Tensor to scalar ratio and large scale power suppression from pre-slow roll initial conditions

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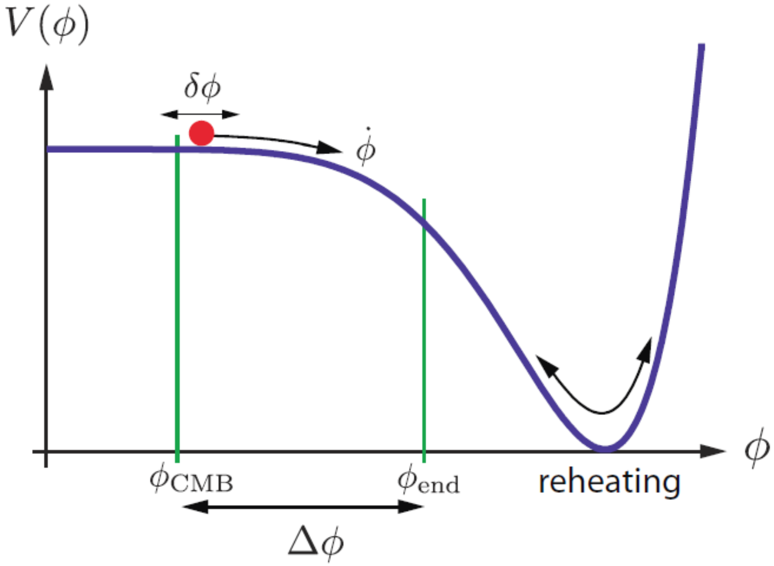
Slow Roll Inflation - Review



Generic slow roll type potential, Baumann **arXiv:0907.5424**

Inflation implemented to fix problems of standard big bang.

Slowly rolling gives correct scale factor for exponential expansion. Small acceleration parameter needed too.

Potential must be fairly flat to ensure enough inflation to correct horizon problem. 60 efolds worth.

Provides a mechanism that produces quantum fluctuations.

Leads to the slow roll paradigm: two small quantities.

Slow roll parameters constrained by Planck.

$$\epsilon_V = \frac{M_{Pl}^2}{2}\left[\frac{V'_{sr}(\Phi)}{V_{sr}(\Phi)}\right]^2$$

$$\eta_V = M_{Pl}^2 \frac{V''_{sr}(\Phi)}{V_{sr}(\Phi)}$$

Perturbations - Review

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_\alpha(\eta)\right] S_\alpha(k;\eta) = 0$$

Perturbations obey mode equations in field expansion with potential from inflation.

Mode equations provide a power spectrum for these perturbations.

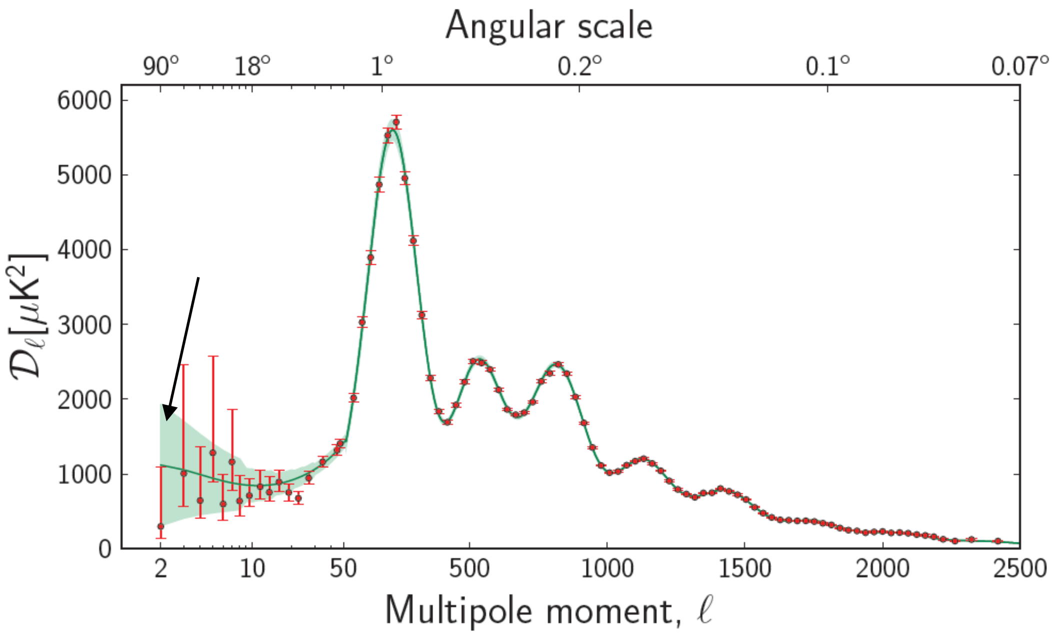
$$\mathcal{P}_\mathcal{R}(k) = \frac{k^3}{2\pi^2}\left|\frac{S_\mathcal{R}(k;\eta)}{z(\eta)}\right|^2 \longrightarrow \frac{\delta T}{T}$$

