Raising the Higgs Mass in SUSY with $t - t'$ Mixing

A new way to address the Little Hierarchy Problem

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Outline

The Little Hierarchy Problem

- In the MSSM, vacuum stability (in D-flat directions) + EWSB lead to the tree level upper bound:

$$m_h^0 < m_Z \cos(2\beta) \leq 91\text{GeV}$$

- Need radiative corrections δm_h to raise m_h^0 to 125 GeV.

Radiative Corrections in MSSM

- Radiative corrections go as (M. Carena, et al.):

$$[\delta m_h^2] \subset \frac{3}{2\pi^2} y_{33}^4 v_u^2 [t + \frac{X_{33}}{2} + \frac{1}{16\pi^2}\left(\frac{3}{2}y_{33}^2 - 32\pi\alpha_s\right)(X_{33}t + t^2)]$$

with

$$t = \log(\frac{\tilde{m}_{\tilde{t}}^2}{m_t^2}), X_{33} \propto (A_{33}^u)^2.$$

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with

$$t = \log(\frac{\tilde{m}_{\tilde{t}}^2}{m_t^2}), X_{33} \propto (A_{33}^u)^2.$$

- To raise the Higgs Mass in the MSSM:
  1. need large A_{33}^u
  2. need large $\tilde{m}_{\tilde{t}}$ ($\gtrsim$ 3TeV)
- Grows only *logarithmically* with $\tilde{m}_{\tilde{t}}$.

What else can be done to raise m_h?

Add a 4th chiral generation of quarks?

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Instead, make it vector-like: MSSM$+\mathbf{10}+\bar{\mathbf{10}}$.

- New quarks now get most of their mass from $\mu_{10}\mathbf{10}\ \bar{\mathbf{10}}$
- $\mathbf{10}$'s, $\bar{\mathbf{10}}$'s reps of $SU(5) \rightarrow$ maintain unification.

What else can be done to raise m_h?

- New terms in superpotential:

$$W \subset y^u_{44} Q_4 H_u U^c_4 + \mu_Q \bar{Q}^c_4 Q_4 + \mu_U \bar{U}_4 U^c_4$$

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- For $m_h = 125$ GeV, danger of Landau poles in Yukawas marginally above EWK scale

Our Extension

- Keep MSSM+$\mathbf{10}$+$\overline{\mathbf{10}}$.
- Allow mixing between 3rd and 4th generation:
  1. Increases top Yukawa y_{33} up to 6%
  2. Raises m_h quickly since $\delta m_h^2 \propto y_{33}^4$
  3. E.g. $1.06^4 = 1.26 \rightarrow$ stop contribution increases by up to 26%!

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  3. E.g. $1.06^4 = 1.26 \rightarrow$ stop contribution increases by up to 26%!
- Can get $m_h = 125$ GeV and push Landau poles up to GUT scale while keeping soft terms $<$ TeV.

The MSSM + 10 + $\bar{10}$ with mixing

- Mixing between 3rd and 4th generations:

$$\begin{aligned} W &\subset +y^u_{44} Q_4 H_u U^c_4 + \mu_4 \bar{Q}^c_4 Q_4 + \mu_4 \bar{U}_4 U^c_4 \\ &+ y^u_{34} Q_3 H_u U^c_4 + y^u_{43} Q_4 H_u U^c_3 \end{aligned}$$

1. y_{34}, $y_{43} \sim \mathcal{O}(1)$ and $y_{44} = 0$ to emphasize mixing

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Also

1. Set $\mu_Q = \mu_U \equiv \mu_4$
2. Same soft mass Δm for all squarks
3. Large $\tan\beta$
4. Ignore all leptons, 1st and 2nd generation quarks

Particle Content

Supermultiplet	Scalars	Fermions
Q_3	$(\tilde{u}_3, \tilde{d}_3)$	(u_3, d_3)
U_3^c	$\tilde{u}_3^c$	u_3^c
D_3^c	$\tilde{d}_3^c$	d_3^c
Q_4	$(\tilde{u}_4, \tilde{d}_4)$	(u_4, d_4)
U_4^c	$\tilde{u}_4^c$	u_4^c
$\bar{Q}_4^c$	$(\tilde{\bar{d}}_4^c, \tilde{\bar{u}}_4^c)$	$(\bar{d}_4^c, \bar{u}_4^c)$
$\bar{U}_4$	$\tilde{\bar{u}}_4$	$\bar{u}_4$

- Top block: MSSM fields
- Bottom block: new fields
- Barred fields in $\bar{\mathbf{10}}$ rep of $SU(5)$

Particle Content - Mass Eigenstates

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D_3^c	$\tilde{d}_3^c$	d_3^c
Q_4	$(\tilde{u}_4, \tilde{d}_4)$	(u_4, d_4)
U_4^c	$\tilde{u}_4^c$	u_4^c
$\bar{Q}_4^c$	$(\tilde{\bar{d}}_4^c, \tilde{\bar{u}}_4^c)$	$(\bar{d}_4^c, \bar{u}_4^c)$
$\bar{U}_4$	$\tilde{\bar{u}}_4$	$\bar{u}_4$

- Mass eigenstates
  1. Fermions: t, b, new quarks $t'_{1,2}$ and b'
  2. Scalars: $\tilde{t}_{1,2}$, $\tilde{b}_{1,2}$, non-MSSM squarks $\tilde{t}'_{1,2,3,4}$, and $\tilde{b}'_{1,2}$

Mixing and the top Yukawa

- In the MSSM, $y_{33} = \frac{m_t}{v \sin \beta}$.

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- With $y_{44} = 0$ in our model:

$$y_{33} \approx 1 + \frac{1}{2}\left(\frac{\Delta^2}{1-\Delta^2}\right)\left(y_{43}^2 + y_{34}^2\right) + \mathcal{O}(\Delta^4)$$

with $\Delta \equiv v/\mu_4$.

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with $\Delta \equiv v/\mu_4$.

1. $\Delta > 0$ increases top Yukawa
2. Mass bounds require $\Delta \lesssim 1/4$
3. $\rightarrow$ Increase $y_{33} \sim 6\%$
4. $y_{44} \neq 0$ corrections are negligible

Corrections to m_h

- To calculate δm_h we use the one-loop effective potential:

$$\Delta V = \frac{3}{32\pi^2}[\sum_{i=1}^{6} \tilde{m}_i^2 \left(\log \frac{\tilde{m}_i^2}{Q^2} - \frac{3}{2}\right) - 2\sum_{i=1}^{3} m_i^2 \left(\log \frac{m_i^2}{Q^2} - \frac{3}{2}\right)]$$

- Masses $m_i, \tilde{m}_i$ are functions of y_{ij}, μ_4, A_{ij} and Δm.

Corrections to m_h

- The Higgs mass in the decoupling limit is:

$$m_h^2 = M_Z^2 \cos^2 2\beta + \frac{1}{2}\left(\frac{\partial^2(\Delta V)}{\partial v_u^2} - \frac{1}{v_u}\frac{\partial(\Delta V)}{\partial v_u}\right)$$

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- So m_h is also a function of $\mu_4, \Delta m, A_{ij}, y_{ij}$.
- Fixing $m_h, \mu_4, \Delta m, A_{ij}$, and a relation between the y_{ij}'s, the $|y_{ij}|$ required are uniquely fixed.

What weak-scale Yukawas yield m_h?

"Fixing $m_h, \mu_4, \Delta m, A_{ij}$, and a relation between the y_{ij}'s, the $|y_{ij}|$ required are uniquely fixed."

- Relations between y_{ij}'s:
  1. $|y_{34}| = |y_{43}|$ large, $y_{44} = 0$
  2. y_{43} large, others 0
  3. y_{34} large, others 0
  4. y_{44} large, others 0 (for comparison)

What weak-scale Yukawas yield m_h?

