Theory of Lepton Flavors

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Where Do We Stand?

- Exciting Time in ν Physics: recent hints/evidences of large θ_{13} from T2K, MINOS, Double Chooz, Daya Bay and RENO
- Latest 3 neutrino global analysis (including recent results from reactor experiments and T2K):

Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo (2013, updated March 2014)

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.37 – 2.49	2.30 – 2.55	2.23 – 2.61
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.38	2.32 – 2.44	2.25 – 2.50	2.19 – 2.56
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34	2.15 – 2.54	1.95 – 2.74	1.76 – 2.95
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40	2.18 – 2.59	1.98 – 2.79	1.78 – 2.98
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.37	4.14 – 4.70	3.93 – 5.52	3.74 – 6.26
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.55	4.24 – 5.94	4.00 – 6.20	3.80 – 6.41
δ/π (NH)	1.39	1.12 – 1.77	0.00 – 0.16 $\oplus$ 0.86 – 2.00	—
δ/π (IH)	1.31	0.98 – 1.60	0.00 – 0.02 $\oplus$ 0.70 – 2.00	—

➡ Evidence of $\theta_{13} \neq 0$
➡ hints of $\theta_{23} \neq \pi/4$
➡ expectation of Dirac CP phase δ

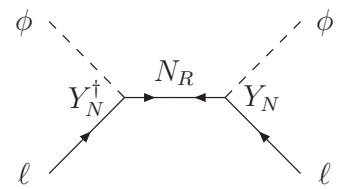
➡ no clear preference for hierarchy

➡ Majorana vs Dirac

Theoretical Challenges

(i) Absolute mass scale: Why $m_\nu << m_{u,d,e}$?

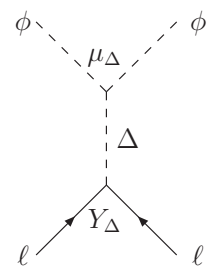
- seesaw mechanism: most appealing scenario ⇒ Majorana
- UV completions of Weinberg operators HHLL
  - Type-I seesaw: exchange of singlet fermions



Minkowski, 1977; Yanagida, 1979;
Glashow, 1979;
Gell-mann, Ramond, Slansky,1979;
Mohapatra, Senjanovic, 1979;

N_R: $SU(3)_c$ x $SU(2)_w$ x $U(1)_Y$
~(1,1,0)

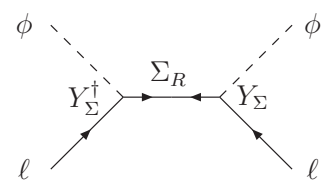
  - Type-II seesaw: exchange of weak triplet scalar



Lazarides, 1980; Mohapatra, Senjanovic, 1980

Δ: $SU(3)_c$ x $SU(2)_w$ x $U(1)_Y$
~(1,3,2)

  - Type-III seesaw: exchange of weak triplet fermion



Foot, Lew, He, Joshi, 1989; Ma, 1998

Σ_R: $SU(3)_c$ x $SU(2)_w$ x $U(1)_Y$
~(1,3,0)

Theoretical Challenges

(i) Absolute mass scale: Why $m_\nu << m_{u,d,e}$?

- seesaw mechanism: most appealing scenario ⇒ Majorana
  - GUT scale (type-I, II) vs TeV scale (type-III, inverse seesaw)
- TeV scale new physics (SUSY, extra dimension, U(1)′) ⇒ Dirac or Majorana

(ii) Flavor Structure: Why neutrino mixing large while quark mixing small?

- neutrino anarchy: no parametrically small number
  - near degenerate spectrum, large mixing
  - predictions strongly depend on choice of statistical measure
  - still alive and kicking

Hall, Murayama, Weiner (2000); de Gouvea, Murayama (2003)

de Gouvea, Murayama (2012)

- family symmetry: there's a structure, expansion parameter (~~symmetry~~ effect)
  - mixing result from dynamics of underlying symmetry
  - for leptons only (normal or inverted)
  - for quarks and leptons: quark-lepton connection ↔ GUT (normal)
- Alternative?

- In this talk: assume 3 generations, no LSND/MiniBoone/Reactor Anomaly
- **These scenarios have drastically different predictions**
- **precision measurements allow for distinguishing models**

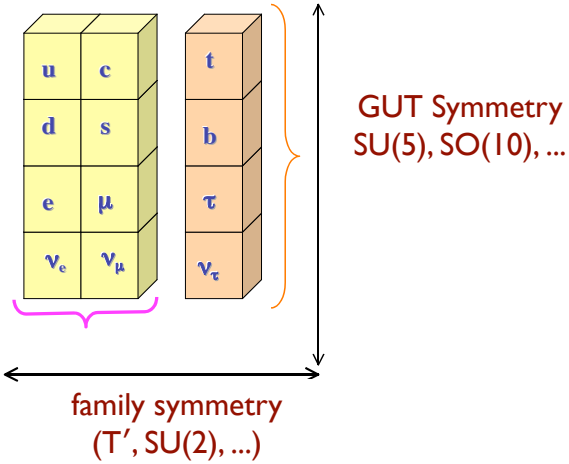
Origin of Mass Hierarchy and Mixing

- In the SM: 22 physical quantities which seem unrelated
- Question arises whether these quantities can be related
- No fundamental reason can be found in the framework of SM
- less ambitious aim ⇒ reduce the # of parameters by imposing symmetries
  - SUSY Grand Unified Gauge Symmetry
    - GUT relates quarks and leptons: quarks & leptons in same GUT multiplets
      - one set of Yukawa coupling for a given GUT multiplet ⇒ intra-family relations
    - seesaw mechanism naturally implemented
  - Family Symmetry
    - relate Yukawa couplings of different families
      - inter-family relations ⇒ further reduce the number of parameters

⇒ Experimentally testable *correlations* among physical observables

Origin of Flavor Mixing and Mass Hierarchy

- Several models have been constructed based on
  - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F
- Family Symmetries G_F based on continuous groups:
  - U(1)
  - SU(2)
  - SU(3)



- Recently, models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T´ (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_4

Motivation: Tri-bimaximal (TBM) neutrino mixing

Discrete gauge anomaly: Araki, Kobayashi, Kubo, Ramos-Sanchez, Ratz, Vaudrevange (2008)

Anomaly-free discrete R-symmetries: simultaneous solutions to mu problem and proton decay problem, naturally small Dirac neutrino mass, M.-C.C, M. Ratz, C. Staudt, P. Vaudrevange, (2012); M.-C.C, M. Ratz, A. Trautner (2013)

Tri-bimaximal Neutrino Mixing

- Tri-bimaximal Mixing Pattern

L. Wolfenstein (1978); Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3 \qquad \sin\theta_{13,\mathrm{TBM}} = 0.$$

- General approach:
  - PMNS = LO prediction (TBM, BM, ...) + corrections
  - corrections:
    - higher order terms in super potential (family symmetry)
    - contributions from charged lepton sector (GUT symmetry)

Non-Abelian Finite Family Symmetry A4

- TBM mixing matrix: can be realized with finite group family symmetry based on A_4 Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); ...

