Accommodating θ_{13} within SU(5)

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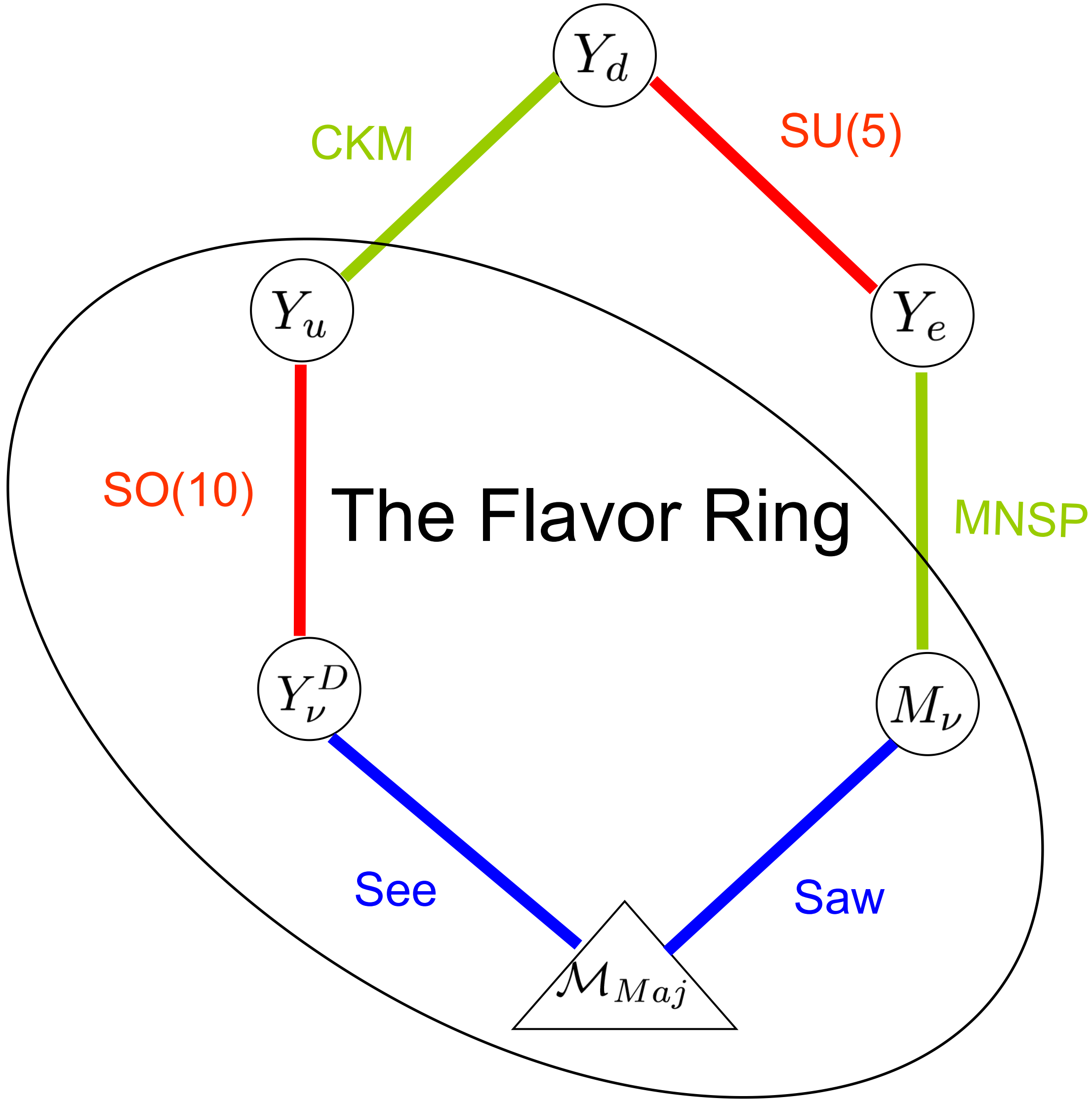
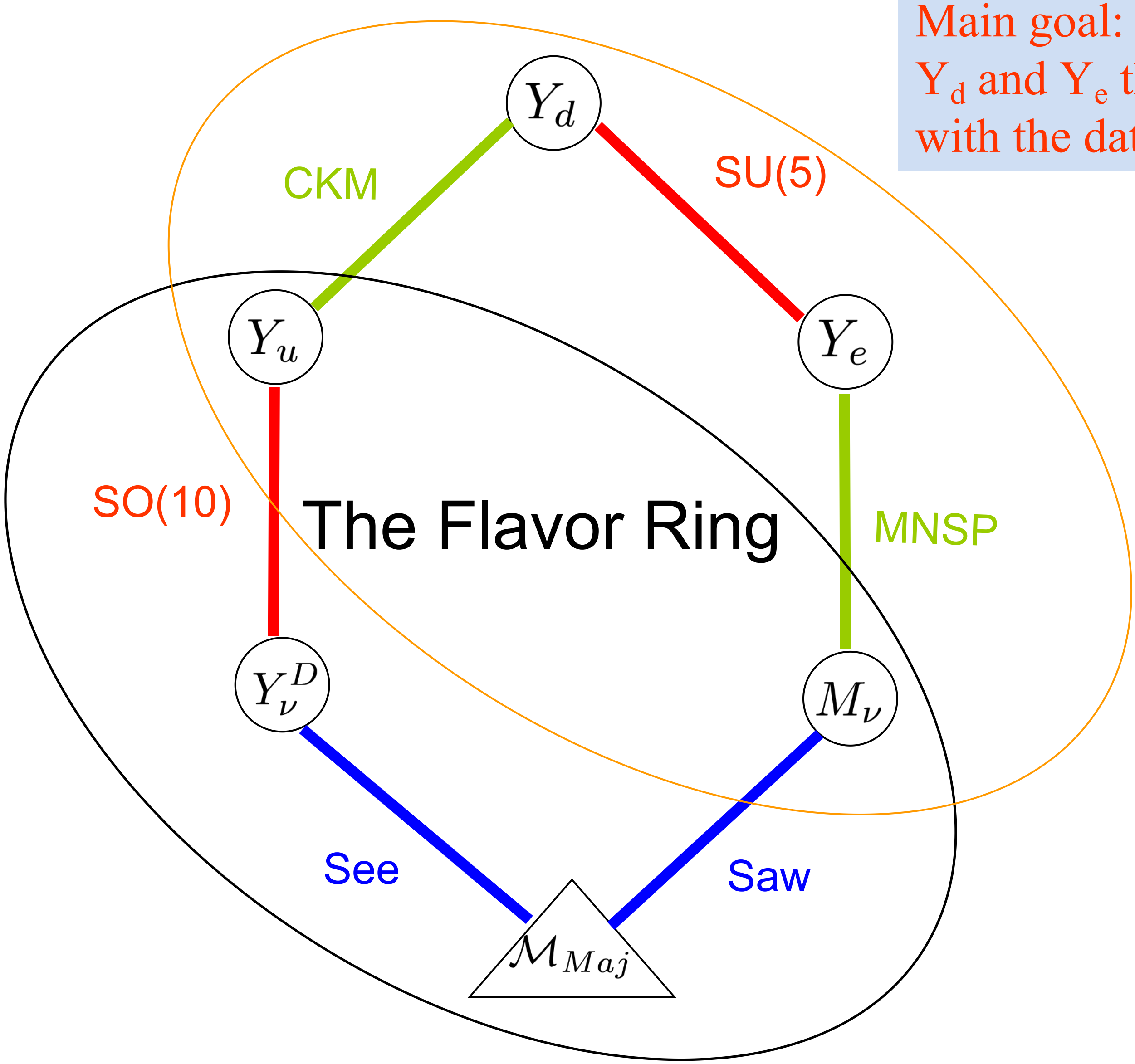


