Taming the Goldstone contributions to the Standard Model Higgs effective potential

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The Standard Model effective potential can be used to relate the VEV of the Higgs field to the Lagrangian parameters:

- λ = Higgs self-coupling
- m^2 = Higgs (mass)^2 parameter
- g_3, g, g', y_t, etc. = gauge and Yukawa couplings

Other uses include testing stability of the vacuum, characterize phase transitions.

Some references:

- Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.
- Sher, Phys. Rept. 179 (1989) 273
- Ford, Jack, Jones, Nucl. Phys. B387 (1992) 373, `hep-ph/0111190`
  2-loop Standard Model effective potential
- SPM, `hep-ph/0111209`
  2-loop effective potential for general theories, including SUSY
- SPM, `1310.7553`
  3-loop Standard Model effective potential, $g_3^4 y_t^4$ and $g_3^2 y_t^6$ and y_t^8 terms

Tree-level Higgs potential:

$$V(\phi) \quad = \quad \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

Tree-level minimum at

$$m^2 + \lambda\phi^2 \quad = \quad 0$$

Include radiative corrections from effective potential in loop expansion:

$$\Delta V_{\rm eff} \quad = \quad \sum_{\ell} \frac{1}{(16\pi^2)^{\ell}} V^{(\ell)} \;=\; \text{sum of 1PI vacuum diagrams}$$

Then

$$m^2 + \lambda\phi^2 \quad = \quad -\frac{1}{\phi}\frac{\partial}{\partial\phi}\Delta V_{\rm eff}.$$

The radiative corrections to the effective potential are usually computed in Landau gauge because it is easier.

The tree-level field-dependent masses are:

$$
\begin{aligned}
H &= m_H^2 = m^2 + 3\lambda\phi^2, \\
G &= m_{G^0}^2 = m_{G^\pm}^2 = m^2 + \lambda\phi^2 \qquad \text{(Goldstone bosons)}, \\
T &= m_t^2 = y_t^2\phi^2/2, \\
W &= m_W^2 = g^2\phi^2/4, \qquad Z = m_Z^2 = (g^2 + g'^2)\phi^2/4.
\end{aligned}
$$

At 1-loop order, Landau gauge, $\overline{\text{MS}}$ scheme:

$$V^{(1)} = f(H) + 3f(G) - 12f(T) + 6f(W) + W^2 + 3f(Z) + \frac{1}{2}Z^2$$

where

$$f(x) \equiv \frac{x^2}{4}\left(\overline{\ln}(x) - 3/2\right)$$

with

$$\overline{\ln}(x) \equiv \ln(x/Q^2), \qquad Q = \text{renormalization scale.}$$

Two related problems of principle associated with the Goldstone boson contributions:

1) G is negative, so $V_{\rm eff}$ is complex, due to $\overline{\ln}(G)$.

   Usually, a complex $V_{\rm eff}$ means an instability, but there is no physical instability here. Traditionally, this unphysical imaginary part of $V_{\rm eff}$ is just dropped.

2) Higher loop contributions become increasingly singular as $G \rightarrow 0$.