- A_4: even permutations of 4 objects

  S: (1234) → (4321)

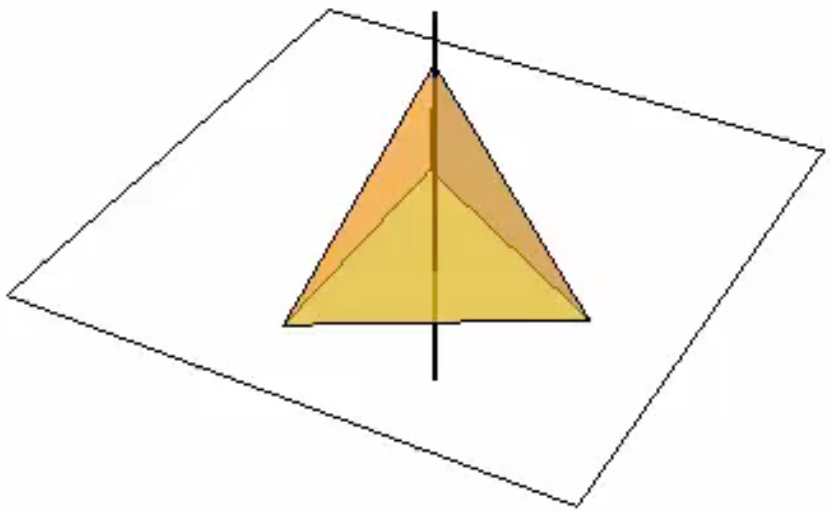
  T: (1234) → (2314)

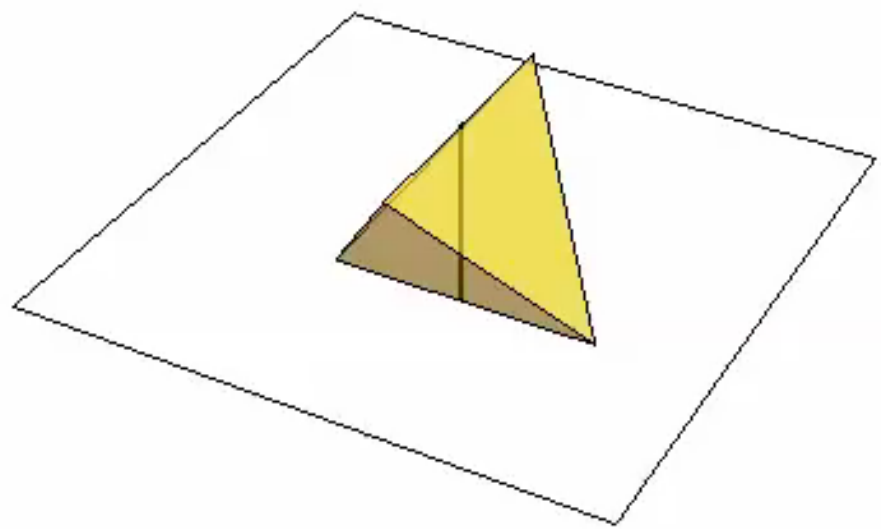
- Group of order 12
- Invariant group of tetrahedron

Invariant Group of Tetrahedron

T: (1234) → (2314)

S: (1234) →(4321)





Tri-bimaximal Neutrino Mixing from A_4

Altarelli, Feruglio (2005)

- fermion charge assignments:

$$L = \begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix} \sim 3, \quad e_R \sim 1, \quad \mu_R \sim 1'', \quad \tau_R \sim 1'$$

- SM Higgs ~ singlet under A_4
- operators for neutrino masses: $\frac{HHLL}{M}\left(\frac{\langle\xi\rangle}{\Lambda} + \frac{\langle\eta\rangle}{\Lambda}\right)$
- two scalar (flavon) fields for neutrino sector: $\xi \sim 3, \quad \eta \sim 1$

$$A_4 \to G_{TST^2}: \quad \langle\xi\rangle = \xi_0\Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad A_4 - \text{invariant:} \quad \langle\eta\rangle = u\Lambda$$

- product rules:

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{1''} \oplus \mathbf{3}_{\rm s} \oplus \mathbf{3}_{\rm a}$$

Tri-bimaximal Neutrino Mixing from A_4

- Neutrino Masses: triplet flavon contribution

Altarelli, Feruglio (2005)

$$3_S = \frac{1}{3}\begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix} \qquad 1 \;= \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2$$

- Neutrino Masses: singlet flavon contribution

$$1 \;= \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2$$

- resulting mass matrix:

$$M_\nu = \frac{\lambda v^2}{M_x}\begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix} \qquad U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$V_\nu^{\rm T} M_\nu V_\nu = {\rm diag}(u + 3\xi_0,\, u,\, -u + 3\xi_0)\frac{v_u^2}{M_x}$$

Form diagonalizable:
-- no adjustable parameters
-- neutrino mixing from CG coefficients!

Tri-bimaximal Neutrino Mixing from A_4

Altarelli, Feruglio (2005)

- charged lepton sector -- without quarks
  - operators for charged lepton masses

$$(\ell\phi)_1 e_R(1) + (\ell\phi)_{1'} \mu_R(1'') + (\ell\phi)_{1''} \tau_R(1')$$

  - scalar sector: flavon triplet for charged lepton masses

$$\begin{aligned} 1 &= \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\ 1' &= \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1 \\ 1'' &= \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \end{aligned}$$

$$A_4 \to G_T : \qquad \langle\phi\rangle = \phi_0\Lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

- resulting charged lepton mass matrix = diagonal
- leptonic mixing matrix = tri-bimaximal

$$V_{MNS} = V_{e,L}^\dagger V_\nu = \mathcal{I} \cdot U_{TBM} = U_{TBM}$$

- seesaw realization with three RH neutrinos (N_1, N_2, N_3) ~ 3

Non-Abelian Finite Family Symmetry A4

- TBM mixing matrix: can be realized with finite group family symmetry based on A_4 Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); ...

