98 $\pm$ $0.9_{\rm exp}$	$\pm$ $0.3_{\rm rad}$	0.01 ppm
HLbL [2]	105 $\pm$ 26		0.22 ppm
EW	154 $\pm$ 2	$\pm$ 1	0.02 ppm
Total SM	116 591 802 $\pm$ 49		0.42 ppm

[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

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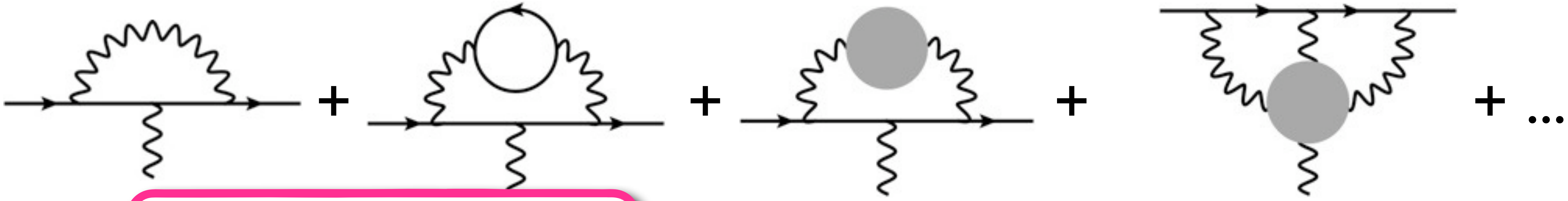
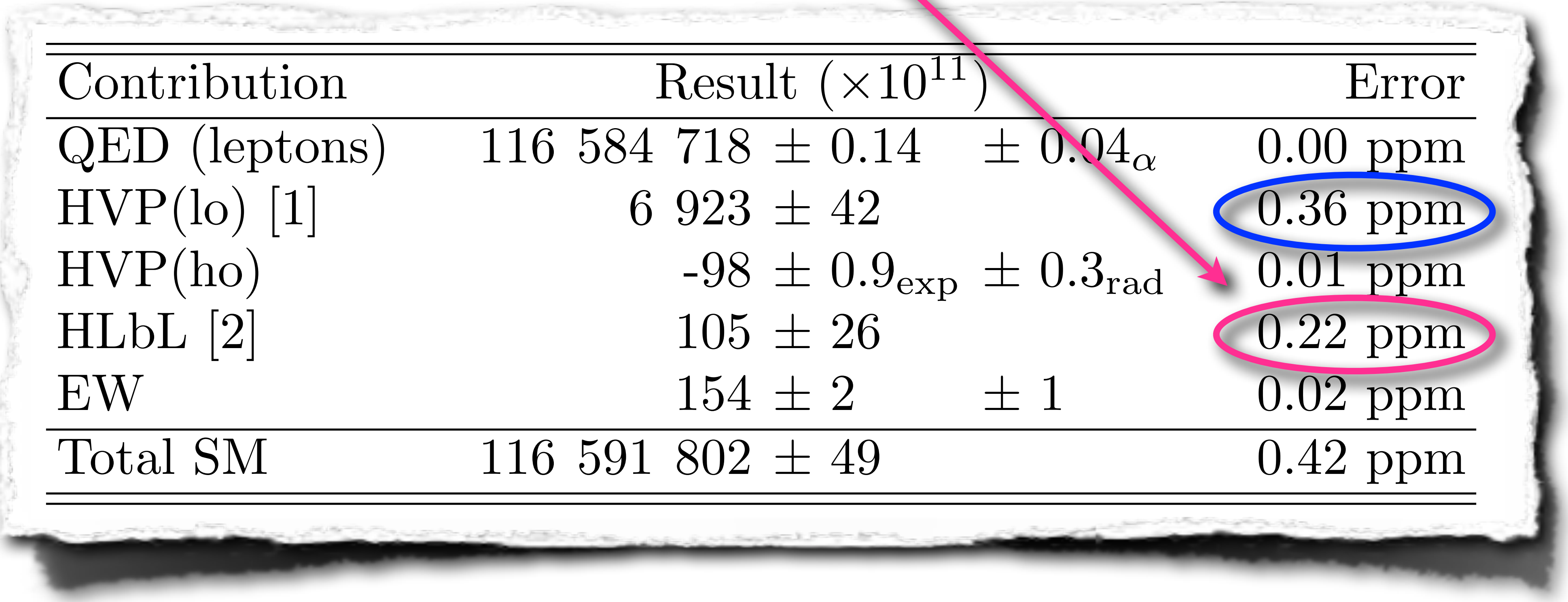


QE

HLbL error more subjective than for other contributions, and somewhat controversial

from experimental result for $e^+e^- \rightarrow$ hadrons plus dispersion relation

estimated from models such as large N_c, χPT, vector meson dominance, etc...



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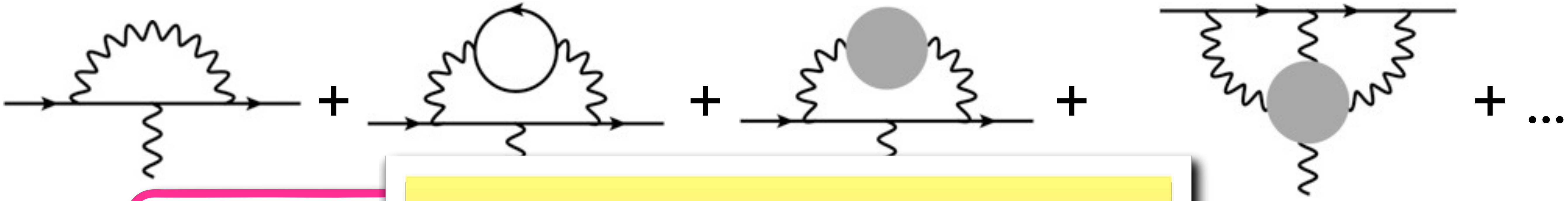

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for other cont
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Precision goals for hadronic contributions set by Muon g-2 Experiment are:

$\delta(a_\mu^{HVP})$~0.2%, $\delta(a_\mu^{HLbL})$~15%

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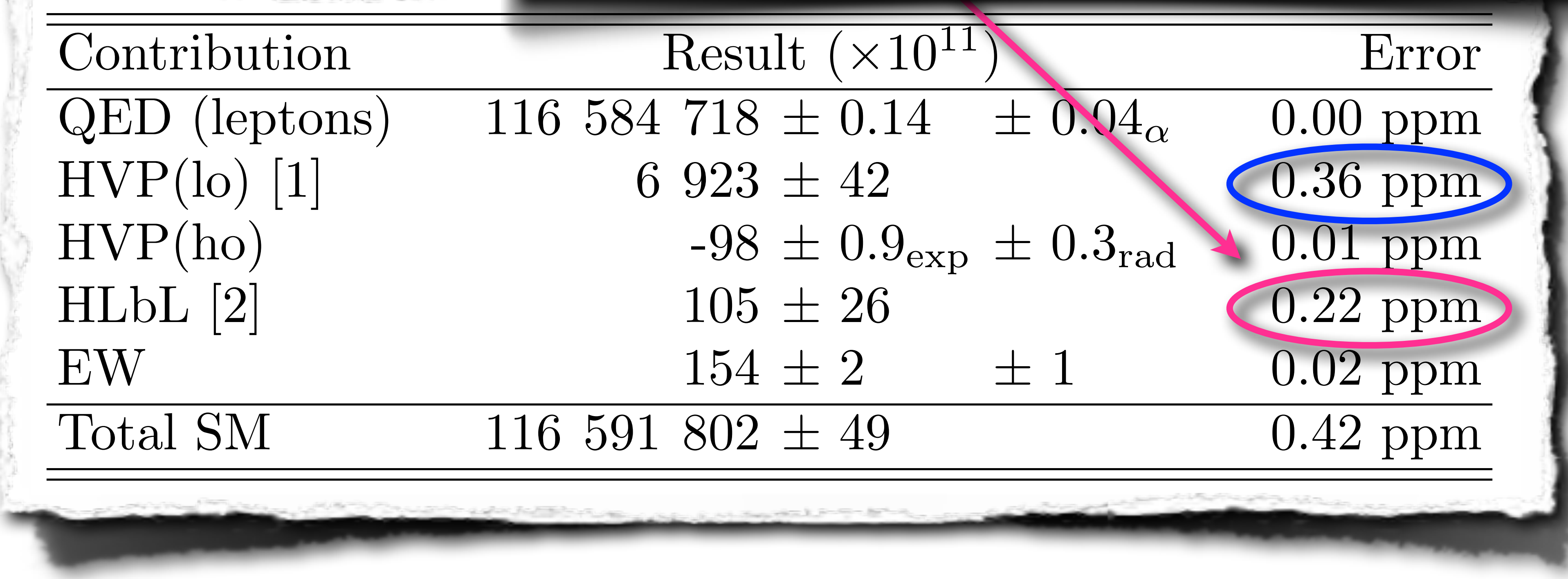

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Standard lattice method for a_μ^{HVP}

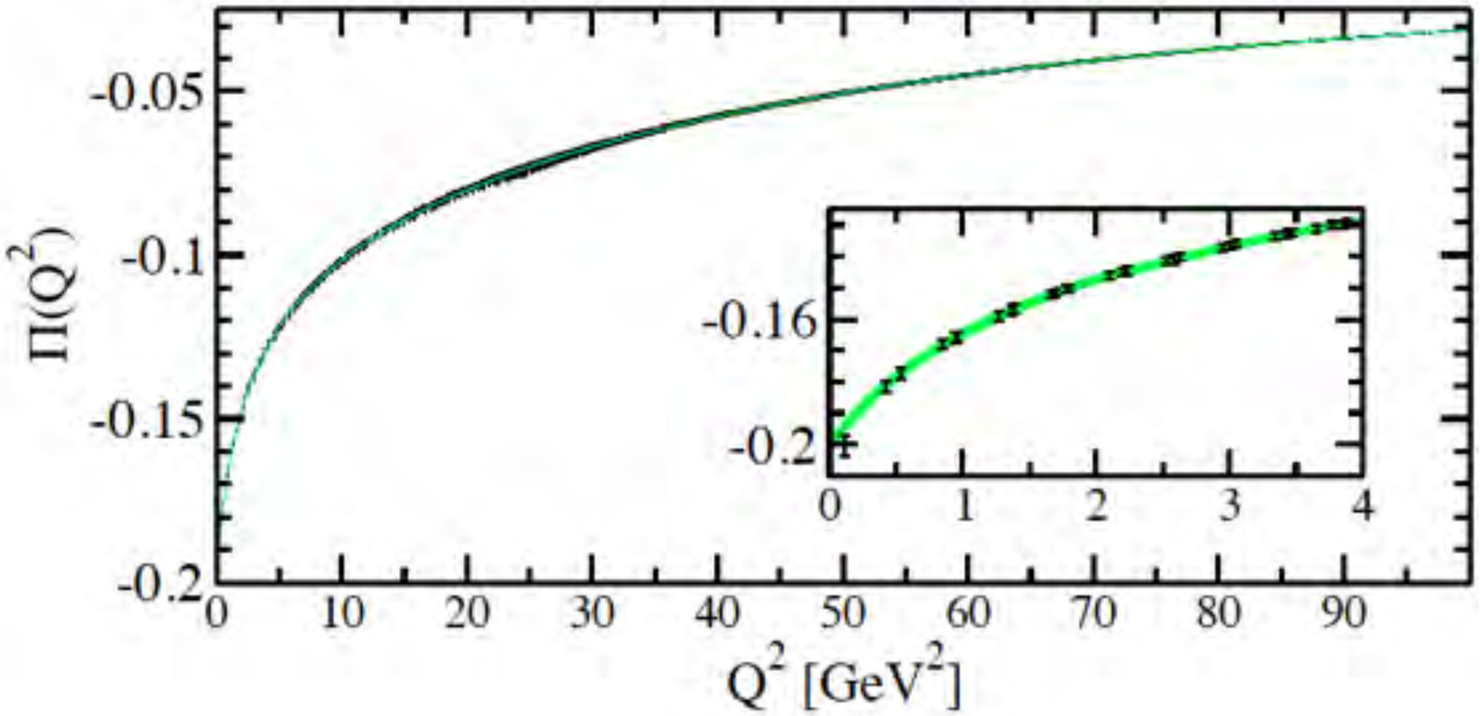
[Blum, Phys.Rev.Lett. 91 (2003) 052001]

- Calculate a_μ^{HVP} directly from Euclidean space vacuum polarization function
- $\Pi(Q^2)$ a simple correlation function of two electromagnetic currents
- In Euclidean space, $\Pi(Q^2)$ has a smooth Q^2 dependence with no resonance structure

$$a_\mu^{\mathrm{HVP(LO)}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2) \left[\Pi(Q^2) - \Pi(0)\right]$$

$$i\Pi_{\mu\nu}(q^2) =$$

[plot from Dru Renner]



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- Integrand $f(Q^2)[\Pi(Q^2)\text{-}\Pi(0)]$, however, peaks around $Q^2 \approx (m_\mu/2)^2$, where lattice data is sparse and noisy ➜ need precise determination of $\Pi(Q^2)$ in this region to obtain precise result for a_μ^{HVP}

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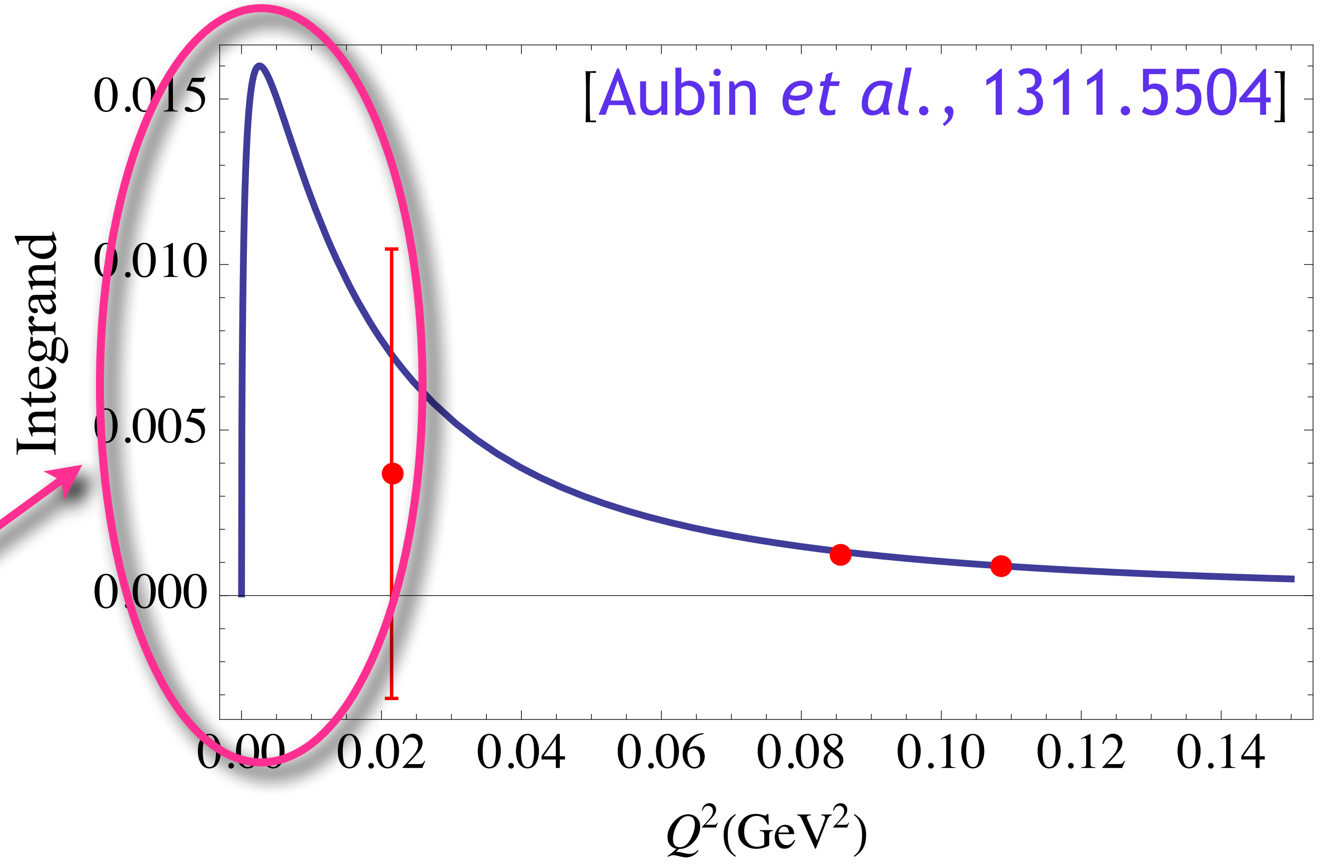

[Aubin *et al.*, 1311.5504]

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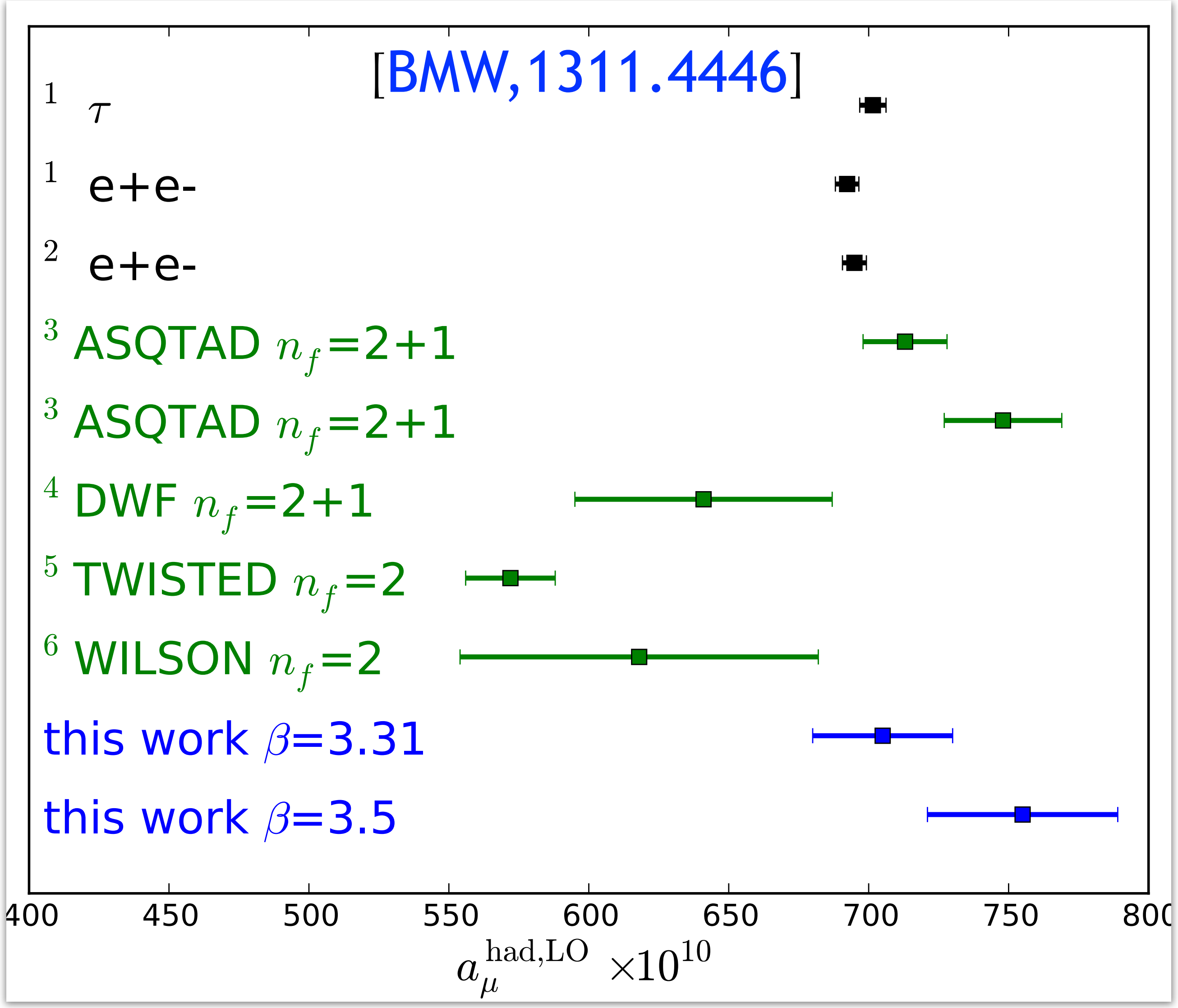
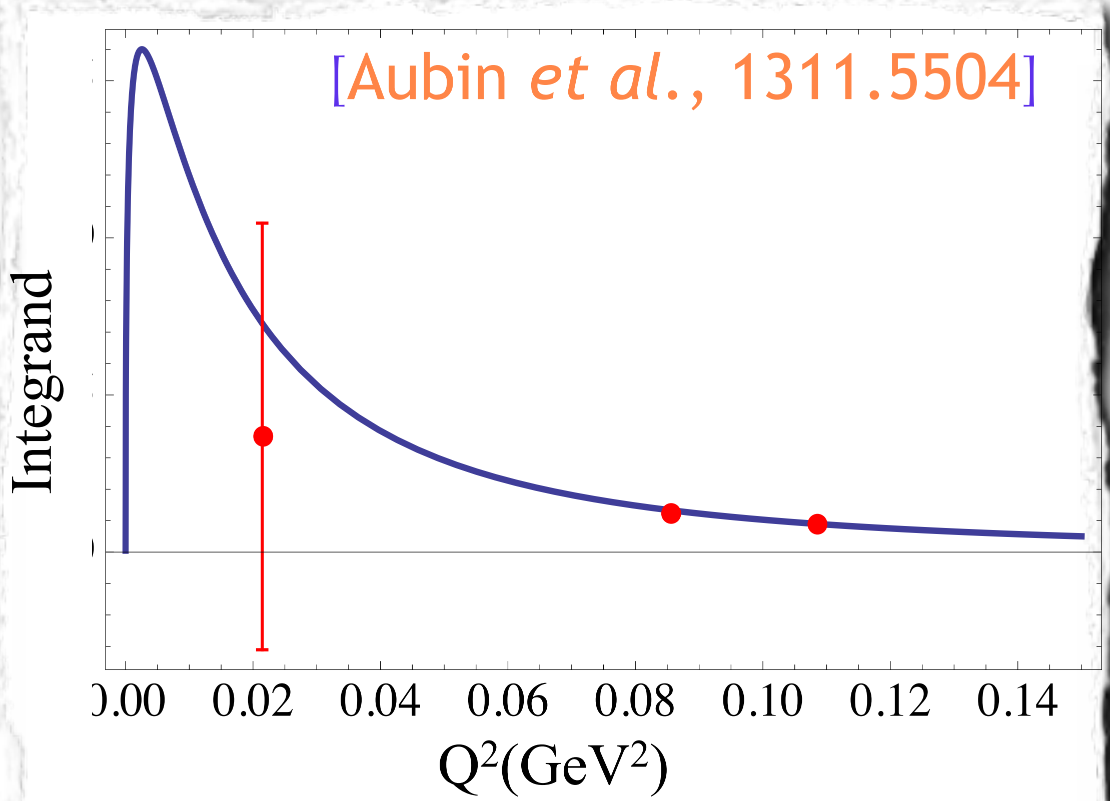
Several independent ongoing efforts using this approach