Y_d
CKM
SU(5)
Y_u
Y_e
SO(10)
The Flavor Ring
MNSP
Y_ν^D
M_ν
See
Saw
$\mathcal{M}_{Maj}$



Main goal: find out Y_d and Y_e that fit with the data!

Leptonic Mixings

Measured Lepton mixing angles

$$\theta_{12} \sim 34^\circ$$

$$\theta_{23} \sim 42^\circ$$

$$\theta_{13} \sim 9^\circ$$

= Corrections +

Leading order contributions:
e.g. Tri-bimaximal (TBM) matrix

$$\theta_{12}^{TBM} = 35.26^\circ$$

$$\theta_{23}^{TBM} = 45^\circ$$

$$\theta_{13}^{TBM} = 0^\circ$$

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$Y_e = U_e D_e V_e^\dagger \qquad M_\nu = U_\nu D_\nu U_\nu^T$$

$$U_{\rm MNSP} = U_e^\dagger U_\nu$$

$\theta_{13}^{\rm MNSP} \sim \frac{\lambda}{\sqrt{2}}$? ⇒ corrections from the quark sector?

Minimal SU(5)

$$\bar{\mathbf{5}} = \{\ \bar{\mathbf{d}},\ \mathbf{L}\}$$

$$\mathbf{10} = \{\ \mathbf{Q},\ \bar{\mathbf{u}},\ \bar{\mathbf{e}}\}$$

$$Y_{10}\ \mathbf{10}\ \mathbf{10}\ \mathbf{5_H} + Y_{\bar{5}}\ \bar{\mathbf{5}}\ \mathbf{10}\ \bar{\mathbf{5}}_\mathbf{H}$$

⇓ ⇓

$$Y_u \sim Y_{10} \qquad \boxed{Y_d \sim Y_e^T} \sim Y_{\bar{5}}$$

Attempt I: θ_{13} within Minimal SU(5)

Diagonal Y_u

sym: $Y_d = U_d^T \, D_d \, U_d$

Minimal SU(5): $Y_e = Y_d^T$

$$U_e = U_d^T = U_{\rm CKM}^* \qquad U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$U_{\rm MNSP} = U_e^\dagger \, U_\nu$$

$$U_{\rm MNSP} = \begin{pmatrix} 1 & -\lambda & \lambda^3 \\ \lambda & 1 & \times \\ \times & \times & 1 \end{pmatrix} \begin{pmatrix} \times & \times & 0 \\ \times & \times & -\frac{1}{\sqrt{2}} \\ \times & \times & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \times & \times & \frac{\lambda}{\sqrt{2}} \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$

$$\sin\theta_{13} = \frac{\lambda}{\sqrt{2}} \sim 9^\circ$$

Georgi-Jarlskog (GJ) Relations

Minimal SU(5): $Y_e = Y_d^T$ $\Rightarrow$

$$m_d = m_e$$
$$m_s = m_\mu$$

$\Updownarrow$

Georgi-Jarlskog relations
$$\begin{cases} m_d = 3m_e \\ m_s = \dfrac{m_\mu}{3} \end{cases}$$

Georgi-Jarlskog Mechanism: Adding 45 Higgs

$$\bar{\mathbf{5}} = \{\ \bar{\mathrm{d}},\ \mathbf{L}\}$$

$$\mathbf{10} = \{\ \mathbf{Q},\ \bar{\mathrm{u}},\ \bar{\mathrm{e}}\}$$

only $\bar{\mathbf{5}}\ \mathbf{10}\ \bar{\mathbf{5}}_{\mathbf{H}}$ ⇨ $Y_e \sim Y_d^T$

only $\bar{\mathbf{5}}\ \mathbf{10}\ \mathbf{45}_{\mathbf{H}}$ ⇨ $Y_e \sim -3Y_d^T$

$\bar{\mathbf{5}}\ \mathbf{10}\ \bar{\mathbf{5}}_{\mathbf{H}} + \bar{\mathbf{5}}\ \mathbf{10}\ \mathbf{45}_{\mathbf{H}}$ ⇨ $Y_e \sim Y_d^T$ **+** some entries being multiplied by -3