Perturbations are small fluctuations in space time: alter photon geodesics. Creates temperature fluctuations. In terms of small quantities in slow roll.

$$\mathcal{P}_T(k) = \frac{4\,k^3}{\pi^2\,M_{pl}^2}\left|\frac{S_T(k;\eta)}{C(\eta)}\right|^2$$

Inflation predicts production of gravitational waves

Motivations: Low quadrupole



Planck fits to temperature variations to model. Scalar power spectrum: 2 parameters.

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}}^2 \left(\frac{k}{aH}\right)^{n_{\mathcal{R}}-1}$$

Gravitational wave spectrum.

$$\mathcal{P}_T(k) = A_T^2 \left(\frac{k}{aH}\right)^{n_T}$$

Prediction of slow roll inflation.

$$r(k) = \frac{\mathcal{P}_T(k)}{\mathcal{P}_{\mathcal{R}}(k)} = -8n_T$$

Fits wonderfully but concerns remain.

Low multipole anomaly? Initial conditions?

Gravitational wave spectrum?

Pre-slow roll stage

Slow roll seems good, minimal excursion – more “natural” initial conditions.

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0$$

Slow roll

$$3H\dot{\Phi} + V'_{sr}(\Phi) \simeq 0$$

Fast roll

$$\ddot{\Phi} + 3H\dot{\Phi} \simeq 0$$

Pre-slow roll stage dominated by kinetic term. Still flat potential, drag reduces to slow roll value.

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_\alpha(\eta)\right] S_\alpha(k;\eta) = 0$$

Modifications enter through mode equations.

Allow large kinetic term. Alters mode equations, power spectrum.

$$\frac{\dot{\Phi}_i^2}{2V_{sr}} = \kappa$$

Corrections to mode equations, merging to slow roll

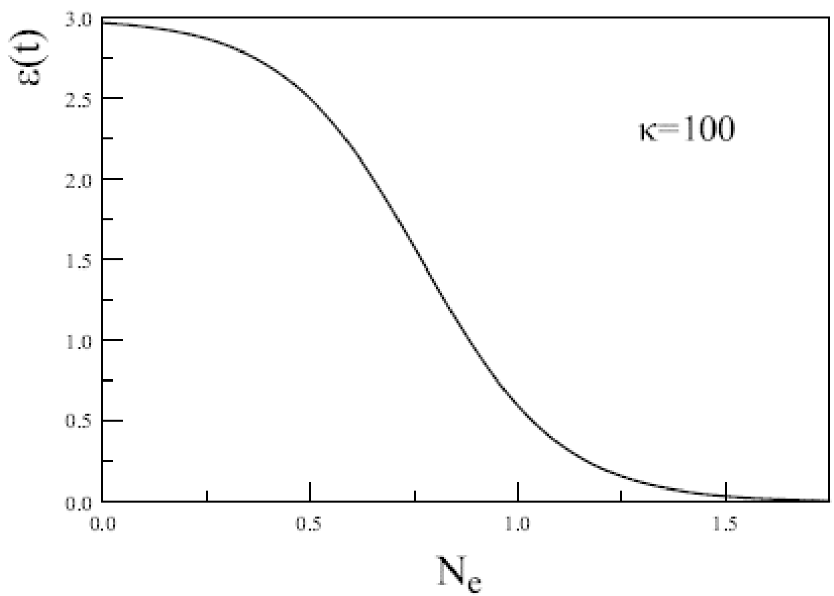
$$\left[\frac{d^2}{d\eta^2}+k^2-V_{SR}(\eta)\right]S(k;\eta)=\Delta V(\eta)S(k;\eta)$$