Present errors in the ~5–10% percent range (and mostly neglect quark-disconnected contributions)

(Select) Recent progress on a_μ^{HVP}

- **Twisted boundary conditions** for fermion fields to access momentum values below the minimum discrete lattice momentum (2π/L) [spatial lattice volume=L^3] [Della Morte *et al.*, JHEP 1203 (2012) 055; Aubin *et al.*, PRD88 (2013) 7, 074505]
- **All-mode averaging** to reduce statistical errors [Blum *et al*, PRD 88 (2013) 094503]



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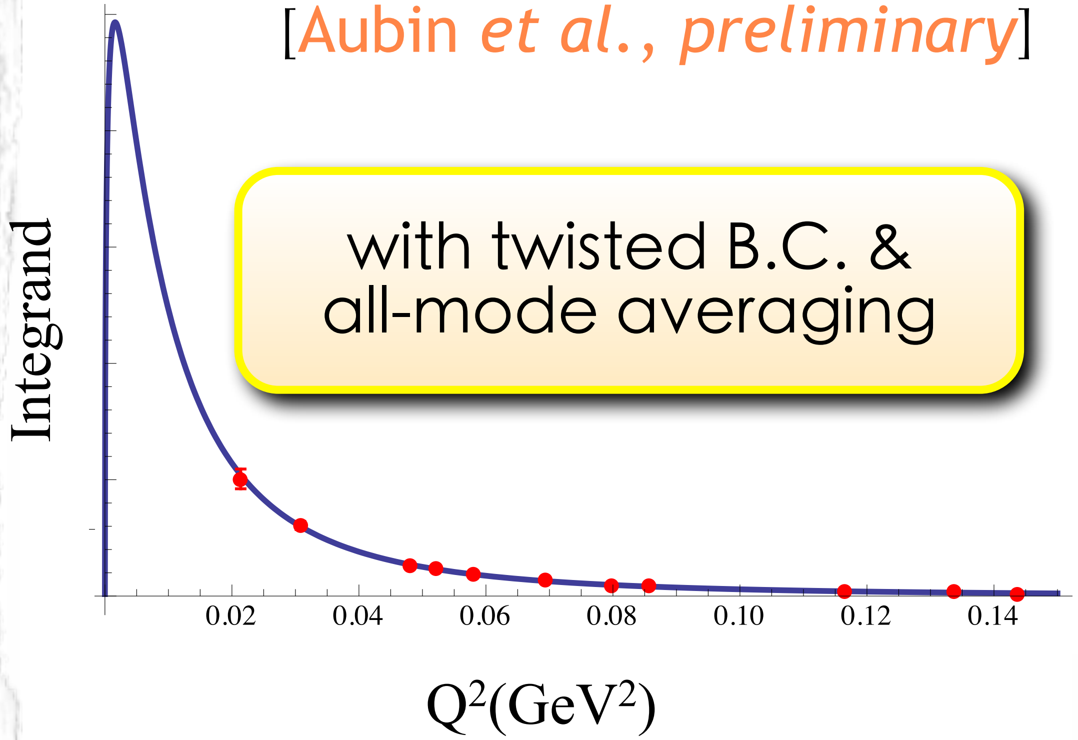

[Aubin *et al., preliminary*]

with twisted B.C. &
all-mode averaging

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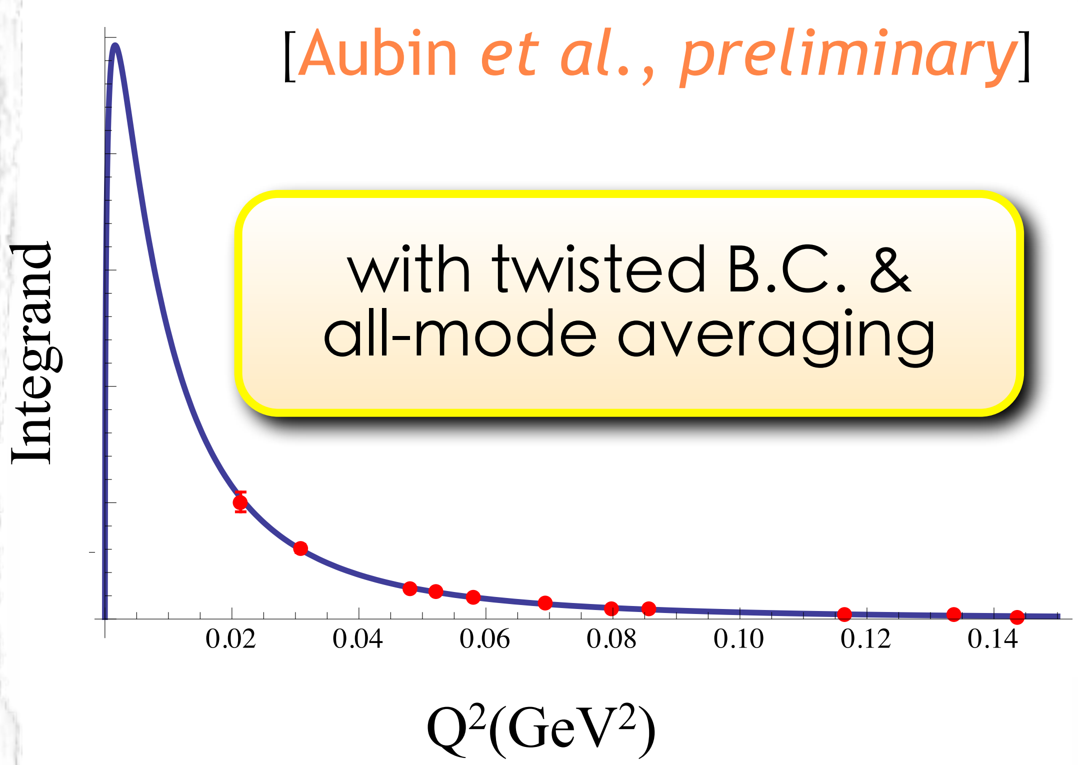
- **All-mode averaging** to reduce statistical errors [Blum *et al*, PRD 88 (2013) 094503]
- **New HPQCD method sidesteps $q^2 \to 0$ extrapolation** by expressing a_μ^{HVP} in terms of derivatives of vacuum polarization function $\Pi(q^2)$ at $q^2=0$ [Chakraborty *et al.*, 1403.1778]
    - Illustrate with strange and charm contributions and obtain a_μ^s to ~1%
    - Correlator noisier for light quarks, but estimate that similar precision can be obtained for $a_\mu^{u,d}$ with 10× larger gauge-field ensembles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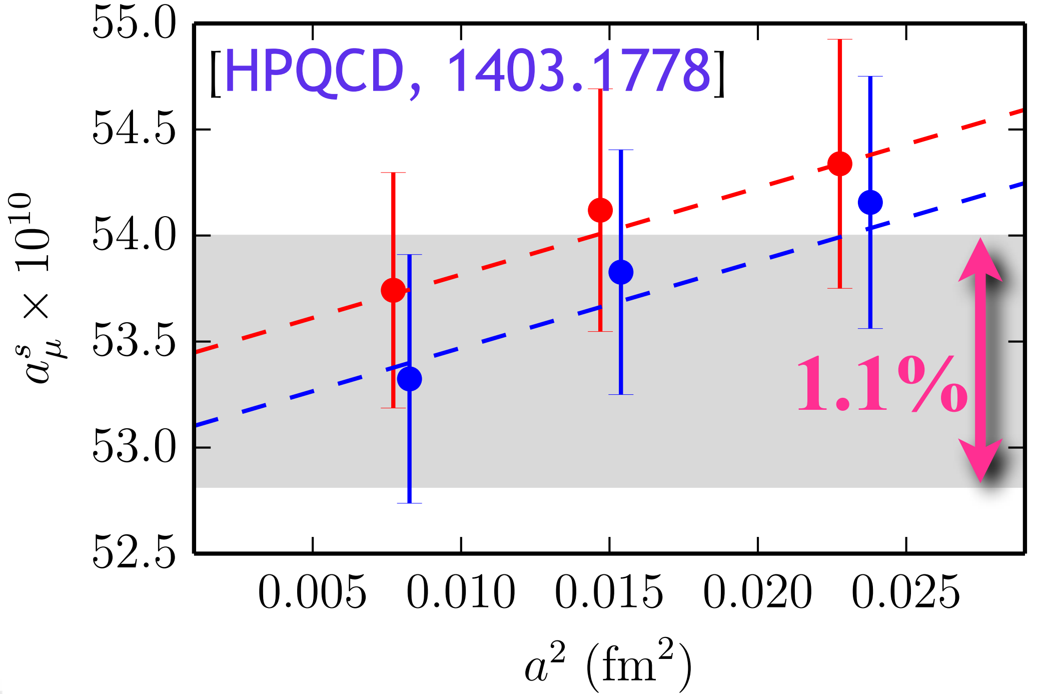
[HPQCD, 1403.1778]

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- **All-mode av**
[Blum *et al*

- **New HPQC**
extrapolati
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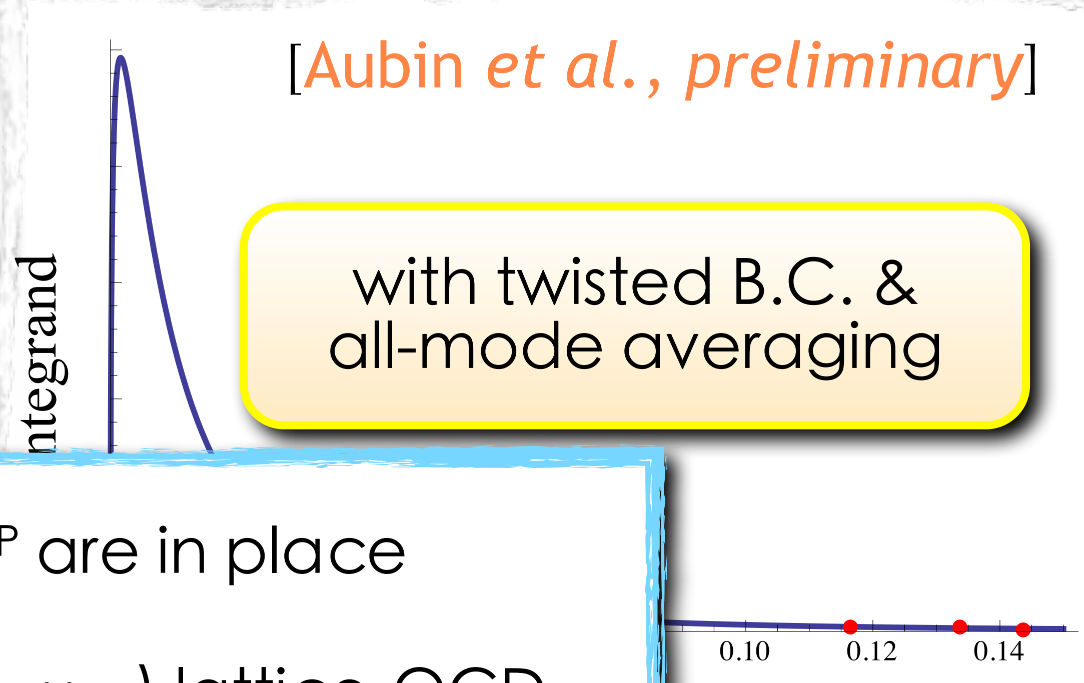
Theoretical methods for a_μ^{HVP} are in place

➜

should enable a few percent (or better) lattice-QCD calculation on the timescale of the coming experiment



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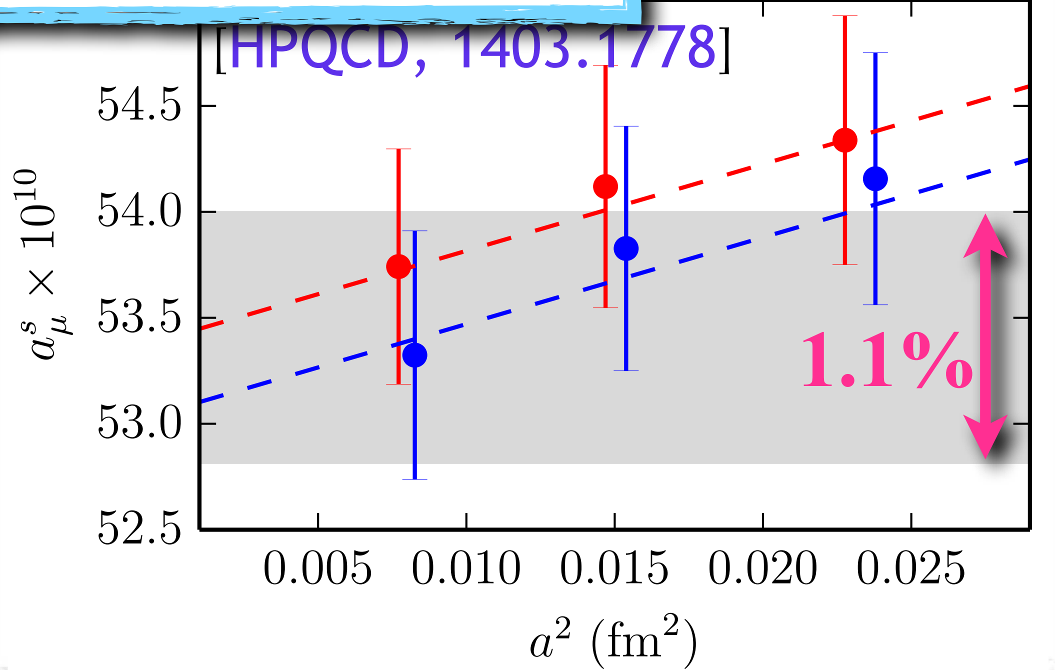
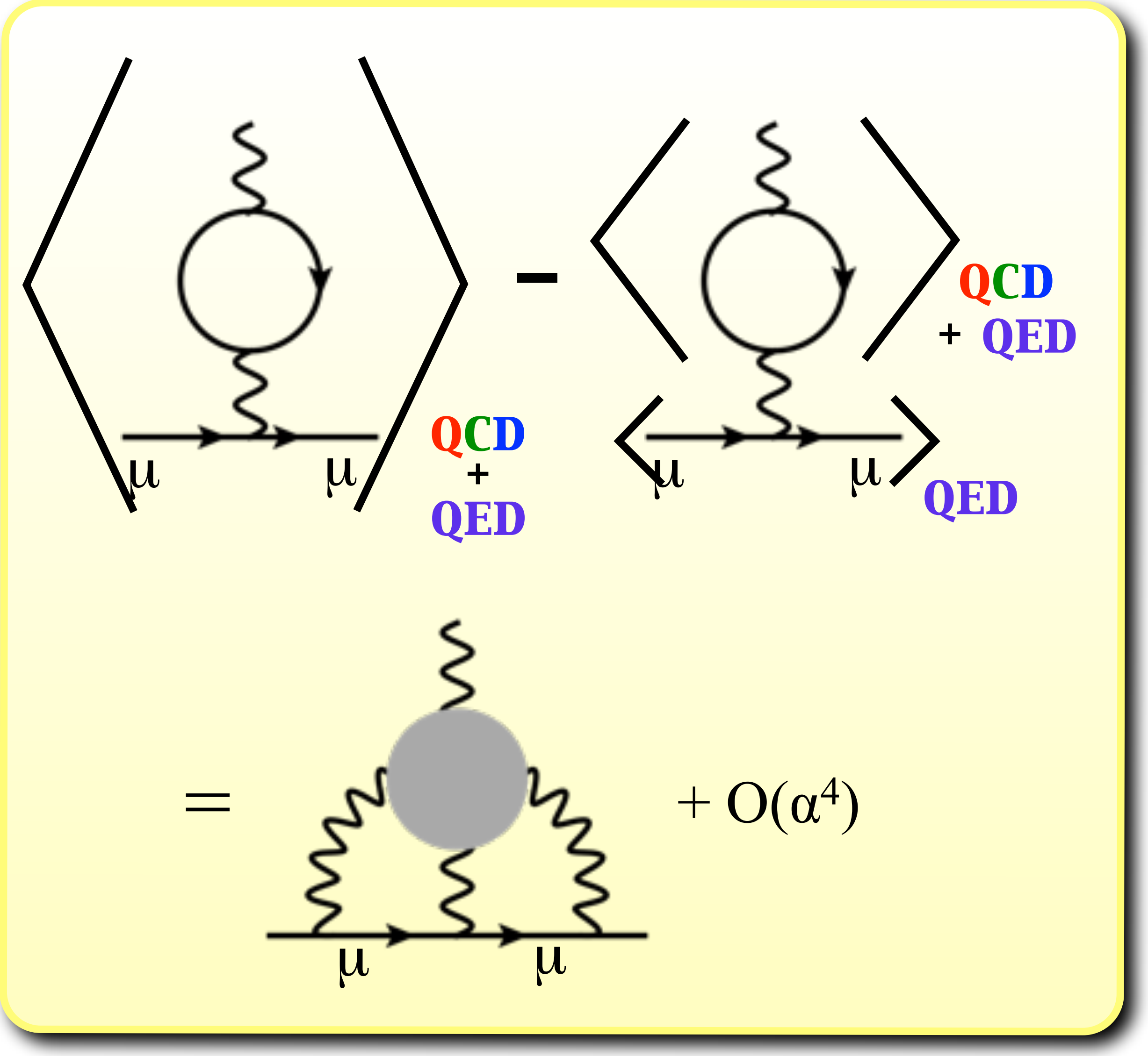


[HPQCD, 1403.1778]

Lattice calculation of a_μ^{HLbL}

[Hayakawa et al., PoS LAT2005 (2006) 353]

- Analogous approach to HVP calculation inserting correlation function of 4 EM currents into 2-loop QED integral (prohibitively?) complicated and costly
- **Promising approach to compute full hadronic amplitude nonperturbatively**
  - Include photon field along with gluon field in gauge link, so simulation & analysis follow conventional lattice-QCD calculation
  - Using all-mode-averaging, Blum *et al.* obtain statistically-significant signal emerging in the ballpark of model estimates [PoS LATTICE2012 (2012) 022]
  - Still much to do for realistic result with controlled uncertainty: larger spatial volumes, extrapolation to physical pion mass and continuum, momentum extrapolation $Q^2 \rightarrow 0$, quark-disconnected contributions, ...



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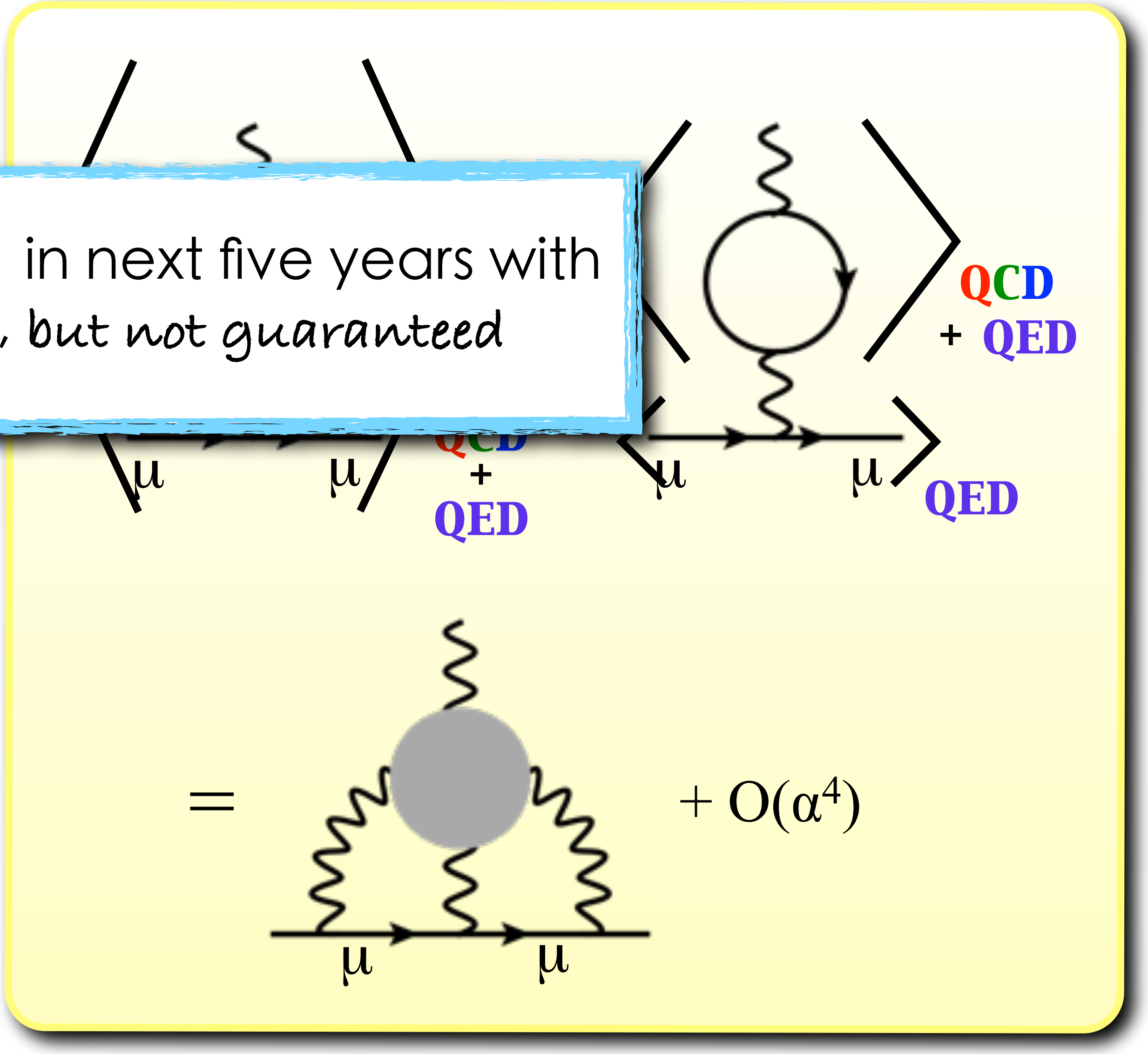
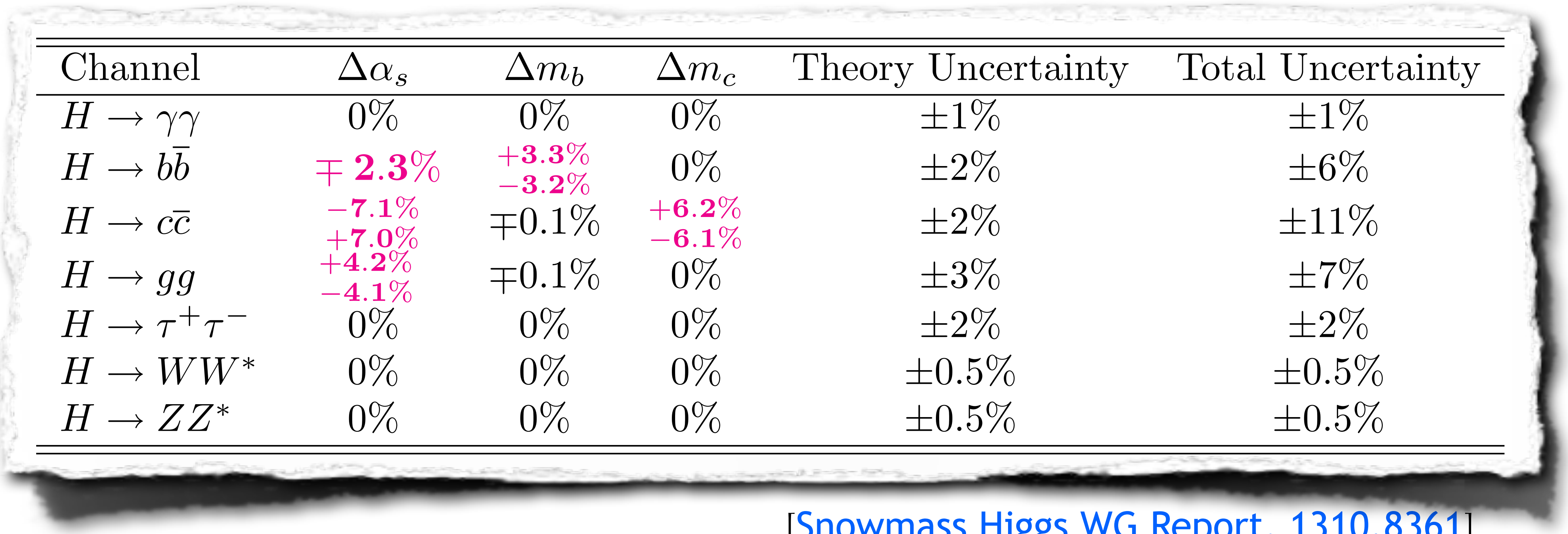