Original GJ Mechanism

$$Y_{\bar{5}}\ \mathbf{\bar{5}\ 10\ \bar{5}_H}\ +\ Y_{45}\ \mathbf{\bar{5}\ 10\ 45_H}$$

$$Y_{\bar{5}} = \begin{pmatrix} 0 & a' & 0 \\ a & 0 & 0 \\ 0 & 0 & c \end{pmatrix} \qquad Y_{45} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Downarrow$$

$$Y_d = \begin{pmatrix} 0 & a' & 0 \\ a & b & 0 \\ 0 & 0 & c \end{pmatrix} \qquad Y_e = \begin{pmatrix} 0 & a & 0 \\ a' & -3b & 0 \\ 0 & 0 & c \end{pmatrix}$$

$$\Downarrow \quad a \sim a' \ll b \ll c$$

$$D_d = \begin{pmatrix} -\frac{aa'}{b} & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \qquad D_e = \begin{pmatrix} \frac{aa'}{3b} & 0 & 0 \\ 0 & -3b & 0 \\ 0 & 0 & c \end{pmatrix}$$

Attempt II: θ$_{13}$ *in the Original GJ Mechanism*

$$U_e = \begin{pmatrix} 1 & \frac{\lambda}{3} & \times \\ -\frac{\lambda}{3} & 1 & \times \\ \times & \times & 1 \end{pmatrix}$$

$$\begin{aligned} U_{\mathrm{MNSP}} &= \begin{pmatrix} 1 & -\frac{\lambda}{3} & \times \\ \frac{\lambda}{3} & 1 & \times \\ \times & \times & 1 \end{pmatrix} \begin{pmatrix} \times & \times & 0 \\ \times & \times & -\frac{1}{\sqrt{2}} \\ \times & \times & \frac{1}{\sqrt{2}} \end{pmatrix} \\ &= \begin{pmatrix} \times & \times & \frac{\lambda}{3\sqrt{2}} \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \end{aligned}$$