- A_4: even permutations of 4 objects

  S: (1234) → (4321)

  T: (1234) → (2314)

- Group of order 12
- Invariant group of tetrahedron
- Problem: A_4 does not seem to give rise to quark mixing

Example: T′ Family Symmetry

- SU(5) compatibility ⇒ Double Tetrahedral Group T´ M.-C.C, K.T. Mahanthappa (2007, 2009)
- In-equivalent representations of T′

A4: $1,\ 1',\ 1'',\ 3$ → vectorial representations for neutrinos → TBM for neutrinos

other: $2,\ 2',\ 2''$ → spinorial representations for quarks → 2 +1 assignments for quarks

- Symmetries ⇒ 9 parameters in Yukawa sector ⇒ 22 physical observables
- neutrino mixing angles from group theory (CG coefficients)
- TBM: misalignment of symmetry breaking patterns
  - neutrino sector: T' → G_{TST2} , charged lepton sector: T' → G_T
- GUT symmetry ⇒ contributions to mixing parameters from charged lepton sector

⇒ deviation from TBM related to Cabibbo angle θ_c ,consequence of Georgi-Jarlskog relations

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$

← CG's of SU(5) & T´

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c\cos\delta$$

- large θ_{13} possible with one additional singlet flavon

M.-C. C., J. Huang, J. O'Bryan, A. Wijiangco, F. Yu (2012); M.-C. C., J. Huang, K.T. Mahanthappa, A. Wijiangco (2013)

before θ_{13} discovery

Sum Rules: Quark-Lepton Complementarity

Quark Mixing

mixing parameters	best fit	3σ range
θ^q_{23}	2.36°	2.25° - 2.48°
θ^q_{12}	12.88°	12.75° - 13.01°
θ^q_{13}	0.21°	0.17° - 0.25°

Lepton Mixing

mixing parameters	best fit	3σ range
θ^e_{23}	42.8°	35.5° - 53.5°
θ^e_{12}	34.4°	31.5° - 37.6°
θ^e_{13}	5.6°	≤ 12.5°

- QLC-I

$$\theta_c + \theta_{sol} \cong 45^o$$

Raidal, '04; Smirnov, Minakata, '04

(BM) $\theta^q_{23} + \theta^e_{23} \cong 45^o$

- QLC-II

$$\tan^2\theta_{sol} \cong \tan^2\theta_{sol,TBM} + (\theta_c / 2) * \cos \delta_e$$

Ferrandis, Pakvasa; King; Dutta, Mimura; M.-C.C., Mahanthappa

(TBM) $\theta^e_{13} \cong \theta_c / 3\sqrt{2}$

- testing sum rules: a *more* robust way to distinguish different classes of models

measuring leptonic mixing parameters to the precision of those in quark sector

after θ_{13} discovery

Sum Rules: Quark-Lepton Complementarity

Quark Mixing

mixing parameters	best fit	3σ range
θ^q_{23}	2.36°	2.25° - 2.48°
θ^q_{12}	12.88°	12.75° - 13.01°
θ^q_{13}	0.21°	0.17° - 0.25°

Lepton Mixing

mixing parameters	best fit	3σ range
θ^e_{23}	38.4° ↓	35.1° - 52.6°
θ^e_{12}	33.6°	30.6° - 36.8°
θ^e_{13}	8.9° ↑	7.5° -10.2°

- QLC-I

$$\theta_c + \theta_{sol} \cong 45^o$$

Raidal, '04; Smirnov, Minakata, '04

(BM) $\theta^q_{23} + \theta^e_{23} \cong 45^o$ ☞ **inconsistent @ 2σ**

- QLC-II

$$\tan^2\theta_{sol} \cong \tan^2\theta_{sol,TBM} + (\theta_c / 2) * \cos \delta_e$$

Ferrandis, Pakvasa; King; Dutta, Mimura; M.-C.C., Mahanthappa

(TBM) $\theta^e_{13} \cong \theta_c / 3\sqrt{2}$ ☞ **Too small**

- testing sum rules: a *more* robust way to distinguish different classes of models

measuring leptonic mixing parameters to the precision of those in quark sector

“Large” Deviations from TBM in A_4

- Generically: corrections on the order of $(\theta_c)^2$
  - from charged lepton sector:
    - through GUT relations
  - from neutrino sector:
    - higher order holomorphic contributions in superpotential

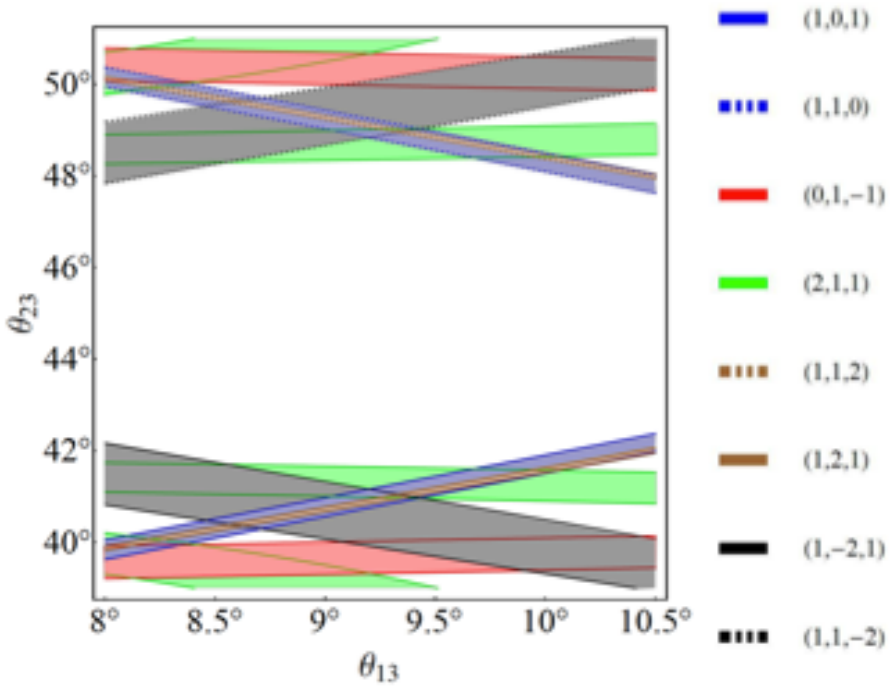
- Modifying the Neutrino sector: Different symmetry breaking patterns
  - TBM: misalignment of
    - $A4 \rightarrow G_{TST2}$ and $A4 \rightarrow G_T$
  - A4: group of order 12 $\Rightarrow$ many subgroups
  - systematic study of breaking into other A4 subgroups

M.-C.C, J. Huang, J. O’Bryan, A. Wijangco, F. Yu, (2012)

