Taming the Goldstone contributions to the Standard Model Higgs effective potential

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The Standard Model effective potential can be used to relate the VEV of the Higgs field to the Lagrangian parameters:

- $\lambda$ = Higgs self-coupling
- $m^2$ = Higgs (mass)$^2$ parameter
- $g_3$, $g$, $g'$, $y_t$, etc. = gauge and Yukawa couplings

Other uses include testing stability of the vacuum, characterize phase transitions.

Some references:
- Sher, Phys. Rept. 179 (1989) 273
  2-loop Standard Model effective potential
- SPM, hep-ph/0111209
  2-loop effective potential for general theories, including SUSY
- SPM, 1310.7553
  3-loop Standard Model effective potential, $g_3^4 y_t^4$ and $g_3^2 y_t^6$ and $y_t^8$ terms
Tree-level Higgs potential:

\[ V(\phi) = \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 \]

Tree-level minimum at

\[ m^2 + \lambda \phi^2 = 0 \]

Include radiative corrections from effective potential in loop expansion:

\[ \Delta V_{\text{eff}} = \sum_{\ell} \frac{1}{(16\pi^2)^{\ell}} V^{(\ell)} = \text{sum of 1PI vacuum diagrams} \]

Then

\[ m^2 + \lambda \phi^2 = -\frac{1}{\phi} \frac{\partial}{\partial \phi} \Delta V_{\text{eff}}. \]

The radiative corrections to the effective potential are usually computed in Landau gauge because it is easier.
The tree-level field-dependent masses are:

\[
\begin{align*}
H &= m_H^2 = m^2 + 3\lambda\phi^2, \\
G &= m_{G^0}^2 = m_{G^\pm}^2 = m^2 + \lambda\phi^2 & \text{(Goldstone bosons)}, \\
T &= m_t^2 = y_t^2\phi^2/2, \\
W &= m_W^2 = g^2\phi^2/4, \\
Z &= m_Z^2 = (g^2 + g'^2)\phi^2/4.
\end{align*}
\]

At 1-loop order, Landau gauge, \( \overline{\text{MS}} \) scheme:

\[
V^{(1)} = f(H) + 3f(G) - 12f(T) + 6f(W) + W^2 + 3f(Z) + \frac{1}{2}Z^2
\]

where

\[
f(x) \equiv \frac{x^2}{4} \left( \overline{\ln}(x) - 3/2 \right)
\]

with

\[
\overline{\ln}(x) \equiv \ln(x/Q^2), \quad Q = \text{renormalization scale}.
\]
Two related problems of principle associated with the Goldstone boson contributions:

1) $G$ is negative, so $V_{\text{eff}}$ is complex, due to $\ln(G')$.

   Usually, a complex $V_{\text{eff}}$ means an instability, but there is no physical instability here. Traditionally, this unphysical imaginary part of $V_{\text{eff}}$ is just dropped.

2) Higher loop contributions become increasingly singular as $G \to 0$.

   Not numerically important unless you are unlucky...

   However, “unlucky” can mean “too clever for your own good”.

Both problems can be eliminated by resummation of the leading Goldstone boson contributions, as I will explain.
Goldstone boson tree-level (mass)$^2$

$G$ as a function of renormalization scale $Q$, at minimum of $V_{\text{eff}}$:

$$G = \text{tree-level Goldstone mass}^2$$

At $Q = 173.35$ GeV:
- $v = 247.253$ GeV
- $\lambda = 0.12710$
- $y_t = 0.93697$
- $g_3 = 1.1666$
- $g = 0.6483$
- $g' = 0.3587$
- $m^2 = -(93.36 \text{ GeV})^2$

For $Q \gtrsim 100$ GeV, we have $G < 0$. 
The Goldstone Boson Catastrophe

The leading behavior as $G \to 0$ is:

$V^{(1)} \sim \frac{3}{4} G^2 \ln G,$ \hspace{1cm} 2nd derivative singular as $G \to 0$

$V^{(2)} \sim -3 N_c y_t^2 T [\ln T - 1] G \ln G,$ \hspace{1cm} 1st derivative singular as $G \to 0$

$V^{(3)} \sim 3 [N_c y_t^2 T (\ln T - 1)]^2 \ln G.$ \hspace{1cm} singular as $G \to 0$

with $T, G =$ squared masses of top, Goldstone. These come from diagrams:

At higher loop orders, the $G \to 0$ singularities get worse...
From $\ell$-loop diagrams with $\ell - 1$ top or top/bottom one-loop subdiagrams:

For $\ell \geq 4$, power-law singularity as $G \to 0$.

$$V^{(\ell)} \sim (N_c y_t^2)^{\ell-1} T^2 \left( \frac{T}{G} \right)^{\ell-3}$$

For a generic choice of renormalization scale, $G \neq 0$, and there is no true singularity.

However, one might have expected that choosing $Q$ so that $G = 0$ would be a good choice, since, at all orders, the Goldstone is massless in Landau gauge. (In the Standard Model, $G = 0$ at roughly $Q = 100$ GeV.)