$$\sin\theta_{13} = \frac{\lambda}{3\sqrt{2}} \sim 3^\circ$$

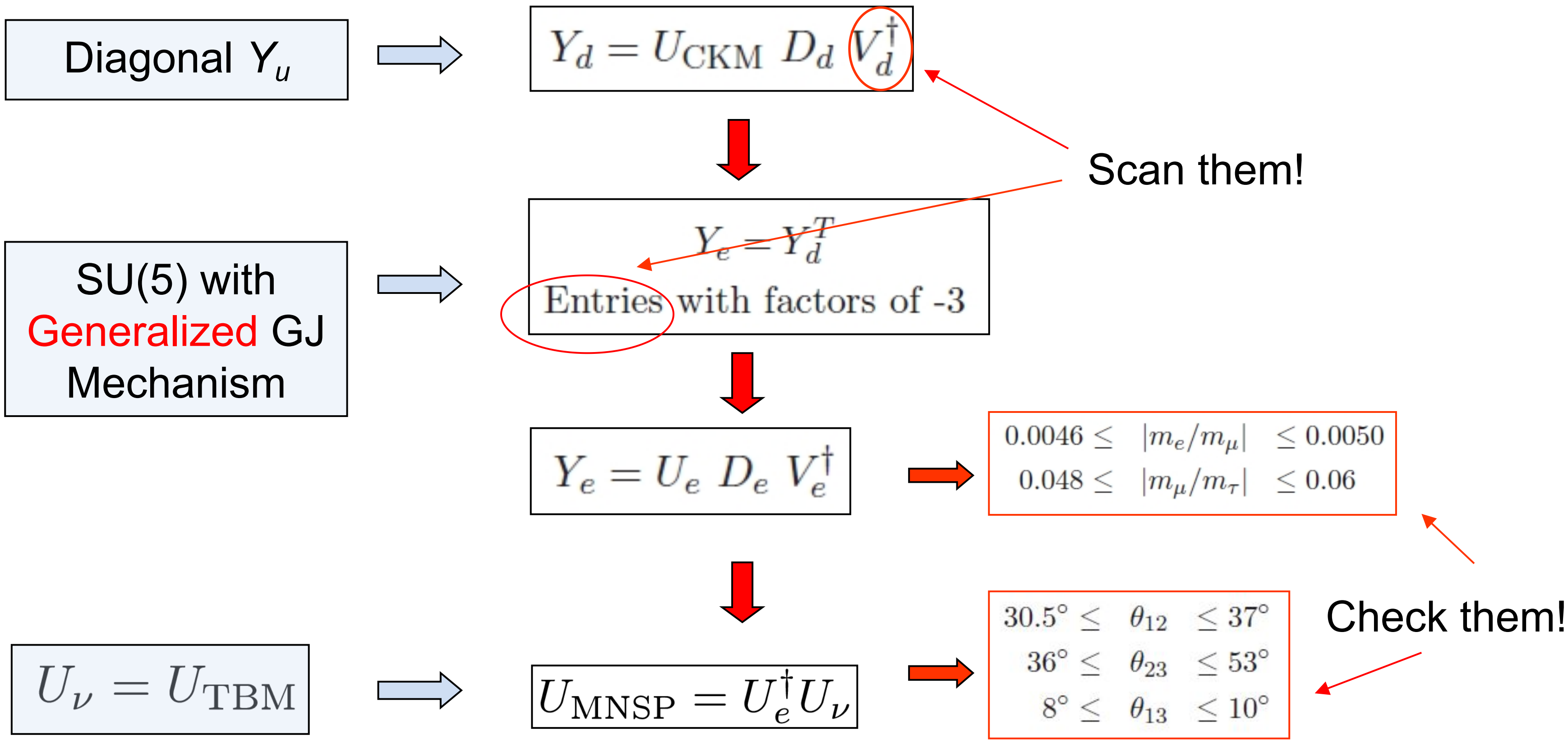
Two Failed Attempts

	Lepton masses	θ_{13}
Minimal SU(5)	WRONG	CORRECT
SU(5) with Original GJ Mechanism	CORRECT	WRONG

Hard to get both lepton masses and θ_{13} correct!

But is it impossible?

A Numerical Search for Allowed Y_d and Y_e



Search Results

$$Y_d = U_{CKM} D_d V_d^\dagger$$

- No solutions with symmetric Y_d !

⇓

- Asymmetric Y_d (Parametrize V_d as a rotation matrix)
  - Solutions for one angle in V_d
  - Solutions for two angles in V_d
  - Solutions for three angles in V_d

Example: One-Angle Solution

$$V_d = \begin{pmatrix} \cos\beta_{13} & 0 & \sin\beta_{13} \\ 0 & 1 & 0 \\ -\sin\beta_{13} & 0 & \cos\beta_{13} \end{pmatrix} \qquad \beta_{13} \sim 3^\circ \approx \lambda^2$$

Entries with GJ factors

$$Y_d \sim \begin{pmatrix} -\frac{1}{3}\lambda^4 + A\lambda^5(\rho - i\eta) & \lambda^3/3 & A\lambda^3(\rho - i\eta) \\ A\lambda^4 + \lambda^5/3 & \boxed{\frac{1}{3}\lambda^2(1-\lambda^2/2)} & \boxed{A\lambda^2} \\ \lambda^2 & -A\lambda^4/3 & 1 \end{pmatrix}.$$

Summary

- Hard to get both lepton masses and θ_{13} correct within SU(5).
- A numerical search was performed.
- Y_d is asymmetric, assuming a diagonal Y_u!
- Ongoing work: flavor model building based on those asymmetric solutions.

Thank you for your attention!