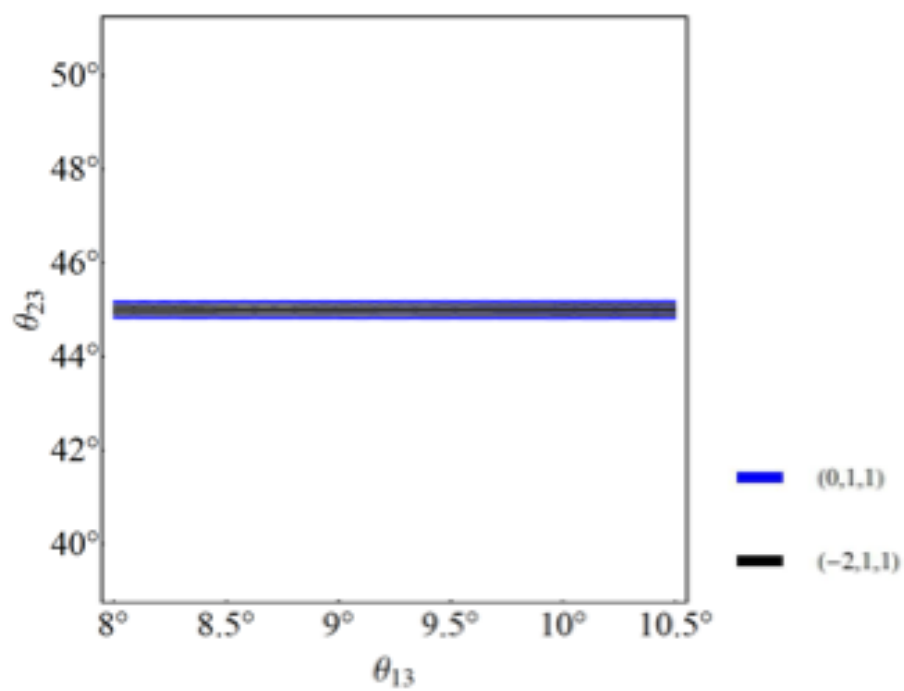
“Large” Deviations from TBM in A_4

M.-C.C, J. Huang, J. O’Bryan, A. Wijangco, F. Yu, (2012)

- other A4 breaking patterns:



normal

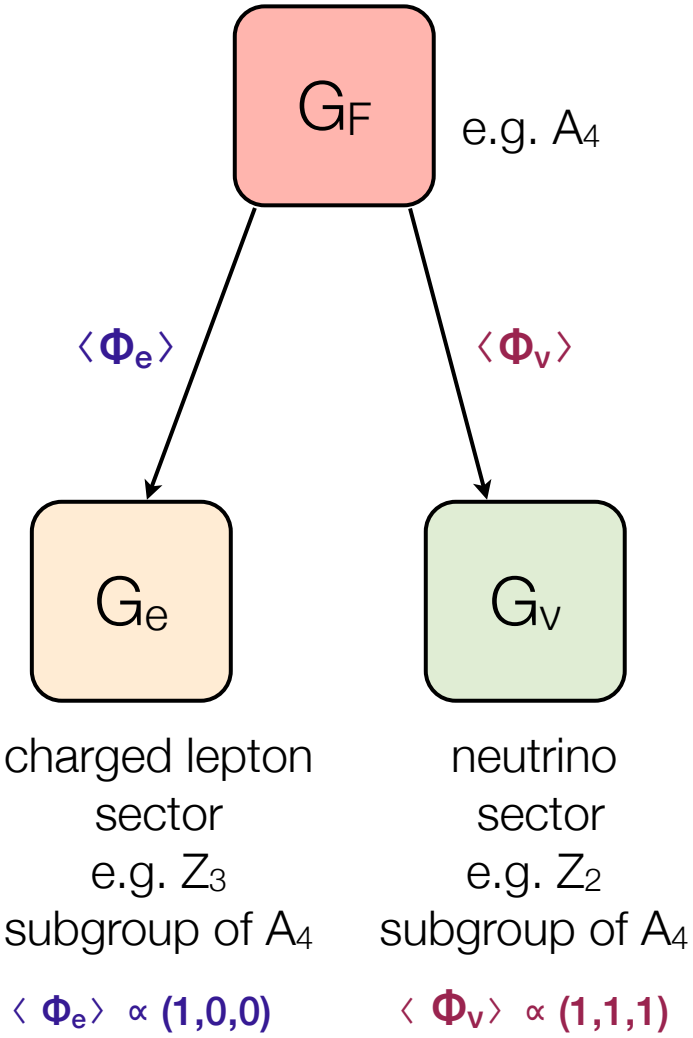


inverted

deviations correlated

non-maximal θ_{23} ⇨ normal hierarchy

Flavor Model Structure: A_4 Example



- interplay between the symmetry breaking patterns in two sectors lead to lepton mixing (BM, TBM, ...)
- symmetry breaking achieved through flavon VEVs
- each sector preserves different residual symmetry
- full Lagrangian does not have these residual symmetries
- general approach: include high order terms in holomorphic superpotential
- possible to construct models where higher order holomorphic superpotential terms vanish to ALL orders
- quantum correction?
  $\Rightarrow$ uncertainty in predictions due to

  Kähler corrections Leurer, Nir, Seiberg (1993); Dudas, Pokorski, Savoy (1995); Dreiner, Thomeier (2003);

Kähler Corrections

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Superpotential: holomorphic

$$\mathscr{W}_{\text{leading}} = \frac{1}{\Lambda}(\Phi_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{\Lambda\,\Lambda_\nu}(\Phi_\nu)_{gf}\, L^g\, H_u\, L^f\, H_u$$

$$\longrightarrow \mathscr{W}_{\text{eff}} = (Y_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{4}\kappa_{gf}\, L^g\, H_u\, L^f\, H_u$$

order parameter
<flavon vev> / Λ ~ θc

- Kähler potential: non-holomorphic

$$K = K_{\text{canonical}} + \Delta K$$

- Canonical Kähler potential

$$K_{\text{canonical}} \supset \left(L^f\right)^\dagger \delta_{fg}\, L^g + \left(R^f\right)^\dagger \delta_{fg}\, R^g$$

- Correction

$$\Delta K = \left(L^f\right)^\dagger (\Delta K_L)_{fg}\, L^g + \left(R^f\right)^\dagger (\Delta K_R)_{fg}\, R^g$$

- can be induced by flavon VEVs
- important for order parameter ~ θc
- can lead to non-trivial mixing

Kähler Corrections

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Consider infinitesimal change, x :

$$K \;=\; K_{\text{canonical}} + \Delta K \;=\; L^\dagger\,(1 - 2x\,P)\,L$$

- rotate to canonically normalized L':

$$L \;\to\; L' \;=\; (1 - x\,P)\,L$$

⇒ corrections to neutrino mass matrix

$$\begin{aligned}
\mathcal{W}_\nu &= \frac{1}{2}(L\cdot H_u)^T \kappa_\nu (L\cdot H_u) \\
&\simeq \frac{1}{2}[(\mathbb{1}+xP)L'\cdot H_u]^T \kappa_\nu [(\mathbb{1}+xP)L'\cdot H_u] \\
&\simeq \frac{1}{2}(L'\cdot H_u)^T \kappa_\nu L'\cdot H_u + x(L'\cdot H_u)^T (P^T\kappa_\nu + \kappa_\nu P)L'\cdot H_u
\end{aligned}$$

with $\quad \kappa\cdot v_u^2 \;=\; 2m_\nu$

Kähler Corrections

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Consider infinitesimal change, x :

$$K \;=\; K_{\text{canonical}} + \Delta K \;=\; L^\dagger \left(1 - 2x\, P\right) L$$

