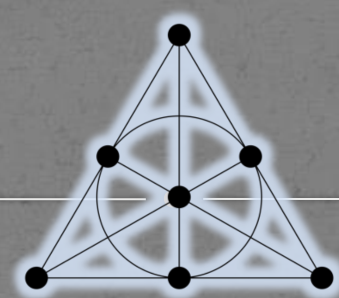
Majorana Physics in the Flavor Ring

arXiv: 1311.4553



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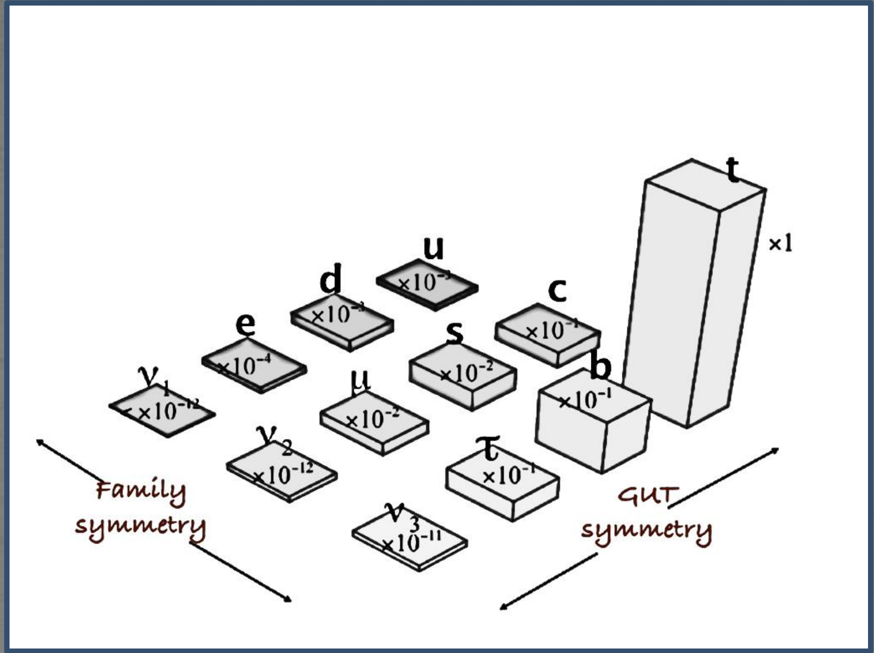
Outline

- The Flavor Puzzle
- The Flavor Ring: neutrino sector
- A Special Majorana Matrix
- Flavor Group : Frobenius group basics
- Underlying Theory
- Summary and Conclusion