Back-up Slides

Two angles in V_d

- Wide ranges of angles
- Diverse sets of GJ patterns
  (model building)

Examples of GJ patterns:

$$\begin{pmatrix} & & \\ & \times & \\ & & \end{pmatrix} \quad \begin{pmatrix} & & \\ \times & & \times \\ & & \end{pmatrix} \quad \begin{pmatrix} \times & \times & \\ & \times & \times \\ \times & \times & \end{pmatrix}$$

β_{12}	β_{13}	β_{23}	Y_d with GJ Patterns	m_e/m_μ	θ^e_{12}	θ^e_{13}	θ^e_{23}	δ^e_{12}	δ^e_{13}	δ^e_{23}	θ_{12}	θ_{13}	θ_{23}	J
			1: (22)											
168.3	351.0	0	(5.0, 4.0, 1.3)	0.0048	4.2	9.0	0.2	0.0	180.0	-0.3	31.8	9.3	45.1	2.1×10^{-8}
348.3	351.0	0	(5.1, 3.1, 1.3)	0.0048	4.2	9.0	0.2	0.0	-180.0	179.8	31.9	9.3	44.8	4.5×10^{-7}
			2: (21,22)											
0.0	183.1	0	(4.7, 4.2, 2.0)	0.0047	9.1	3.1	0.2	180.0	0.0	-0.3	31.0	8.6	44.6	3.7×10^{-7}
170.5	354.8	0	(5.2, 5.0, 1.6)	0.0048	6.4	5.2	0.2	0.0	180.0	-0.3	36.1	8.2	44.8	1.1×10^{-7}
350.5	185.2	0	(5.2, 5.0, 1.6)	0.0048	6.4	5.2	0.2	180.0	0.0	-0.3	34.4	8.2	44.8	-5.9×10^{-8}
			2: (22,23)											
180.0	182.8	0	(4.8, 4.1, 2.0)	0.0048	9.0	2.7	0.3	-180.0	0.0	0.1	30.8	8.3	44.8	4.1×10^{-7}
			3: (22,23,32)											
10.5	174.8	0	(4.4, 3.8, 3.8)	0.0048	6.8	5.2	0.2	0.0	-180.0	0.7	36.4	8.5	44.8	2.1×10^{-6}
190.5	5.2	0	(4.4, 3.8, 3.8)	0.0048	6.8	5.2	0.2	-180.0	0.0	0.7	34.1	8.5	44.8	-2.2×10^{-6}
159.0	352.2	0	(5.1, 4.9, 1.3)	0.0047	3.9	7.8	0.1	0.0	-180.0	0.8	32.5	8.3	45.1	3.4×10^{-6}
			3: (11,13,22)											
164.6	186.0	0	(5.1, 3.2, 1.5)	0.0047	5.5	6.0	0.1	180.0	0.0	-179.2	35.6	8.1	44.6	1.7×10^{-6}
164.6	354.0	0	(5.4, 5.2, 1.5)	0.0047	5.5	6.0	0.1	0.0	-180.0	0.8	34.4	8.2	44.9	4.4×10^{-6}
344.6	186.0	0	(5.4, 5.2, 1.5)	0.0047	5.5	6.0	0.1	-180.0	0.0	0.8	35.6	8.2	44.9	-4.5×10^{-6}
344.6	354.0	0	(5.1, 3.2, 1.5)	0.0047	5.5	6.0	0.1	0.0	-180.0	-179.2	34.9	8.2	44.6	-2.1×10^{-6}
			3: (22,23,31)											
174.7	178.8	0	(5.1, 4.7, 2.6)	0.0048	9.6	3.6	0.3	179.9	0.0	0.1	31.0	9.3	44.7	-7.3×10^{-6}
			3: (11,22,31)											
343.2	178.9	0	(5.4, 3.5, 2.7)	0.0048	9.6	3.2	0.1	180.0	0.0	-0.3	30.8	9.0	44.5	2.6×10^{-6}
			4: (11,13,21,22)											
164.3	353.0	0	(5.3, 5.9, 1.4)	0.0048	6.3	7.0	0.1	0.0	-180.0	-0.8	34.8	9.4	44.8	4.7×10^{-6}
			4: (11,21,22,31)											
171.7	358.3	0	(5.3, 4.4, 2.4)	0.0047	9.0	5.1	0.1	180.0	0.0	-0.3	32.5	10.0	44.5	-6.6×10^{-7}
351.7	358.3	0	(5.6, 3.8, 2.4)	0.0047	9.0	5.1	0.1	-180.0	0.0	179.8	32.4	9.9	44.2	-4.9×10^{-7}
			4: (11,12,22,31)											
175.5	1.6	0	(5.1, 4.0, 2.4)	0.0047	6.8	4.8	0.2	0.0	-180.0	0.8	36.7	8.2	44.8	1.1×10^{-7}
342.3	183.1	0	(5.5, 3.9, 2.0)	0.0048	4.1	9.3	0.1	0.0	-180.0	0.8	31.6	9.4	45.1	6.1×10^{-6}
355.5	178.4	0	(5.1, 4.0, 2.4)	0.0047	6.8	4.8	0.2	-180.0	0.0	0.8	33.8	8.2	44.8	-1.2×10^{-7}

