OPE Methods for the Holomorphic Higgs Portal

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Based on P.K., Li, Poland, Stergiou (1401.7690)

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Outline

• Motivation & Introduction

• Basic Framework – Generalities

• Computations of Higgs Parameters

• Reproducing known Results – (Old Problems from a New Perspective)

• (New) Possibilities for Solutions

• Summary & Future Directions
• Discovery of a Higgs Particle @ 125 GeV

• Signal consistent with that of a SM-like Higgs
At Crossroads.....

- Electroweak-Tuned
  but “minimal”

- Electroweak-Natural
  but “elaborate”

Higgs @ ~125 GeV

More Data will Decide!

For concreteness, stay within the SUSY paradigm

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“Holomorphic” Higgs Portal

\[ W = \lambda_u H_u O_u + \lambda_d H_d O_d. \]

- \( H_u \) and \( H_d \) assumed to have dimension close to unity.
- \( \lambda_u, \lambda_d \) taken to be perturbative (taking into account dimensional dependence)
- But Extra sector (to which \( O_u \) and \( O_d \) belong) could be strongly coupled in general.
  -- superconformal in the UV, but develops a mass gap in the IR

Examples – “Single Sector Models” in Gauge mediation.
Messengers coupled to strong Dynamical SUSY (DSB) sector.

“Integrate-out” dynamics of the Extra sector
carry out the path integral over \( O_u \) and \( O_d \)

Azatov et al 1106.3646,1106.4815; Kitano et al 1206.4053; Stancato et al 0807.3961
Gherghetta, Pomarol 1107.4697; Komargodski, Seiberg 0812.3900; Craig et al 1302.2642,
Knapen et al 1311.7107, …
Goal: Study Effective Higgs Lagrangian from general perspective

This Work: To quadratic order \((p = 0)\),

Future: To quartic order

\[
\mathcal{L} = \mathcal{Z}_u F_{H_u}^\dagger F_{H_u} + \mathcal{Z}_d F_{H_d}^\dagger F_{H_d} + \left( \mu \int d^2 \theta \mathcal{H}_u \mathcal{H}_d + \text{c.c.} \right) - V_{\text{Higgs}}^{(\text{soft})} - V_{\text{Higgs}}^{(\text{other})},
\]

\[
V_{\text{Higgs}}^{(\text{soft})} = m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + (B_{\mu} H_u H_d + \text{c.c.}) + (A_{\mu} H_u F_{H_u}^\dagger + A_{d} H_d F_{H_d}^\dagger + \text{c.c.}),
\]

\[
V_{\text{Higgs}}^{(\text{other})} = (a_u' H_u F_{\mathcal{H}_d} + \text{c.c.}) + (a_d' F_{\mathcal{H}_u} H_d + \text{c.c.}) + (\gamma F_{\mathcal{H}_u} F_{\mathcal{H}_d} + \text{c.c.}).
\]

- Usual soft terms and \(\mu\) term.
- Terms in \(V_{\text{higgs}}^{(\text{other})}\) present in general as well.
  Example: Wrong Higgs Trilinears \(a_u', a_d'\).

If \(|F|/M^2 < 1\),

Can show that parameters in \(V_{\text{higgs}}^{(\text{other})}\) suppressed by powers of \(|F|/M^2\).

Distler, Robbins 0807.2006; Komargodski, Seiberg 0812.3900

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“Integrating out” procedure yields for $V_{\text{higgs}}^{\text{(soft)}}$:

$$
\mu = \frac{i}{8} \lambda_u \lambda_d \left\langle \int d^4 x e^{-ip \cdot x} Q^\alpha (O_u(x)) Q_\alpha (O_d(0)) \right\rangle \bigg|_{p \to 0},
$$

$$
B_\mu = \frac{i}{2^5} \lambda_u \lambda_d \left\langle \int d^4 x e^{-ip \cdot x} Q^2 (O_u(x)) Q^2 (O_d(0)) \right\rangle \bigg|_{p \to 0},
$$

$$
\delta A_{u,d} = -\frac{i}{8} |\lambda_{u,d}|^2 \left\langle \int d^4 x e^{-ip \cdot x} Q^2 [O_{u,d}(x) O_{u,d}^\dagger(0)] \right\rangle \bigg|_{p \to 0},
$$

$$
\delta m_{H_u,d}^2 = -\frac{i}{2^5} |\lambda_{u,d}|^2 \left\langle \int d^4 x e^{-ip \cdot x} Q^2 \bar{Q}^2 [O_{u,d}(x) O_{u,d}^\dagger(0)] \right\rangle \bigg|_{p \to 0}.
$$

