Black Holes

- Most interesting and intriguing objects in our Universe
- Hawking (1974): black holes radiate
- Treatment only semi-classical
- Many questions left unanswered (information loss paradox)
- Recent developments:

Firewall Paradox!
• Black holes – basics

• The setup of the Firewall Paradox

• Assumptions made in FP

• No need for firewalls

• *Unitary evolution perhaps implies “icezones”*

• *Interactions can purify thermal density matrix*
Firewall Paradox

The Setup
We Believe that

(1) Hawking evaporation is information preserving

(2) Low energy effective field theory should be valid beyond some microscopic distance from the horizon

(3) Infalling observer does not see anything unusual at the horizon

$L - \text{mode of late Hawking radiation}$

*Unitarity*: $L$ is entangled with earlier radiation $E$

*Smooth horizon*: $L$ is also entangled with mode $L'$ inside the horizon
L ↔ E and simultaneously L ↔ L’ which is impossible!

**AMPS**: The least painful option - FIREWALL

AMPS gave us a free choice:
abandon unitarity and/or QFT, or accept the firewalls

The illusion of free choice.
Option 1:

Dismiss firewalls altogether!

Old School (*Bill Unruh, Bob Wald...*)

*Large BH can have arbitrarily small curvature at the horizon*

- We tested GR in the low curvature region very well
- Quantum corrections to classical GR solutions negligible
The same argument implies: superconductivity or superfluidity can’t exist

We tested Maxwell’s equations at the scale of cm many times.

*We know that electrons interact with EM force with ions in the crystal lattice – finite resistance*

**HOWEVER**

Macroscopic quantum phenomena like SC and SF still possible
Option 2:

Dissect the firewalls paradox!
To have a paradox, someone has to observe it

Asymptotic observer can’t see all three modes

Infalling can, but he can never compare his findings with asymptotic

Bousso: Infalling guy comes close to the horizon, uses his theory to infer entanglement between L and L’ and then comes back to infinity
In QM two modes are either entangled or not: $\psi = \alpha \uparrow \downarrow + \beta \downarrow \uparrow$

In GR, accelerated observer sees **entanglement degradation**

**Reason:** flux of Hawking radiation (interactions destroy entanglement)

*I. Fuentes-Schuller and R. B. Mann, Phys. Rev. Lett. 95, 120404 (2005)*

*E. Martin-Martinez, L. J. Garay and J. Leon, Phys. Rev. D 82, 064006 (2010)*

Infalling guy becomes accelerated when he turns back!
He will witness entanglement degradation
• Formulation of the paradox heavily relies on the statement about the strong correlations between the large and small subsystems.

• If we divide a system into small and large subsystems,
  - All the information is in the correlations between the systems,
  - Large and small systems are maximally entangled.
AMPS: Unitarity implies $L \leftrightarrow E$

- All the information is in the correlations between the small and large systems
- Large and small systems are maximally entangled

- Large subsystem is early Hawking radiation $E$
- Small subsystem is late Hawking radiation $L$

- AMPS interpretation: early radiation and a mode of the late radiation must be maximally entangled.
Divide a system into two subsystems of sizes $m$ and $n$, $m<n$

After long enough time there is almost no information left in the small subsystem

$$I_m = S_{\text{max}} - \langle S \rangle \approx \frac{m}{2n}, \quad \text{for } 1 \ll m < n$$

All the information is in the correlations between the systems

**Problem with AMPS interpretation:**

1. Note that $1 \ll m$, so the “small system” can’t be $L$

2. Not clear why BH is taken out of the picture
   (you can do it only at the end of evaporation, not before)
Unitary process of particle decay

I)

II)

- It is incorrect to say that $E$ is strongly entangled with $L$
- $E$ is strongly entangled with the system $B + L$
Step 1: black hole $B_0$ emits spin $\frac{1}{2}$ particle and changes its state into $B_1$

$$\psi_{B_1 E} = \alpha \left| \uparrow_{B_1} \downarrow_E \right> + \beta \left| \downarrow_{B_1} \uparrow_E \right>$$

Step 2: black hole $B_1$ emits spin $\frac{1}{2}$ particle and changes its state into $B_2$

$$\psi_{B_2 EL} = \alpha \left( \gamma \left| \uparrow_{B_2} \downarrow_E \downarrow_L \right> + \delta \left| 0_{B_2} \downarrow_E \uparrow_L \right> \right) + \beta \left( \lambda \left| \downarrow_{B_2} \uparrow_E \uparrow_L \right> + \sigma \left| 0_{B_2} \uparrow_E \downarrow_L \right> \right)$$
If we know the state of E (say $E=\uparrow$) then we know the state of the system ($B+L$)

$$\psi_{B_2L} = \alpha\left|\downarrow_{B_2} \uparrow_L\right> + \delta\left|0_{B_2} \downarrow_L\right>$$

If we know the state of L (say $L=\downarrow$) then we know the state of the system ($B+E$)

$$\psi_{B_2E} = \alpha\left|\uparrow_{B_2} \downarrow_E\right> + \beta\left|0_{B_2} \uparrow_E\right>$$

But we never know separate states of all three members!
But we can’t say that L is correlated with E and B(L’) separately!