- rotate to canonically normalized L':

$$L \;\to\; L' \;=\; \left(1 - x\, P\right) L$$

⇒ corrections to neutrino mass matrix

$$m_\nu(x) \;\simeq\; m_\nu + x\, P^T m_\nu + x\, m_\nu\, P$$

⇒ differential equation

$$\frac{\mathrm{d}m_\nu}{\mathrm{d}x} \;=\; P^T m_\nu + m_\nu\, P$$

- same structure as the RG evolutions for neutrino mass operator
- analytic understanding of evolution of mixing parameters

S. Antusch, J. Kersten, M. Lindner, M. Ratz (2003)

- size of Kähler corrections can be substantially larger (no loop suppression)

Back to A_4 Example

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Kähler corrections due to flavon field:
  - linear in flavon: can be forbidden with additional symmetries
  - quadratic in flavon

$$\Delta K^{\text{quadratic}}_{\phi^{(\prime)}} \supset \frac{1}{\Lambda^2} \sum_{X}^{6} \kappa^{X}_{\phi^{(\prime)},\text{quadratic}} \left(L\phi^{(\prime)}\right)^{\dagger}_{X} \left(L\phi^{(\prime)}\right)_{X} + \text{h.c.}$$

$$(L\Phi_\nu)^\dagger(L\Phi_\nu) \quad \text{and} \quad (L\Phi_e)^\dagger(L\Phi_e)$$

  - such terms cannot be forbidden by any (conventional) symmetry
  - Kähler corrections once flavon fields attain VEVs
  - additional parameters $\kappa^{X}_{\phi^{(\prime)}}$ reduce predictivity of the scheme
  - possible to forbid all contributions from RH sector as well as $(L\Phi_\nu)^\dagger(L\Phi_e)$ with additional symmetries in the example considered

Back to A_4 Example

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Contributions from Flavon VEVs (1,0,0) and (1,1,1)
  - five independent "basis" matrices

$$P_{\rm I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\rm II} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\rm III} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_{\rm IV} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad P_{\rm V} = \begin{pmatrix} 0 & {\rm i} & -{\rm i} \\ -{\rm i} & 0 & {\rm i} \\ {\rm i} & -{\rm i} & 0 \end{pmatrix}$$

  - RG correction: essentially along P_{III} = diag(0,0,1) direction due to y_τ dominance
  - Kähler corrections can be along different directions than RG

Enhanced θ_{13}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- consider change due to correction along P_V direction
- Kähler metric:

$$\mathcal{K}_L = 1 - 2xP \qquad \text{with} \qquad P_{\rm V} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}$$

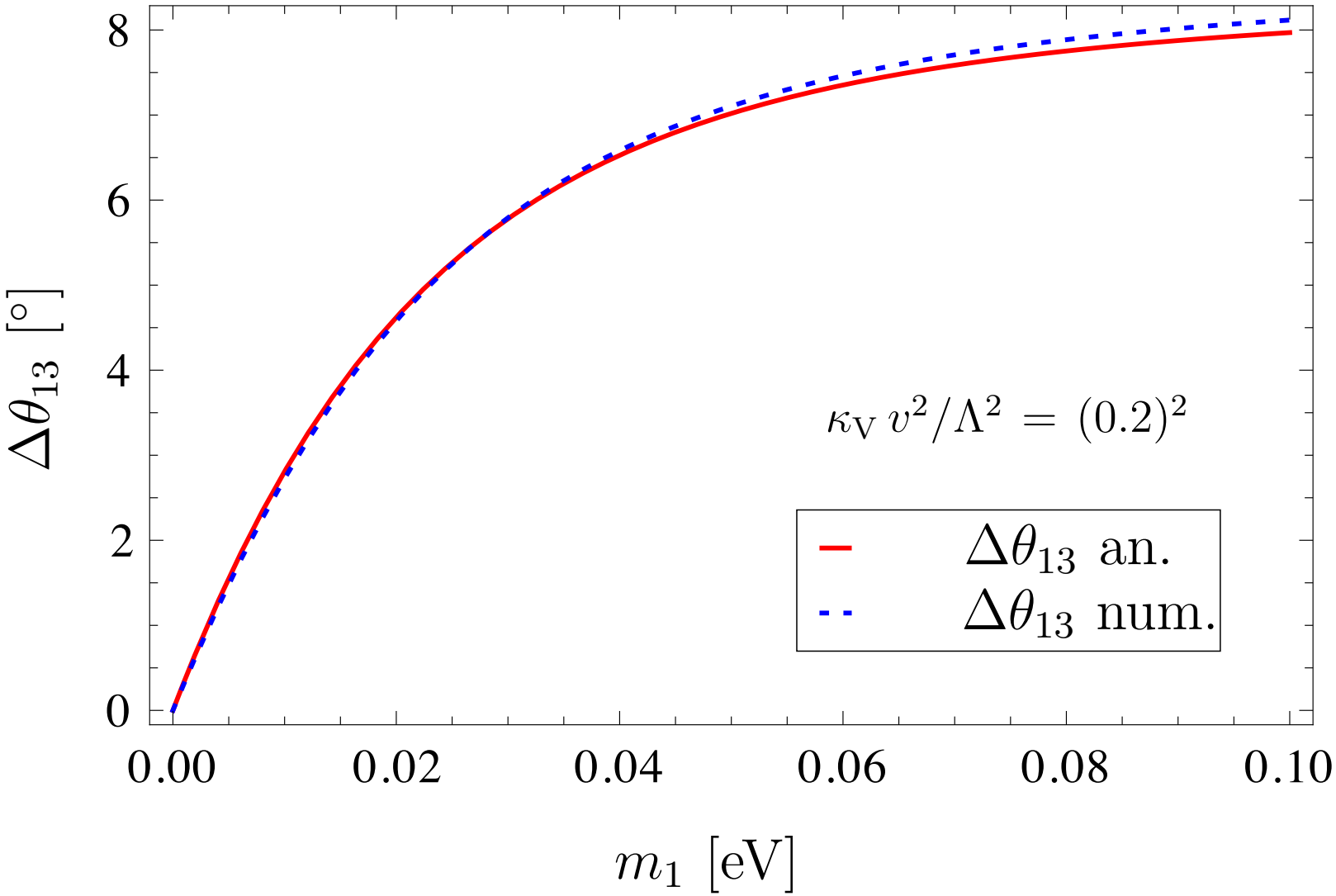
- Contributions of flavon VEV: $\langle \Phi \rangle = (1, 1, 1)\ \upsilon$
- Corrections to the leading order TBM prediction ($m_e \ll m_\mu \ll m_\tau$)

$$\Delta\theta_{13} \simeq \kappa_{\rm V} \cdot \frac{v^2}{\Lambda^2} \cdot 3\sqrt{6}\, \frac{m_1}{m_1 + m_3}$$