Lattice calculation of a_μ^{HLbL} in next five years with ~15% precision is possible, but not guaranteed

Precision Higgs measurements

- Now that the Higgs mass is known, can predict all Higgs-boson couplings and properties within the Standard Model and look for deviations
- **Future high-energy/luminosity colliders will measure Higgs partial widths to sub-percent precision**, but commensurate theoretical uncertainties on Standard-Model predictions needed to fully exploit measurements
- Parametric errors from m_c, m_b, and α_s are largest sources of uncertainty in SM width predictions for the dominant Higgs decay mode H→bb, many other Higgs decay channels, and the Higgs total width [LHC Higgs X-Section WG, EPJ C71 (2011) 1753]



Channel	$\Delta\alpha_s$	Δm_b	Δm_c	Theory Uncertainty	Total Uncertainty
$H \to \gamma\gamma$	0%	0%	0%	±1%	±1%
$H \to b\bar{b}$	$\mp$ **2.3%**	$^{+3.3\%}_{-3.2\%}$	0%	±2%	±6%
$H \to c\bar{c}$	$^{-7.1\%}_{+7.0\%}$	∓0.1%	$^{+6.2\%}_{-6.1\%}$	±2%	±11%
$H \to gg$	$^{+4.2\%}_{-4.1\%}$	∓0.1%	0%	±3%	±7%
$H \to \tau^+\tau^-$	0%	0%	0%	±2%	±2%
$H \to WW^*$	0%	0%	0%	±0.5%	±0.5%
$H \to ZZ^*$	0%	0%	0%	±0.5%	±0.5%

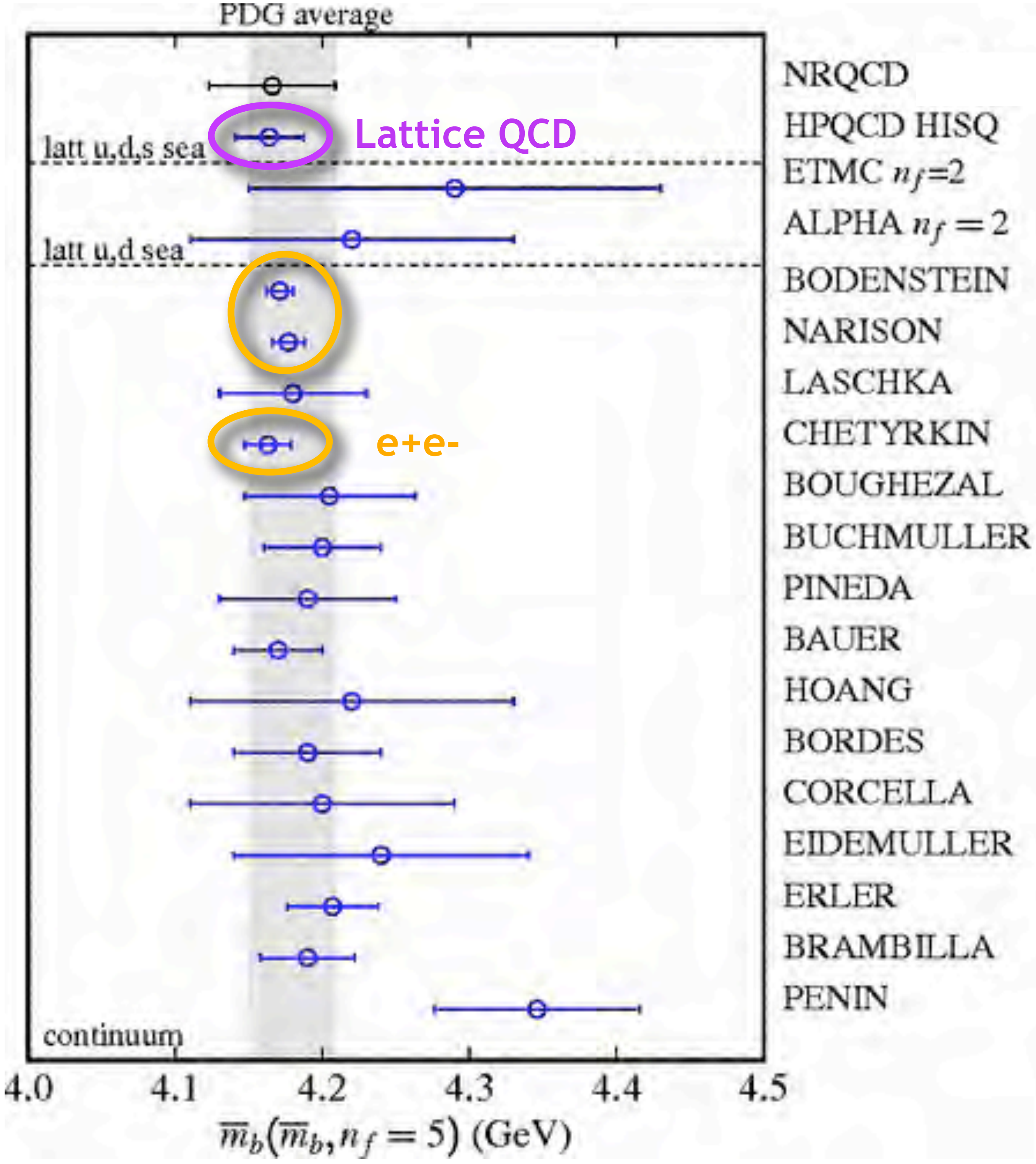
[Snowmass Higgs WG Report, 1310.8361]

Heavy-quark masses from lattice QCD

[McNeile et al., PRD82 (2010) 034512]

- Most precise m_c and m_b obtained by fitting moments of correlation functions of the quarks' electromagnetic current to $O(\alpha_S^3)$ perturbative expressions

- Moments can be obtained from experimental e+e- annihilation data, and also computed numerically with lattice-QCD simulations
  - **Lattice moments have negligible statistical uncertainties**, so cleaner than e+e- data
  - Can vary lattice quark-mass between m_c and m_b to control and estimate errors
- **HPQCD obtains m_c and m_b to about a half percent precision** and finds good agreement with non-lattice determinations
  - m_c will only improve modestly without higher-order PT calculation, but **m_b will improve significantly with simulations using finer lattice spacings**

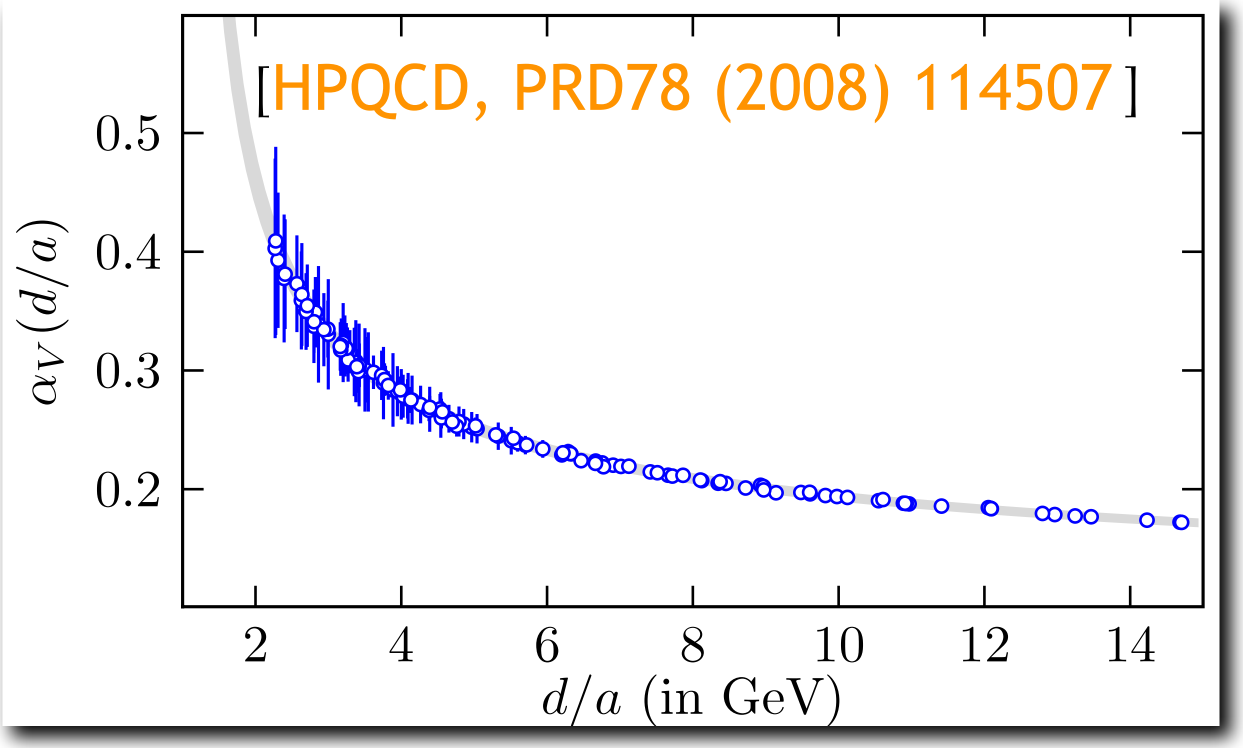


Lattice QCD

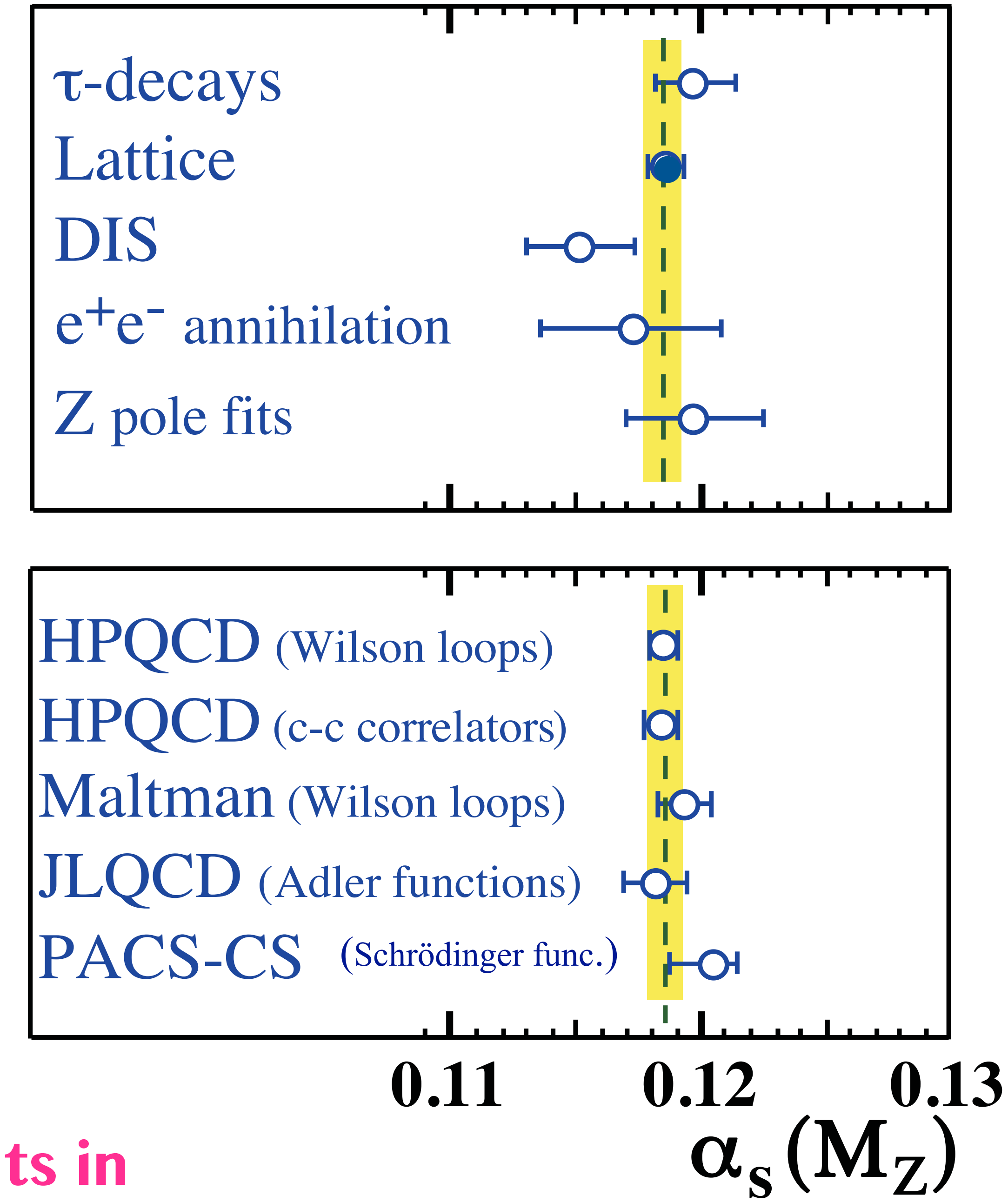
e+e-

Strong coupling from lattice QCD

- ✦ **There are several good independent lattice methods available to obtain α_s**
  - ❖ Most precise result from fitting NNNLO QCD β-function to 22 short-distance lattice quantities built from Wilson loops (*current correlators give α_s with similar precision*)


[HPQCD, PRD78 (2008) 114507]

[PDG (2013)]



  - ❖ Different approaches consistent, and each is more precise than from non-lattice methods

- ✦ In the coming years, **anticipate both improvements in individual calculations and increased corroboration from new ones**

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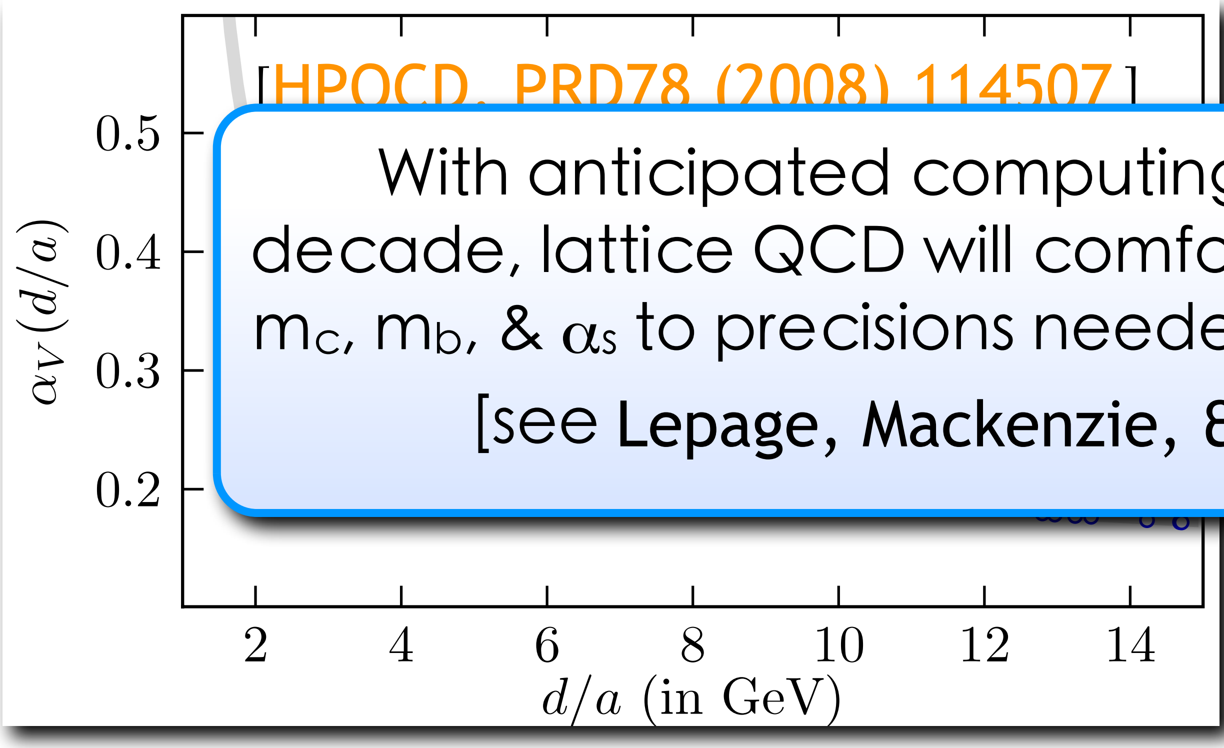

With anticipated computing resources in the next decade, lattice QCD will comfortably be able to achieve m_c, m_b, & α_s to precisions needed by a high-luminosity ILC [see **Lepage, Mackenzie, & Peskin, 1404.0319**]

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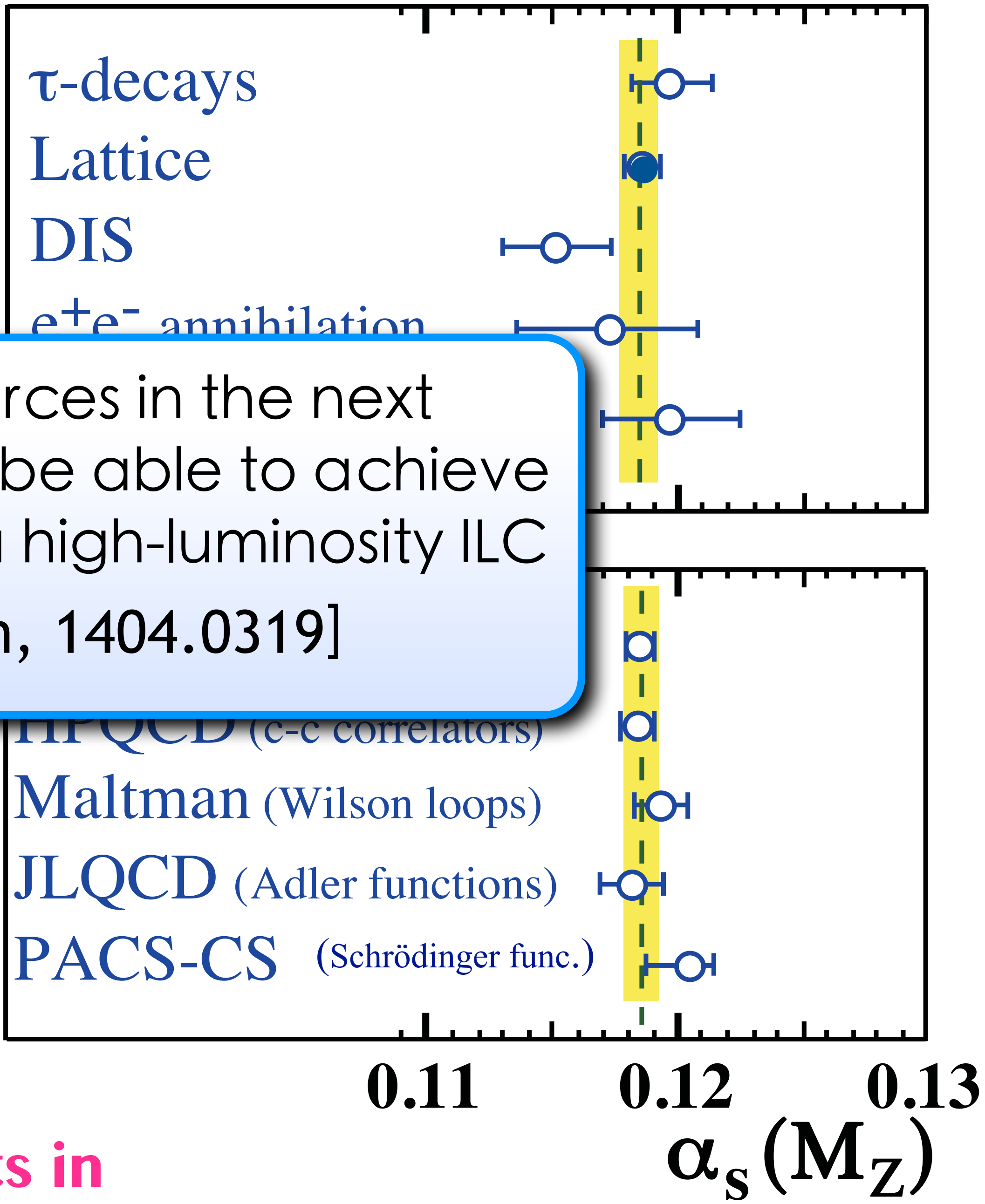
[PDG (2013)]