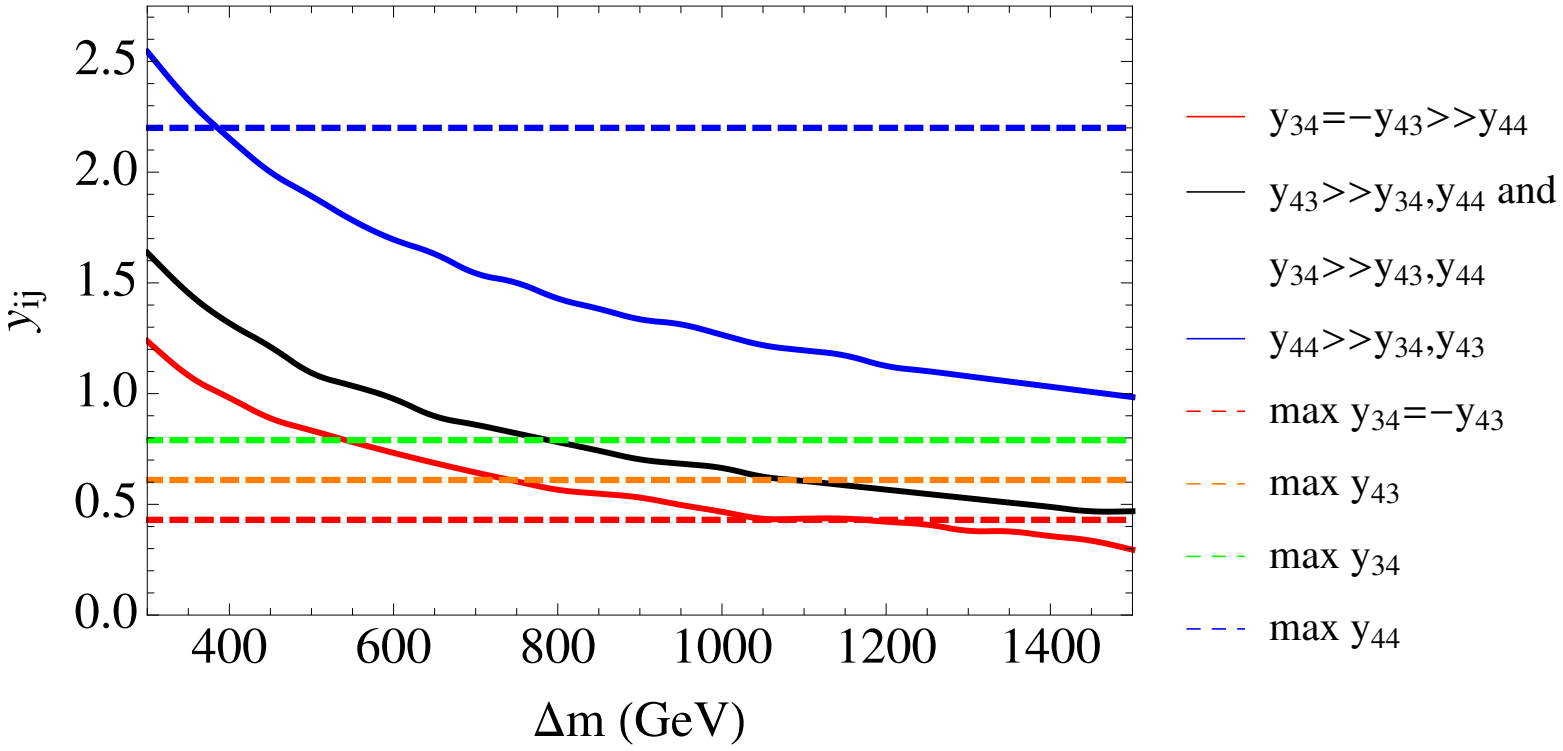

Figure: $A = \Delta m$, $\mu_4 = 900$ GeV. Above the dotted lines requires Yukawas larger than allowed by EWPM and is thus ruled out. This gives a lower bound on Δm.

Great. What about Landau poles?

Study effects on Λ_L from:

- Mixing scenarios:
  1. $|y_{34}| = |y_{43}|$ large, $y_{44} = 0$
  2. y_{43} large, others 0
  3. y_{34} large, others 0
  4. y_{44} large, others 0
- A-terms
- Vector-like mass μ_4

Top Yukawa Landau Poles: Mixing

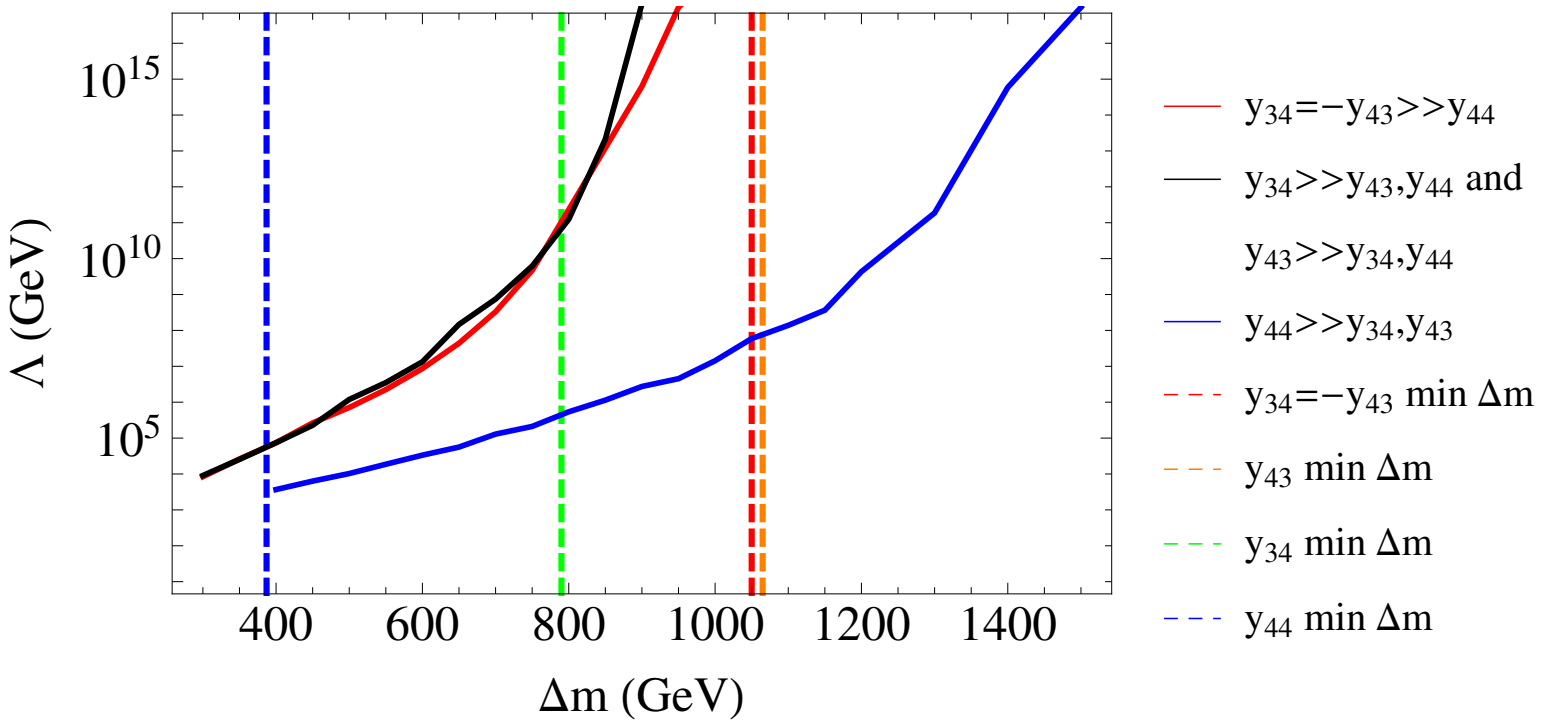

Figure: $A = \Delta m$, $\mu_4 = 900$ GeV. Soft masses left of the dotted lines require y_{ij} larger than allowed by EWPM. $\Lambda_L < 1$ TeV is not plotted.

Mixing pushes Λ_L above GUT scale with soft parameters $\lesssim 1$ TeV.

Top Yukawa Landau Poles: $A-$terms

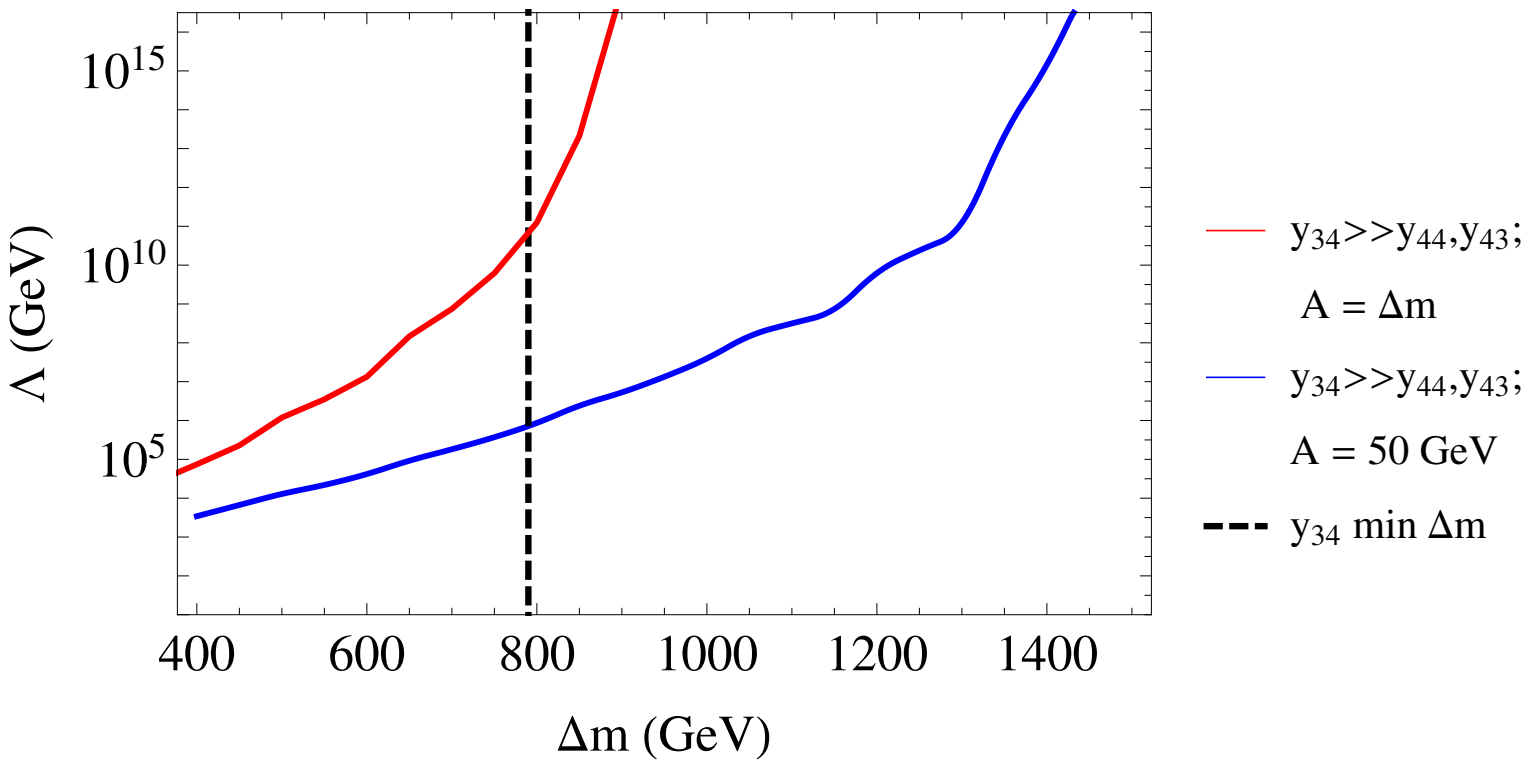

Figure: y_{34} large, $\mu_4 = 900$ GeV, $y_{44} = y_{43} = 0$. Soft masses left of the dotted lines require y_{ij} larger than allowed by EWPM.