(Table 1 continued on the next page)

One angle result (analytic)

$$\mathbf{Y}_d = \begin{pmatrix} \frac{\lambda^4}{3}(1-\frac{\lambda^2}{2}) & \frac{\lambda^3}{3} & A\lambda^3(\rho - \mathrm{i}\eta) \\ -\frac{\lambda^5}{3} & \frac{\lambda^2}{3}(1-\frac{\lambda^2}{2}) & A\lambda^2 \\ \frac{1}{3}A\lambda^7(1-\mathrm{i}\eta-\rho) & -A\frac{\lambda^4}{3} & 1 \end{pmatrix} \mathbf{V}_d^\dagger$$

$$= \begin{pmatrix} \frac{\lambda^4}{3}(1-\frac{\lambda^2}{2}) - Ae\lambda^4(\rho-\mathrm{i}\eta) & \frac{\lambda^3}{3} & A\lambda^3(\rho-\mathrm{i}\eta) + \frac{1}{3}e\lambda^5 + \mathcal{O}(\lambda^7) \\ -\frac{\lambda^5}{3} - Ae\lambda^3 & \frac{\lambda^2}{3}(1-\frac{\lambda^2}{2}) & A\lambda^2 + \mathcal{O}(\lambda^6) \\ -e\lambda + \mathcal{O}(\lambda^7) & -A\frac{\lambda^4}{3} & 1 + \mathcal{O}(\lambda^8) \end{pmatrix}$$

$$\mathbf{V}_d \approx \begin{pmatrix} 1 & 0 & -e\lambda \\ 0 & 1 & 0 \\ e\lambda & 0 & 1 \end{pmatrix}$$

$$\mathbf{Y}_e = \begin{pmatrix} \frac{\lambda^4}{3}(1-\frac{\lambda^2}{2}) - Ae\lambda^4(\rho-\mathrm{i}\eta) & -\frac{\lambda^5}{3} - Ae\lambda^3 & -e\lambda + \mathcal{O}(\lambda^7) \\ \frac{\lambda^3}{3} & -\lambda^2(1-\frac{\lambda^2}{2}) & -A\frac{\lambda^4}{3} \\ A\lambda^3(\rho-\mathrm{i}\eta) + \frac{1}{3}e\lambda^5 + \mathcal{O}(\lambda^7) & -3A\lambda^2 + \mathcal{O}(\lambda^6) & 1 + \mathcal{O}(\lambda^8) \end{pmatrix}$$

$$\mathbf{U}_e \approx \begin{pmatrix} 1 & -4Ae\lambda & e\lambda \\ * & 1 & * \\ * & * & 1 \end{pmatrix}$$

$$m_e \approx (\frac{1}{3} - \frac{4}{3}eA)\lambda^4 \approx 0.109\lambda^4$$

$$\theta_{13}^{MNSP} \approx \frac{(1+4A)e\lambda}{\sqrt{2}} \approx 0.89\frac{\lambda}{\sqrt{2}} \approx 8.2^o$$