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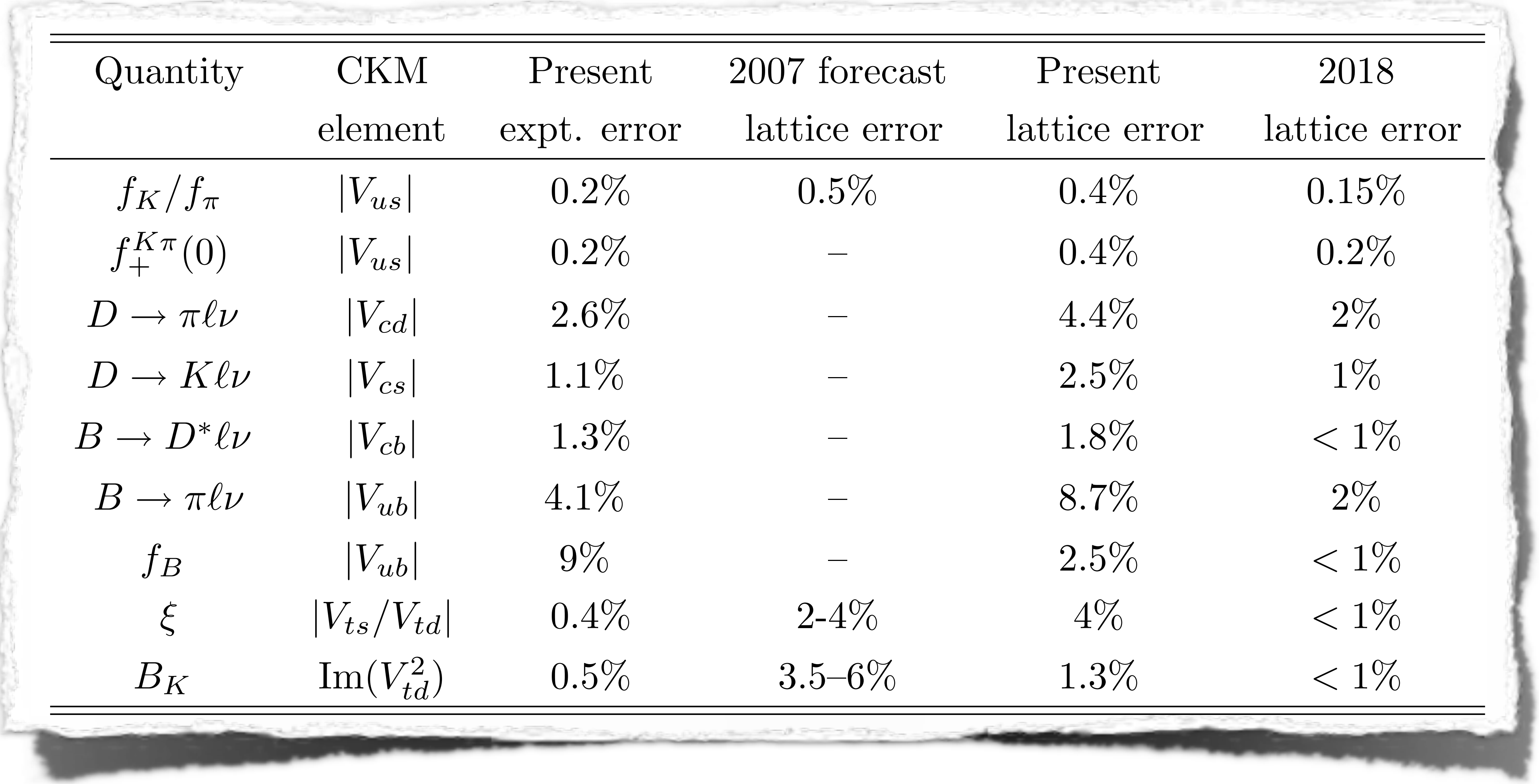
Summary and outlook

> “[An] area of striking progress has been lattice gauge theory. … It is now possible to compute the spectrum of hadrons with high accuracy, and lattice computations have been crucial in the measurement of the properties of heavy quarks. Continuing improvements in calculational methods are anticipated in coming years.”
>
> – **Snowmass Executive Summary**

Forecasts and plans

- Petascale computing resources will enable simulations with lighter pions, finer lattice spacings, and larger volumes, thereby helping most sources of uncertainty

[Snowmass Quark-flavor WG report, 1311.1076]



Quantity	CKM element	Present expt. error	2007 forecast lattice error	Present lattice error	2018 lattice error
f_K/f_π	$\lvert V_{us}\rvert$	0.2%	0.5%	0.4%	0.15%
$f_+^{K\pi}(0)$	$\lvert V_{us}\rvert$	0.2%	–	0.4%	0.2%
$D \to \pi\ell\nu$	$\lvert V_{cd}\rvert$	2.6%	–	4.4%	2%
$D \to K\ell\nu$	$\lvert V_{cs}\rvert$	1.1%	–	2.5%	1%
$B \to D^*\ell\nu$	$\lvert V_{cb}\rvert$	1.3%	–	1.8%	$< 1\%$
$B \to \pi\ell\nu$	$\lvert V_{ub}\rvert$	4.1%	–	8.7%	2%
f_B	$\lvert V_{ub}\rvert$	9%	–	2.5%	$< 1\%$
ξ	$\lvert V_{ts}/V_{td}\rvert$	0.4%	2-4%	4%	$< 1\%$
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$

Forecasts and plans

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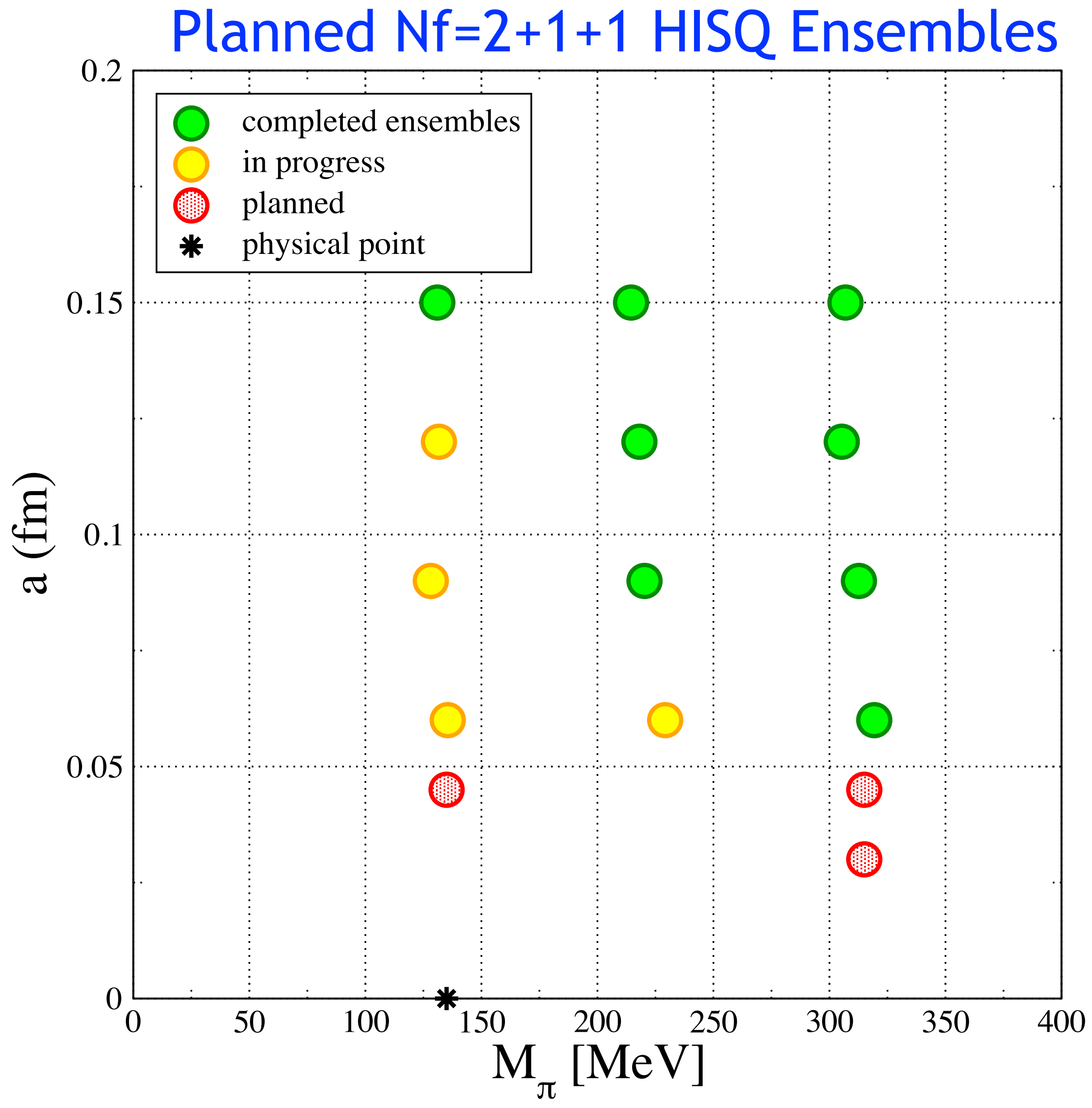
- The following improvements will become widespread over the next five years

  (1) Simulations with physical-mass pions

  (2) Systematic inclusion of isospin-breaking and EM

  (3) Dynamical charm quarks



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  - K→ππ decays (ΔI=1/2 rule and ϵ'/ϵ)
  - Hadronic contributions to muon g-2
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Improved algorithms and analysis methods being pursued, but difficult to predict

Outlook

- Success of future experimental high-energy physics program hinges on reliable theoretical predictions on same time scale as experiments and with commensurate uncertainties
- Lattice-QCD calculations are needed throughout the HEP program
  - For precision measurements of rare kaon and B decays, muon g-2, neutrino oscillation parameters, Higgs properties, ...
  - For searches for μ→e conversion, dark matter, proton decay, nucleon EDMs, ...
- Lattice community expanding program to meet needs of current and upcoming experiments:
  - **Increasing precision** in parameters of QCD Lagrangian and simplest quark flavor-changing and nucleon matrix elements
  - **Addressing new challenges** such as rare decays, muon g-2, long-distance amplitudes, and multi-hadron final states

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    - ❖ I o
      c
    - ❖ F s, ...

Continued support for lattice-QCD hardware and software is essential to achieve scientific goals and fully capitalize on enormous investments in the HEP (and NP) experimental programs

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    - ❖ **Increasing precision** in parameters of QCD Lagrangian and simplest quark flavor-changing and nucleon matrix elements
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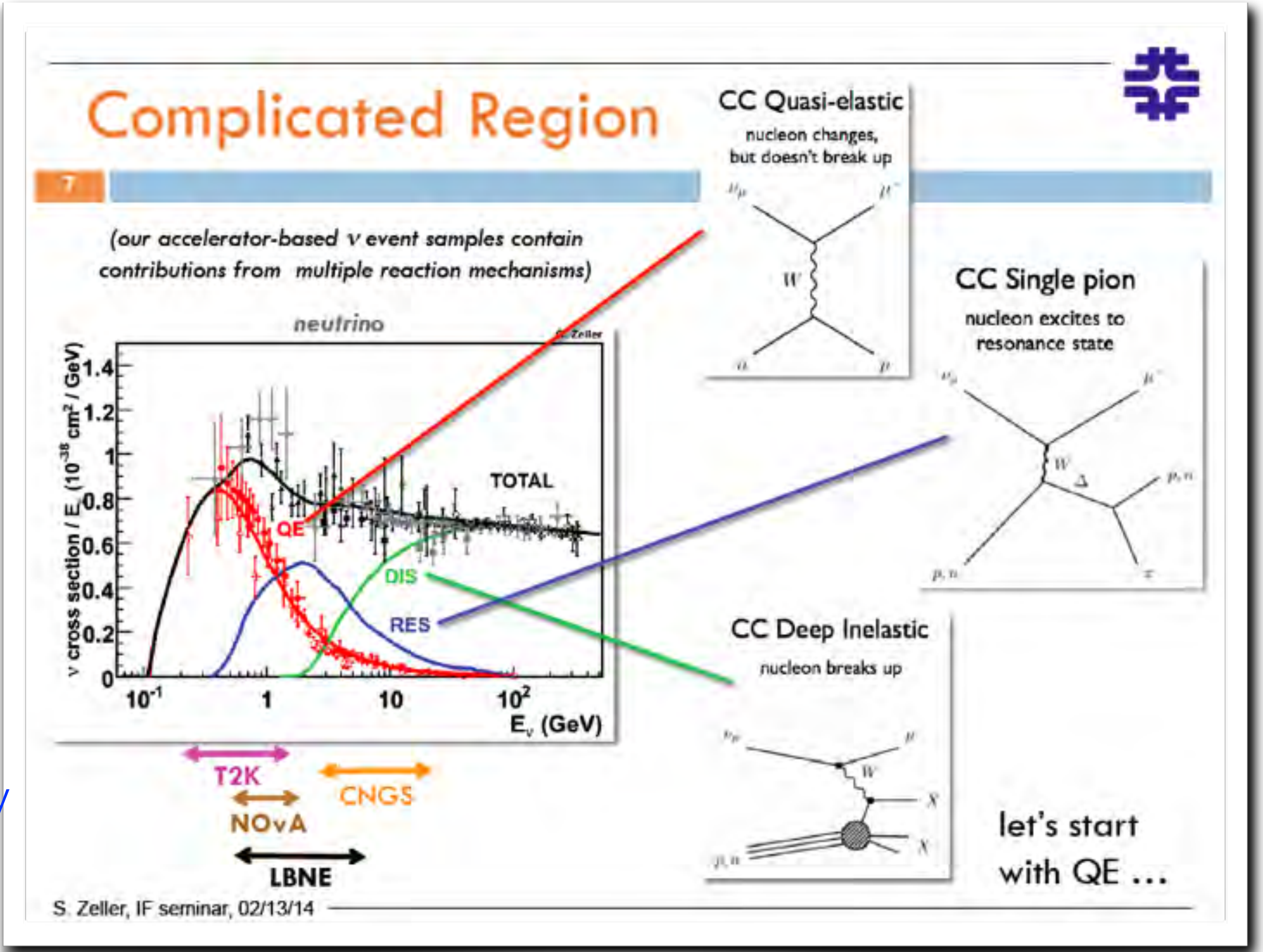
References (with hyperlinks)

- 2013 USQCD white paper “Lattice QCD at the Intensity Frontier”
- Snowmass reports:
  - “Charged Leptons”
  - “Higgs Working Group Report of the Snomass 2013 Community Planning Study”
  - “Lattice field theory for the energy and intensity frontiers: Scientific goals and computing needs”
  - “Neutrinos”
  - “Report of the Snowmass 2013 Energy Frontier QCD Working Group”
  - “Report of the Quark Flavor Physics Working Group”
- 2013 Argonne Intensity Frontier Workshop
- 2012 Project X workshop report “Project X: Physics Opportunities”
- 2011 Rockville workshop report “Fundamental Physics at the Intensity Frontier”

Extras

Neutrino physics

- Accelerator-based ν experiments in low-energy regime complicated by nuclear environment
- Largest contribution to signal sample in most oscillation experiments from charged-current quasielastic (CCQE) scattering on bound neutron
- Measurement of ν oscillation parameters and possible discovery of new ν states limited by understanding of CCQE cross section



Complicated Region

7

(our accelerator-based ν event samples contain contributions from multiple reaction mechanisms)

CC Quasi-elastic

nucleon changes, but doesn't break up

CC Single pion

nucleon excites to resonance state

CC Deep Inelastic

nucleon breaks up

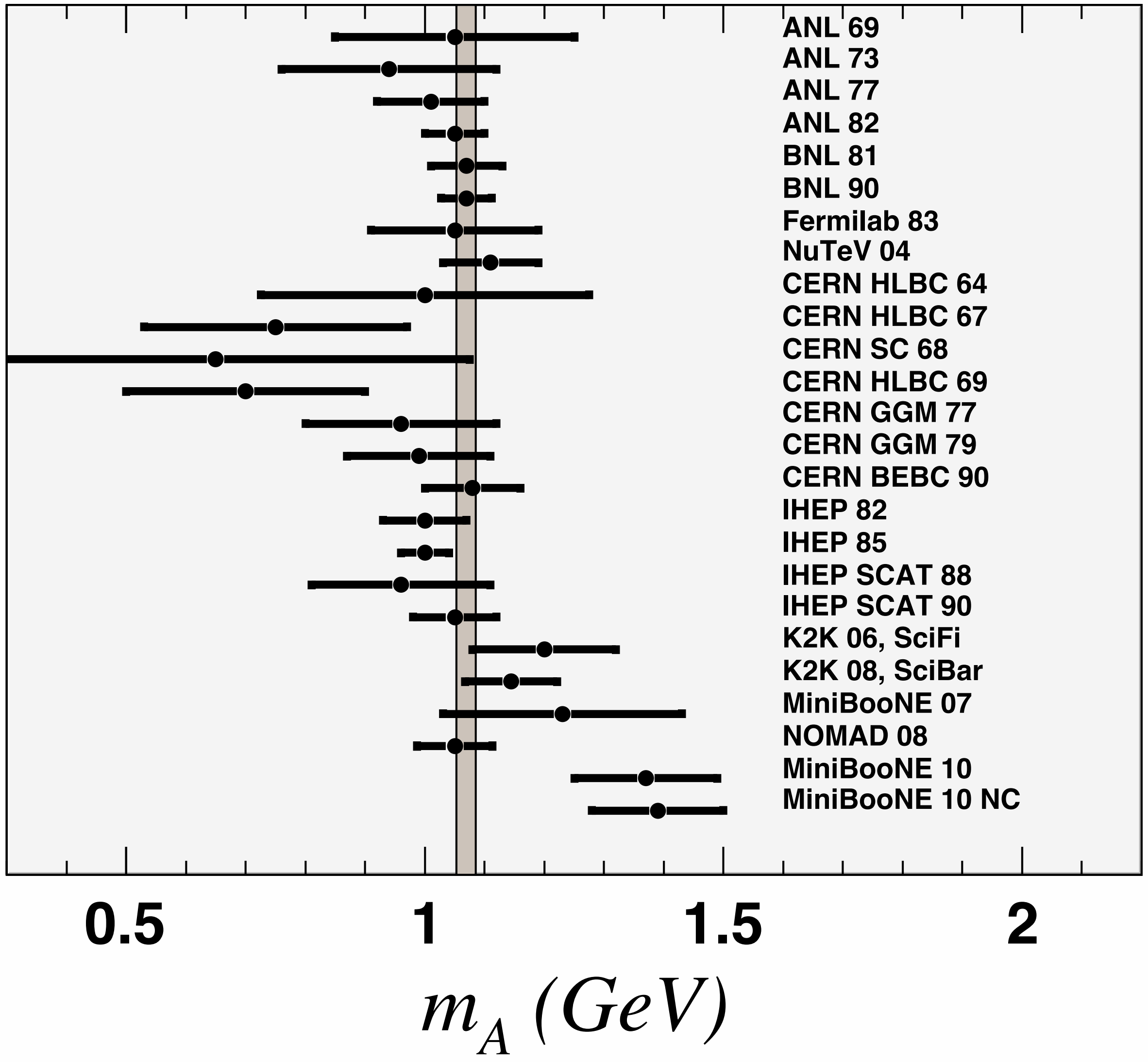
let's start with QE ...