Landau poles get significantly pushed up by larger A-terms.

Top Yukawa Landau Poles: μ_4

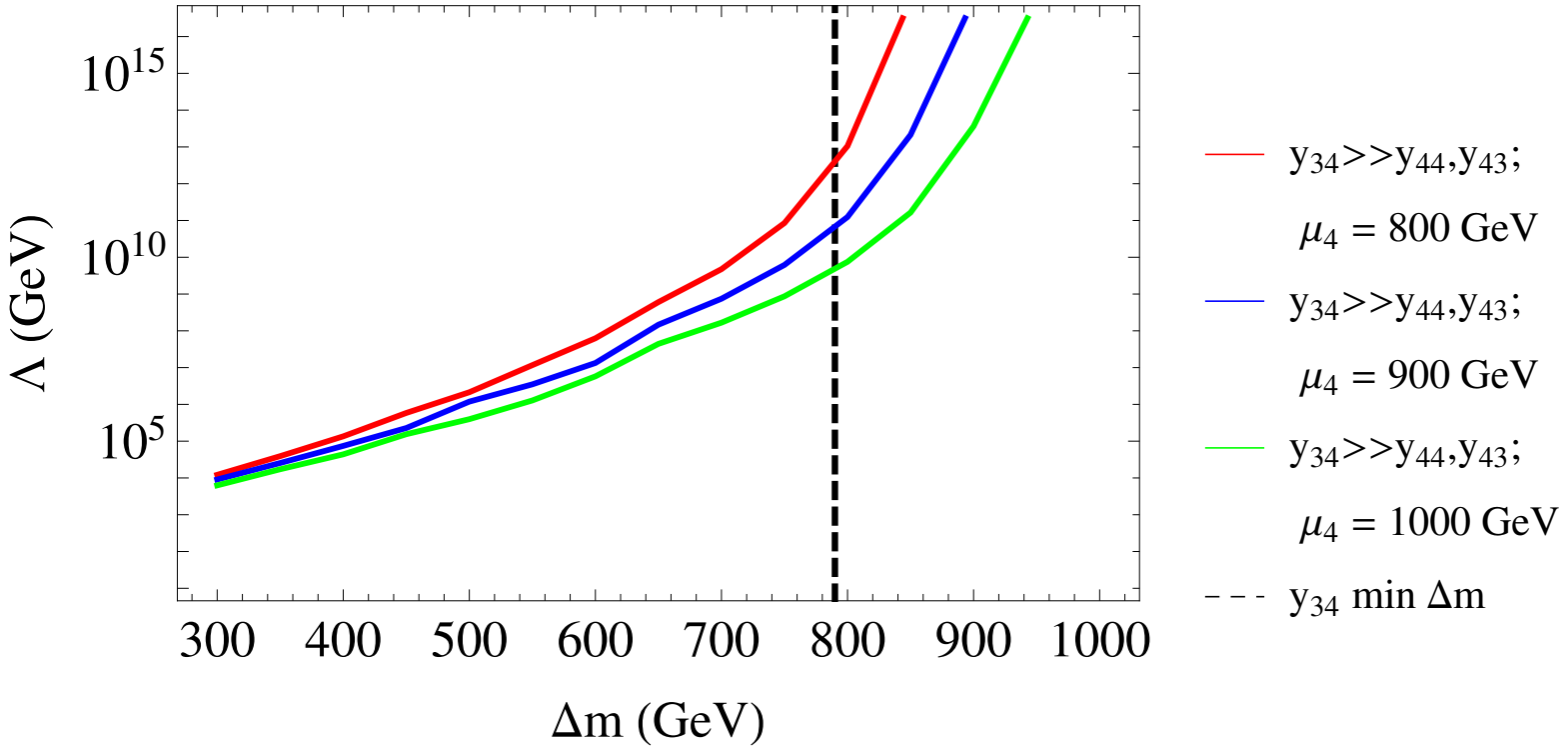

Figure: y_{34} large, $\mu_4 = 900$ GeV, $y_{44} = y_{43} = 0$. Soft masses left of the dotted lines require y_{ij} larger than allowed by EWPM.

For a given soft mass, Λ_L increases as μ_4 decreases.

Constraints from Experiments

Studied constraints from

- Electroweak precision measurements (S and T parameters)
- Measurements of V_{tb}^{CKM}
- Higgs production
- LHC direct searches

Constraints from Experiments

EWPM and LHC direct searches are most constraining.

- LHC: $m_{t'}, m_{b'} \gtrsim 700 - 800$ GeV
- $\rightarrow$ take $\mu_4 \gtrsim 800$ GeV

EWPM constraints

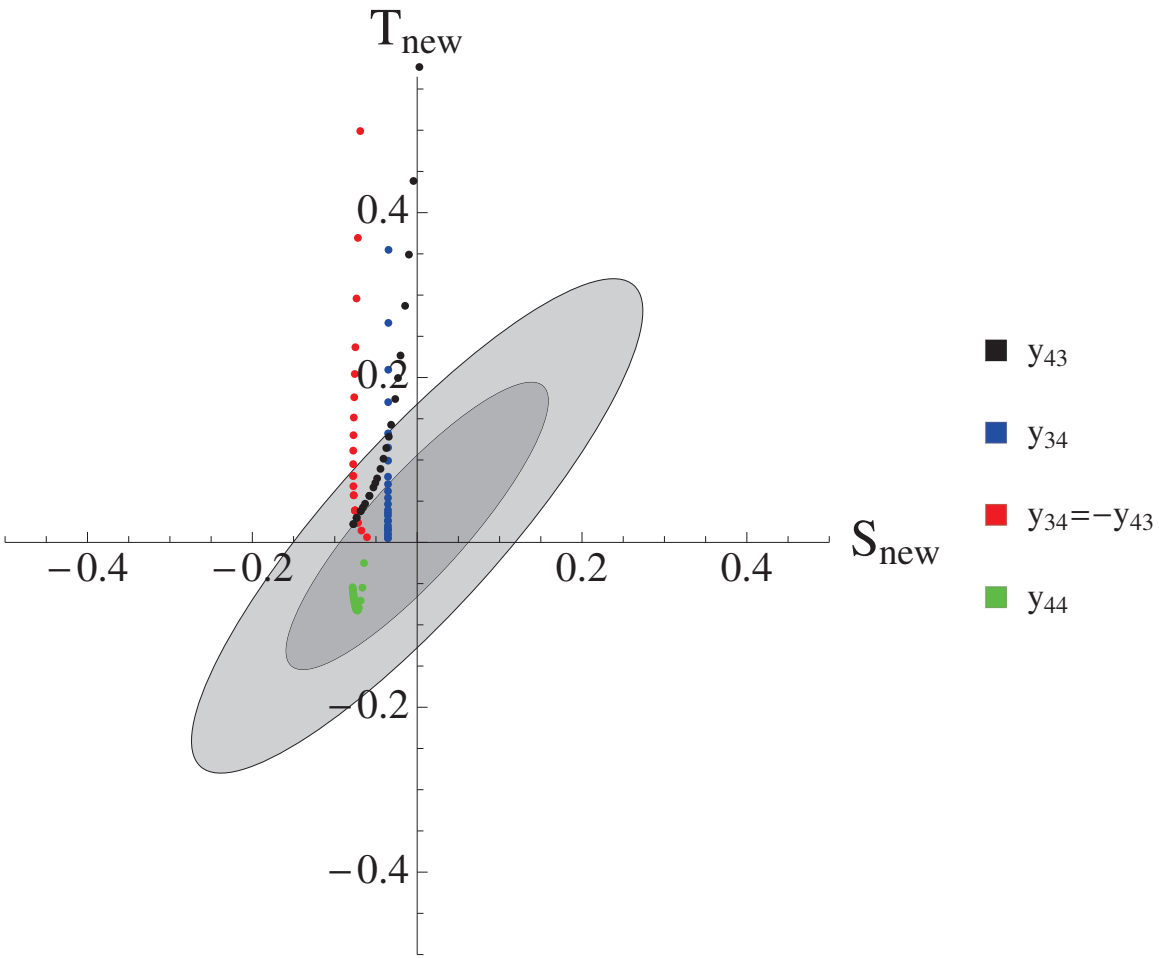

Figure: S_{new} and T_{new} for each of our mixing scenarios. For each, $\mu_4 = 900$ GeV, $A = 600$ GeV, and Δm varies from 300 to 1500 GeV.