The Flavor Puzzle



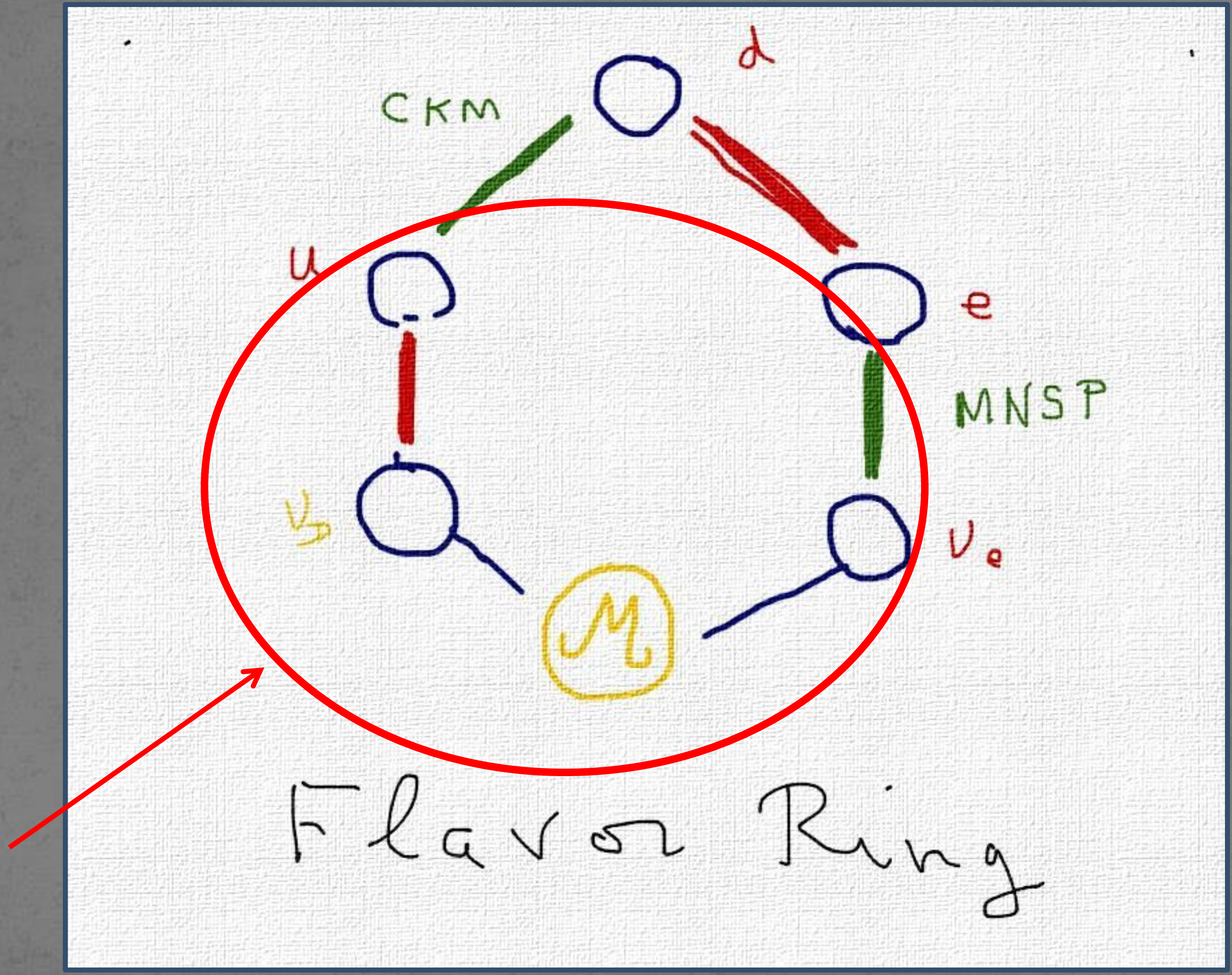
Quarks

$$\theta_{12} \sim 13^\circ \qquad \theta_{23} \sim 2.4^\circ \qquad \theta_{13} \sim 0.2^\circ \qquad \delta_{CKM} \sim 60^\circ$$

Leptons

$$\theta_{12} \sim 35^\circ \qquad \theta_{23} \sim 45^\circ \qquad \theta_{13} \sim (8-10)^\circ$$

The Flavor Ring



Up-Quarks and Neutrinos

- Up-quarks display large hierarchy

$$Y_u \sim y_t \begin{pmatrix} \lambda^8 & & \\ & \lambda^4 & \\ & & 1 \end{pmatrix}$$

$$\lambda = \sin \theta_c$$

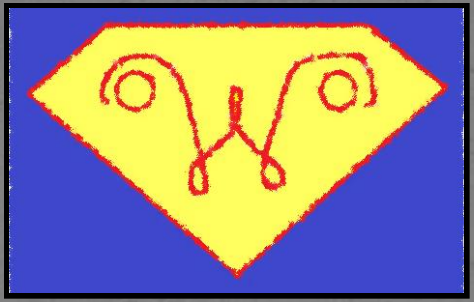
- GUT Puzzle ?

$$SO(10): \quad Y_\nu^D = Y_u \sim \begin{pmatrix} \lambda^8 & & \\ & \lambda^4 & \\ & & 1 \end{pmatrix}$$

- Seesaw

$$M_\nu \sim - Y_\nu^D \frac{1}{\mathcal{M}} Y_\nu^{DT}$$

Majorana matrix to the rescue



- Can majorana mass undo the hierarchy?
- Would need to have a severe hierarchy
- What sets the physics of Majorana matrix?
  - Should “know” what sets the hierarchy in $Y^{(0)}$
  - Natural candidate: Family symmetry!