Komargodski, Seiberg 0812.3900

Assumption: $\mu$ forbidden in the SUSY limit

Comments:

- Parameters defined in the zero-momentum limit

* $\mu, B_\mu, \text{-- “CHIRAL – CHIRAL” Two-point function}$

* $A_u, A_d, m_{H_u}^2, m_{H_d}^2 \text{-- “CHIRAL – ANTICHIRAL” Two-point function}$

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OPE Methods & Dispersion Relations

- Apply the Operator Product Expansion (OPE) to the two-point functions.
  \[ \mathcal{O}_i(x) \mathcal{O}_j(0) = \sum_k c_{ij}^k(x) \mathcal{O}_k(0), \]
  UV \hspace{1cm} IR

- Identify operators that can obtain vevs consistent with Poincare & Gauge invariance.
- Compute Wilson coefficients \( c_{ij}^k(x) \) or \( \sim C_{ij}^k(p^2) \) using superconformal symmetry.

  Analytically continue to large time-like momenta (physical region \( s' > 0 \)).

- Use
  \[
  A(s) = \frac{1}{2\pi i} \int_{s_0}^\infty ds' \text{Disc} \frac{A(s')}{s' - s} = \frac{1}{\pi} \int_{s_0}^\infty ds' \frac{\text{Im} A(s')}{s' - s}.
  \]

  to compute Wilson coefficients at any other \( s \) (even at \( s = 0 \))

Assumption:

*Branch-cut starting at some threshold \( s_0 = (2M)^2 \), no other singularities*
• Compute Chiral-Chiral OPE $O_u^* O_d$ and Chiral-Antichiral OPE $O_{u,d}^* O_{u,d}^\dagger$

• For $O_u^* O_d$ OPE to contain $O^I$ and $O_{u,d}^* O_{u,d}^\dagger$ OPE to contain $O^I$, need to compute

\[
\langle O_u O_d O^I \dagger \rangle \text{ and } \langle O_{u,d} O_{u,d}^\dagger O^I \dagger \rangle
\]

• 3-point functions for primary superfields computed for a general $\mathcal{N}=1$, 4D Superconformal Field Theory (SCFT) in

Osborn hep-th/9808041; Poland, Simmons-Duffin 1009.2087

RESULTS

$B_\mu$:

\[
B_\mu = \frac{i}{2}\lambda_u \lambda_d \left< \int d^4x e^{-ip \cdot x} Q^2(O_u(x))Q^2(O_d(0)) \right|_{p \to 0},
\]

\[
Q^2O_u(x)Q^2O_d(0) = \sum_i c_i B_\mu Q^2O_{3,0;i}(0), \quad c_i B_\mu = -2\lambda u \lambda_d \Delta O_{3,0;i} - \Delta O_u - \Delta O_d - 1,
\]

Only one type of operator : $Q^2(O_{3,0})$ contributes.
\[ \mu = \frac{i}{8} \lambda_u \lambda_d \left\langle \int d^4x e^{-ipx} Q^\alpha(O_u(x))Q_\alpha(O_d(0)) \right\rangle \bigg|_{p \to 0}, \]

\[ Q^\alpha O_u(x)Q_\alpha O_d(0) = c_{\mu;1} Q^2 O_{1,0}(0) + c_{\mu;2} Q^\alpha O_{2,1 \alpha}(0) \]

\[ + \sum_i (c_{\mu;3}^i O_{3,0;i}(0) + c_{\mu;4}^i [Q^2 Q^2 O_{3,0;i}]_P(0) + c_{\mu;5}^i [Q O_\mu Q O_{3,1;i}]_P(0)) + \cdots, \]

\[ c_{\mu;1} = 2^4 \frac{\lambda_u \lambda_d O_{o,0} O_{1,0}}{C_{O_{1,0}}} \frac{\Delta O_u \Delta O_d}{(\Delta O_u + \Delta O_d)(\Delta O_u + \Delta O_d - 1)}, \quad c_{\mu;2} = \frac{\lambda_u \lambda_d O_{o,1} O_{2,1}}{C_{O_{2,1}}} \frac{\Delta O_u - \Delta O_d}{\Delta O_u + \Delta O_d - 2}, \]

\[ c_{\mu;3}^i = \tilde{c}_{\mu;3}^i \Delta O_{3,0;i} - \Delta O_u - \Delta O_d - 1, \quad c_{\mu;4}^i = \tilde{c}_{\mu;4}^i \Delta O_{3,0;i} - \Delta O_u - \Delta O_d + 1, \quad c_{\mu;5}^i = \tilde{c}_{\mu;5}^i \Delta O_{3,1;i} - \Delta O_u - \Delta O_d. \]