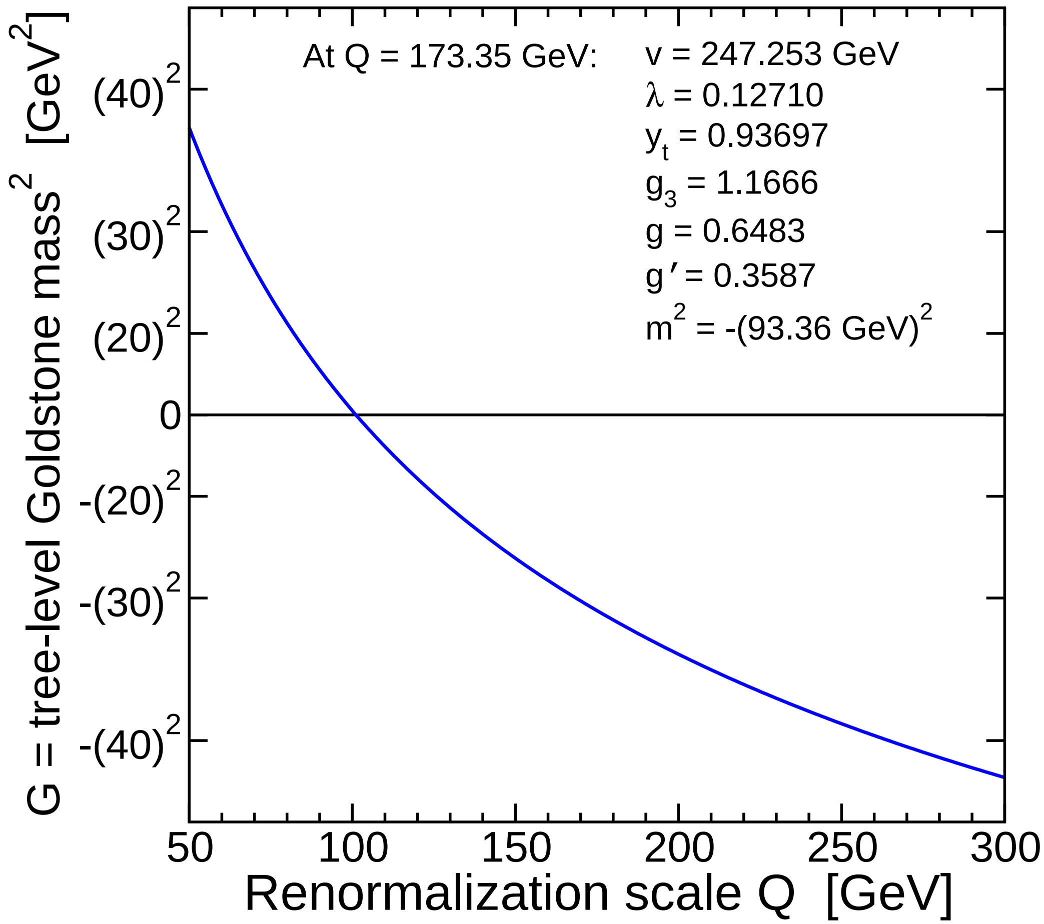
   Not numerically important unless you are unlucky. . .

   However, “unlucky” can mean “too clever for your own good”.

Both problems can be eliminated by resummation of the leading Goldstone boson contributions, as I will explain.

Goldstone boson tree-level $(\text{mass})^2$

G as a function of renormalization scale Q, at minimum of $V_{\rm eff}$:



For $Q \gtrsim 100$ GeV, we have $G < 0$.

The Goldstone Boson Catastrophe

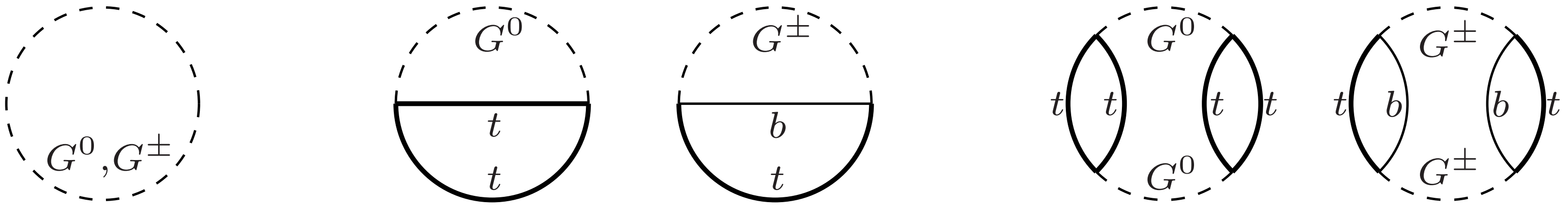
The leading behavior as $G \to 0$ is:

$$V^{(1)} \sim \frac{3}{4} G^2 \,\overline{\ln} G,$$ 2nd derivative singular as $G \to 0$

$$V^{(2)} \sim -3 N_c y_t^2 T \left[\overline{\ln} T - 1\right] G \,\overline{\ln} G,$$ 1st derivative singular as $G \to 0$