S. Zeller, IF seminar, 02/13/14

CCQE and the axial form factor

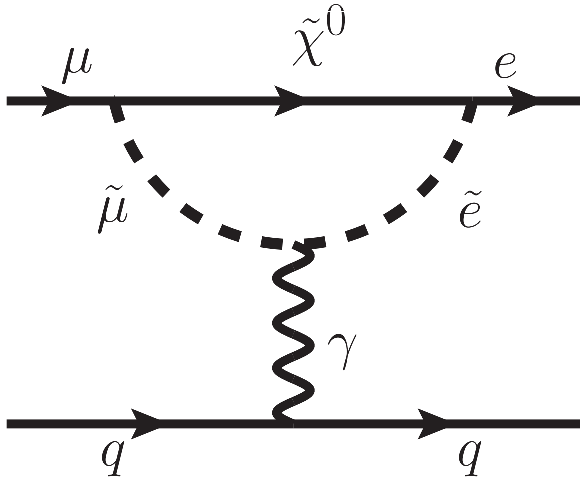
- **CCQE described by axial-vector form-factor of nucleon $F_A(q^2)$**
- Typically q^2 dependence modeled by dipole form, $F_A(q^2) = \frac{g_A}{(1+q^2/m_A^2)^2}$, with g_A taken from neutron decay:
- Fits to dipole form over different q^2 ranges and by different experiments lead to inconsistent determinations of axial mass m_A
  - Difference may stem from nuclear effects, inadequate model parameterization, or both
- Shape of $F_A(q^2)$ can be calculated from first principles by merging constraints from analyticity [Bhattacharya *et al.*, PRD84 (2011) 073006] with lattice QCD
- Axial-vector form factor also enters Standard-Model prediction for neutrinoless double β-decay [see, *e.g.*, Barea, Kotila, Iachelo, PRC87 (2013) 014315]

[Hill, "Lattice Meets Experiment" 2014]

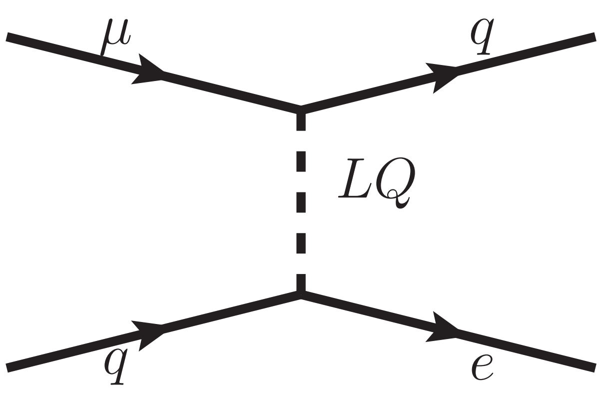


Muon-to-electron conversion

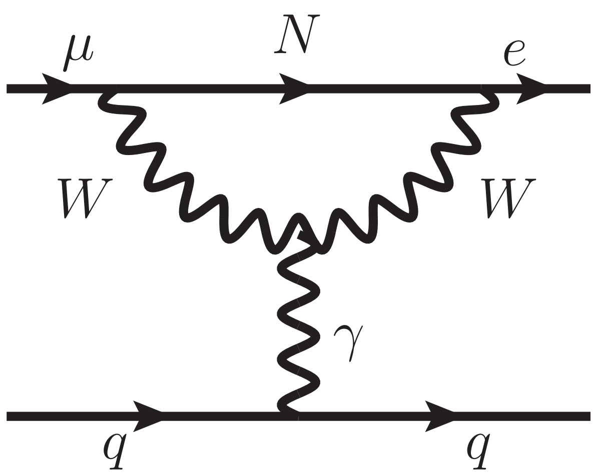
SUSY



LEPTOQUARKS



TYPE-I SEESAW



- Charged-lepton flavor violation so highly suppressed in the Standard Model that any observation would be unambiguous evidence of new physics

- Several experiments searching for μ → e conversion running or on the horizon, motivated in part by new-physics models that predict measurable rates close to current limits:

  - **MEG@PSI** searching for μ → eɣ, while **Mu3e** proposes improved search for μ → eee

  - **Mu2e @ Fermilab** aims to search for μN → eN (where N is a nucleus) with a sensitivity four orders of magnitude below the current best limit

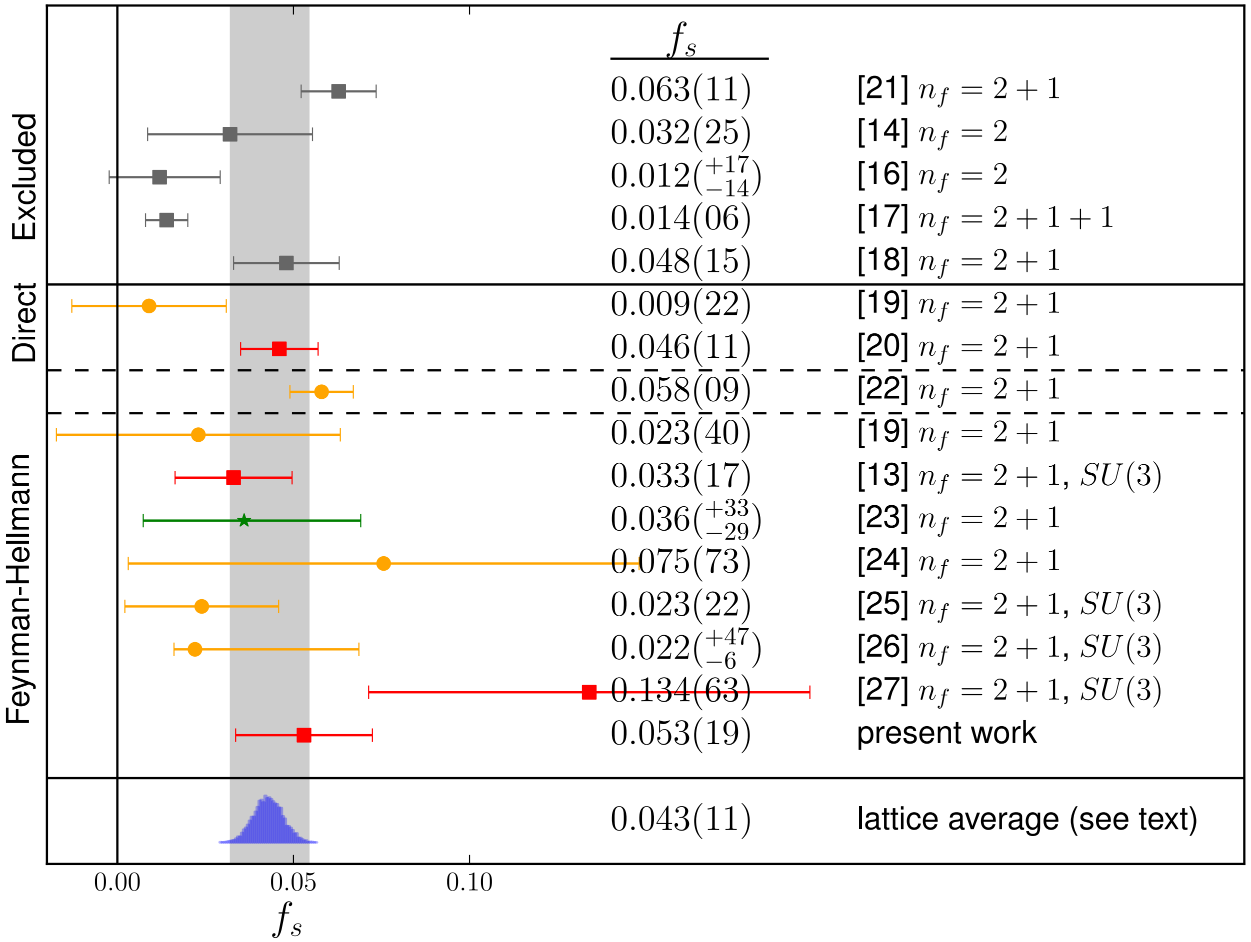
Model discrimination in CLFV

- If observed, combining measured μ → eɣ and μ → e conversion rates on different target nuclei can distinguish between models and reveal information on the underlying theory
    - **Model predictions depend upon nucleon light- and strange-quark contents**

- Lattice calculations of $\sigma_{\pi N}$ and $f_s=m_s\langle N|ss|N\rangle/m_N$ have improved significantly in recent years, and already rule out large f_s favored by early non-lattice estimates

- Present lattice uncertainty in $y=2\langle p|ss|p\rangle/\langle p|uu+dd|p\rangle$ sufficient to test models in which a single operator dominates, but improvement is needed to test two-operator models [Cirigliano *et al.*, PRD80 (2009) 013002]
    - Pinning down values with ~10-20% errors in the next five years is both realistic and sufficient

[Junnarkar & Walker-Loud, PRD87 (2013) 11, 114510]



Rare kaon decays

- Standard-Model branching ratios for **"GOLDEN" MODES** $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ known to a **precision unmatched by any other quark FCNC processes**

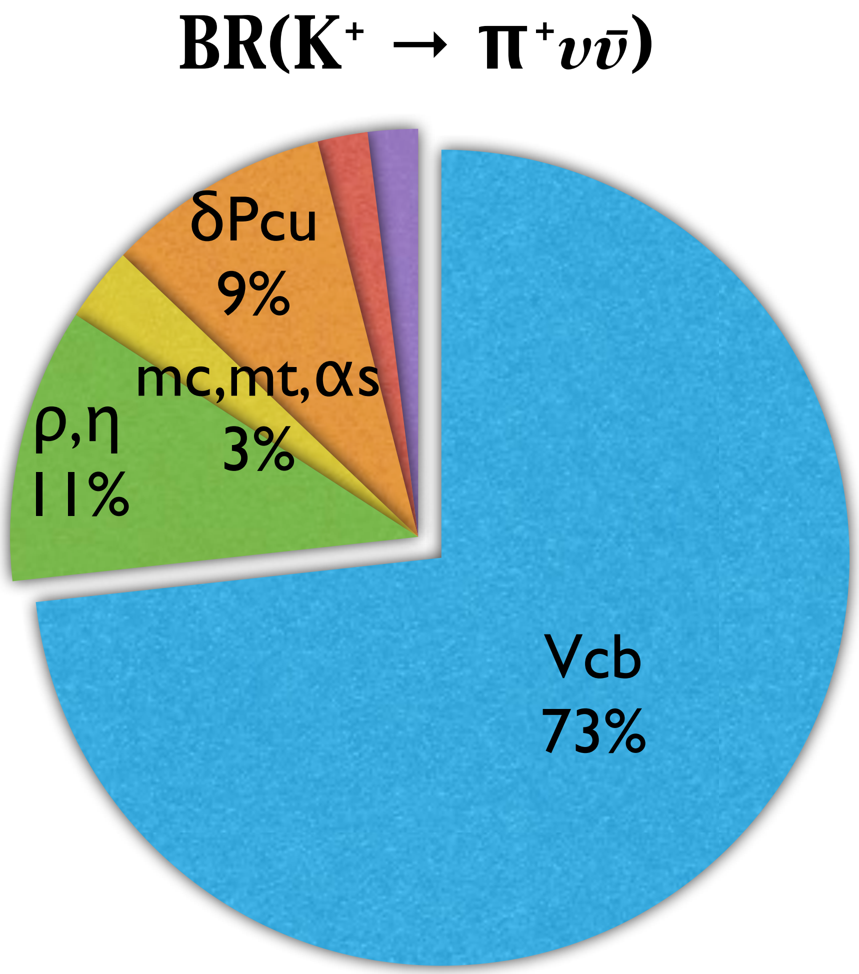
- Within this decade, **NA62 @ CERN SPS** will measure $\mathcal{O}(100)$ K^+ events (assuming the SM), and **KOTO @ J-PARC** will collect first K^0_L events

- Hadronic form factor can be obtained precisely using experimental $K \to \pi \ell \nu$ data and chiral perturbation theory [Mescia & Smith, PRD76 (2007) 034017]

  ➡ Limited by parametric uncertainty in $A^4 \propto |V_{cb}|^4$

- With calculations of $B \to D^{(*)} l \nu$ at *nonzero* recoil in the next few years, expect to reduce error in $|V_{cb}|$ to ~1.5%, and in the Standard-Model branching fractions to ~6%

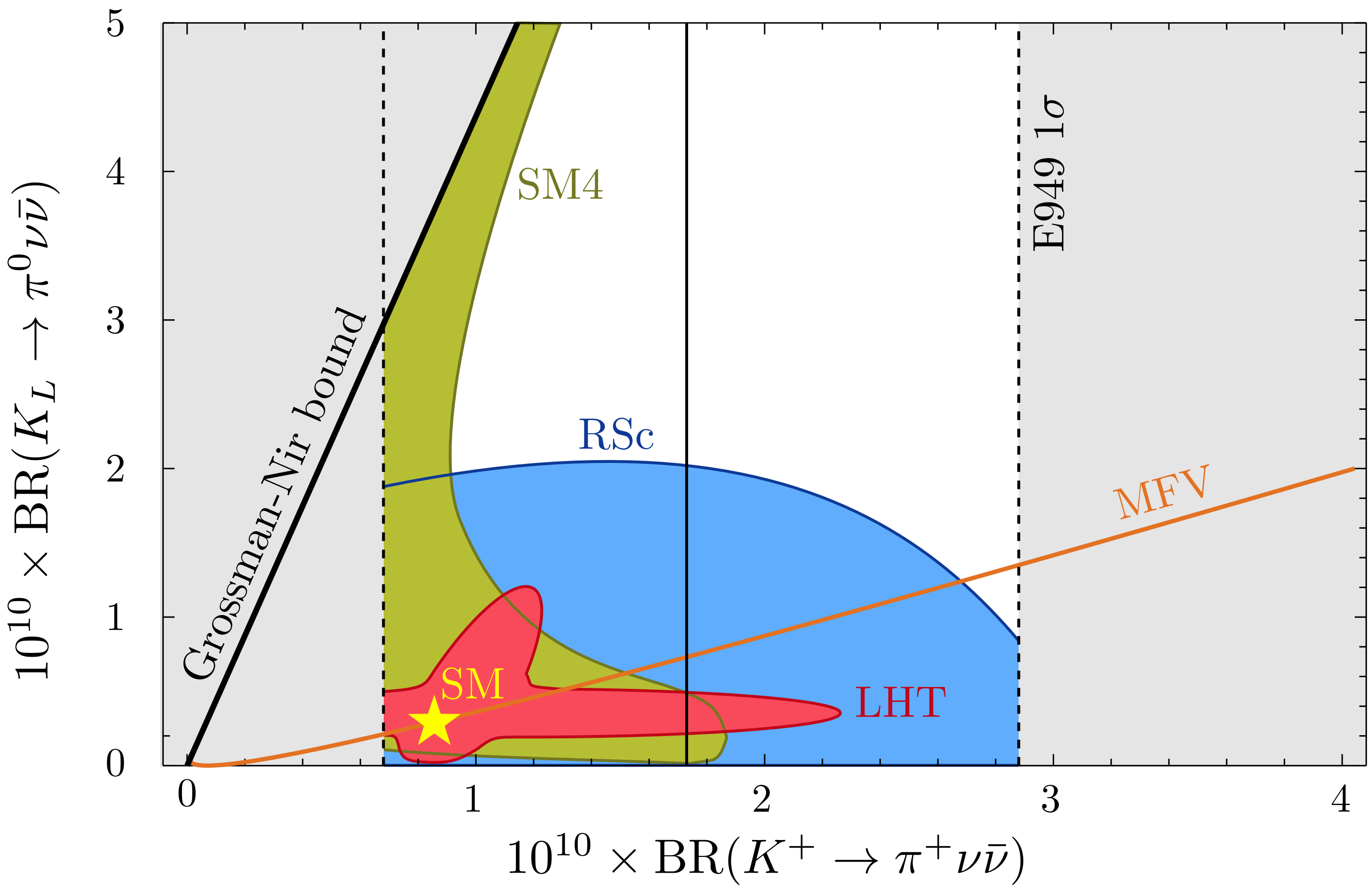
  ➡ Theory error in Standard-Model predictions will be commensurate with expected experimental error



[Brod & Gorbahn PRD83 (2011) 034030]

Room for new physics

- Sensitive to Little Higgs models, warped extra dimensions, and 4th generation [Buras, Acta Phys.Polon.B41:2487-2561,2010]



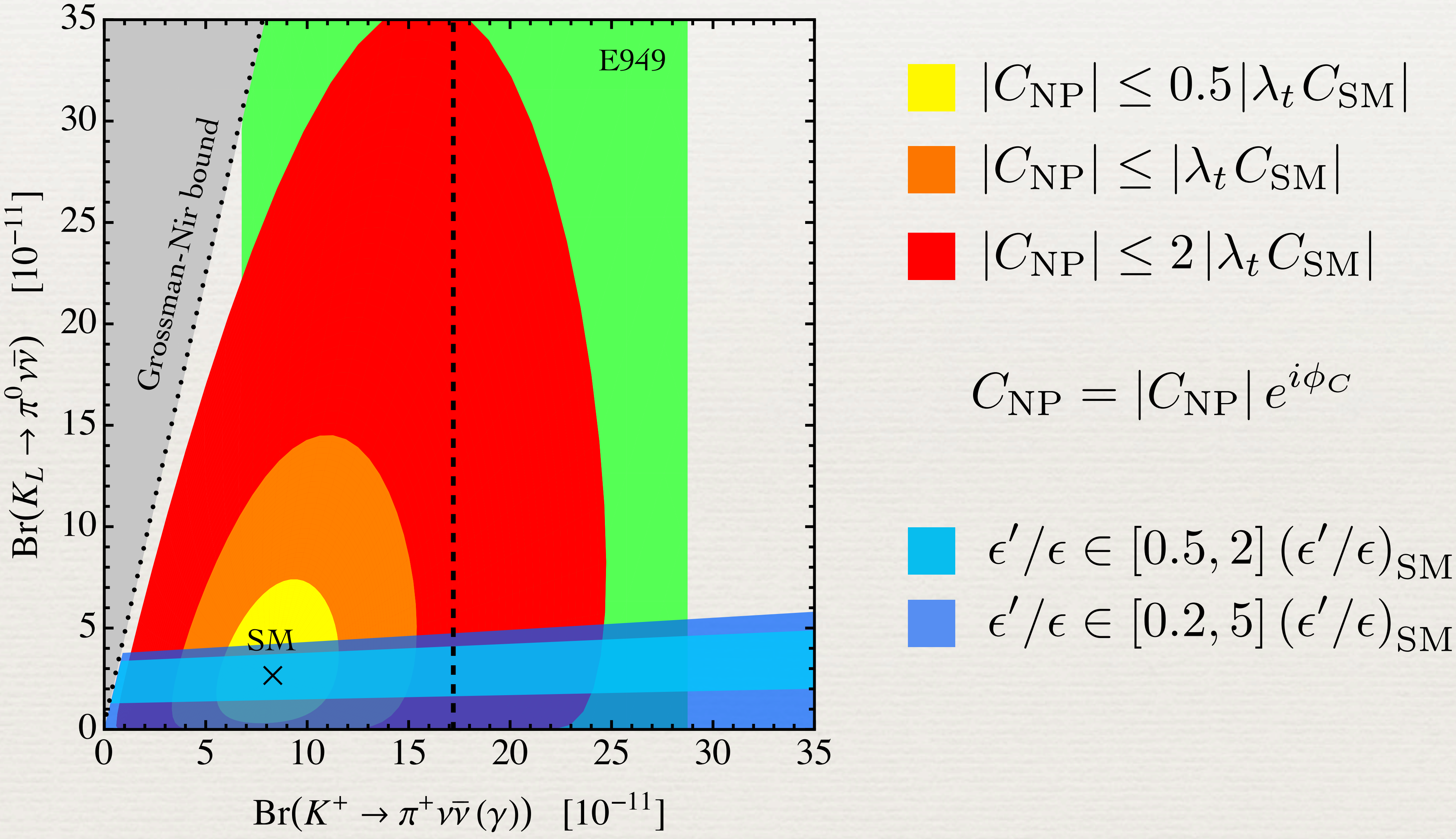
[D. Straub, arXiv:1012.3893 (CKM 2010)]

- **Spectacular deviations from the Standard Model are possible in many new physics scenarios**

- Correlations between the two channels can help distinguish between models

ε'/ε Strikes Back

[U. Haisch, 2012 Project X Physics Study]



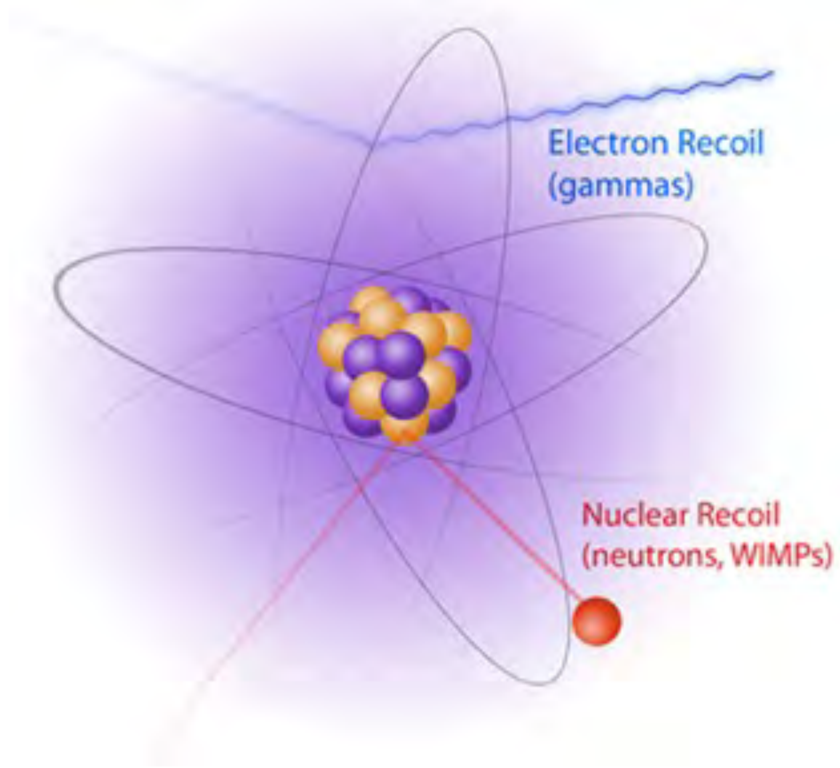
[see S. Jäger, talk at NA62 Physics Handbook Workshop; M. Bauer et al., arXiv:0912.1625 [hep-ph]]

- With the anticipated lattice-QCD improvements from ongoing $K\to\pi\pi$ calculation by RBC/UKQCD, combining the pattern of results $\varepsilon'_K/\varepsilon_K$ with $K\to\pi\nu\nu$ decays can further distinguish between new-physics scenarios [Buras *et al.*, Nucl.Phys. B566 (2000)]

Proton decay & other new interations

- Interpretation of many experimental measurements as constraints on TeV-scale or GUT-scale new physics requires knowledge of nucleon matrix elements

  - **PROTON DECAY:** large underground detectors for neutrino physics also sensitive to proton decay; GUT model predictions for proton lifetime depend upon expectation values $<\pi,K,\eta,\ldots \mid \mathcal{O}_{NP} \mid p>$ of new-physics operators



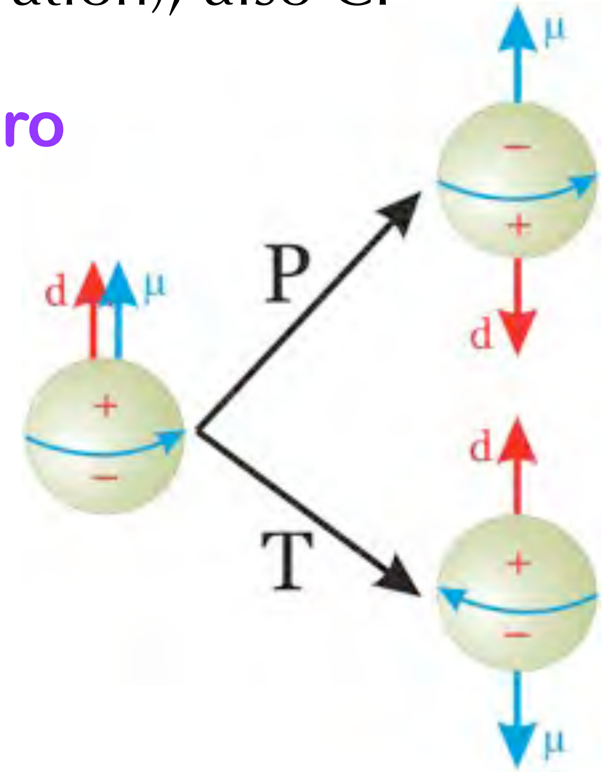
  - **DARK-MATTER DETECTION:** for spin-independent dark matter (*e.g.* mediated by Higgs exchange), cross-section for DM-nucleon scattering depends upon the light- and strange-quark contents of the nucleon (same matrix elements as for μ → e)

  - **NEUTRON BETA DECAY:** constraints on new TeV-scale interactions depend on the neutron scalar and tensor charges g_S and g_T

- For all of these matrix elements, **lattice calculations with 10–20% precision are sufficient for the time being** and can be achieved in the next five years

Neutron electric dipole moment

- Nonzero particle EDM violates P, T and, (assuming CPT conservation), also CP
  - **New CP-violating interactions could show up in nonzero EDMs of leptons and nucleons**
  - Standard-Model contribution to neutron EDM from *CP*-odd phase in CKM matrix $d_N \sim 10^{-30}$ e•cm
  - Contribution from QCD θ-term could in principle be larger, but experimental limit combined with theoretical estimates of d_N/θ set bounds $|\theta| < 10^{-10}$