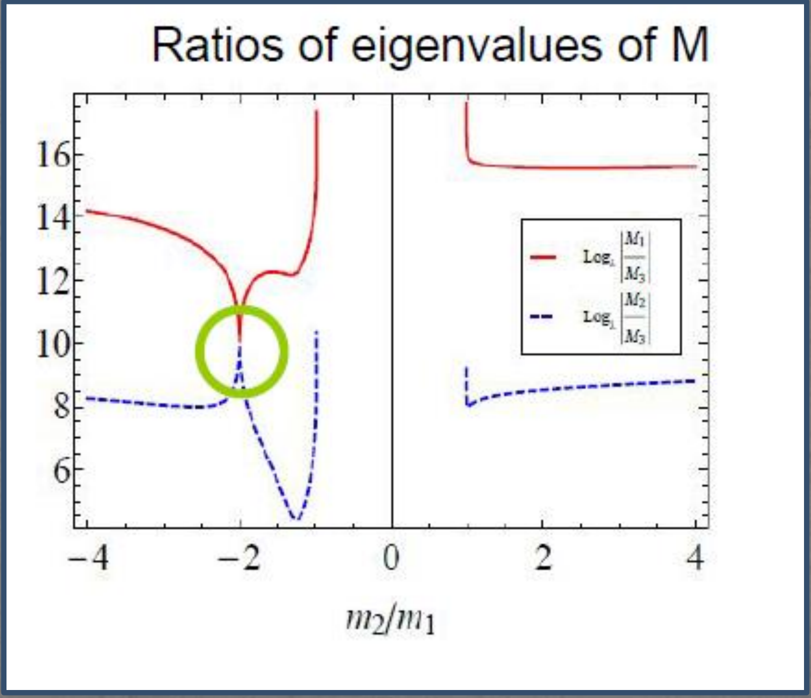
Hierarchical Majorana Matrix

$$\mathcal{M} \sim \begin{pmatrix} \sim \lambda^{16} & \sim \lambda^{12} & \sim \lambda^{8} \\ \sim \lambda^{12} & \sim \lambda^{8} & \sim \lambda^{4} \\ \sim \lambda^{8} & \sim \lambda^{4} & \sim 1 \end{pmatrix}$$

- Generic Eigenvalues :

$$1 \;:\; \sim \lambda^{8} \;:\; \sim \lambda^{16}$$

$$1 \;:\; \sim 10^{-5} \;:\; \sim 10^{-10}$$



A Special Majorana Matrix

- At this special point, a vanishing sub-determinant

$$\mathcal{M} = \begin{pmatrix} r\lambda^{16} & r\lambda^{12} & r\lambda^{8} \\ r\lambda^{12} & \lambda^{8} & -\lambda^{4} \\ r\lambda^{8} & -\lambda^{4} & 1 \end{pmatrix}, \qquad r = \frac{m_3}{m_1}$$

$$1 : r\lambda^{12} : r\lambda^{12} \sim 1 : 10^{-10} : 10^{-10}$$

- Can we produce it naturally with a family symmetry?

Predictions

- Gatto-like Relation

$$\boxed{\tan^2\theta_{12} = -\frac{m_1}{m_2}}$$

- Neutrino masses for a particular mixing scheme

- Ex: TBM Mixing

$$\boxed{\frac{m_1}{m_2} = -\frac{1}{2}} \Rightarrow \boxed{\begin{aligned} m_1 &\approx 0.005\ \text{eV} \\ m_2 &\approx 0.01\ \text{eV} \\ m_3 &\approx 0.05\ \text{eV} \end{aligned}}$$