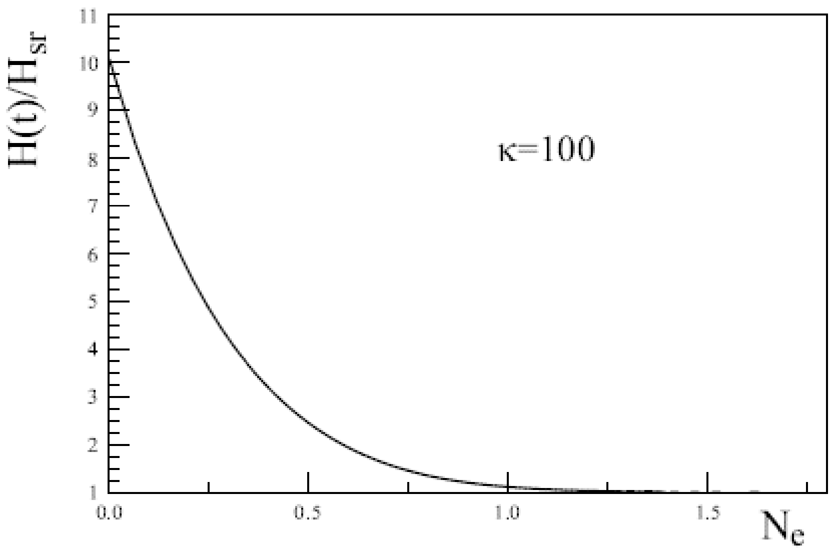
Have shown corrections to SR can be written as a Schrodinger equation.

Can implement powerful tools from scattering theory: Green’s function + perturbative expansion .

Technical details: **arXiv:1312.4251**

Merging happens very quickly (1 expansion time). Leading order only but changes mode equations with dramatic effect!



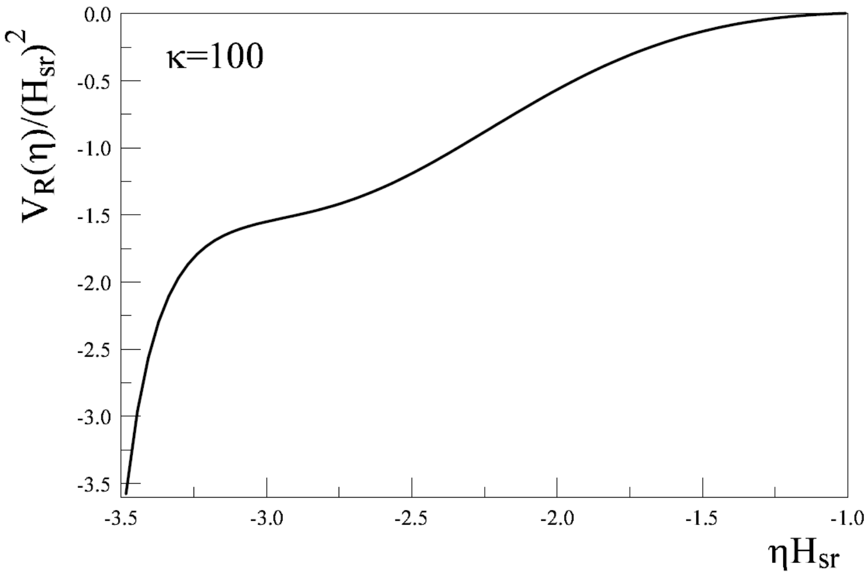


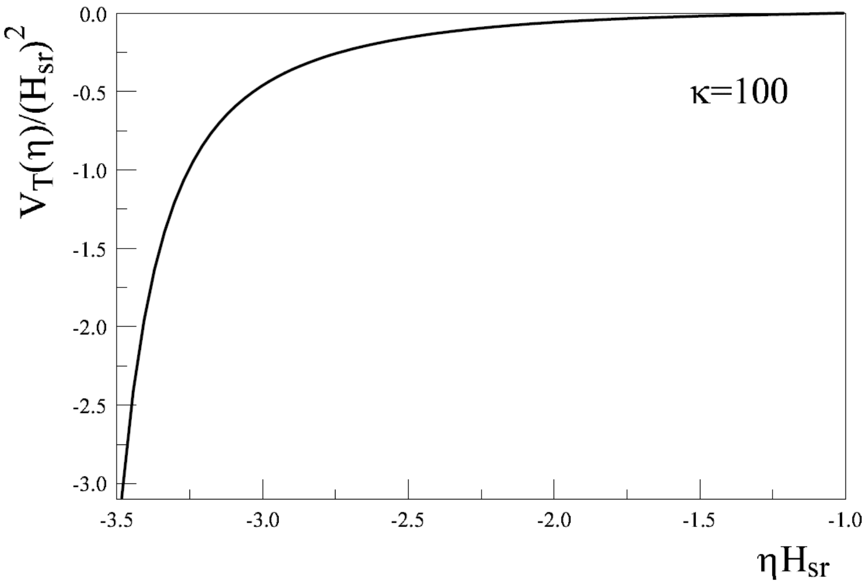
Acceleration parameter and Hubble factor as function of number of e-folds

Corrections to potentials

Alterations to potentials calculated from fast stage.
Returns to slow roll after scale factor grows by e.