$\mu_Q \neq \mu_U$ can hurt or help

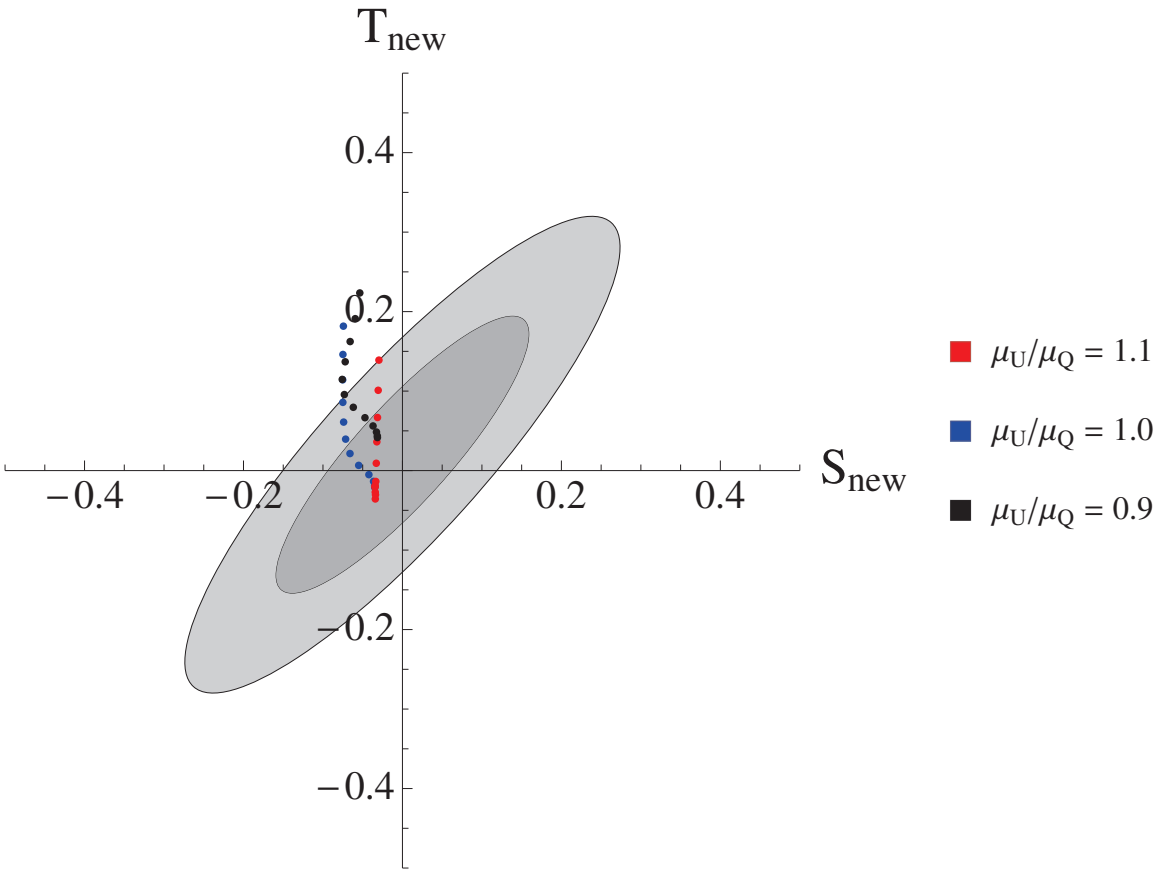

Figure: S_{new}, T_{new} for ratios $\mu_U/\mu_Q = 0.9, 1.0, 1.1$, and Yukawa values $y_{34} = -y_{43}$ ranging from 0.01 to 0.56 in steps of 0.05.

$\mu_Q \neq \mu_U$ can hurt or help

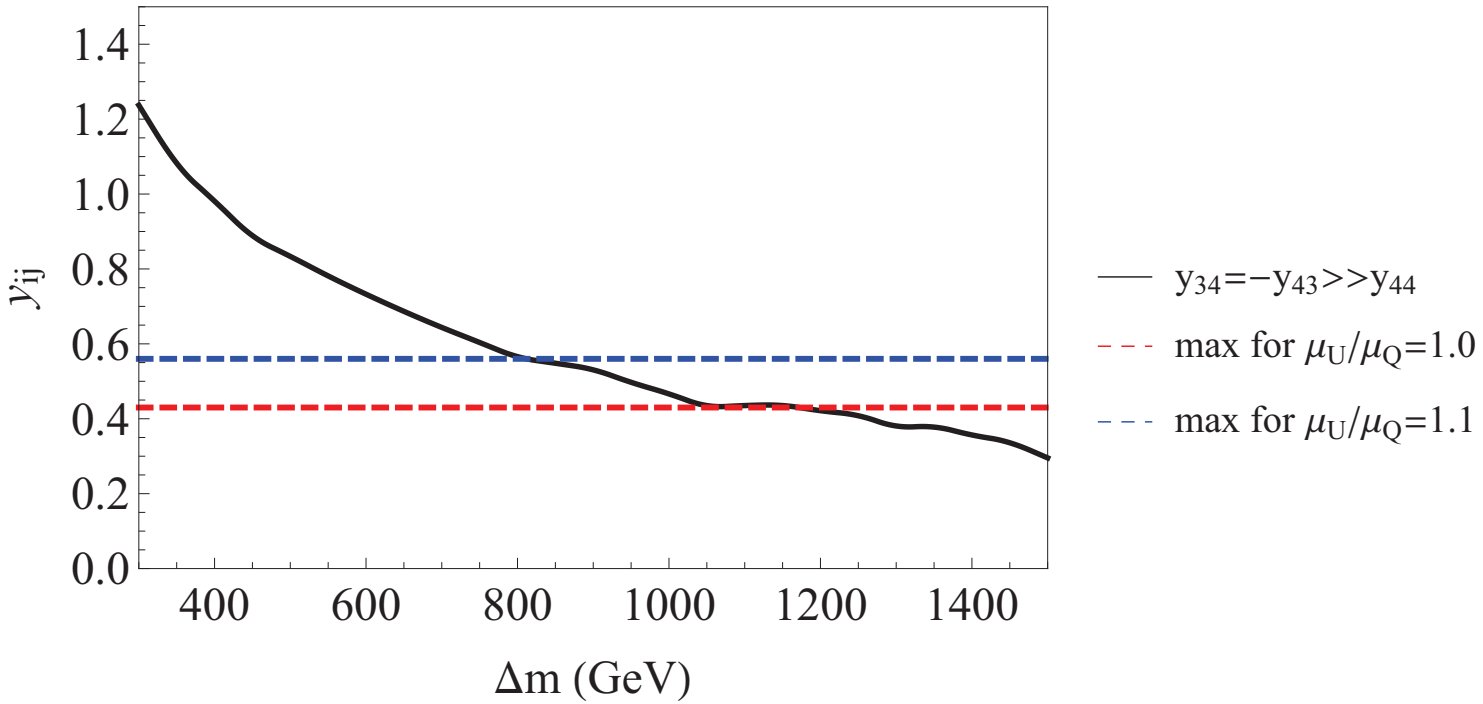

Figure: Yukawa value $y_{34} = -y_{43}$ required for $m_h = 125$ GeV. Dashed lines are max allowed for $\mu_U/\mu_Q = 1.0, 1.1$.

- $|y_{34}| = |y_{43}| \lesssim 0.43 \rightarrow |y_{34}| = |y_{43}| \lesssim 0.56$
- soft terms $\lesssim 1100$ GeV $\rightarrow$ soft terms $\lesssim 800$ GeV

Conclusions

- Mixing raises m_h by increasing y_{33}.
- Landau poles can be pushed up with $\mu_4, \Delta m$, A-terms $<$ TeV
  1. $\Lambda_L \sim 10^{10}$ for soft masses ~ 800 GeV
  2. $\Lambda_L \gtrsim M_{GUT}$ for soft masses ~ 900 GeV
- Constraints allow sufficiently large y_{ij} to obtain $m_h = 125$ GeV with $\mu_4, \Delta m, A <$ TeV.

Conclusions

A large parameter space exists for SUSY models with a vector-like 4th generation that passes all experimental tests. It is predictive and within the LHC's reach. The models have a moderate soft SUSY breaking scale and therefore address the little hierarchy problem.

BACKUP SLIDES

Quark Mass Matrices

$$W \quad \subset \quad y^u_{ij} Q_i H_u U^c_j + \mu_4 \bar{Q}^c_4 Q_4 + \mu_4 \bar{U}_4 U^c_4 + \mu H_u H_d$$

Yukawa terms in W lead to the following fermion mass matrices:

$$m^u_f \equiv \begin{pmatrix} y_{33} v_u & y_{34} v_u & 0 \\ y_{43} v_u & y_{44} v_u & \mu_Q \\ 0 & \mu_U & 0 \end{pmatrix}, \qquad \text{and} \qquad m^d_f \equiv \begin{pmatrix} m_{\text{bot}} & 0 \\ 0 & \mu_Q \end{pmatrix}$$

Up Squark Squared Mass Matrix

After the $SU(2)_L \times U(1)_Y$ gauge symmetry is broken, Yukawa terms in W, soft terms, F terms, and D terms lead to:

$$(M_s^u)^2 = (M_f^u)^2 + \begin{pmatrix} Y_{u_3} & 0 & 0 & -y_{33}v_u X_u & -y_{34}v_u X_u & 0 \\ 0 & \mu_Q^2 + Y_{u_4} & 0 & -y_{43}v_u X_u & -y_{44}v_u X_u & B\mu \\ 0 & 0 & \mu_U^2 + Y_{\bar{u}_4} & 0 & B\mu & 0 \\ -y_{33}v_u X_u & -y_{43}v_u X_u & 0 & Y_{u_3^c} & 0 & 0 \\ -y_{34}v_u X_u & -y_{44}v_u X_u & B\mu & 0 & \mu_U^2 + Y_{u_4^c} & 0 \\ 0 & B\mu & 0 & 0 & 0 & \mu_Q^2 + Y_{\bar{u}_4^c} \end{pmatrix}$$

- $X_u = A + \mu \cot\beta$ and $X_d = A + \mu \tan\beta$.
- $Y_q \equiv \Delta m^2 + D_a$
- $D_a = (T_a^3 - Q_a \sin^2\theta_w)\cos(2\beta) m_Z^2$ (D-terms)