- Normal Hierarchy

Frobenius Group of order 21 : Basics

$$\boxed{\mathcal{Z}_7 \rtimes \mathcal{Z}_3}$$

- Smallest Non-Abelian discrete subgroup of SU(3) that is not a subgroup of SO(3)
- Also called T7 in the literature
- Contains 21 Elements

$$\boxed{Irreps: \qquad \mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}', \bar{\mathbf{1}}', \mathbf{1}}$$

Model Building with $\mathcal{Z}_7 \rtimes \mathcal{Z}_3$

- Step I : Choose Rep's of Fields
  - Want to work within SU(5) :

$$\overline{\mathbf{5}}(\psi) \quad \mathbf{10}(\chi) \quad \mathbf{1}(N)$$

$$SO(10)$$

- Choose all matter fields to transform as family triplets
- Also introduce Familons which are family triplets or anti-triplets but gauge singlets

Producing the Majorana Matrix

- One linear combination of dimension-five Invariants

$$\boxed{(N\bar{\varphi})_{\mathbf{1}'}(N\bar{\varphi}')_{\bar{\mathbf{1}}'} + (N\bar{\varphi})_{\bar{\mathbf{1}}'}(N\bar{\varphi}')_{\mathbf{1}'}}$$

$$\Rightarrow \begin{pmatrix} 2\bar{\varphi}_1\bar{\varphi}'_1 & -(\bar{\varphi}_1\bar{\varphi}'_2 + \bar{\varphi}_2\bar{\varphi}'_1) & -(\bar{\varphi}_1\bar{\varphi}'_3 + \bar{\varphi}_3\bar{\varphi}'_1) \\ & 2\bar{\varphi}_2\bar{\varphi}'_2 & -(\bar{\varphi}_2\bar{\varphi}'_3 + \bar{\varphi}_3\bar{\varphi}'_2) \\ & & 2\bar{\varphi}_3\bar{\varphi}'_3 \end{pmatrix}$$

$$\bar{\varphi} \sim \begin{pmatrix} \bar{\alpha}\lambda^8 \\ \lambda^4 \\ 1 \end{pmatrix}, \qquad \bar{\varphi}' \sim \begin{pmatrix} \bar{\alpha}'\lambda^8 \\ \lambda^4 \\ 1 \end{pmatrix}$$

gives zero sub-determinant!

- Hierarchies carried by Familon Fields
- Can also be done through single dimension-six

Underlying Theory

- Invariant is singled out by additional symmetries

	N	$\overline{\Phi}$	S	$\bar{S}$
$\mathcal{Z}_7 \rtimes \mathcal{Z}_3$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{1}'$	$\bar{\mathbf{1}}'$
$\mathcal{S}_3$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$
$\mathcal{Z}_2$	$-$	$-$	$+$	$+$

$$\begin{aligned}\mathcal{W}_u &= aN\overline{\Phi}S + bN\overline{\Phi}\bar{S} + M_S S\bar{S} + \lambda_S S^3 + \lambda_{\bar{S}}\bar{S}^3 \\ &= aN(\bar{\varphi}S_2 + \bar{\varphi}'S_1) + bN(\bar{\varphi}\bar{S}_2 + \bar{\varphi}'\bar{S}_1) + M_S(S_1\bar{S}_2 + S_2\bar{S}_1) + \ldots\end{aligned}$$

$$\Downarrow$$

$$\frac{ab}{M_S}\left[(N\bar{\varphi})_{\mathbf{1}'}(N\bar{\varphi}')_{\bar{\mathbf{1}}'} + (N\bar{\varphi})_{\bar{\mathbf{1}}'}(N\bar{\varphi}')_{\mathbf{1}'}\right]$$

Conclusions

- Flavor Ring: tying Grand Unification to the flavor puzzle can lead to new insights
- Using family symmetry to give Majorana physics begins to give a more connected picture
- Found a compelling and predictive Majorana matrix
- Natural in terms of a family symmetry
- Stay tuned – more to come!