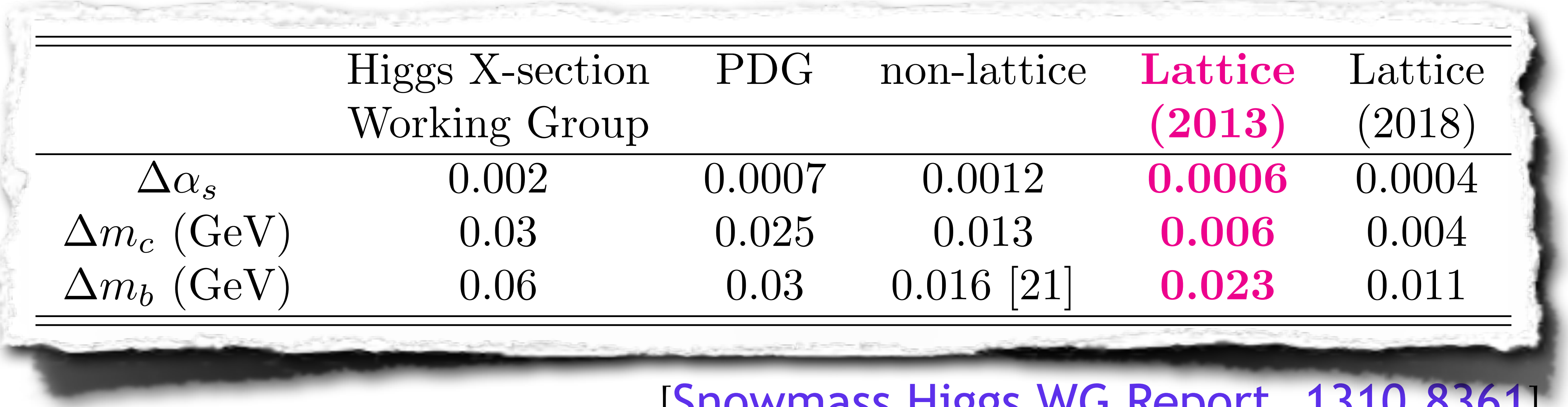
[A. Knecht]

- Lattice-QCD can provide first-principles calculations of d_N/θ, as well as matrix elements of non-Standard Model EDM-inducing operators
  - Test calculations have been carried out for the QCD θ-term contribution to neutron and proton EDMs and statistical errors are ~30%
  - Calculation of matrix elements of dimension-6 BSM operators also underway
  - Expect errors at the ~10-20% level in the next five years

Expected precision of SM Higgs couplings

- Uncertainties in m_c, m_b, and α_s have led some to conclude that (sub)percent measurements of Higgs properties may never be useful [Almeida *et al.*, PRD89 (2014) 033006]

- In fact, however, **lattice calculations have already determined m_c, m_b, and α_s more precisely than is currently being assumed in discussions of Higgs decay channels**



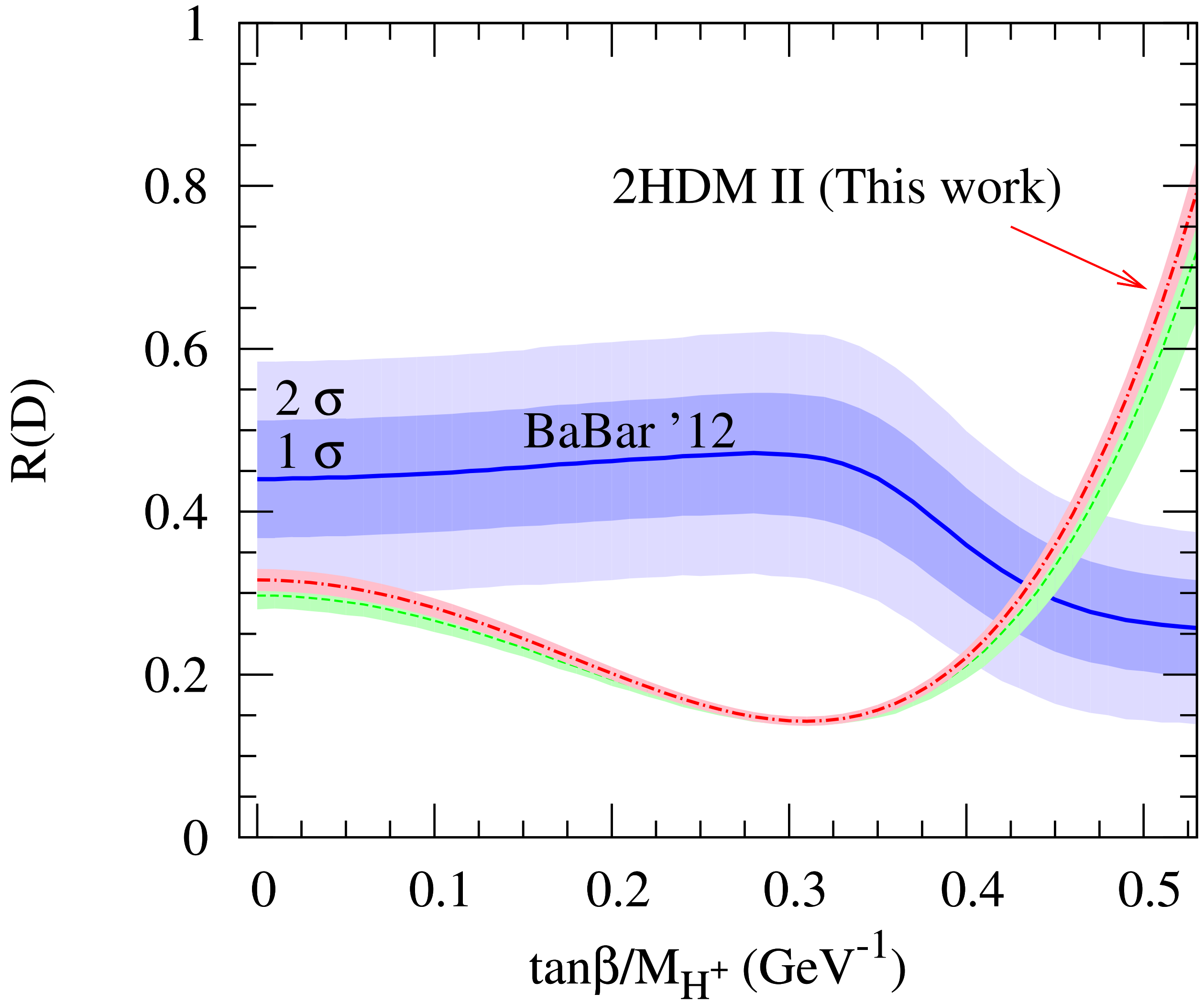
	Higgs X-section Working Group	PDG	non-lattice	**Lattice (2013)**	Lattice (2018)
$\Delta\alpha_s$	0.002	0.0007	0.0012	**0.0006**	0.0004
Δm_c (GeV)	0.03	0.025	0.013	**0.006**	0.004
Δm_b (GeV)	0.06	0.03	0.016 [21]	**0.023**	0.011

[Snowmass Higgs WG Report, 1310.8361]

- Lepage, Mackenzie, & Peskin [1404.0319] use toy Monte-Carlo calculations to estimate how much the uncertainties in m_c, m_b, and α_s from lattice QCD could be decreased over the next decade given the anticipated ~100x growth in computing resources
    - Show that **reducing lattice spacing to 0.023 fm with current analysis methods sufficient to bring parametric errors in SM Higgs couplings to below errors expected from full ILC**

Recent highlight:
R(D) from unquenched lattice QCD

- $B \rightarrow D^{(*)}\tau\nu$ decays sensitive to new-physics contributions such as from charged Higgs bosons
- Recently BaBar measured the ratios R(D) = BR(B → Dτν)/BR(B → Dlν), R(D^*) = BR(B → D^*τν)/BR(B → D^*lν) and observed excesses in both channels that disagree with the Standard Model by 3.4σ [PRL 109 (2012) 101802]



- Fermilab Lattice and MILC Collaborations quickly followed with **first Standard-Model calculation of R(D) from *ab initio* lattice-QCD** [PRL 109 (2012) 071802]
- Uncertainty smaller than previous model estimate from dispersive bounds, heavy-quark symmetry, and quenched lattice QCD
- Lattice calculation of R(D^*) in progress...

Recent highlight: f_K/f_π at the physical point

- The SU(3) flavor-breaking ratio f_K/f_π allows a determination of $|V_{ud}|/|V_{us}|$ [Marciano]

$$\frac{\Gamma(K \to l\bar{\nu}_l)}{\Gamma(\pi \to l\bar{\nu}_l)} = \left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K \left(1 - \frac{m_l^2}{m_K^2}\right)^2}{m_\pi \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2} \left[1 + \frac{\alpha}{\pi}(C_K - C_\pi)\right]$$

- MILC collaboration recently obtained the **first lattice-QCD determination of f_K/f_π (1) including dynamical charm** and **(2) at the physical pion mass** with highly-improved staggered (HISQ) quarks [Bazavov *et al.* PRL**110**, 172003]
  - **Eliminate error from extrapolation to physical u- and d-quark masses**

Source	f_{K^+}/f_{π^+}
Monte-Carlo statistics	0.22%
Continuum extrapolation	0.28%
Finite-volume corrections	0.14%
EM corrections	0.02%
Total	0.38%

- Combined with $|V_{ud}|$ from nuclear β-decay, enables sub-percent test of unitarity of 1st row of CKM matrix

$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = 0.0003(6)$$

Coming soon:
B→Dℓν form factor at nonzero recoil

- B → Dℓν semileptonic form factor allows determination of $|V_{cb}|$ via:

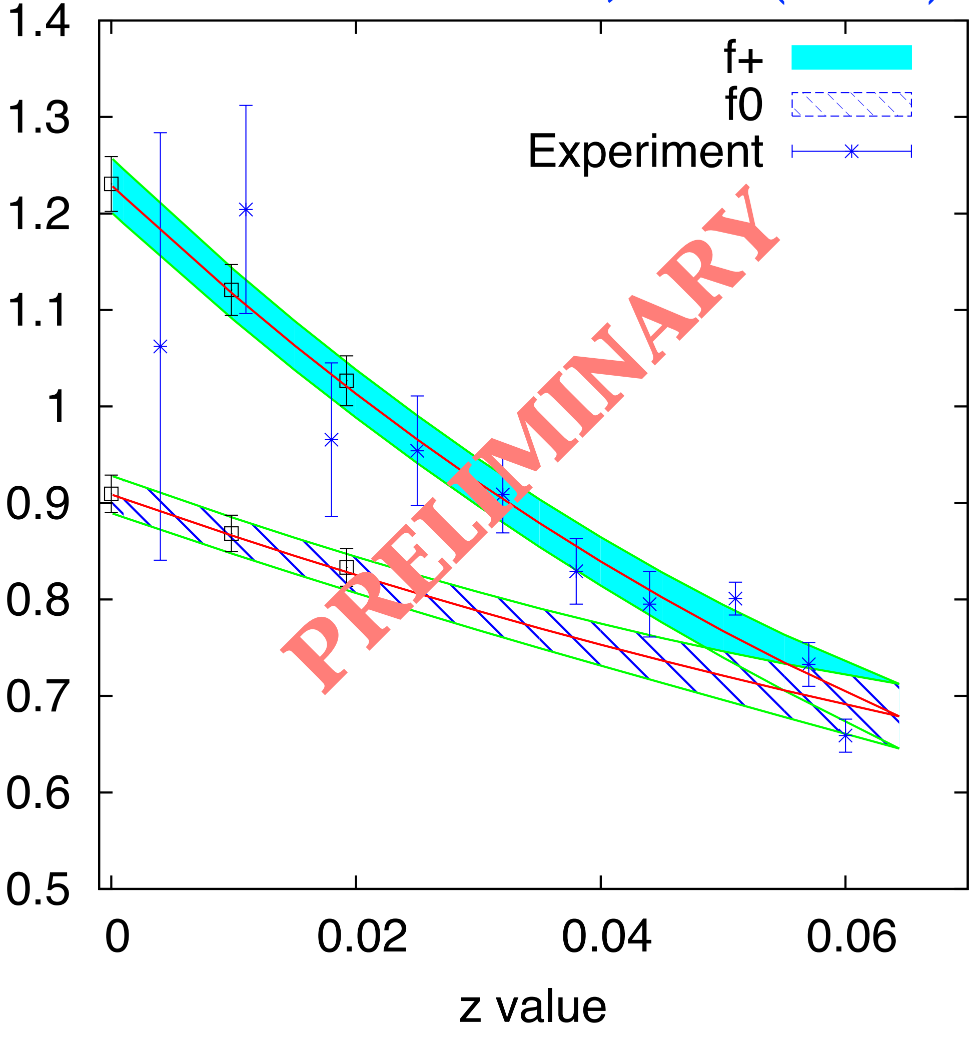
$$\frac{d\Gamma(B\to Dl\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B+m_D)^2 (w^2-1)^{3/2} |\mathbf{V_{cb}}|^2 |\mathcal{G}_{\mathbf{B\to D}}(\mathbf{w})|^2 \quad \Big\} \, w \equiv v_B \cdot v_D$$

- Common practice comparing theory and experiment at zero recoil (w=1) leads to large experimental errors in $|V_{cb}|$ because decay rate kinematically suppressed at low recoil momentum

- Fermilab/MILC presented **first unquenched results for G(w) over full kinematic range at Lattice 2013**, and analysis is now almost finalized

- Following method now standard for B→π exclusive decays, **obtain $|V_{cb}|$ with reduced uncertainties from combined fit of lattice and experimental data to model independent "z-parameterization"** based on analyticity and unitarity [Boyd, Grinstein, Lebed, PRL74 (1995) 4603-4606]

[Qiu *et al.*, PoS LATTICE 2013, 385 (2013)]



Highlight: the kaon mixing parameter B_K

- Indirect CP-violation in the neutral kaon system (ϵ_K) constrains the apex of the CKM unitarity triangle via

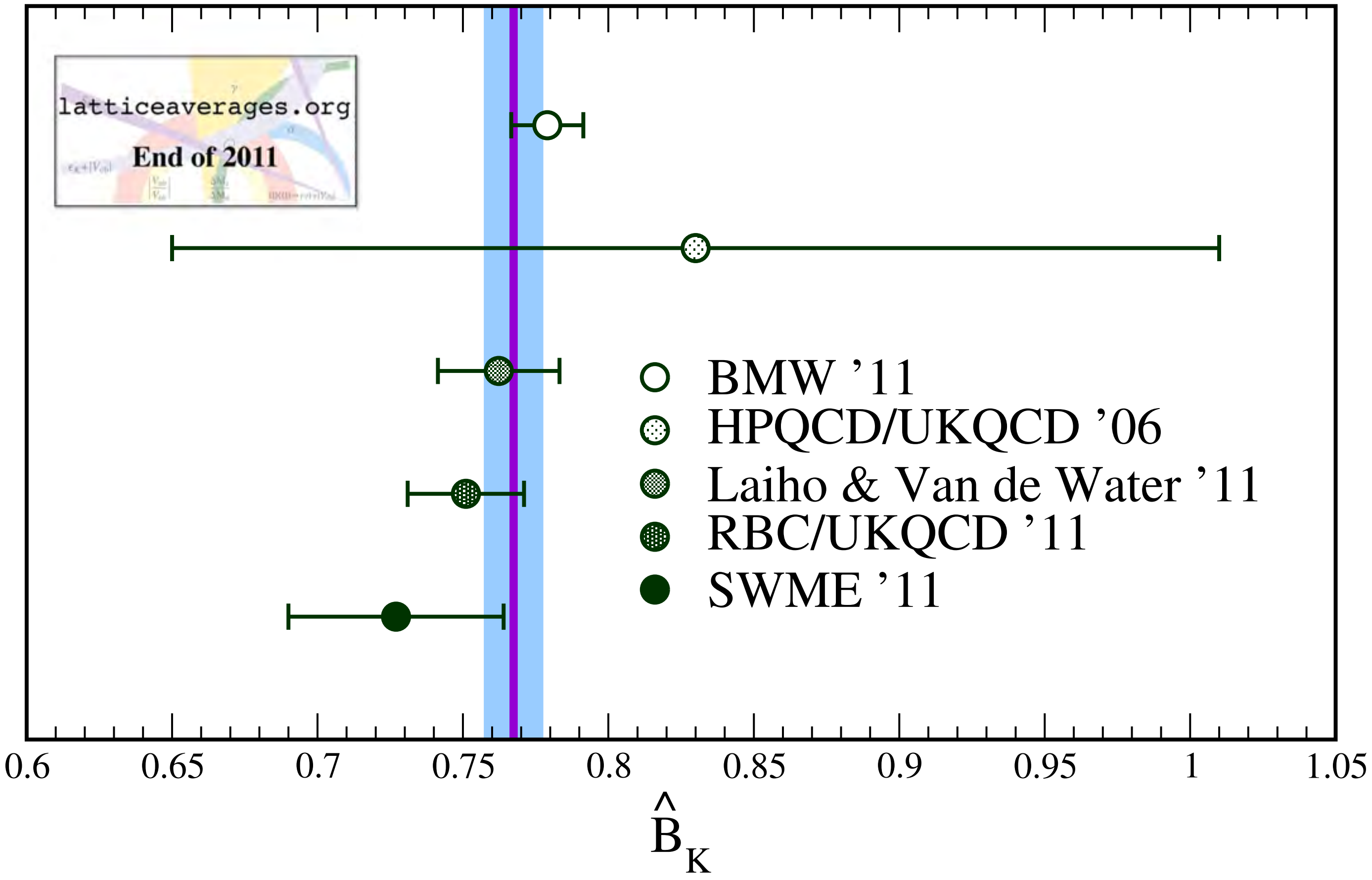
$$|\epsilon_K| = C_\epsilon B_K A^2 \bar{\eta} \{ -\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\bar{\rho}) \}$$

  - C_ε, η_i and S_0 known to NLO (in some cases NNLO) in perturbation theory
  - B_K parameterizes the nonperturbative hadronic contributions

- Until recently, the unitarity- triangle constraint from ϵ_K was limited by the ~20% uncertainty in B_K

- Significant theoretical and computational effort has been devoted to improving B_K, and there are **now several independent lattice results in good agreement**

~1.3% error in average



Status of the | ϵK | band

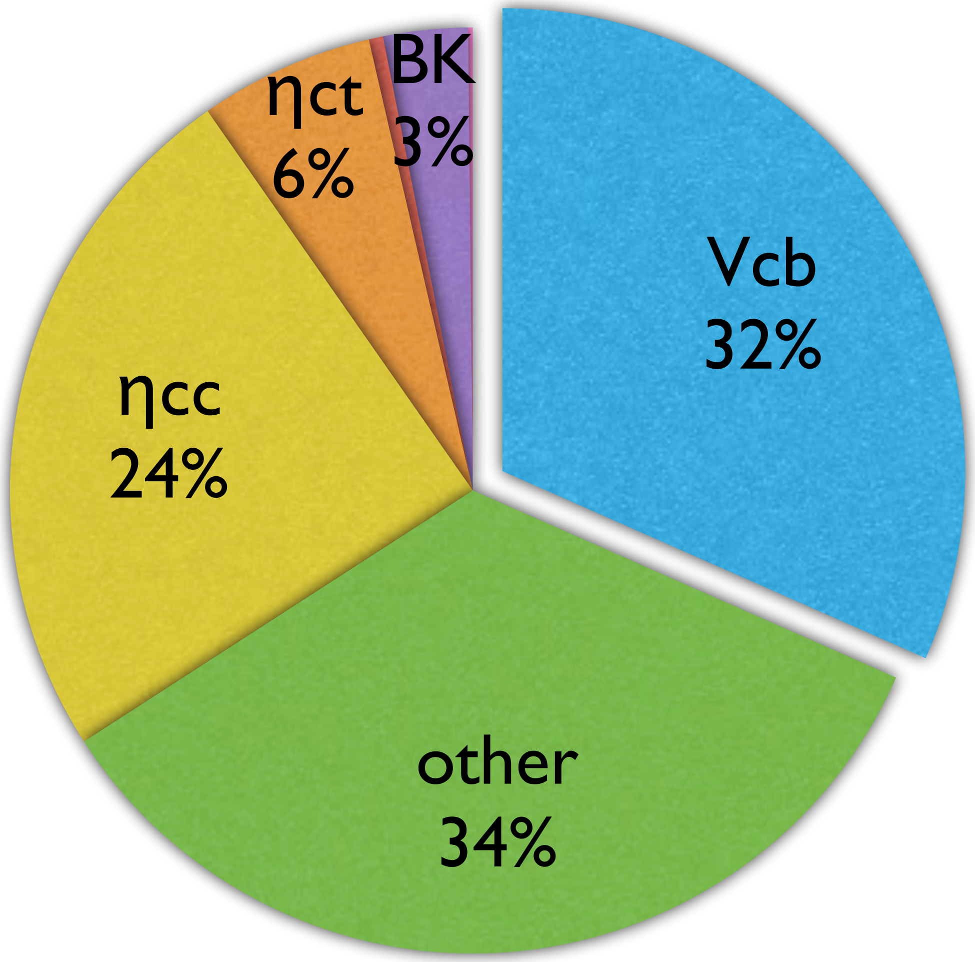
- Recent calculation by Brod & Gorbahn [Phys.Rev.Lett. 108 (2012) 121801] gives the following error breakdown for $|\epsilon_K|$ in the Standard Model:

$$|\varepsilon_K| = (1.81 \pm 0.14_{\eta_{cc}} \pm 0.02_{\eta_{tt}} \pm 0.07_{\eta_{ct}} \pm 0.05_{\rm LD} \pm 0.23_{\rm parametric}) \times 10^{-3}$$

(1) Largest individual uncertainty is from ~10% parametric error in $A^4 \propto |V_{cb}|^4$

(2) η_{cc} and η_{ct} are both known to 3-loops (NNLO)

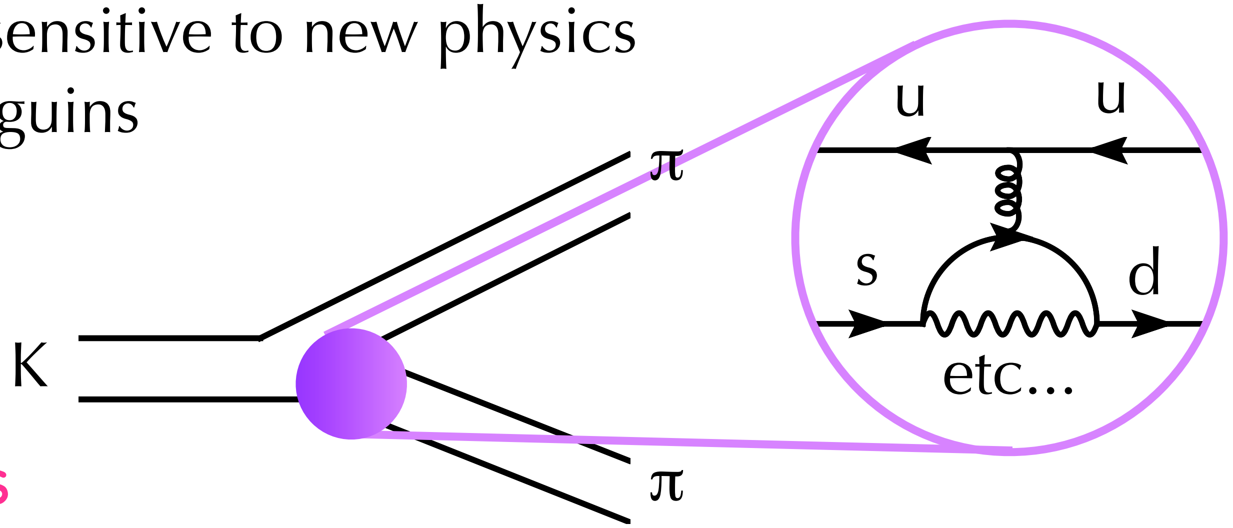
(3) Error from B_K only fourth-largest individual contribution



- Lattice community is moving on to other more challenging kaon physics quantities ...