Down Squark Squared Mass Matrix

$$(M_s^d)^2 = (M_f^d)^2 + \begin{pmatrix} m_{\text{bot}}^2 + Y_{d_3} & 0 & -m_{\text{bot}} X_d & 0 \\ 0 & \mu_Q^2 + Y_{d_4} & 0 & B\mu \\ -m_{\text{bot}} X_d & 0 & m_{\text{bot}}^2 + Y_{d_3^c} & 0 \\ 0 & B\mu & 0 & \mu_Q^2 + Y_{\bar{d}_4^c} \end{pmatrix}$$

- $X_d = A + \mu \tan\beta$
- $Y_q \equiv \Delta m^2 + D_a$
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Top Yukawa Landau Poles: $A-$terms, case 2

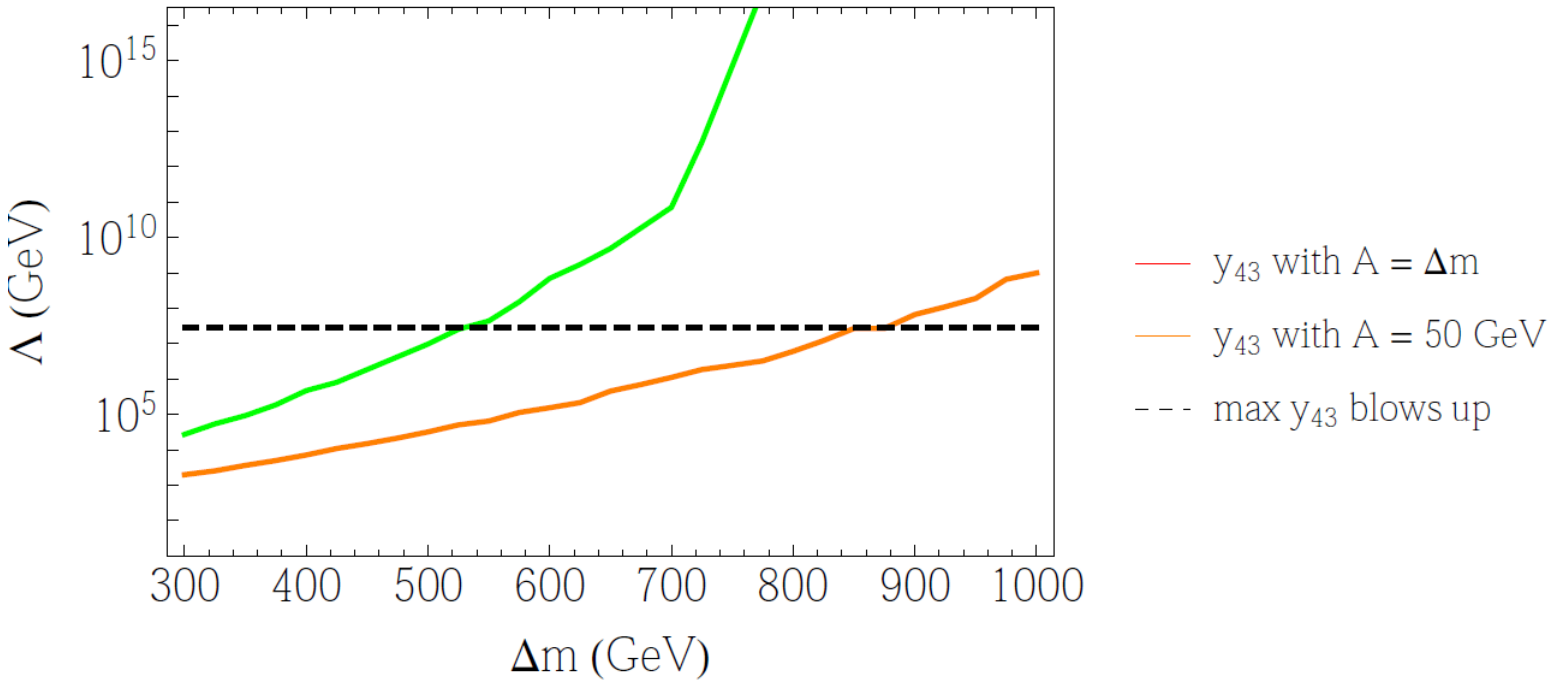

Figure: $y_{43} \gg y_{44}, y_{34}$, $\mu_4 = 700$ GeV, and $n_5 = 0$. y_{33} becomes non-perturbative below the dotted lines.

- For a given soft mass, the implied Landau poles get significantly pushed up by the presence of A-terms.

LHC phenomenology

Initial State	Intermediate state	Final State	Initial State	Intermediate State	Final State
t'	ht	$bbWb$	b'	hb	bbb
t'	Zt	$ffWb$	b'	Zb	ffb
t'	Wb	Wb	b'	Wt	WWb
$t't$	htt	$bbWbWb$	$b'b$	hb	$bbbb$
$t't$	Ztt	$ffWbWb$	$b'b$	Zb	$ffbb$
$t't$	Wbt	$WbWb$	$b'b$	Wtb	$WWbb$
$t'bj$	$htbj$	$bbWbbj$	$b'tj$	$hbWbj$	$bbbWbj$
$t'bj$	$Ztbj$	$ffWbbj$	$b'tj$	$ZbWbj$	$ffbWbj$
$t'bj$	$Wbbj$	$Wbbj$	$b'tj$	$WtWbj$	$WWbWbj$

Table: Possible event topologies with initial state singly produced t' or b'.

- t' decays through three decay chanels: ht, Zt, or Wb.
- Single production $t'bj$ via t-channel W, can have a larger cross section than $t't$.

LHC phenomenology continued ...

Initial State	Intermediate state	Final State	Initial State	Intermediate State	Final State
$t't'$	$htht$	$bbWbbbWb$	$b'b'$	$hbhb$	$bbbbbb$
$t't'$	$htZt$	$bbWbffWb$	$b'b'$	$hbZb$	$bbbffb$
$t't'$	$htWb$	$bbWbWb$	$b'b'$	$hbWt$	$bbWWb$
$t't'$	$ZtZt$	$ffWbffWb$	$b'b'$	$ZbZb$	$ffbffb$
$t't'$	$ZtWb$	$ffWbWb$	$b'b'$	$ZbWt$	$ffbWWb$
$t't'$	$WbWb$	$WbWb$	$b'b'$	$WtWt$	$WWbWWb$

Table: Possible event topologies with initial state pair produced t' or b'.

- As many as six b jets.
- As many as six W's (if Higgs decays via the WW^*).
- $t'bj \to Wbbj$ and $t't' \to WbWb$ good for discovery since m_{Wb} would reconstruct to $m_{t'}$ and the signals are relatively clean.

Contribution to S and T from new particles

- Standard test of any BSM model

$$S \propto \left(\Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0) - \frac{c_W^2}{c_W s_W}\Pi_{Z\gamma}(M_Z^2) - \Pi_{\gamma\gamma}(M_Z^2)\right)$$

- T is sensitive to isospin violation $(m_{t'} - m_{b'})$.

$$T = \frac{1}{\alpha}\left(\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right)$$

The Π's are the vector boson self-energies.

Self-energy diagrams needed for S and T

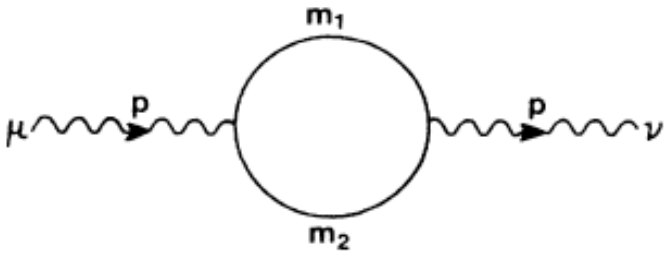

Figure: vector boson self-energies from fermions.

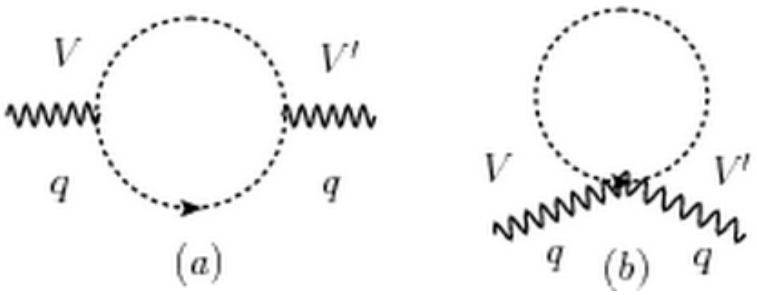

Figure: Vector boson self-energies from scalars.

Need to find all Feynan rules and new couplings

Higgs Production

- The loop amplitude can be shown to scale as

$$A_{gg\to h} \propto \frac{1}{\det m_f^2} \frac{\partial \det^2 m_f}{\partial v}$$

$$m_f = \begin{pmatrix} y_{33} v \sin\beta & y_{34} v \sin\beta & 0 \\ y_{43} v \sin\beta & y_{44} v \sin\beta & \mu_4 \\ 0 & \mu_4 & \bar{y}_{44} v \cos\beta \end{pmatrix}$$

- In our model, $\bar{y}_{44} = 0$ and we get $A_{gg\to h} \propto -2/v$
- This is Suppressed and has no μ_4 dependence so we can neglect this effect.

Constraints from V_{tb}^{CKM}

- $(K_L^{ud})_{1,1}$ should lie within the margin of error of the measured value of V_{tb}^{CKM}.
- We neglect mixing between the first two generations and the higher generations.
- CMS (unitary of V^{CKM} not assumed): $|V_{tb}^{\text{CKM}}| = 1.14 \pm 0.22$. We therefore require $0.92 < (K_L^{ud})_{1,1} < 1.36$.
- We scan over parameter space. This restriction is always satisfied!
- These constraints are negligible.