Choice of Family Group

- Extreme hierarchy dictates a subgroup of SU(3)

$$Y \sim y \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} + \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$\mathcal{SO}(3) : \mathbf{3} \otimes \mathbf{3} = \mathbf{3}_a + \mathbf{5}_s + \mathbf{1}$$

$$\mathcal{SU}(3) : \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}}_a + \mathbf{6}_s$$

- Neutrino mixing seems to indicate a finite subgroup

Kronecker Products

$$(\mathbf{3}\otimes\mathbf{3})_{+} \quad\longrightarrow\quad \mathbf{3}: \begin{cases} |3\rangle|3'\rangle \\ |1\rangle|1'\rangle \\ |2\rangle|2'\rangle \end{cases} ; \qquad \longrightarrow \quad \overline{\mathbf{3}}: \begin{cases} \frac{1}{\sqrt{2}}(|3\rangle|2'\rangle + |2\rangle|3'\rangle) \\ \frac{1}{\sqrt{2}}(|1\rangle|3'\rangle + |3\rangle|1'\rangle) \\ \frac{1}{\sqrt{2}}(|2\rangle|1'\rangle + |1\rangle|2'\rangle) \end{cases}$$

$$(\mathbf{3}\otimes\mathbf{3})_{-} \quad\longrightarrow\quad \overline{\mathbf{3}}: \begin{cases} \frac{1}{\sqrt{2}}(|3\rangle|2'\rangle - |2\rangle|3'\rangle) \\ \frac{1}{\sqrt{2}}(|1\rangle|3'\rangle - |3\rangle|1'\rangle) \\ \frac{1}{\sqrt{2}}(|2\rangle|1'\rangle - |1\rangle|2'\rangle) \end{cases} .$$

$$\mathbf{3}\otimes\overline{\mathbf{3}} \quad\longrightarrow\quad \mathbf{3}: \begin{cases} |2\rangle|\overline{1}'\rangle \\ |3\rangle|\overline{2}'\rangle \\ |1\rangle|\overline{3}'\rangle \end{cases} ; \qquad \longrightarrow \quad \overline{\mathbf{3}}: \begin{cases} |1\rangle|\overline{2}'\rangle \\ |2\rangle|\overline{3}'\rangle \\ |3\rangle|\overline{1}'\rangle \end{cases} .$$

$$\mathbf{3}\otimes\overline{\mathbf{3}} \quad\longrightarrow\quad \mathbf{1}: \frac{1}{\sqrt{3}}\left(|1\rangle|\overline{1}'\rangle + |2\rangle|\overline{2}'\rangle + |3\rangle|\overline{3}'\rangle\right) ,$$

$$\mathbf{3}\otimes\overline{\mathbf{3}} \quad\longrightarrow\quad \mathbf{1}': \frac{1}{\sqrt{3}}\left(|1\rangle|\overline{1}'\rangle + \omega^2|2\rangle|\overline{2}'\rangle + \omega|3\rangle|\overline{3}'\rangle\right) ,$$

$$\mathbf{3}\otimes\overline{\mathbf{3}} \quad\longrightarrow\quad \overline{\mathbf{1}}': \frac{1}{\sqrt{3}}\left(|1\rangle|\overline{1}'\rangle + \omega|2\rangle|\overline{2}'\rangle + \omega^2|3\rangle|\overline{3}'\rangle\right) , \quad \omega = \exp(2i\pi/3)$$

- A single Dim-6 Operator

$$\boxed{((N\varphi)_{\bar{\mathbf{3}}_+}(N\varphi')_{\bar{\mathbf{3}}_+})_{\mathbf{3}_-}\bar{\varphi}}$$

$$\Rightarrow \begin{pmatrix} 2\bar{\varphi}_1 B_{23} & \bar{\varphi}_1 B_{13} - \bar{\varphi}_2 B_{23} & -\bar{\varphi}_1 B_{12} - \bar{\varphi}_3 B_{23} \\ & -2\bar{\varphi}_2 B_{13} & -\bar{\varphi}_2 B_{12} + \bar{\varphi}_3 B_{13} \\ & & 2\bar{\varphi}_3 B_{12} \end{pmatrix}$$

Similar to the Dim-5 case.

$$B_{ij} = \varphi_i \varphi'_j - \varphi'_i \varphi_j$$

The form of nesting is quite suggestive.