Breakthrough: K→ππ decay

- Direct CP-violation in K→π π decays (ϵ'_K/ϵ_K) sensitive to new physics because it receives contributions from EW penguins
  - Measured experimentally to <10% precision, but utility for testing SM handicapped by large uncertainty in corresponding weak matrix elements



- In the past two years, RBC-UKQCD collaboration made significant progress in resolving theoretical issues associated with computing K→ππ amplitudes via the "direct" Lellouch-Lüscher approach [Blum et al., PRD 84 (2011) 114503, PRL 108, 141601 (2012)]
  - Computed ΔI = 3/2 matrix elements with nearly physical pion and kaon masses, and obtained Re(A_2) & Im(A_2) with ~20% errors [Phys.Rev.Lett. 108 (2012) 141601]
  - Studied ΔI = 1/2 matrix elements with ~330 MeV pions, demonstrating ability to perform power-divergent subtractions and tackle expensive disconnected diagrams [Phys.Rev. D84 (2011) 114503]
- **Simulations with physical pions underway on new BG/Q supercomputers, and should soon yield first *ab initio* QCD calculation of ΔI=1/2 rule and calculation of $\varepsilon'_K/\varepsilon_K$ with ~20-30% precision**

a_μ^{HVP} from $e^+e^-\to$hadrons

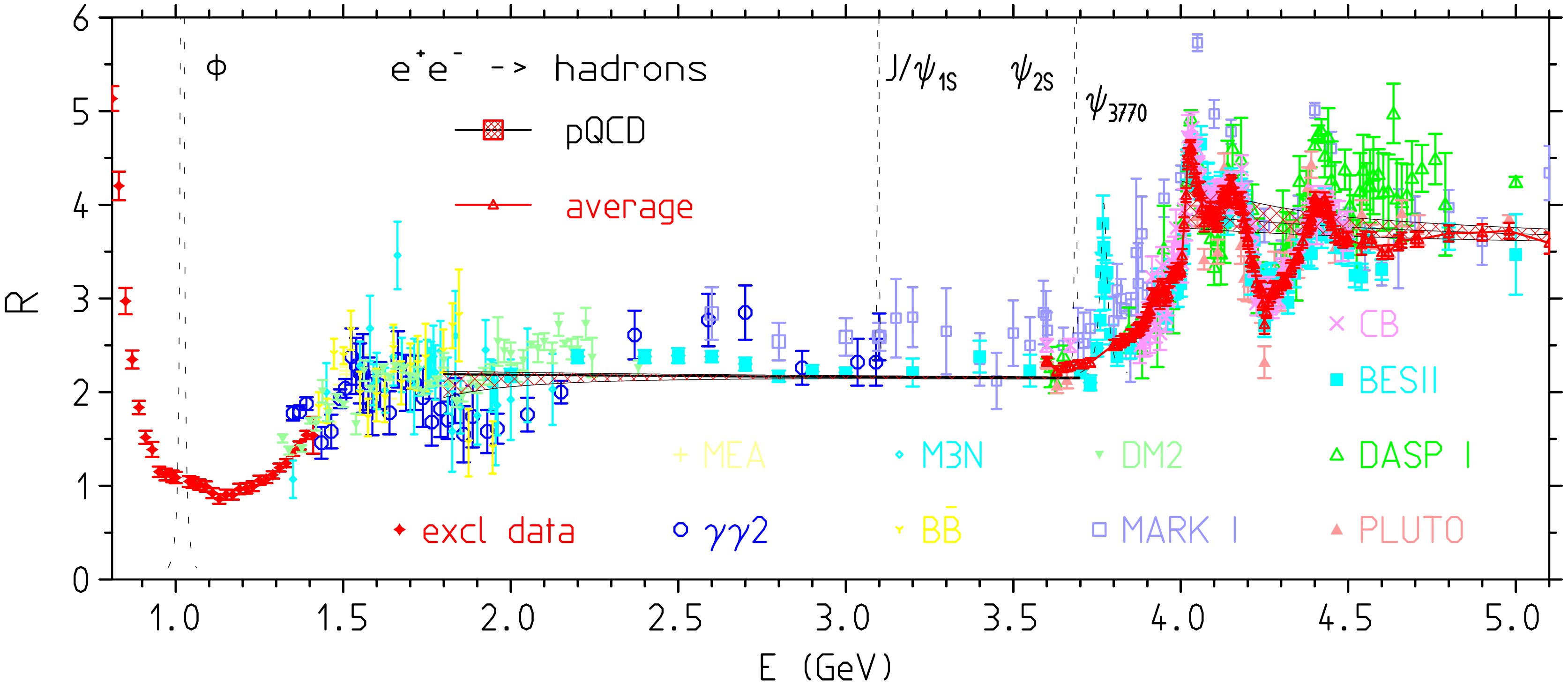
- Standard-Model value for a_μ^{HVP} obtained from experimental measurement of $\sigma_{\mathrm{total}}(e^+e^-\to\text{hadrons})$ via optical theorem:

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} \mathrm{d}s \frac{R(s)K(s)}{s^2} \qquad R \equiv \frac{\sigma_{\mathrm{total}}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

  - (Away from quark thresholds, use four-loop pQCD)

- Includes >20 multi-particle channels with up to six final-state hadrons

- Multi-hadron channels represent a small absolute contribution to a_μ^{HVP}, but contribute a significant fraction of the total uncertainty



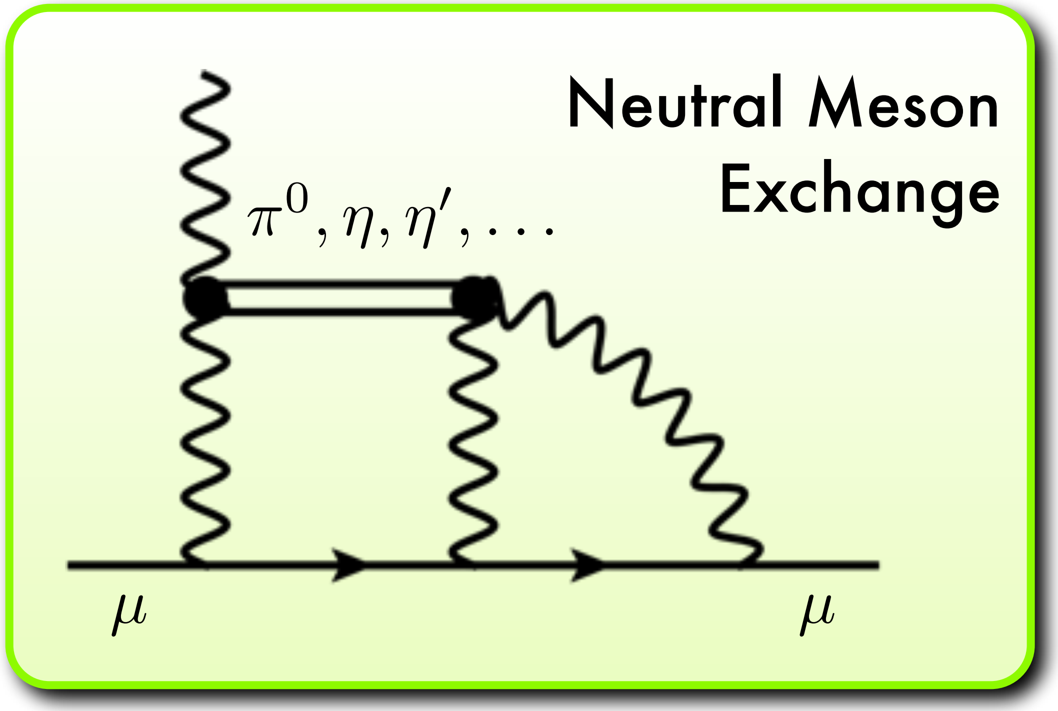
[Jegerlehner and Nyffeler, Phys.Rept. 477 (2009) 1-110]

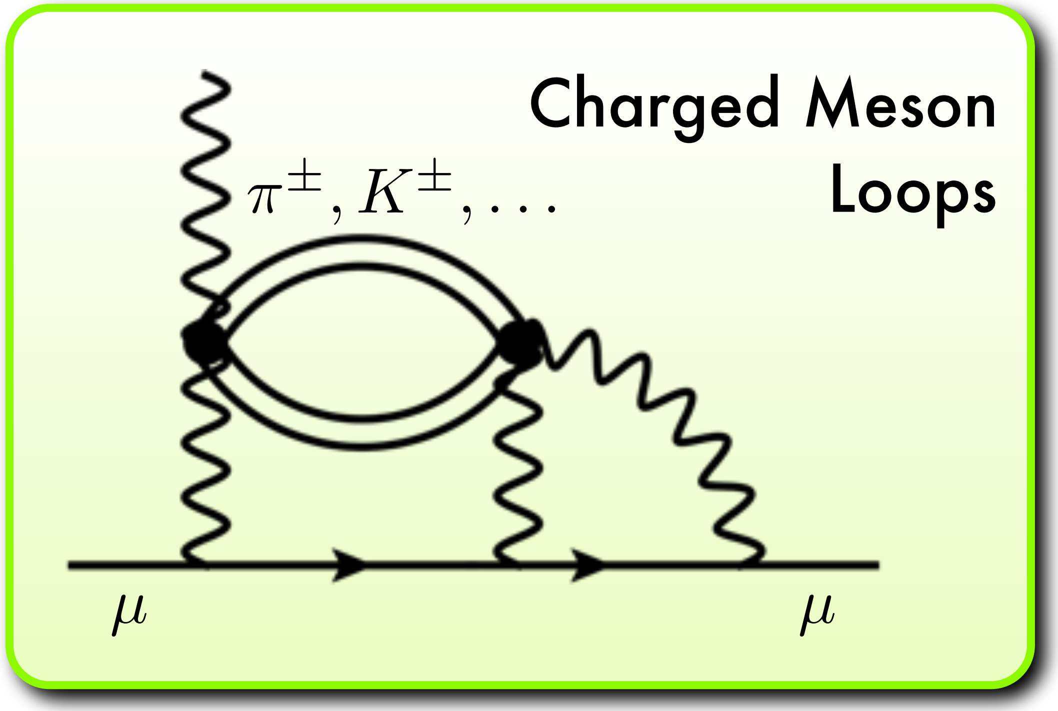
a_μ^{HLbL} from QCD models

[Jegerlehner & Nyffeler, Phys.Rept. 477 (2009) 1-110; Prades, de Rafael, Vainshtein, 0901.0306]

- Hadronic light-by-light contribution cannot be expressed in terms of experimental quantities and must be obtained from theory: present model estimates report errors in the 25-40% range

- All recent calculations compatible with constraints from large-N_c and chiral limits and normalize dominant π^0-exchange contribution to measured $\pi^0 \rightarrow \gamma\gamma$ decay width
  - Differ for form factor shape due to different QCD-model assumptions such as vector-meson dominance, chiral perturbation theory, and the large N_c limit





- Error estimate more subjective than for HVP and somewhat controversial

The Glasgow consensus for a_μ^{HLbL}

[Prades, de Rafael, Vainshtein, 0901.0306]

- Quoted error for a_μ^{HLbL} is based on model estimates, but does not cover spread of values
  - π^0-exchange contribution estimated to be ~10 times larger than others
  - Largest contribution to uncertainty ($\pm 1.9\times 10^{-10}$) attributed to charged pion and kaon loop contributions

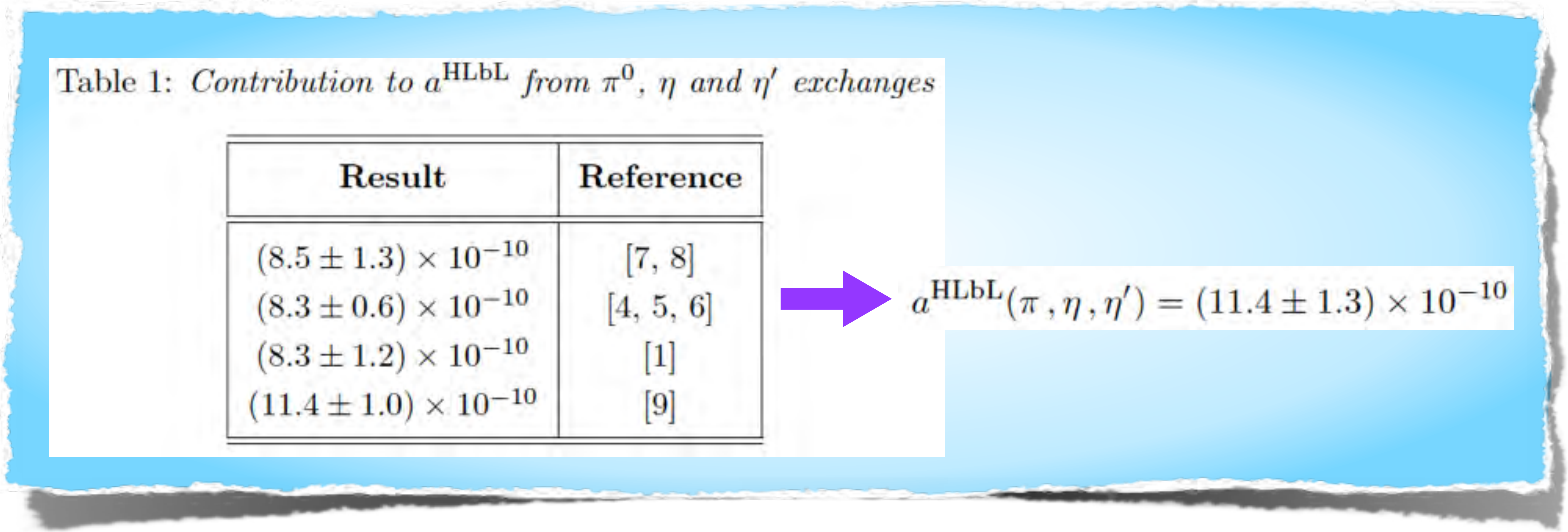


Table 1: *Contribution to a^{HLbL} from π^0, η and η' exchanges*

Result	**Reference**
$(8.5 \pm 1.3) \times 10^{-10}$	[7, 8]
$(8.3 \pm 0.6) \times 10^{-10}$	[4, 5, 6]
$(8.3 \pm 1.2) \times 10^{-10}$	[1]
$(11.4 \pm 1.0) \times 10^{-10}$	[9]

$$\rightarrow \quad a^{\mathrm{HLbL}}(\pi\,,\eta\,,\eta') = (11.4 \pm 1.3) \times 10^{-10}$$

- Error estimate more subjective than for HVP and somewhat controversial

Lattice efforts on a_μ^{HVP}

- Several independent efforts ongoing (plus additional ones without quotable results...):

Collaboration	N_f	Fermion action	$a_\mu^{\rm HVP} \times 10^{10}$
HPQCD	2+1+1	HISQ	strange: $53.41(59)_{\rm tot}$ charm: $14.42(39)_{\rm tot}$
ETMC	2+1+1	twisted-mass	$674(21)_{\rm stat}(18)_{\rm sys}$
Aubin & Blum	2+1	Asqtad staggered	$713(15)_{\rm stat}(31)_{\chi\rm PT}(??)_{\rm other}$
Edinburgh	2+1	domain-wall	$641(33)_{\rm stat}(32)_{\rm sys}$
ETMC	2	twisted-mass	$572(16)_{\rm tot}$
Mainz	2	$\mathcal{O}(a)$ improved Wilson	$618(64)_{\rm tot}$

- Most use the same general approach
- **Errors typically in the 5–10% percent range**, and (mostly) neglect quark-disconnected contributions

[1] Chakraborty *et al.*, JHEP 1402 (2014) 099
[2] Feng *et al.*, JHEP 1402 (2014) 099
[3] Aubin & Blum, PRD 75 (2007) 114502
[4] Boyle *et al.*, PRD 85 (2012) 074504
[5] Feng *et al.*, PRL 107 (2011) 081802
[6] Della Morte *et al.*, JHEP 1203 (2012) 055

New lattice method for a_μ^{HVP}

[Chakraborty et al. (HPQCD), 1403.1778]

- Sidestep $q^2 \to 0$ extrapolation by expressing a_μ^{HVP} in terms of derivatives of vacuum polarization function $\Pi(q^2)$ at $q^2=0$

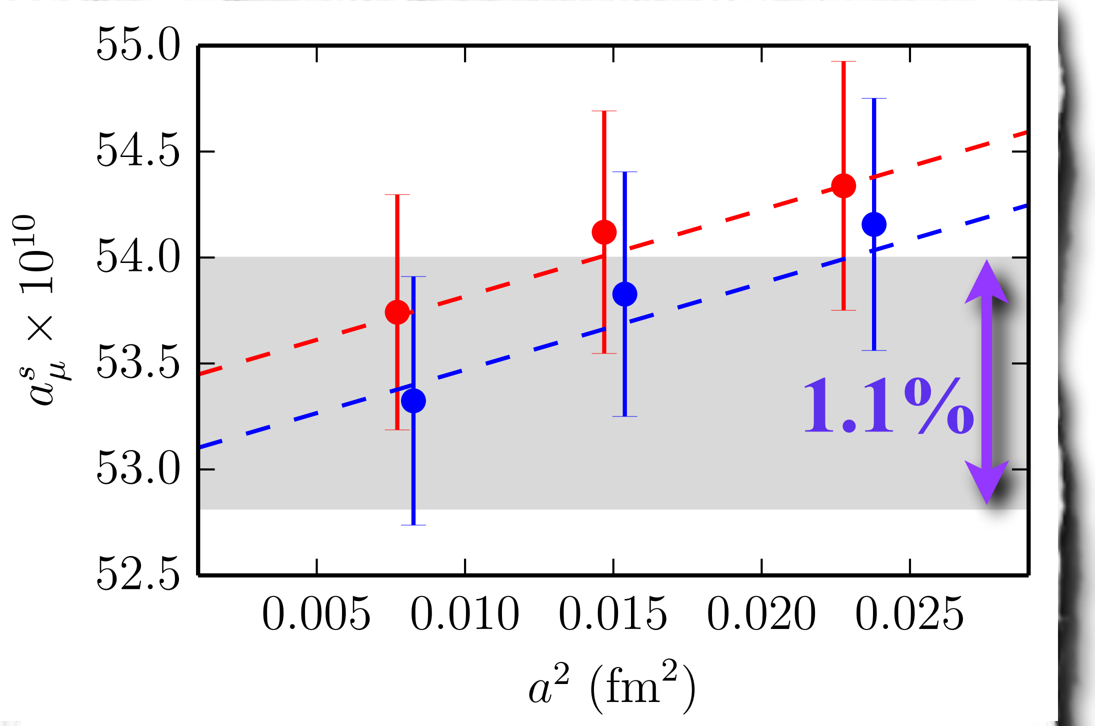
- Derivatives easily computed on lattice to high statistical precision from time-moments of the electromagnetic current-current correlator at $q^2 = 0$

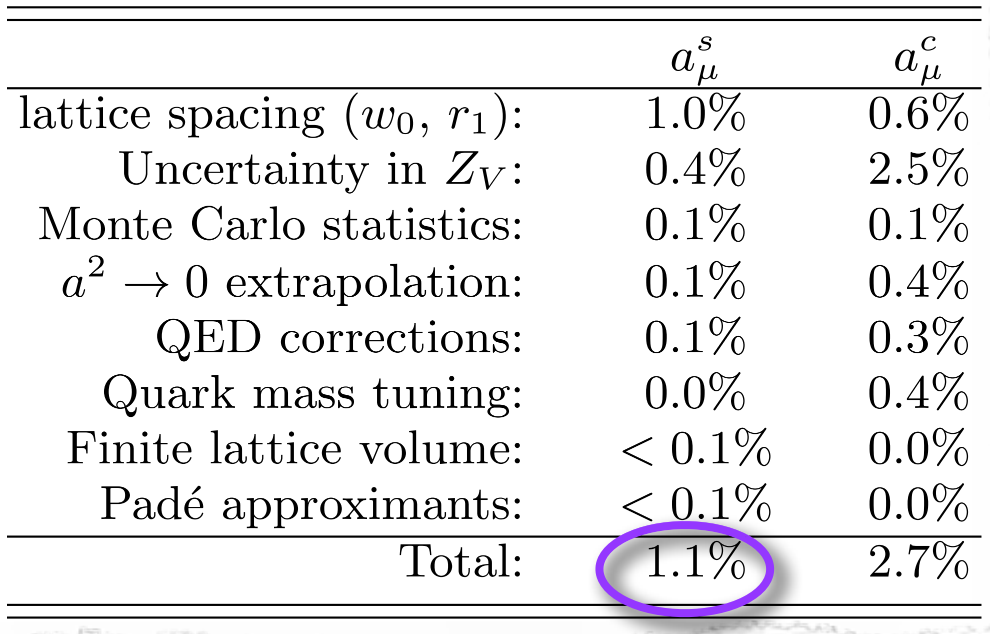
$$
\begin{aligned}
G_{2n} &\equiv a^4 \sum_t \sum_{\mathbf{x}} t^{2n} Z_V^2 \langle j^i(\mathbf{x},t) j^i(\mathbf{0},0)\rangle \\
&= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 [\Pi(q^2) - \Pi(0)] \Big|_{q^2=0}
\end{aligned}
$$

- Illustrate method with strange and charm-quark contributions and obtain a_μ^s to ~1%

- Correlator noisier for light quarks, but **estimate that similar precision can be obtained for $a_\mu^{u,d}$ with 10× larger gauge-field ensembles**

- Beyond ~1%, likely need to directly include EM and isospin-breaking in simulations