Sanity checks for S an T

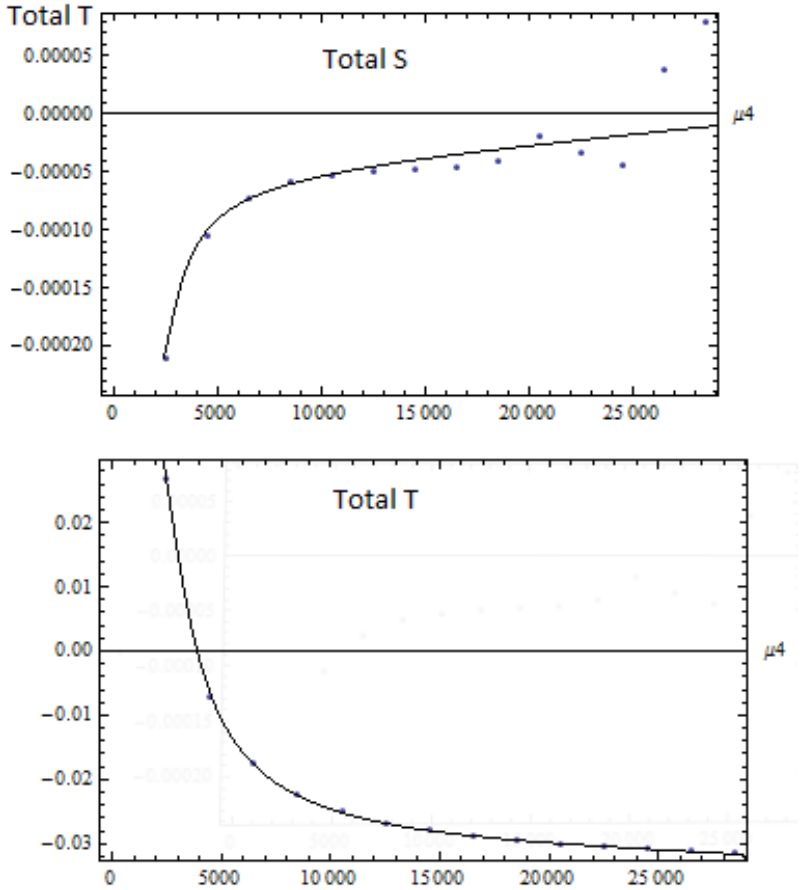

Figure: The μ_4 dependence of S and T for $y_{34} = -y_{43} \sim 0.8 \gg y_{44}$, (hence $y_{33} \sim 1.04$). Both S and T remain very small as $\mu_4 \to \infty$.

Mass Bounds from LHC Direct Searches: CMS

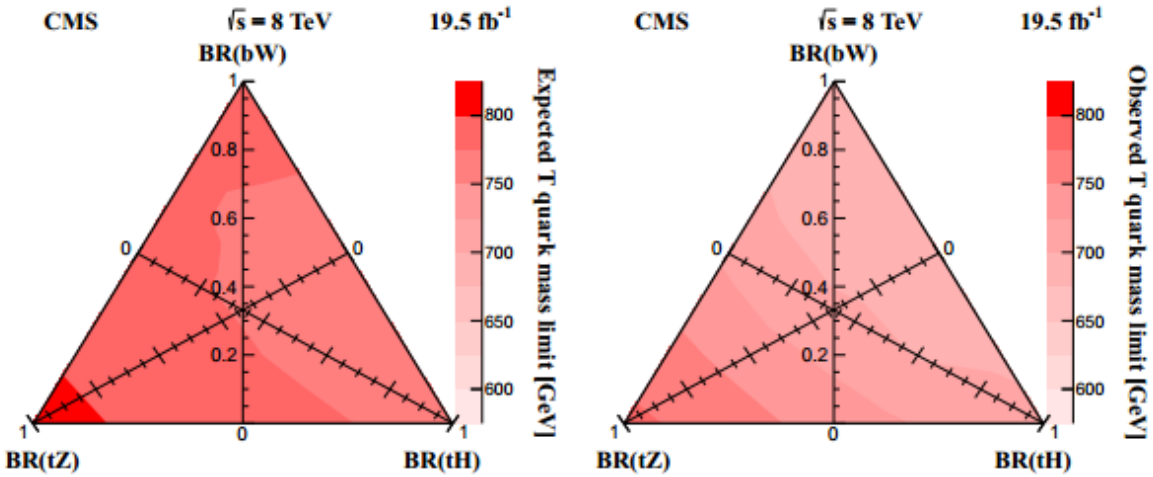

Figure: Present status of heavy vector-like top searches with 19.5 fb^{-1} of 8 TeV data with the CMS detector (Figure taken from CMS).

CMS: Model independent lower limits for $m_{t'}$ between 687 and 782 GeV for all possible branching fractions.

Mass Bounds from LHC Direct Searches: ATLAS (t')

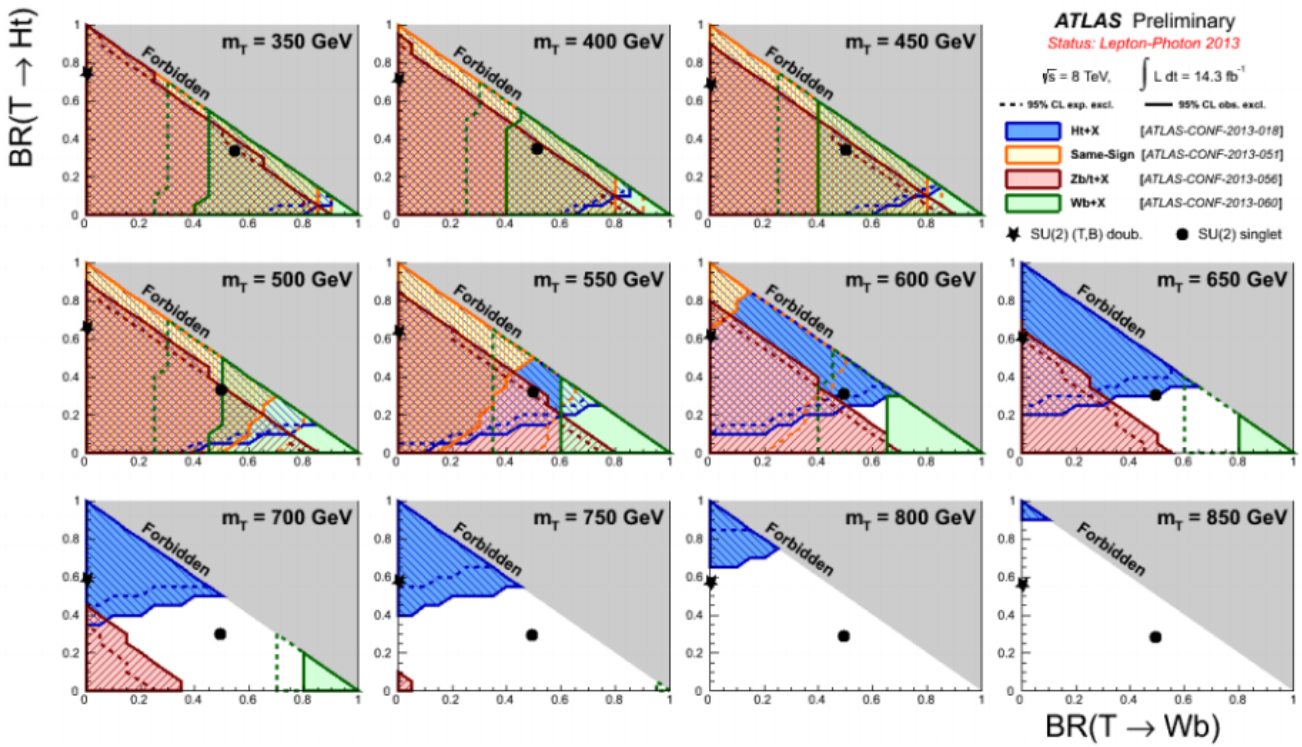

Figure: Present status (Lepton-Photon June 2013) of vector-like top searches with 14.3 fb^{-1} of 8 TeV data with the ATLAS detector.

ATLAS: lower limits $m_{t'} \sim 750$ GeV as well.