- Complex matrix P $\Rightarrow$ CP violation induced
- for the example considered: $\delta \approx \pi/2$

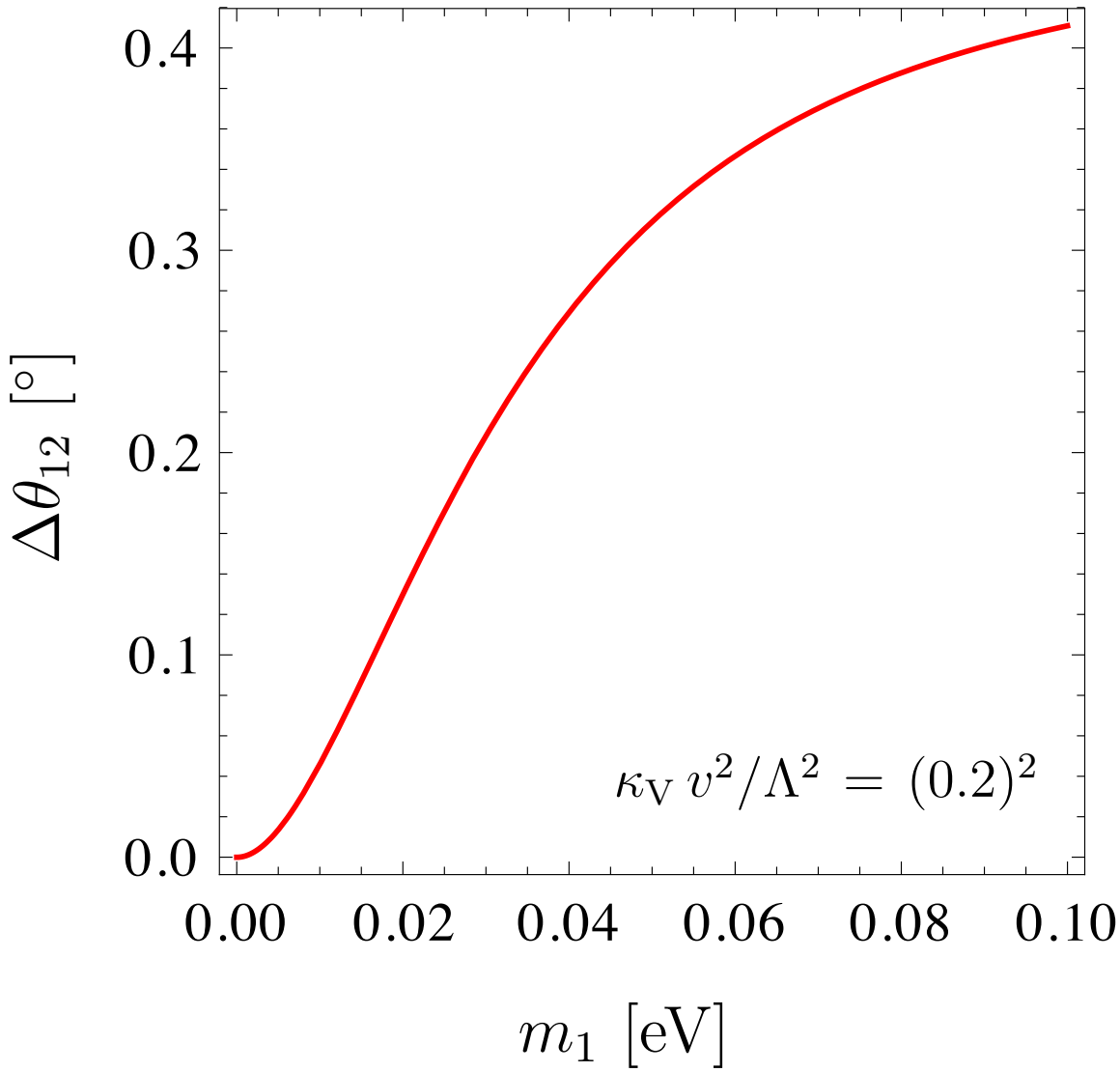
An Example: Enhanced θ_{13}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



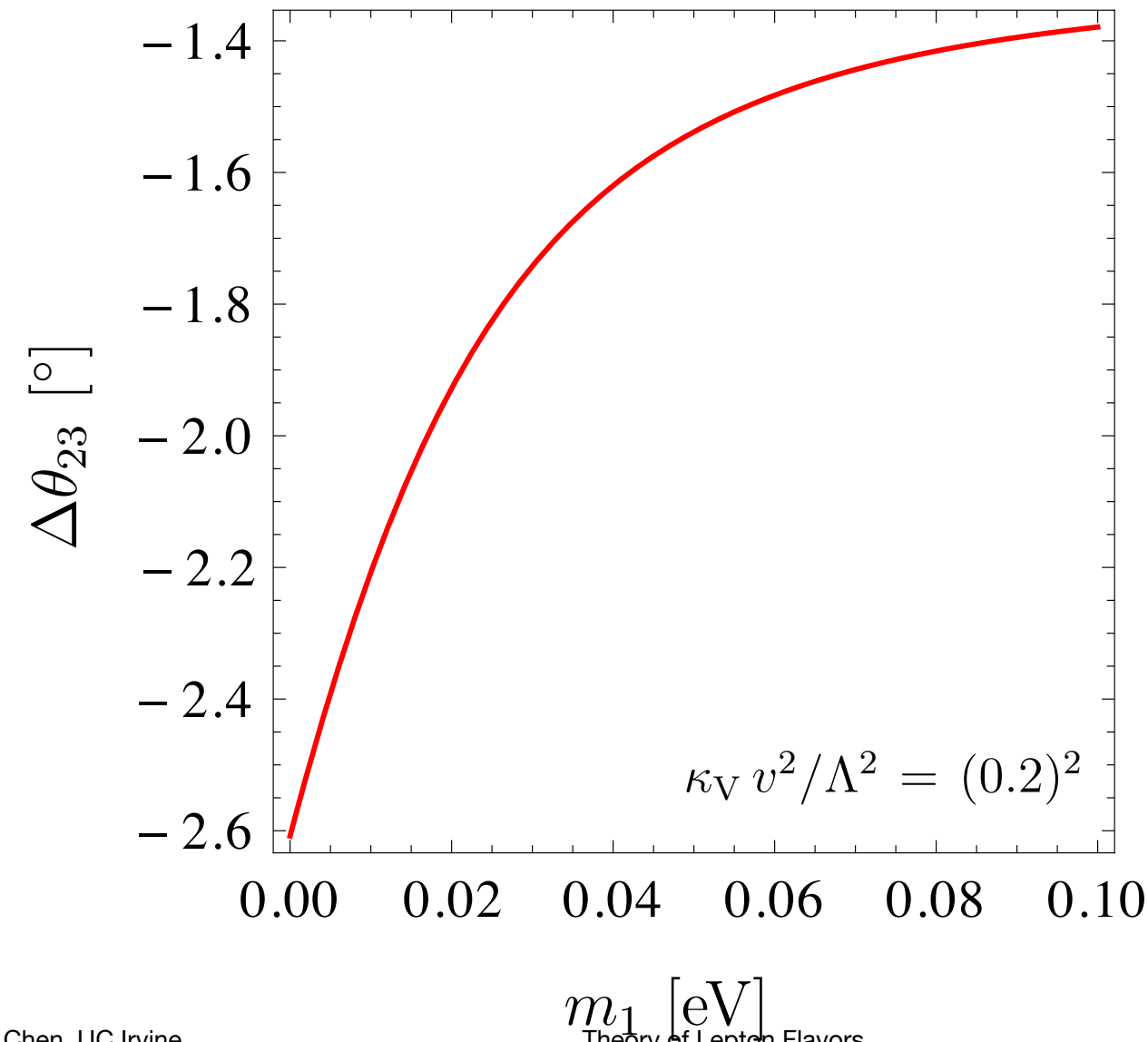
Corresponding Change in θ_{12}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