Potential is localized before SR. Can treat like a scattering problem!





Corrections to tensor/scalar mode functions as function of conformal time.

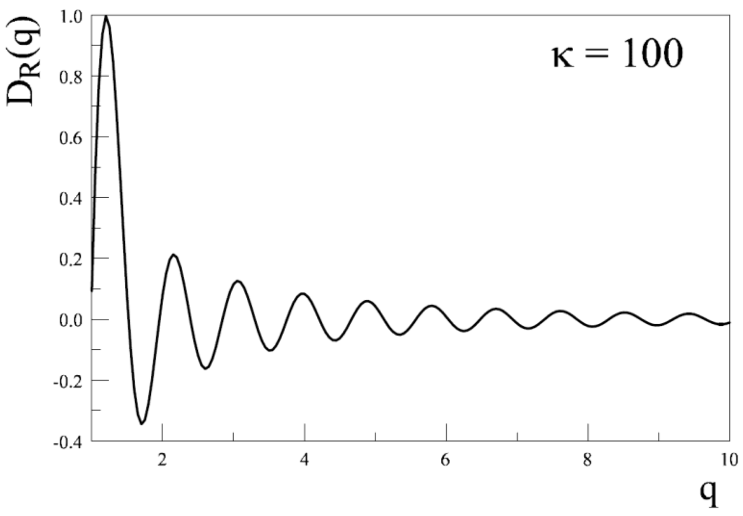
Corrections to power spectra

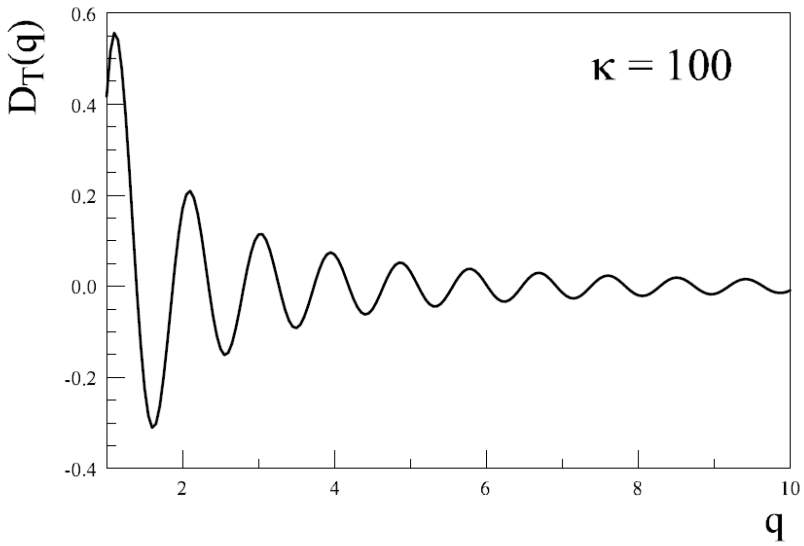
Power spectra corrected relevant for modes relevant today.

$$\mathcal{P}_\alpha(k) = \mathcal{P}_\alpha^{BD}(k)\,\mathcal{T}_\alpha(k)$$

Transfer function is akin to transmission coefficient.

$$\mathcal{T}_\alpha(k) = 1 + \frac{1}{k}\int_{\eta_i}^{\eta_{sr}} V_\alpha(\eta)\left[\frac{2\cos(2k\eta)}{k\eta} + \sin(2k\eta)\left(1 - \frac{1}{k^2\eta^2}\right)\right] d\eta$$





Corrections to tensor/scalar power spectra as function of wavevector.