This is the one choice you must not make!
A parable: consider the function that arises in the quantity $\frac{1}{\phi} \frac{\partial V^{(1)}}{\partial \phi}$:

$$A(x) = x \ln(x) - x.$$ 

Taylor expand it (corresponds to the loop expansion):

$$A(G + \delta) = A(G) + \delta A'(G) + \frac{\delta^2}{2} A''(G) + \frac{\delta^3}{6} A'''(G) + \ldots$$

$$= [G \ln(G) - G] + \delta \ln(G) + \frac{\delta^2}{2G} - \frac{\delta^3}{6G^2} + \ldots$$

At the minimum of the potential,

$$\delta = \frac{1}{\phi} \frac{\partial V^{(1)}}{\partial \phi} = -G,$$

so the perturbative contribution to the $V_{\text{eff}}$ from Goldstone loops punctuated by heavy particle loops is proportional to:

$$A(0) = [G \ln(G) - G] + [-G \ln(G')] + \frac{G}{2} + \frac{G}{6} + \frac{G}{24} + \ldots$$

$$= -\frac{G}{n} \quad \text{if stopped after } n\text{th order term (loop order } \ell = n + 1)$$

However, the correct, resummed, value of $A(0)$ is just 0.
The sum of all diagrams with Goldstone bosons punctuated by heavy particle loops gives the leading order in $G \to 0$.

The correct, resummed contribution to the $V_{\text{eff}}$ minimization condition from these diagrams is, at leading order in $G \to 0$:

$$\frac{1}{\phi} \frac{\partial V_{\text{eff}}}{\partial \phi} = \frac{3\lambda}{16\pi^2} A(0) = 0.$$

However, the perturbative expansion of the same quantity, as found in the standard calculation, and verified explicitly to 3-loop order, is:

$$\frac{1}{\phi} \frac{\partial V_{\text{eff}}}{\partial \phi} = \frac{3\lambda}{16\pi^2} \left[ A(G) + \delta A'(G) + \frac{\delta^2}{2} A''(G) + \ldots \right]$$

where

$$\delta = \frac{1}{16\pi^2} \left( 3\lambda A(H) - 6y_t^2 A(T) + g^2 \left[ \frac{3}{2} A(W) + W \right] + \frac{g^2 + g'^2}{2} \left[ \frac{3}{2} A(Z) + Z \right] \right)$$

To resum: subtract the red expression from the usual perturbative calculation.
Result for the condition of minimization of the Landau gauge $\overline{MS}$ effective potential, after this resummation:

\[ 0 = m^2 + \lambda \phi^2 + \frac{1}{16\pi^2} \delta_1 + \frac{1}{(16\pi^2)^2} \delta_2 + \frac{1}{(16\pi^2)^3} \delta_3 + \ldots \]

where:

\[ \delta_1 = 3\lambda A(G) + 3\lambda A(H) - 6y_t^2 A(T) + g^2 \left[ \frac{3}{2} A(W) + W \right] + g^2 + g' \left[ \frac{3}{2} A(Z) + Z \right] \]

\[ \delta_2 = (\text{doesn’t depend on } G) \]

\[ \delta_3 = (\text{doesn’t depend on } G) \]

See forthcoming paper for details and the explicit results.

Trivially, no more problems from negative $G$, or from $G \to 0$.

Future computer code implementing this, with David G. Robertson.
Numerical impact is very small, unless you carefully choose $Q$ so that $G = 0$.

Run Lagrangian parameters to scale $Q$, find VEV $v(Q)$.

At $Q = 173.35$ GeV:

- $m^2 = -(93.36 \text{ GeV})^2$
- $\lambda = 0.12710$
- $y_t = 0.93697$
- $g_3 = 1.1666$
- $g = 0.6483$
- $g' = 0.3587$

2-loop = Ford, Jack, Jones

hep-ph/0111190

3-loop, leading Goldstone contributions resummed, no $G$ in minimization condition

3-loop = SPM 1310.7553
Alternate view: run VEV \( v(Q) \) and Lagrangian parameters other than \( m^2 \) to scale \( Q \), then find \( m^2 \) that minimizes the potential.

Figure shows resulting \( \sqrt{-m^2} \) as a function of \( Q \):

\[ \begin{align*} 
\text{At } Q = 173.35 \text{ GeV:} \\
\nu &= 247.253 \text{ GeV} \\
\lambda &= 0.12710 \\
\gamma_t &= 0.93697 \\
g_3 &= 1.1666 \\
g &= 0.6483 \\
g' &= 0.3587 
\end{align*} \]

2-loop = Ford, Jack, Jones
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3-loop = SPM 1310.7553

3-loop, leading Goldstone contributions resummed, no \( G \) in minimization condition
Outlook

- After resumming contributions from Goldstone loops, eliminate problems of principle with $G < 0$ (spurious imaginary part) and $G \rightarrow 0$ (spurious singularities) in the Landau gauge effective potential.

- Numerical impact very small, unless $Q$ is chosen so that $G$ is very close to 0.

- Why bother? Resummed result is actually easier to implement than the usual effective potential minimization. Besides, we should be attempting to make all theory errors **ridiculously** small, so that all uncertainties can be reliably blamed on experimentalists.

- Same resummation method is useful for multi-loop calculation of $m_H$. 