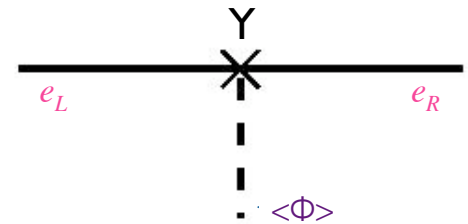


Corresponding Change in θ_{23}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



A Novel Origin of CP Violation



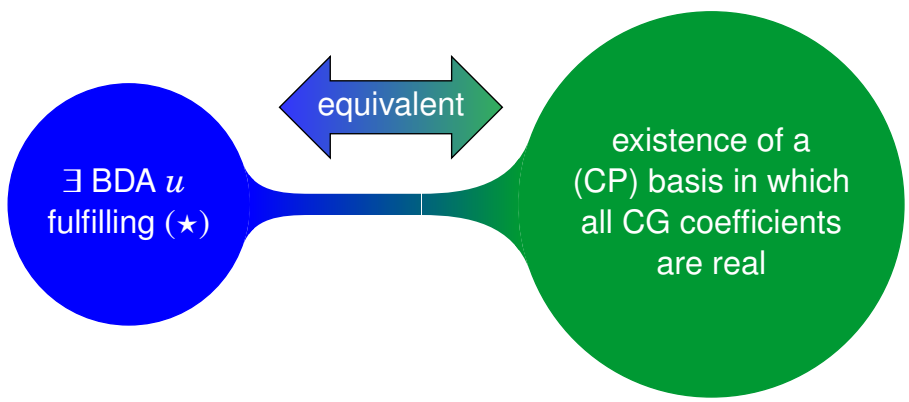
- Conventionally:
  - explicit CP violation: complex Yukawa couplings
  - spontaneous CP violation: complex Higgs VEVs
- Complex CG coefficients in discrete groups ⇒ *explicit* CP violation in quark and lepton sectors (e.g. $\delta \neq 0$)

M.-C.C, K.T. Mahanthappa, Phys. Lett. B681, 444 (2009);
M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, Nucl. Phys. B (2014)

- Conditions for a discrete group to admit real CG's

Bickerstaff, Damhus, 1985

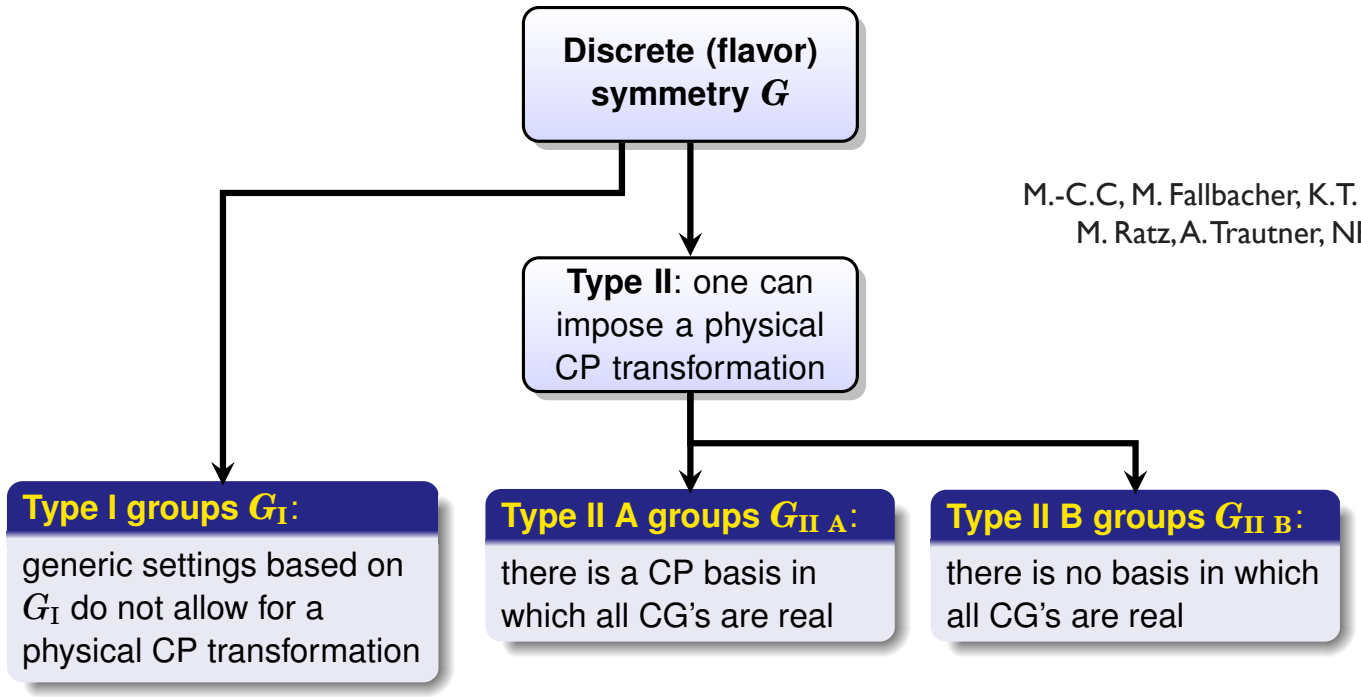
$\exists$ automorphism u, such that $\lambda_k(R) = \lambda_k(u(R))^*$ for all $R \in G$



A Novel Origin of CP Violation

- **more generally, for discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ physical CP violation**

CP Violation from Group Theory!