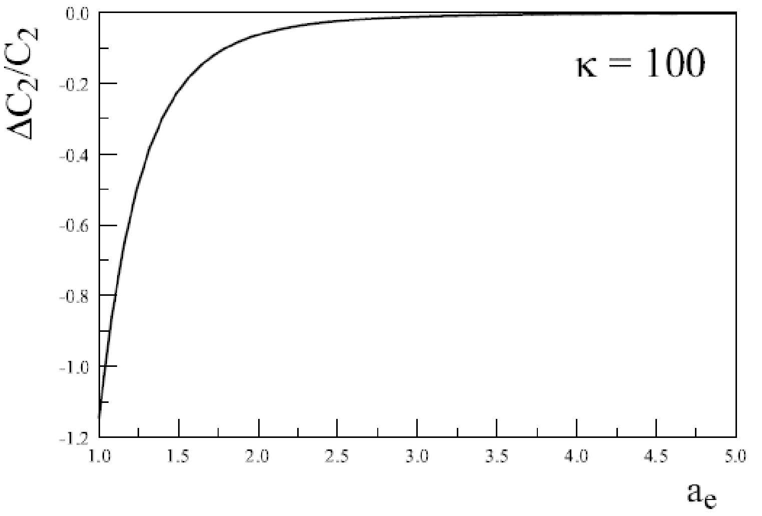
Corrections to multipoles

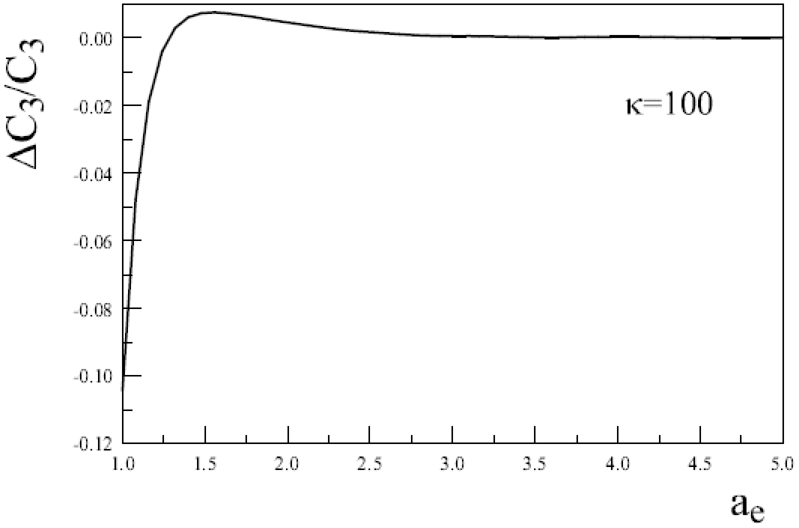
Multipoles corrected by transfer function.

Quadrupole modes exiting at beginning of SR are suppressed.

Corrections vanish deep in SR.

$$\frac{\Delta C_l}{C_l} = 2l(l+1)\int_0^{\infty}\frac{dq}{q}\,D_R(q)\,j_l^2\left[\frac{3.12}{a_e}q\right]$$





Corrections to multipoles as function of slow roll horizon crossing.

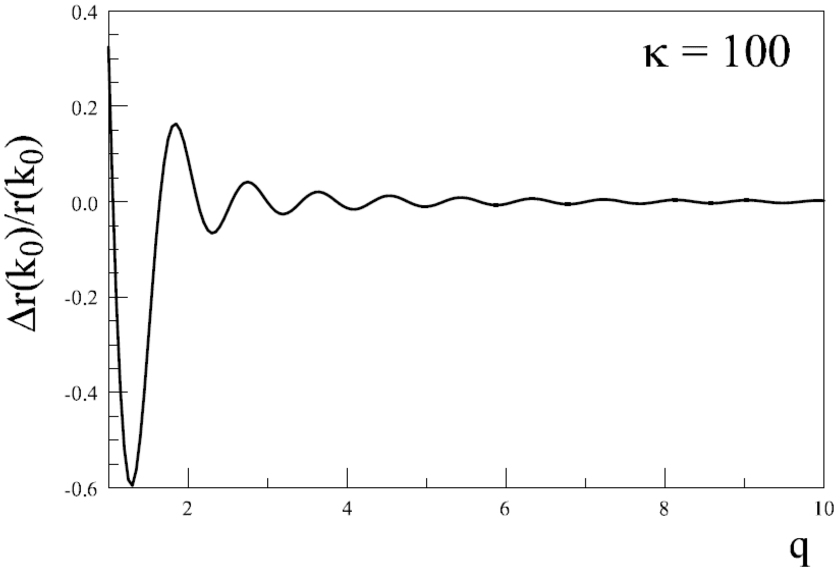
Corrections to tensor to scalar

Consistency relation modified by transfer function.

New potentials lead to change in tensor/scalar ratio.

$$r(k_0) = -8n_T(k_0)\left[\frac{\mathcal{T}_T(k_0)}{\mathcal{T}_\mathcal{R}(k_0)}\right]$$

$$\frac{\Delta r(k_0)}{r(k_0)} = D_T(q) - D_R(q)$$



Corrections to tensor/scalar ratio as function of wavevector.

Modifications lead to oscillatory behavior in modes.

At time of completion, skepticism of measuring gravitational waves. Now a real possibility exists.

Summary

- Fast roll stage merges smoothly to slow roll. Can be carried out systematically to desired accuracy.

- Fast roll leads to modified power spectra, could explain low quadrupole.

- Fast roll corrections lead to oscillatory behavior: potential signature.

Thanks for your attention!