• All we can say is that

  • E is strongly correlated with (L+B)
    or
  • L is strongly correlated with (E+B)
    or
  • B is strongly correlated with (L+E)
Correct interpretation of Don Page’s result

at infinity

No black hole

\[ L_R \leftrightarrow E \]
\[ L \leftrightarrow E + L_R - L \]

Black hole still exists

\[ L_R \equiv L + B \]
\[ L \leftrightarrow E + B \]
\[ \text{unitarity} \]
\[ E \leftrightarrow L + B \]

We never have \( L \leftrightarrow E \) and \( L \leftrightarrow L' \)
• Interesting case

\[ L \text{ is strongly correlated with } (E+B) \]
\[ L \leftrightarrow L' \quad \text{Rindler approximation fails} \]

Extended entanglement needed!
All the outgoing modes will eventually be entangled

“Icezone” consists of a sea of quanta that mediate interactions (either perturbative or non-perturbative) between the thermal Hawking quanta
Density Matrix

\[ \rho = \sum_i p_i |\psi_i\rangle \langle \psi_i| \]

Pure state

\[ \rho_{\text{pure}} = \begin{pmatrix} p_1 & 0 & 0 & \ldots \\ 0 & 0 & 0 & \ldots \\ \end{pmatrix} \]

Mixed state

\[ \rho_{\text{mixed}} = \begin{pmatrix} p_1 & p_2 & \ldots \\ p_2 & p_3 & \ldots \\ \end{pmatrix} \]

You can’t convert mixed thermal state of Hawking radiation into a pure state
Small corrections do not help

$$\rho = \begin{pmatrix} p_1 + \varepsilon & 0 & 0 & 0 \\ 0 & p_2 + \varepsilon & 0 & 0 \\ 0 & 0 & p_3 + \varepsilon & 0 \\ 0 & 0 & 0 & p_4 + \varepsilon \end{pmatrix}$$
If we include interactions?

\[ \rho = \begin{pmatrix}
  p_{11} & \varepsilon p_{12} & \varepsilon p_{13} & \varepsilon p_{14} \\
  \varepsilon p_{21} & p_{22} & \varepsilon p_{23} & \varepsilon p_{24} \\
  \varepsilon p_{31} & \varepsilon p_{32} & p_{33} & \varepsilon p_{34} \\
  \varepsilon p_{41} & \varepsilon p_{42} & \varepsilon p_{43} & p_{44}
\end{pmatrix} \]
Interactions can purify thermal density matrix!

YOU WANT PROOF?
I'LL GIVE YOU PROOF!
Including interactions

Hawking: after black hole emits $K$ quanta its density matrix is thermal

$$\rho_0 = \frac{1}{2^K} \mathbf{1}_K$$

Spin $\frac{1}{2}$ particles

# of states is $2^K$

Superoperator $G$ gives corrections to $\rho_0$ due to interactions

$$\rho(t) = G(t)\rho_0 = (\mathbf{1} + A)\rho_0$$

Interactions:

$$A = a\mathbf{1} = a \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$
Coefficient $a$ gives the magnitude of the interactions

Non-perturbative:

$$a = e^{-S} \approx \frac{1}{2^N - K}$$

Total number of quanta that a BH can emit

$$N = \left( \frac{M_{bh}}{M_{Pl}} \right)^2$$

$K$ - Number of quanta emitted at a given moment

$$\rho_K = (1 + A) \rho_0 = \left( \frac{1}{K} + \frac{1}{2^N - K} \frac{1}{K} \right) \frac{1}{2^K} \frac{1}{K}$$

$$= \rho_{\text{max}} + \frac{1}{2^N} \frac{1}{K}$$
At the end of evaporation, $K=N$

$$\rho_N = \frac{1}{2^N} (\mathbf{1}_N + \mathbf{1}_N^\dagger) = \frac{1}{2^N} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \rho_{pure}$$

$\rho_{pure} = \frac{1}{2^N} \sum_{i,j} |i\rangle\langle j|$ is the density matrix of the pure state

$$\psi = \frac{1}{2^{N/2}} \sum_i |i\rangle$$

Non-perturbative interactions can unitarize Hawking radiation
Perturbative interactions

\[ a = \frac{1}{N!} \frac{1}{(N-K)!K!} \]

Normalized by the number of ways to choose \( K \) quanta out of \( N \) total quanta

\[ \rho_K = \left( 1_K + \frac{1}{N!} \frac{1}{(N-K)!K!} 1_K \right) \frac{1}{2^K} 1_K = \rho_{\text{max}} + \frac{1}{2^K} \frac{1}{N!} \frac{1}{(N-K)!K!} 1_K \]

At the end, \( K=\text{N} \)

\[ \rho_N = \left( 1_1 + 1_N \right) \frac{1}{2N} = \rho_{\text{pure}} \]

Perturbative interactions can unitarize Hawking radiation
Interesting trend

\[ a = \frac{1}{N! \frac{(N-K)!K!}{(N-K)!K!}} \]

If \( K=1 \), then \( a = 1/N \rightarrow \) suppression is large

\( a \) stays small until \( K=N/2 \)

When \( K=N \), then \( a=1 \)

Agrees well with

Page’s result:
black hole only starts to emit information at significant rate after half of the black hole has been radiated away
No need for firewalls whatsoever
Conclusions

1. Useful to keep questioning well established truths

2. Firewall paradox made us carefully re-examine some old statements

3. Firewall paradox not formulated consistently

4. Icezones are more likely than firewalls (small corrections due to standard physics)
THANK YOU