- **Operators appearing**: \( Q^2(O_{1,0}), Q^\alpha(O_{2,1 \alpha}), O_{3,0}, Q^2 Q^2 O_{3,0} \), \( Q \sigma^\mu Q(O_{3,1 \mu}) \)

- **Dominant contribution**: \( O_{3,0} \)

Similarly, \( A_{u,d} : Q^2(O^{u,d}_{o}); \) \& \( \delta m^2_{Hu,d} : Q^2 Q^2(O^{u,d}_{o}) \)

**Important**: Different Operators contribute to different parameters.
Old problems from a New perspective

- Reproduce $\mu/ \, B\mu$ problem of weakly coupled models from this formalism.

  -- in particular original Model studied in Dvali et al hep-ph/9603238

\[ \mu : \]
\[ i \int d^4x e^{-ipx} Q^\alpha(\Phi_1 \Phi_2(x)) Q_\alpha(\bar{\Phi}_1 \bar{\Phi}_2(0)) = \tilde{c}_{X+F}(s) X^\dagger F^\dagger(0) + \cdots, \quad s = -p^2. \]

Higher-order contributions -- $X^* \, F^* \, (X^* \, X)^n \, (F^* \, F)^m$

\[ \tilde{c}_{X+F}(s) = \frac{(\lambda*)^2 \, 1}{2 \pi^2} \ln \frac{-s}{\mu^2} + \cdots, \]

\[ \mu \approx -\frac{1}{2} \frac{\lambda_u \lambda_d \lambda*}{16 \pi^2 \lambda} \, F^\dagger \]

Compute contributions from higher orders $X^* \, F^* \, (X^* \, X)^n \, (F^* \, F)^m$ – resum them

Result matches precisely!  Similarly for $B\mu$, $A_{u,d}$, $M^2_{hu,d}$
(New) Possibilities for Solutions

**Basic Idea:**
1) Different Operators contribute to different Higgs Parameters.
2) Their vevs determined by non-trivial IR dynamics

-- “simple” parametrics may not hold.

**Possibilities:**

i) EWSB with $B\mu/\mu^2 < \sim 1$ & $m_{H_u,d}^2 / A_{u,d}^2 < \sim 1$

\[
\frac{\langle Q^2 O_{3,0} \rangle}{|\langle O_{3,0} \rangle|^2} \sim \frac{|c_\mu|^2}{c_{B\mu}} ; \quad \frac{\langle Q^2 \bar{Q}^2 O^u_0 \rangle}{|\langle Q^2 O^u_0 \rangle|^2} \sim \frac{|c_{A_{u,d}}|^2}{c_{m_{H_u,d}}^2}
\]

ii) EWSB with “Lopsided Gauge Mediation”    
*Csaki et al 0809.4492; DeSimone et al 1103.6033; Schafer-Nameki et al 1005.0841*

\[
c_{m_{H_u}}^2 \langle Q^2 \bar{Q}^2 O^u_0 \rangle \ll c_{B\mu} \langle Q^2 O_{3,0} \rangle \ll c_{m_{H_d}}^2 \langle Q^2 \bar{Q}^2 O^d_0 \rangle
\]

iii) Conformal Sequestering:    *Luty, Sundrum hep-th/0105137; hep-th/0111231*
*Schmaltz, Sundrum th/0608051; Roy, Schmaltz. 0708.3593; Murayama, Nomura, Poland 0709.0775*
Summary

• Developed a general and systematic formalism to compute parameters in the Higgs Lagrangian, when Higgs couples to another sector via the superpotential

\[ W = \lambda_u H_u O_u + \lambda_d H_d O_d \]

• Valid even when extra sector strongly coupled in the IR.
  -- superconformal in the UV, mass gap in IR.

OPE methods provide a powerful organizing principle

-- Deeper insight into problems of SUSY breaking & mediation.
-- Possibilities for various solutions of these problems, some qualitatively new, in the sense that cannot be seen in the spurion limit.
-- All of them can be described by the OPE formalism

Future

• Apply results to realistic models of dynamical SUSY and mediation and compute Wilson coefficients and Higgs parameters explicitly.

• Study Quartic terms in the Higgs Potential - requires more advanced techniques