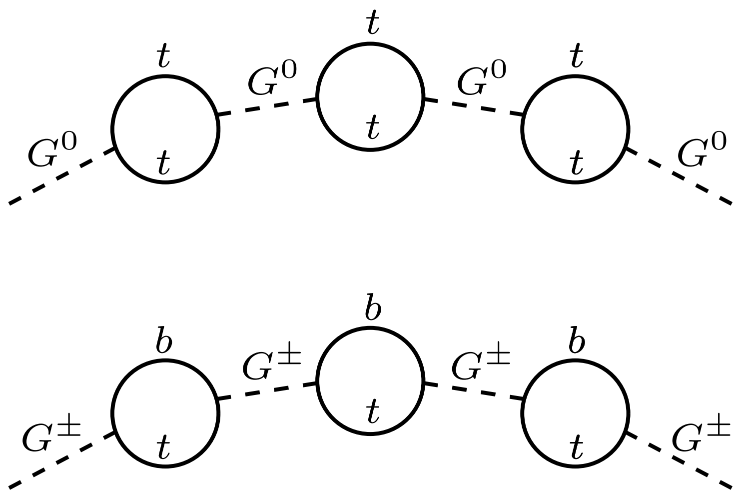
$$V^{(3)} \sim 3 \left[N_c y_t^2 T (\overline{\ln} T - 1)\right]^2 \overline{\ln} G.$$ singular as $G \to 0$

with $T, G =$ squared masses of top, Goldstone. These come from diagrams:



At higher loop orders, the $G \to 0$ singularities get worse...

From ℓ-loop diagrams with $\ell - 1$ top or top/bottom one-loop subdiagrams:



For $\ell \geq 4$,
power-law singularity as $G \to 0$.

$$V^{(\ell)} \sim (N_c y_t^2)^{\ell-1} T^2 \left(\frac{T}{G}\right)^{\ell-3}$$

For a generic choice of renormalization scale, $G \neq 0$, and there is no true singularity.

However, one might have expected that choosing Q so that $G = 0$ would be a *good* choice, since, at all orders, the Goldstone is massless in Landau gauge. (In the Standard Model, $G = 0$ at roughly $Q = 100$ GeV.)

This is the one choice you must <u>not</u> make!

A parable: consider the function that arises in the quantity $\frac{1}{\phi}\frac{\partial V^{(1)}}{\partial \phi}$:

$$A(x) \quad = \quad x\overline{\ln}(x) - x.$$

Taylor expand it (corresponds to the loop expansion):

$$\begin{aligned} A(G+\delta) \quad &= \quad A(G) + \delta A'(G) + \frac{\delta^2}{2} A''(G) + \frac{\delta^3}{6} A'''(G) + \ldots \\ &= \quad [G\overline{\ln}(G) - G] + \delta\overline{\ln}(G) + \frac{\delta^2}{2G} - \frac{\delta^3}{6G^2} + \ldots \end{aligned}$$

At the minimum of the potential,

$$\delta \;=\; \frac{1}{\phi}\frac{\partial V^{(1)}}{\partial \phi} \;=\; -G,$$

so the perturbative contribution to the V_{eff} from Goldstone loops punctuated by heavy particle loops is proportional to:

$$\begin{aligned} A(0) \quad &= \quad [G\ln(G) - G] + [-G\ln(G)] + \frac{G}{2} + \frac{G}{6} + \frac{G}{24} + \ldots \\ &= \quad -\frac{G}{n} \qquad \text{if stopped after } n\text{th order term (loop order } \ell = n+1) \end{aligned}$$

However, the correct, resummed, value of $A(0)$ is just 0.

The sum of all diagrams with Goldstone bosons punctuated by heavy particle loops gives the leading order in $G \to 0$.

The correct, resummed contribution to the $V_{\rm eff}$ minimization condition from these diagrams is, at leading order in $G \to 0$:

$$\frac{1}{\phi}\frac{\partial V_{\rm eff}}{\partial \phi} \;=\; \frac{3\lambda}{16\pi^2} A(0) \;=\; 0.$$

However, the perturbative expansion of the same quantity, as found in the standard calculation, and verified explicitly to 3-loop order, is:

$$\frac{1}{\phi}\frac{\partial V_{\rm eff}}{\partial \phi} \;=\; \frac{3\lambda}{16\pi^2}\left[A(G) + \delta A'(G) + \frac{\delta^2}{2} A''(G) + \ldots\right]$$

where

$$\delta = \frac{1}{16\pi^2}\left(3\lambda A(H) - 6y_t^2 A(T) + g^2\left[\frac{3}{2}A(W) + W\right] + \frac{g^2 + g'^2}{2}\left[\frac{3}{2}A(Z) + Z\right]\right)$$

To resum: subtract the red expression from the usual perturbative calculation.