Mass Bounds from LHC Direct Searches: ATLAS b'

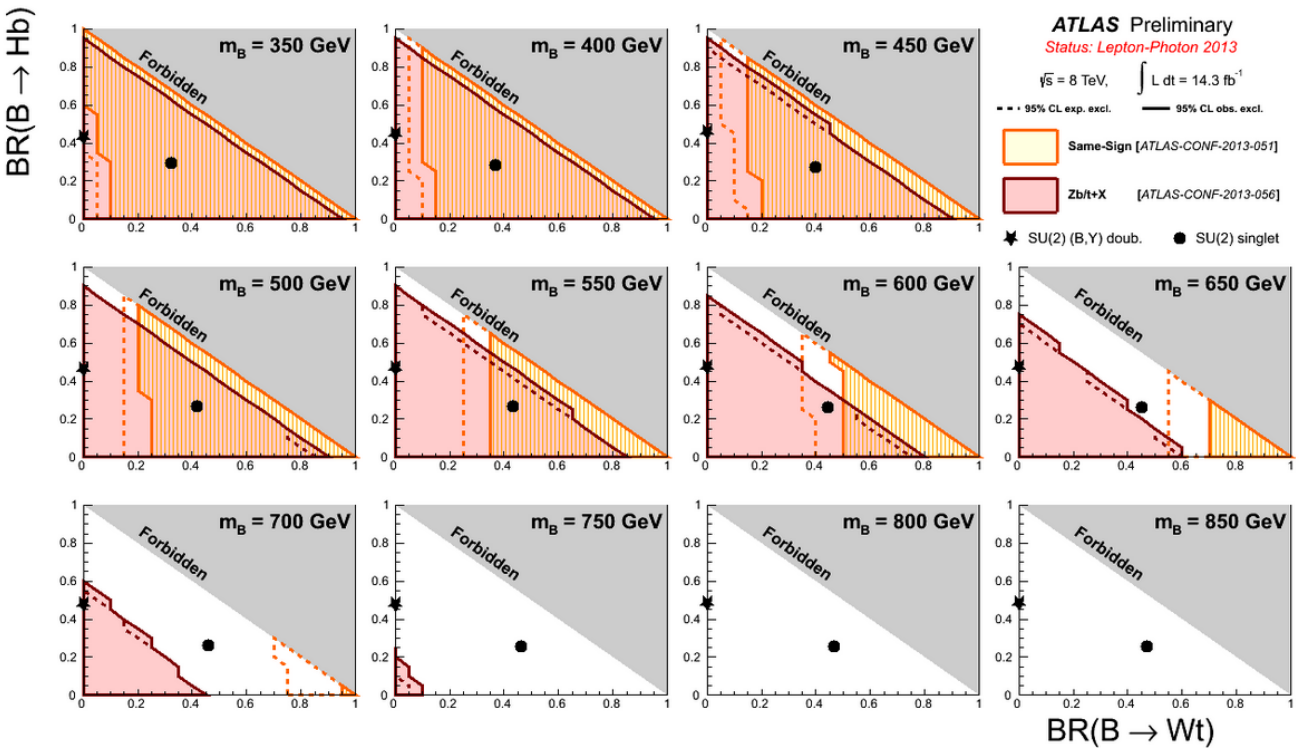

Figure: Present status (Lepton-Photon June 2013) of vector-like bottom searches with 14.3 fb^{-1} of 8 TeV data with the ATLAS detector.

Mass Lagrangian

With the mass matrices defined, the relevant mass gauge eigenstate Lagrangian is:

$$-\mathcal{L}_m = (f_L^{uT} m_f^u f_R^u + f_L^{dT} m_f^d f_R^d + \text{h.c}) + \tilde{f}^{u\dagger}(M_s^u)^2 \tilde{f}^u + \tilde{f}^{d\dagger}(M_s^d)^2 \tilde{f}^d$$

where the gauge basis in family space is

$$\begin{aligned}
f_L^u &= (u_3, u_4, \bar{u}_4)^T \\
f_R^u &= (u_3^c, u_4^c, \bar{u}_4^c)^T \\
f_L^d &= (d_3, d_4)^T \\
f_R^d &= (d_3^c, \bar{d}_4^c)^T \\
\tilde{f}^u &= (\tilde{u}_3, \tilde{u}_4, \tilde{\bar{u}}_4, \tilde{u}_3^c, \tilde{u}_4^c, \tilde{\bar{u}}_4^c)^T \\
\tilde{f}^d &= (\tilde{d}_3, \tilde{d}_4, \tilde{d}_3^c, \tilde{\bar{d}}_4^c)^T
\end{aligned}$$

We need to Diagonalize $\mathcal{L}_m$!

Diagonalizing the Fermion Mass Matrices

Physical masses obtained by bi-diagonalizing the mass matrices using singular value decomposition:

$$\begin{aligned} m_D^u &= V_L^{u\dagger} m_f^u V_R^u \\ m_D^d &= V_L^{d\dagger} m_f^d V_R^d \end{aligned}$$

- $V_L^{u,d}$ and $V_R^{u,d}$ are unitary
- $m_D^{u,d}$ are diagonal
- Singular values of m_f^u (m_f^d) give the physical masses of t,(b), $t'_{1,2}$ (b')
- mass basis is given by $\hat{f}_{L,R}^{u,d} = V_{L,R}^{u,d\dagger} f_{L,R}^{u,d}$

Diagonalizing the Scalar Mass Squared Matrices

Physical masses obtained by diagonalizing the mass squared matrices:

$$\begin{aligned} \tilde{M}_D^u &= W^{u\dagger} m_f^u W^u \\ \tilde{M}_D^d &= W^{d\dagger} m_f^d W^d \end{aligned}$$

- $W^{u,d}$ are unitary
- $\tilde{M}_D^{u,d}$ are diagonal
- Positive square roots of $(\tilde{M}_D^u)^2$ (and $(\tilde{M}_D^d)^2$) give the physical masses of $\tilde{t}_{1,2}$, $\tilde{t}'_{1,2,3,4}$, ($\tilde{b}_{1,2}$, $\tilde{b}'_{1,2}$)
- Mass basis is given by $\hat{\tilde{f}}^{u,d} = W^{u,d\dagger} \tilde{f}^{u,d}$

Interaction Lagrangian

CKM like matrices in every interaction vertex: FCNC's.

$\mathbf{V}_\mu \hat{\mathbf{f}}^{\mathbf{a}\dagger}_\alpha \mathbf{K}^{\mathbf{ab}}_\alpha \bar{\sigma}^\mu \hat{\mathbf{f}}^{\mathbf{b}}_\alpha$	$\mathbf{K}^{\mathbf{ab}}_\alpha$
$W^+_\mu \hat{f}^{u\dagger}_L K^{ud}_L \bar{\sigma}^\mu \hat{f}^d_L$	$V^{u\dagger}_L D^{ud}_L V^d_L$
$W^+_\mu \hat{f}^{d\dagger}_R K^{\bar{u}d\dagger}_R \bar{\sigma}^\mu \hat{f}^u_R$	$V^{u\dagger}_R D^{\bar{u}\bar{d}}_R V^d_R$
$Z^0_\mu \hat{f}^{u\dagger}_L K^{uu}_L \bar{\sigma}^\mu \hat{f}^u_L$	$V^{u\dagger}_L D^{uu}_L V^u_L$
$Z^0_\mu \hat{f}^{u\dagger}_L K^{\bar{u}\bar{u}}_L \bar{\sigma}^\mu \hat{f}^u_L$	$V^{u\dagger}_L S^{\bar{u}\bar{u}}_L V^u_L$
$Z^0_\mu \hat{f}^{u\dagger}_R K^{\bar{u}\bar{u}}_R \bar{\sigma}^\mu \hat{f}^u_R$	$V^{u\dagger}_R D^{\bar{u}\bar{u}}_R V^u_R$
$Z^0_\mu \hat{f}^{u\dagger}_R K^{uu}_R \bar{\sigma}^\mu \hat{f}^u_R$	$V^{u\dagger}_R S^{uu}_R V^u_R$
$Z^0_\mu \hat{f}^{d\dagger}_L K^{dd}_L \bar{\sigma}^\mu \hat{f}^d_L$	$V^{d\dagger}_L D^{dd}_L V^d_L$
$Z^0_\mu \hat{f}^{d\dagger}_R K^{\bar{d}\bar{d}}_R \bar{\sigma}^\mu \hat{f}^d_R$	$V^{d\dagger}_R D^{\bar{d}\bar{d}}_R V^d_R$
$Z^0_\mu \hat{f}^{d\dagger}_R K^{dd}_R \bar{\sigma}^\mu \hat{f}^d_R$	$V^{d\dagger}_R S^{dd}_R V^d_R$