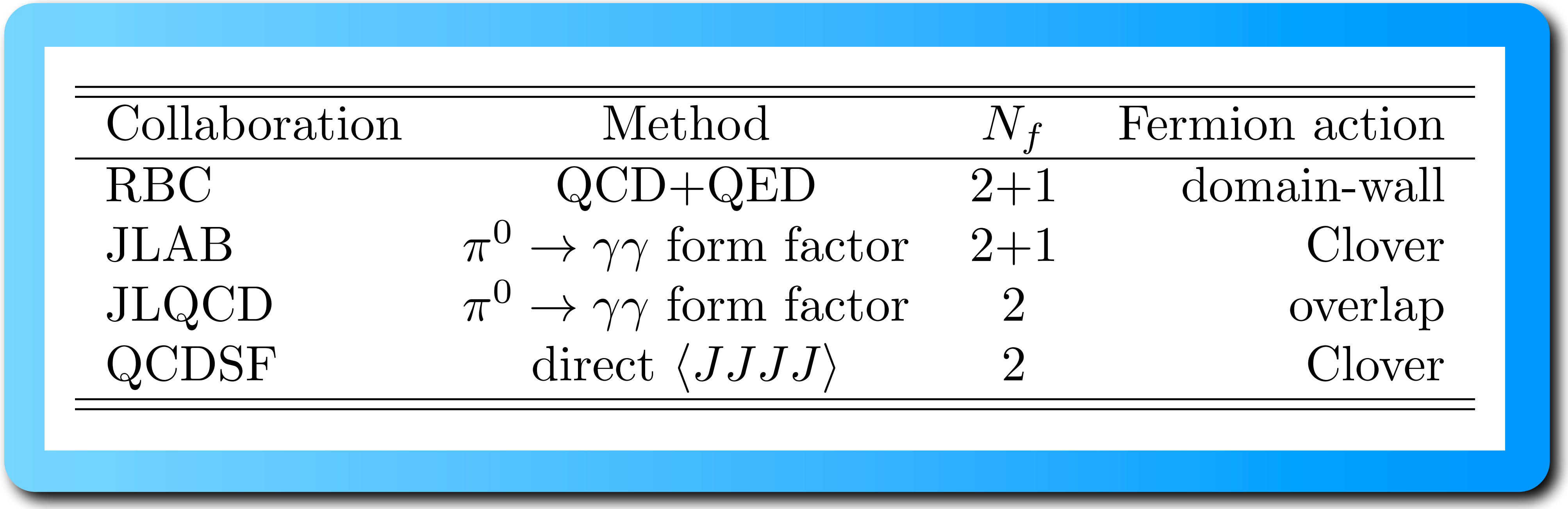




	a_μ^s	a_μ^c
lattice spacing (w_0, r_1):	1.0%	0.6%
Uncertainty in Z_V:	0.4%	2.5%
Monte Carlo statistics:	0.1%	0.1%
$a^2 \to 0$ extrapolation:	0.1%	0.4%
QED corrections:	0.1%	0.3%
Quark mass tuning:	0.0%	0.4%
Finite lattice volume:	$< 0.1\%$	0.0%
Padé approximants:	$< 0.1\%$	0.0%
Total:	1.1%	2.7%

Lattice efforts on a_μ^{HLbL}

- Several efforts ongoing to compute all or part of the light-by-light contribution with different methods



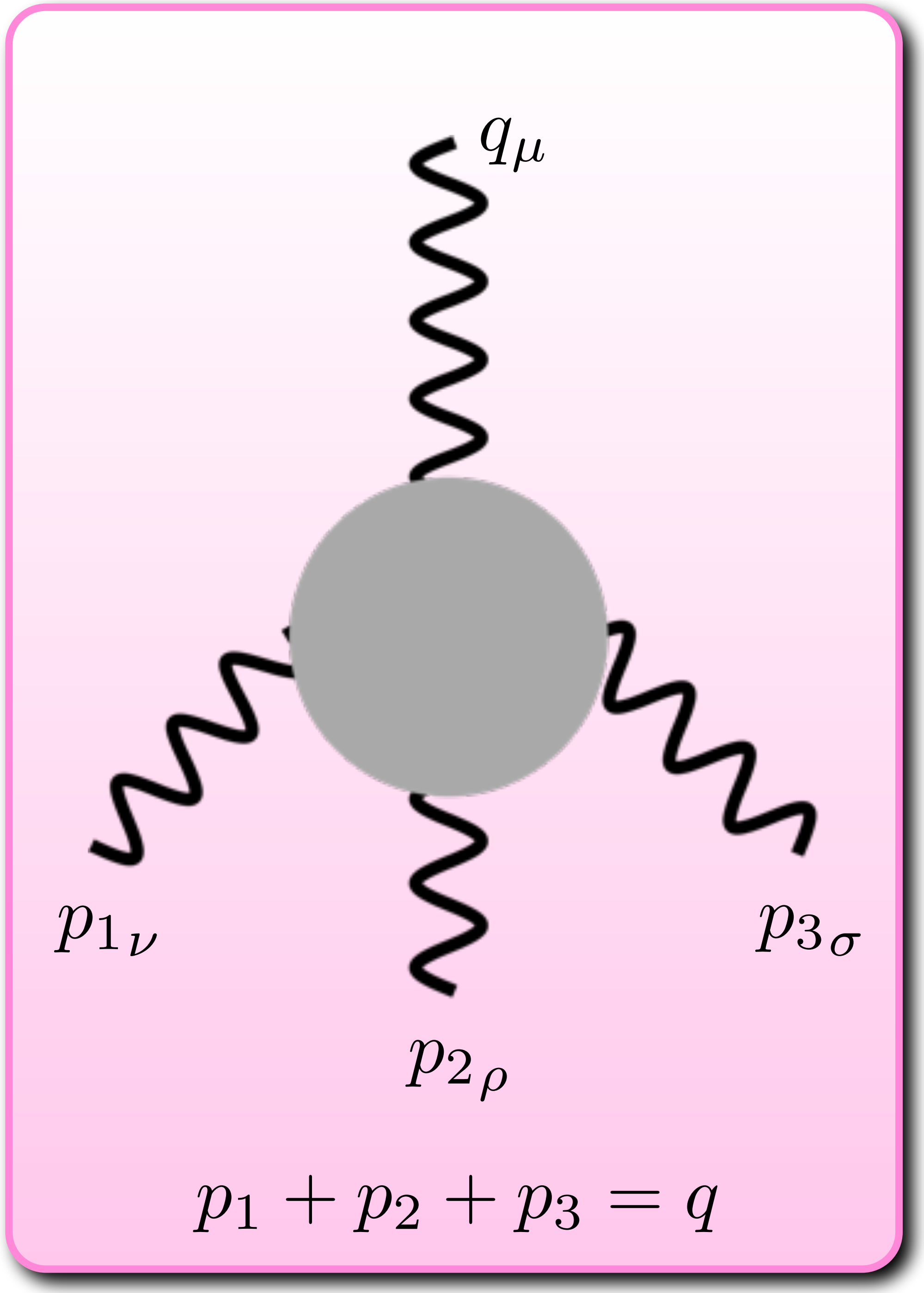
Collaboration	Method	N_f	Fermion action
RBC	QCD+QED	2+1	domain-wall
JLAB	$\pi^0 \to \gamma\gamma$ form factor	2+1	Clover
JLQCD	$\pi^0 \to \gamma\gamma$ form factor	2	overlap
QCDSF	direct $\langle JJJJ \rangle$	2	Clover

- None of them yet have results for a_μ^{HLbL}

[1] Hayakawa *et al.*, PoS LAT2005 (2006) 353; Blum *et al.*, PoS LATTICE2012 (2012) 022; ...
[2] Cohen *et al.*, PoS LATTICE2008 (2008) 159
[3] Feng *et al.*, Phys.Rev.Lett. 109 (2012) 182001
[4] Rakow, Lattice 2008

“Conventional” approach for a_μ^{HLbL}

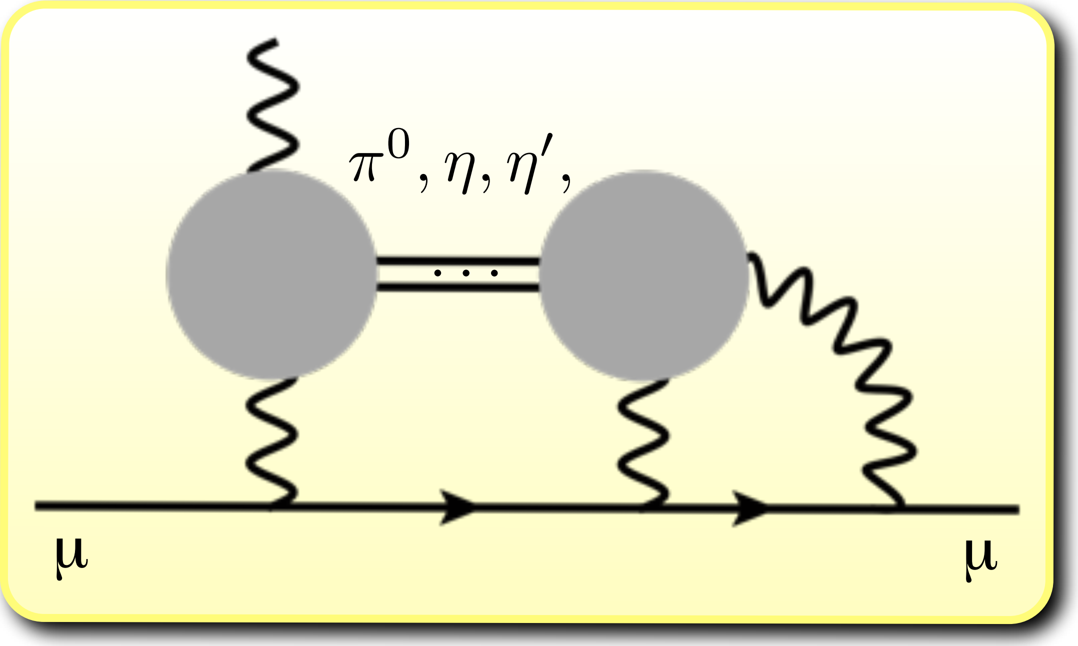
- Can follow a similar approach to that used for HVP
- Calculate the correlation function of four electromagnetic currents and insert into a continuum two-loop QED integral
- Computationally costly because one must compute the four-index tensor for all possible combinations of loop momenta (p_1,p_2) and several values of the external momentum q
- Exploratory calculations under way [Rakow, Lattice 2008], but viability of this method has yet to be demonstrated



$$p_1 + p_2 + p_3 = q$$

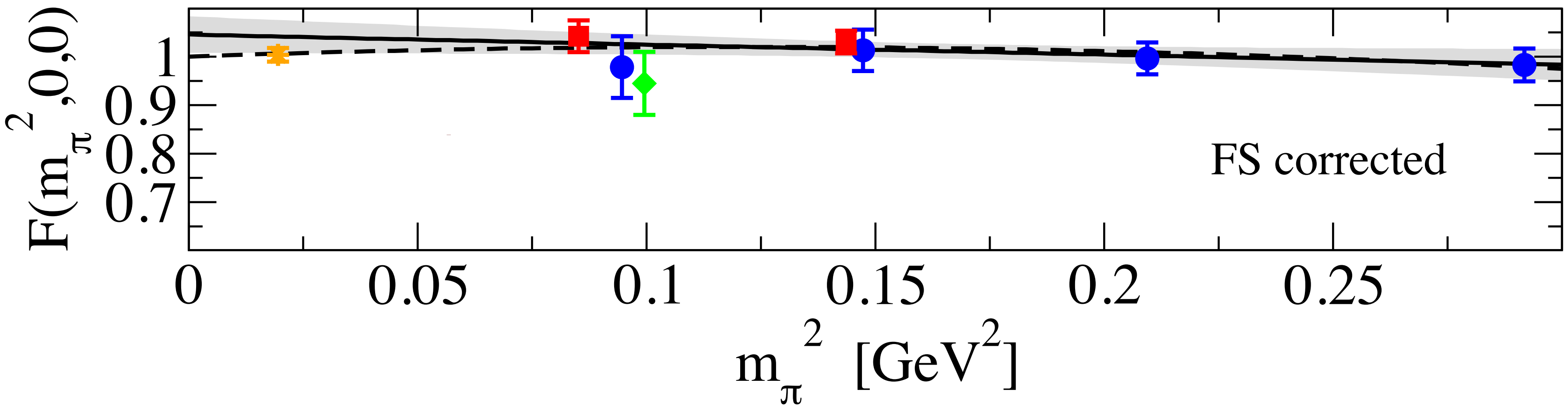
$\pi^0\gamma^*\gamma^*$ form factor

- Dominant contribution to a_μ^{HLbL} from π^0 exchange
- Theoretical estimates of incorporate π^0 exchange contribution modulated by the $\pi^0\gamma^*\gamma^*$ form factor and normalized to the $\pi^0\rightarrow\gamma\gamma$ decay width
    - As a simpler intermediate step, lattice calculations of $F_{\pi0\gamma\gamma}(k_1,k_1)$ and $\Gamma_{\pi0\gamma\gamma}$ can check these inputs to model calculations



- JLQCD recently published the first lattice-QCD result for on-shell $\pi^0\rightarrow\gamma\gamma$ decay with controlled errors $\Gamma_{\pi0\gamma\gamma} = 7.83(31)(49)$eV

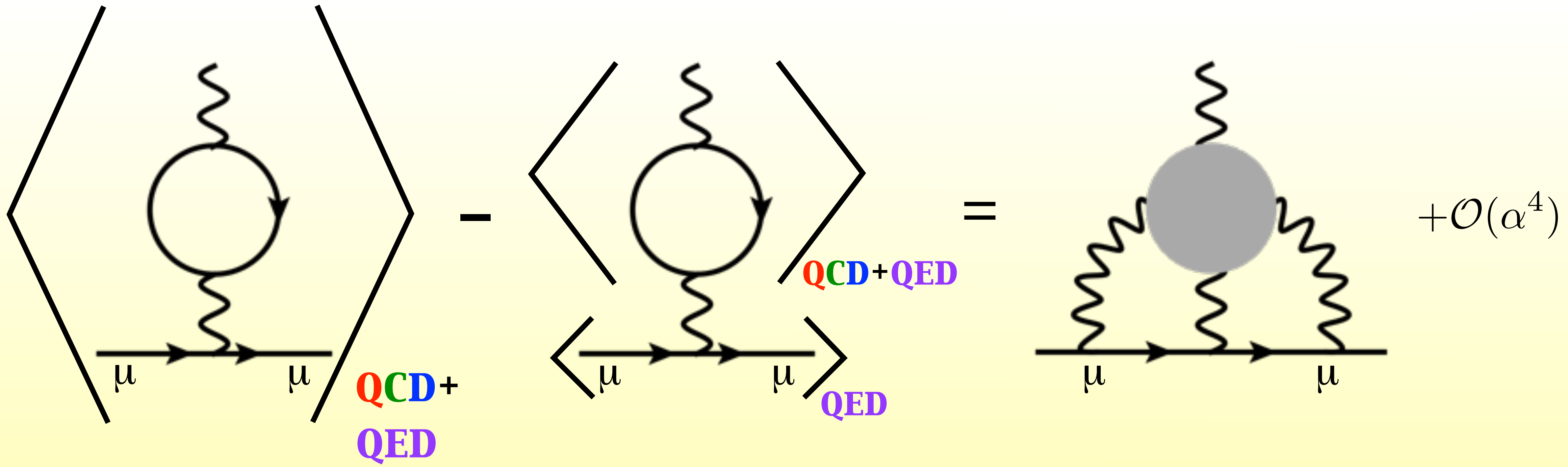
[Feng *et al.*, PRL 109 (2012) 182001]



- Consistent with PrimEx [PRL 106 (2011) 162303], but errors not yet competitive

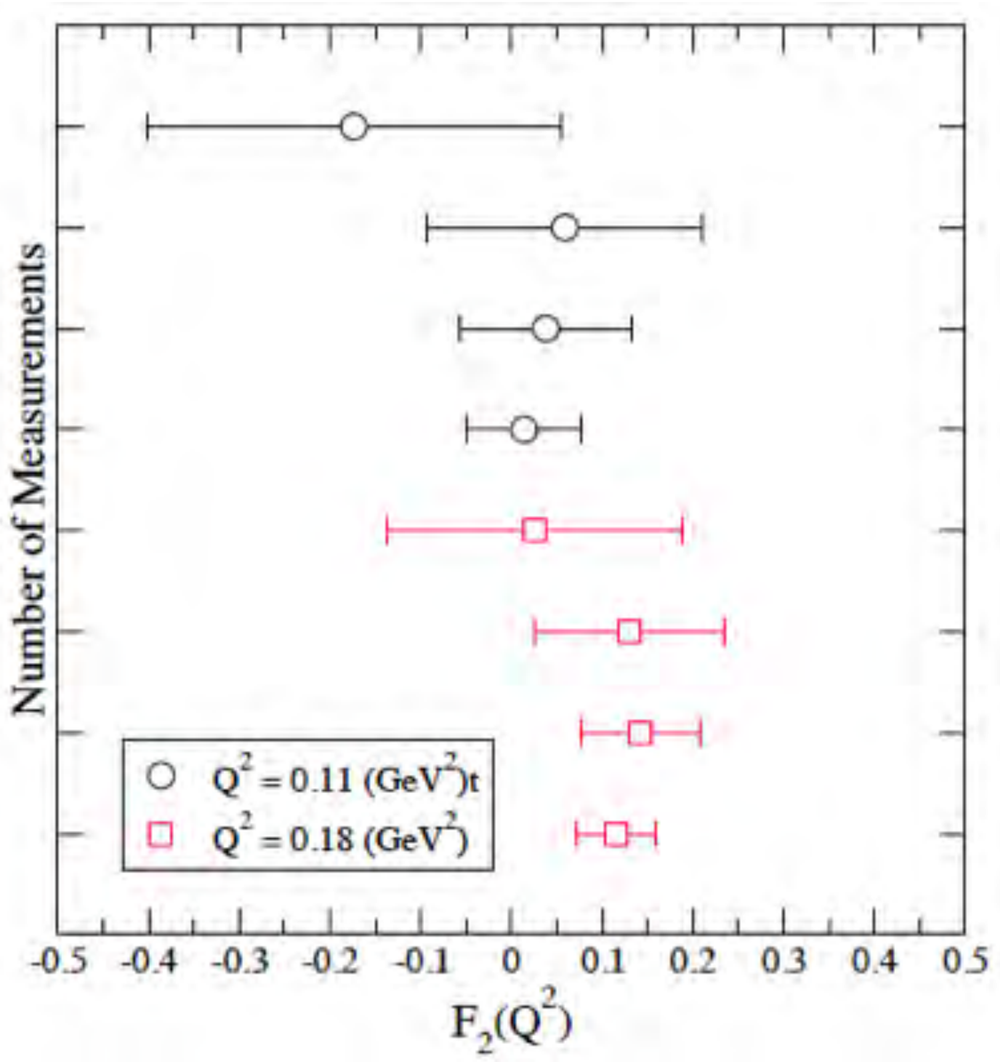
QCD + QED simulations

- Most promising method introduced by Blum and collaborators in which one computes the full hadronic amplitude, including the muon and photons, nonperturbatively [Hayakawa *et al.*, PoS LAT2005 (2006) 353]
- Treat photon field in parallel with gluon field and include in gauge link, so the simulation and analysis follows a conventional lattice-QCD calculation
- In practice, must insert a single valence photon connecting the muon line to the quark loop "by hand" into the correlation function, then perform correlated nonperturbative subtraction to remove the dominant O(α^2) contamination



Preliminary tests

- **Early results appear promising** [Blum *et al.*, PoS LATTICE2012 (2012) 022]
- Stable, statistically-significant signal emerging in the ballpark of model estimates



- a = 0.114 fm; V=$(24 \times a)^3$
- Q^2 = 0.11 and 0.18 GeV^2
- m_π = 329 MeV
- m_μ = 190 MeV
- $\alpha=1/4\pi$ to enhance signal

$a_\mu^{HLbL} = F_2(Q^2 \rightarrow 0) \times (\alpha/\pi)^3$

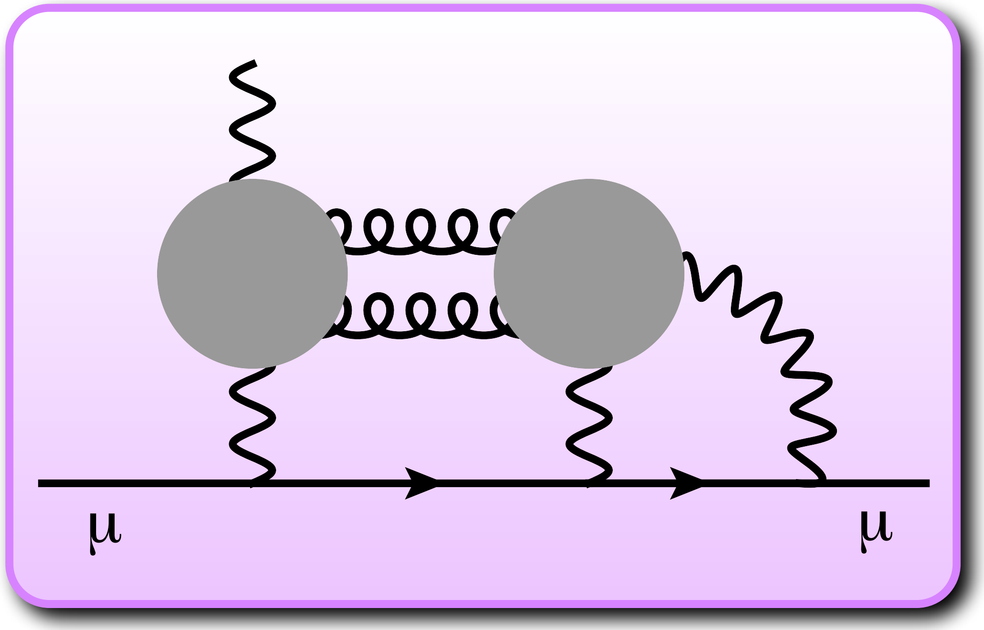
Other outstanding issues

(1) Finite-volume effects

- QED-only calculations suggest that errors due to the finite lattice size may be significant, but increased computing power is allowing the generation of larger lattices

(2) Quark-disconnected contributions

- Preliminary calculations work in the quenched approximation of QED, so contributions from diagrams with two quark loops only connected by a pair of gluons are not included
- Studying various approaches to include these such as directly simulating dynamical photons



(3) Chiral ($m_q \rightarrow m_q^{phys}$) and continuum ($a \rightarrow 0$) extrapolations

- New large-volume lattices being generated have close-to-physical pion masses

(4) Momentum extrapolation ($Q^2 \rightarrow 0$)

Still quite a bit of work to do...