Result for the condition of minimization of the Landau gauge $\overline{\text{MS}}$ effective potential, after this resummation:

$$0 = m^2 + \lambda\phi^2 + \frac{1}{16\pi^2}\delta_1 + \frac{1}{(16\pi^2)^2}\delta_2 + \frac{1}{(16\pi^2)^3}\delta_3 + \ldots$$

where:

$$\begin{aligned}
\delta_1 &= \cancel{3\lambda A(G)} + 3\lambda A(H) - 6y_t^2 A(T) + g^2\left[\frac{3}{2}A(W) + W\right] \\
&\quad + \frac{g^2 + g'^2}{2}\left[\frac{3}{2}A(Z) + Z\right] \\
\delta_2 &= \big(\text{doesn't depend on } G\big) \\
\delta_3 &= \big(\text{doesn't depend on } G\big)
\end{aligned}$$

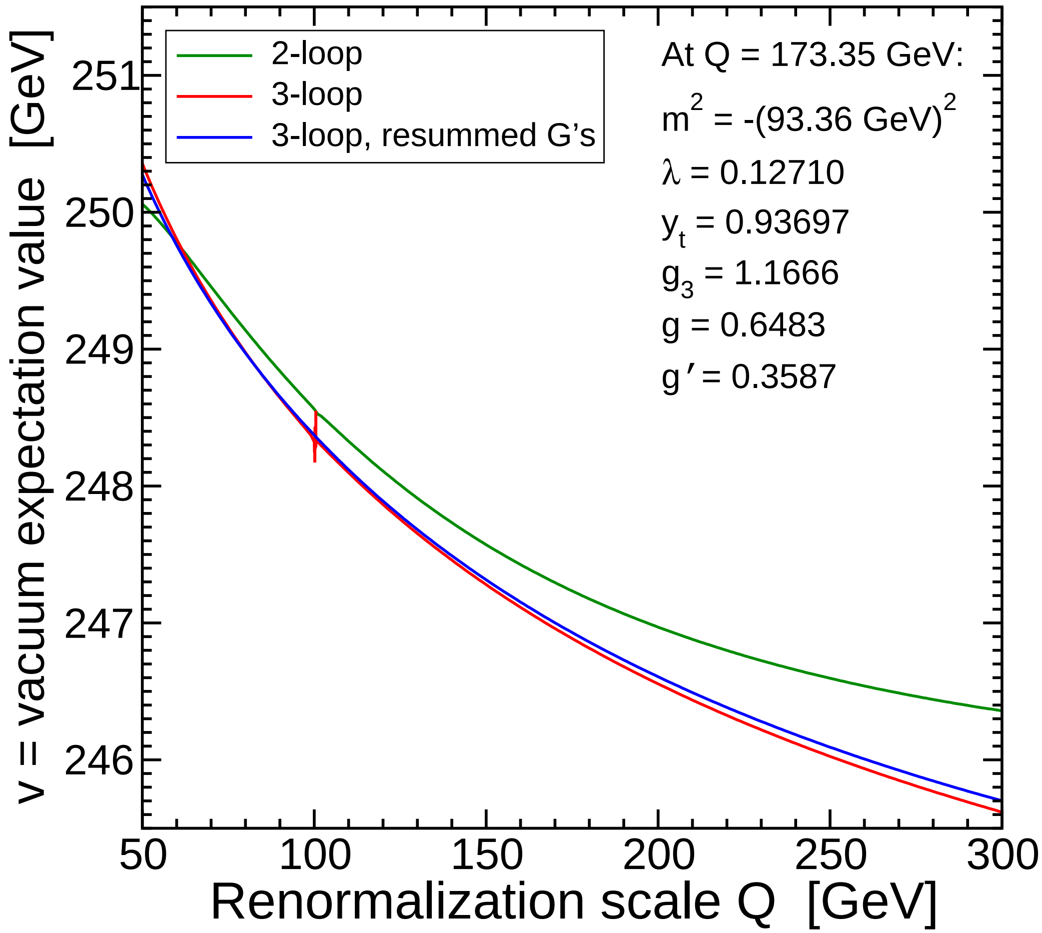
See forthcoming paper for details and the explicit results.

Trivially, no more problems from negative G, or from $G \to 0$.

Future computer code implementing this, with David G. Robertson.

Numerical impact is very small, *unless* you carefully choose Q so that $G = 0$.