$\mathbf{V}_\mu \hat{\tilde{\mathbf{f}}}^{\mathbf{a}\dagger} \tilde{\mathbf{K}}^{\mathbf{ab}}_\alpha \overleftrightarrow{\partial}^\mu \hat{\tilde{\mathbf{f}}}^{\mathbf{b}}$	$\mathbf{V}_\mu \mathbf{V}^\mu \hat{\tilde{\mathbf{f}}}^{\mathbf{a}\dagger} \tilde{\mathbf{K}}^{\mathbf{ab}}_\alpha \hat{\tilde{\mathbf{f}}}^{\mathbf{b}}$	$\tilde{\mathbf{K}}^{\mathbf{ab}}_\alpha$
$W^+_\mu \hat{\tilde{f}}^{u\dagger} \tilde{K}^{ud}_L \overleftrightarrow{\partial}^\mu \hat{\tilde{f}}^d$	$W^+_\mu W^{\mu+} \hat{\tilde{f}}^{u\dagger} \tilde{K}^{ud}_L \hat{\tilde{f}}^d$	$W^{u\dagger} \tilde{D}^{ud}_L W^d$
$W^+_\mu \hat{\tilde{f}}^{d\dagger} \tilde{K}^{\bar{u}\bar{d}\dagger}_R \overleftrightarrow{\partial}^\mu \hat{\tilde{f}}^u$	$W^+_\mu W^{\mu+} \hat{\tilde{f}}^{d\dagger} \tilde{K}^{\bar{u}\bar{d}\dagger}_R \hat{\tilde{f}}^u$	$W^{u\dagger} \tilde{D}^{\bar{u}\bar{d}}_R W^d$
$Z^0_\mu \hat{\tilde{f}}^{u\dagger} \tilde{K}^{uu}_L \overleftrightarrow{\partial}^\mu \hat{\tilde{f}}^u$	$Z^0_\mu Z^{\mu 0} \hat{\tilde{f}}^{u\dagger} \tilde{K}^{uu}_L \hat{\tilde{f}}^u$	$W^{u\dagger} \tilde{D}^{uu}_L W^u$
$Z^0_\mu \hat{\tilde{f}}^{u\dagger} \tilde{K}^{\bar{u}\bar{u}}_L \overleftrightarrow{\partial}^\mu \hat{\tilde{f}}^u$	$Z^0_\mu Z^{\mu 0} \hat{\tilde{f}}^{u\dagger} \tilde{K}^{\bar{u}\bar{u}}_L \hat{\tilde{f}}^u$	$W^{u\dagger} \tilde{S}^{\bar{u}\bar{u}}_L W^u$
$Z^0_\mu \hat{\tilde{f}}^{u\dagger} \tilde{K}^{\bar{u}\bar{u}}_R \overleftrightarrow{\partial}^\mu \hat{\tilde{f}}^u$	$Z^0_\mu Z^{\mu 0} \hat{\tilde{f}}^{u\dagger} \tilde{K}^{\bar{u}\bar{u}}_R \hat{\tilde{f}}^u$	$W^{u\dagger} \tilde{D}^{\bar{u}\bar{u}}_R W^u$
$Z^0_\mu \hat{\tilde{f}}^{u\dagger} \tilde{K}^{uu}_R \overleftrightarrow{\partial}^\mu \hat{\tilde{f}}^u$	$Z^0_\mu Z^{\mu 0} \hat{\tilde{f}}^{u\dagger} \tilde{K}^{uu}_R \hat{\tilde{f}}^u$	$W^{u\dagger} \tilde{S}^{uu}_R W^u$
$Z^0_\mu \hat{\tilde{f}}^{d\dagger} \tilde{K}^{dd}_L \overleftrightarrow{\partial}^\mu \hat{\tilde{f}}^d$	$Z^0_\mu Z^{\mu 0} \hat{\tilde{f}}^{d\dagger} \tilde{K}^{dd}_L \hat{\tilde{f}}^d$	$W^{d\dagger} \tilde{D}^{dd}_L W^d$
$Z^0_\mu \hat{\tilde{f}}^{d\dagger} \tilde{K}^{\bar{d}\bar{d}}_R \overleftrightarrow{\partial}^\mu \hat{\tilde{f}}^d$	$Z^0_\mu Z^{\mu 0} \hat{\tilde{f}}^{d\dagger} \tilde{K}^{\bar{d}\bar{d}}_R \hat{\tilde{f}}^d$	$W^{d\dagger} \tilde{D}^{\bar{d}\bar{d}}_R W^d$
$Z^0_\mu \hat{\tilde{f}}^{d\dagger} \tilde{K}^{dd}_R \overleftrightarrow{\partial}^\mu \hat{\tilde{f}}^d$	$Z^0_\mu Z^{\mu 0} \hat{\tilde{f}}^{d\dagger} \tilde{K}^{dd}_R \hat{\tilde{f}}^d$	$W^{d\dagger} \tilde{S}^{dd}_R W^d$

Coupling Matrices

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Coupling Matrix	Explicit Form
G^W_{ud}	$\frac{g}{\sqrt{2}}K^{ud}_L$
$G^Z_{u_L}$	$g^Z_{(\frac{1}{2},\frac{2}{3})}K^{uu}_L + g^Z_{(0,\frac{2}{3})}K^{\bar{u}\bar{u}}_L$
$G^Z_{d_L}$	$g^Z_{(-\frac{1}{2},-\frac{1}{3})}K^{dd}_L$
$G^A_{u_L}$	$g^A_{\frac{2}{3}}[K^{uu}_L + K^{\bar{u}\bar{u}}_L]$
$G^A_{d_L}$	$g^A_{-\frac{1}{3}}K^{dd}_L$

Coupling Matrix	Explicit Form
$G^W_{\bar{u}\bar{d}}$	$-\frac{g}{\sqrt{2}}K^{\bar{u}\bar{d}}_R$
$G^Z_{u_R}$	$g^Z_{(0,-\frac{2}{3})}K^{uu}_R + g^Z_{(-\frac{1}{2},-\frac{2}{3})}K^{\bar{u}\bar{u}}_R$
$G^Z_{d_R}$	$g^Z_{(0,\frac{1}{3})}K^{dd}_R + g^Z_{(\frac{1}{2},\frac{1}{3})}K^{\bar{d}\bar{d}}_R$
$G^A_{u_R}$	$g^A_{\frac{2}{3}}[K^{uu}_R + K^{\bar{u}\bar{u}}_R]$
$G^A_{d_R}$	$g^A_{-\frac{1}{3}}K^{dd}_R$

TABLE VI: The coupling matrices at the triple vertex between quarks and gauge bosons.

We define $g^Z_{(T^3,Q)} = \frac{g}{\cos\theta_W}(T^3 - Q\sin^2\theta_W)$, $g^A_Q = Qe$

Coupling Matrix	Explicit Form
$\tilde{G}^W_{ud}$	$\frac{g}{\sqrt{2}}\tilde{K}^{ud}_L$
$\tilde{G}^Z_u$	$g^Z_{(\frac{1}{2},\frac{2}{3})}\tilde{K}^{uu}_L + g^Z_{(0,\frac{2}{3})}\tilde{K}^{\bar{u}\bar{u}}_L + g^Z_{(0,-\frac{2}{3})}\tilde{K}^{uu}_R + g^Z_{(-\frac{1}{2},-\frac{2}{3})}\tilde{K}^{\bar{u}\bar{u}}_R$
$\tilde{G}^A_u$	$g^A_{\frac{2}{3}}\tilde{K}^{uu}_L + g^A_{\frac{2}{3}}\tilde{K}^{\bar{u}\bar{u}}_L + g^A_{\frac{-2}{3}}\tilde{K}^{uu}_R + g^A_{\frac{-2}{3}}\tilde{K}^{\bar{u}\bar{u}}_R$
$\tilde{G}^W_{\bar{u}\bar{d}}$	$-\frac{g}{\sqrt{2}}\tilde{K}^{\bar{u}\bar{d}}_R$
$\tilde{G}^Z_d$	$g^Z_{(-\frac{1}{2},-\frac{1}{3})}\tilde{K}^{dd}_L + g^Z_{(0,\frac{1}{3})}\tilde{K}^{dd}_R + g^Z_{(\frac{1}{2},\frac{1}{3})}\tilde{K}^{\bar{d}\bar{d}}_R$
$\tilde{G}^A_d$	$g^A_{-\frac{1}{3}}\tilde{K}^{dd}_L + g^A_{\frac{1}{3}}\tilde{K}^{dd}_R + g^A_{\frac{1}{3}}\tilde{K}^{\bar{d}\bar{d}}_R$

TABLE VII: The coupling matrices at the triple vertex between squarks and gauge bosons.

We define $g^Z_{(T^3,Q)} = \frac{g}{\cos\theta_W}(T^3 - Q\sin^2\theta_W)$, $g^A_Q = Qe$

Perturbativity of Gauge Couplings

- Perturbative gauge coupling unification ($g_{\text{unif}} \lesssim 3$) is verified at 1-loop for MSSM $+ \mathbf{10} + \bar{\mathbf{10}} + \mathbf{5} + \bar{\mathbf{5}}$.
- The 1-loop Beta function is:

$$16\pi^2 \frac{dg_i}{dt} = -b_i g_i^3, \quad t = \ln Q$$

The Beta function coefficients are:

$$\begin{aligned} b1 &= \frac{3(11)}{5} + n_{10} b_{10} + n_5 b_5 \\ b2 &= 1 + n_{10} b_{10} + n_5 b_5 \\ b3 &= -3 + n_{10} b_{10} + n_5 b_5 \end{aligned}$$

with $b_{10} = 3$, $b_5 = 1$ from group theory. The $\mathbf{5}$'s push up Λ_L since they make g_i stronger in the UV, slowing the growth of y_{ij}'s

Top Yukawa Landau Poles: 2-loop Beta function

The 2-loop Beta function of the top Yukawa is (Martin and Vaughn):

$$\begin{aligned}\beta_{Y_u}(t) &= \frac{1}{16\pi^2}[(3\mathrm{Tr}[Y_u(t).Y_u^\dagger(t)]Y_u(t) + 3Y_u(t)Y_u^\dagger(t)Y_u(t) \\ &+ Y_u(t)Y_d^\dagger(t)Y_d(t)) \\ &- (\frac{16}{3}g_3(t)^2 + 3g_2(t)^2 + \frac{13}{15}g_1(t)^2)Y_u(t)]\end{aligned}$$

Here, Y_u is the up-type Yukawa coupling matrix containing y_{33}, y_{34}, y_{43} and y_{44}.

The top and Higgs sector in the MSSM

- The MSSM Superpotential contains

$$W_{MSSM} \subset y^u_{33} Q_3 H_u U^c_3 + \mu H_u H_d$$

- The soft Lagrangian contains

$$\begin{aligned} -L^{MSSM}_{Soft} &\subset (A^u_{33} \tilde{Q}_3 H_u \tilde{U}^c_3 + \text{c.c}) \\ &+ m^2_{Q_3} \tilde{Q}^\dagger_3 \tilde{Q}_3 + m^2_{U^c_3} \tilde{U}^c_3 \tilde{U}^{c\dagger}_3 + m^2_{D^c_3} \tilde{D}^c_3 \tilde{D}^{c\dagger}_3 \\ &+ m^2_{H_u} H^*_u H_u + m^2_{H_d} H^*_d H_d + (B_\mu H_u H_d + \text{c.c}) \end{aligned}$$

- The classical scalar potential for the neutral Higgs is:

$$\begin{aligned} V &= (|\mu|^2 + m^2_{H_u})|H^0_u|^2 + (|\mu|^2 + m^2_{H_d})|H^0_d|^2 \\ &- (B_\mu H^0_u H^0_d + \text{c.c}) + \frac{1}{8}(g^2 + g'^2)(|H^0_u|^2 - |H^0_d|^2)^2 \end{aligned}$$