Run Lagrangian parameters to scale Q, find VEV $v(Q)$.



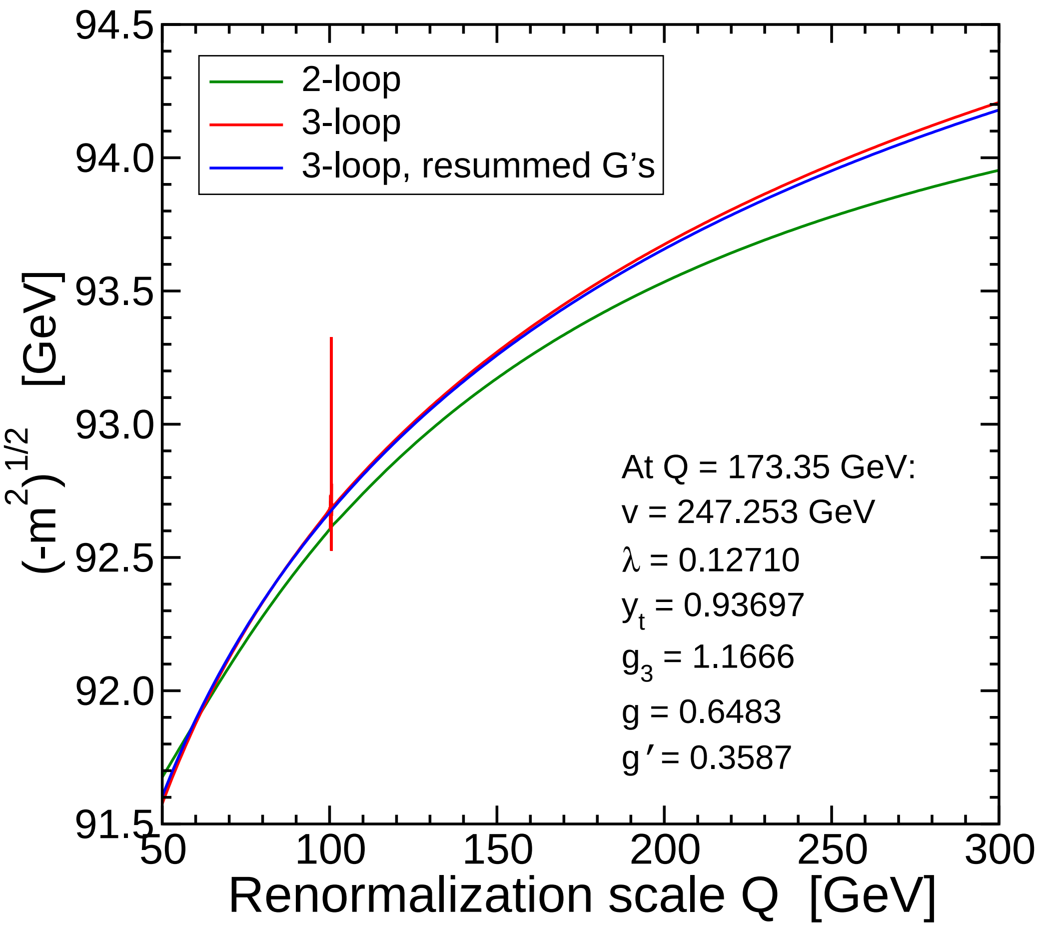
2-loop = Ford, Jack, Jones hep-ph/0111190

3-loop = SPM 1310.7553

3-loop, leading Goldstone contributions resummed, no G in minimization condition

Alternate view: run VEV $v(Q)$ and Lagrangian parameters other than m^2 to scale Q, then find m^2 that minimizes the potential.

Figure shows resulting $\sqrt{-m^2}$ as a function of Q:



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3-loop, leading Goldstone contributions resummed, no G in minimization condition

Outlook

- After resumming contributions from Goldstone loops, eliminate problems of principle with $G < 0$ (spurious imaginary part) and $G \to 0$ (spurious singularities) in the Landau gauge effective potential.
- Numerical impact very small, unless Q is chosen so that G is very close to 0.
- Why bother? Resummed result is actually easier to implement than the usual effective potential minimization. Besides, we should be attempting to make all theory errors **ridiculously** small, so that all uncertainties can be reliably blamed on experimentalists.
- Same resummation method is useful for multi-loop calculation of m_H.