M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

CP conserving models can be constructed by:

- introducing only special subsets of representations
- enlarging the symmetry non–trivially beyond the type I symmetry

Discrete (flavor) symmetry G

Type II: one can impose a physical CP transformation

Type I groups G_{I}:

generic settings based on G_{I} do not allow for a physical CP transformation

Type II A groups $G_{\mathrm{II\ A}}$:

there is a CP basis in which all CG's are real

Type II B groups $G_{\mathrm{II\ B}}$:

there is no basis in which all CG's are real

Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

Discrete (flavor) symmetry G

Type II: one can impose a physical CP transformation

- the CP basis may not be the most convenient choice for performing computations
- CP invariance restricts the phases of coupling coefficients

Type I groups G_{I}:

generic settings based on G_{I} do not allow for a physical CP transformation

Type II A groups $G_{\mathrm{II\,A}}$:

there is a CP basis in which all CG's are real

Type II B groups $G_{\mathrm{II\,B}}$:

there is no basis in which all CG's are real

Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

Discrete (flavor) symmetry G

Type II: one can impose a physical CP transformation

- CP may enlarge the group (trivially) to $G_{\mathrm{II\ B}} \rtimes \mathbb{Z}_N$
- CP invariance may forbid certain coupling coefficients

Type I groups G_{I}:

generic settings based on G_{I} do not allow for a physical CP transformation

Type II A groups $G_{\mathrm{II\ A}}$:

there is a CP basis in which all CG's are real

Type II B groups $G_{\mathrm{II\ B}}$:

there is no basis in which all CG's are real

Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

- Type IIA: dihedral and all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T'	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- Type IIB

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

Example for a type I group:

$$\Delta(27)$$

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory

Toy Model based on Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Field content

fermions

field	S	X	Y	Ψ	Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{3}$
U(1)	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	q_Σ

$q_\Psi - q_\Sigma \neq 0$

- Interactions

$$\mathscr{L}_{\text{toy}} = f\left[S_{\mathbf{1}_0} \otimes \left(\overline{\Psi}\Sigma\right)_{\mathbf{1}_0}\right]_{\mathbf{1}_0} + g\left[X_{\mathbf{1}_1} \otimes \left(\overline{\Psi}\Sigma\right)_{\mathbf{1}_2}\right]_{\mathbf{1}_0}$$

$$+ h_\Psi\left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Psi}\Psi\right)_{\mathbf{1}_6}\right]_{\mathbf{1}_0} + h_\Sigma\left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Sigma}\Sigma\right)_{\mathbf{1}_6}\right]_{\mathbf{1}_0} + \text{h.c.}$$

$$= F^{ij}\, S\, \overline{\Psi}_i \Sigma_j + G^{ij}\, X\, \overline{\Psi}_i \Sigma_j + H_\Psi^{ij}\, Y\, \overline{\Psi}_i \Psi_j + H_\Sigma^{ij}\, Y\, \overline{\Sigma}_i \Sigma_j + \text{h.c.}$$

$$F = f\,\mathbb{1}_3$$

$$G = g\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$H_{\Psi/\Sigma} = h_{\Psi/\Sigma}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

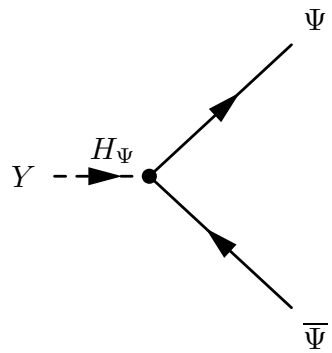
with $\omega := e^{2\pi i/3}$

Toy Model based on Δ(27)

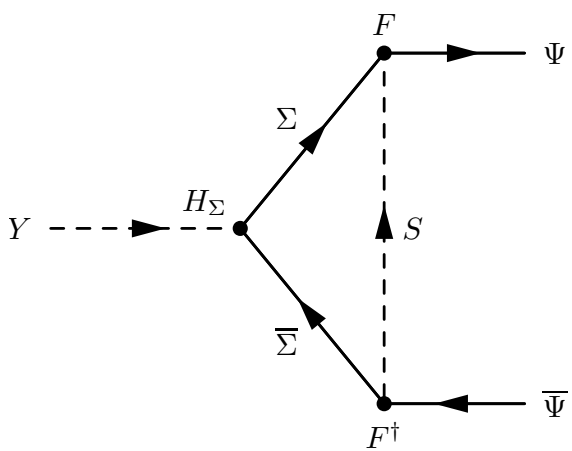
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

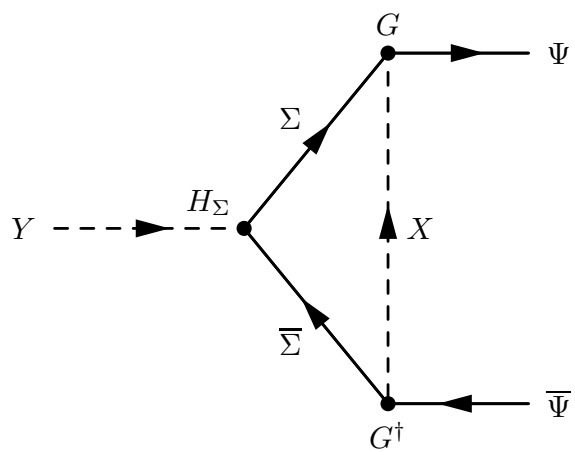
- Particle decay $Y \to \overline{\Psi}\Psi$

interference of



with





Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |f|^2 \ \mathrm{Im}\left[I_S\right] \ \mathrm{Im}\left[h_\Psi\, h_\Sigma^*\right] + |g|^2 \ \mathrm{Im}\left[I_X\right] \ \mathrm{Im}\left[\omega\, h_\Psi\, h_\Sigma^*\right]$$

  - cancellation requires delicate adjustment of relative phase $\varphi := \arg(h_\Psi\, h_\Sigma^*)$
  - for non-degenerate M_S and M_X: $\mathrm{Im}\left[I_S\right] \neq \mathrm{Im}\left[I_X\right]$
    - phase φ unstable under quantum corrections
  - for $\mathrm{Im}\left[I_S\right] = \mathrm{Im}\left[I_X\right]$ & $|f| = |g|$
    - phase φ stable under quantum corrections
    - relations cannot be ensured by outer automorphism of Δ(27)
    - require symmetry larger than Δ(27)

model based on Δ(27) violates CP!

CP Conservation vs Symmetry Enhancement

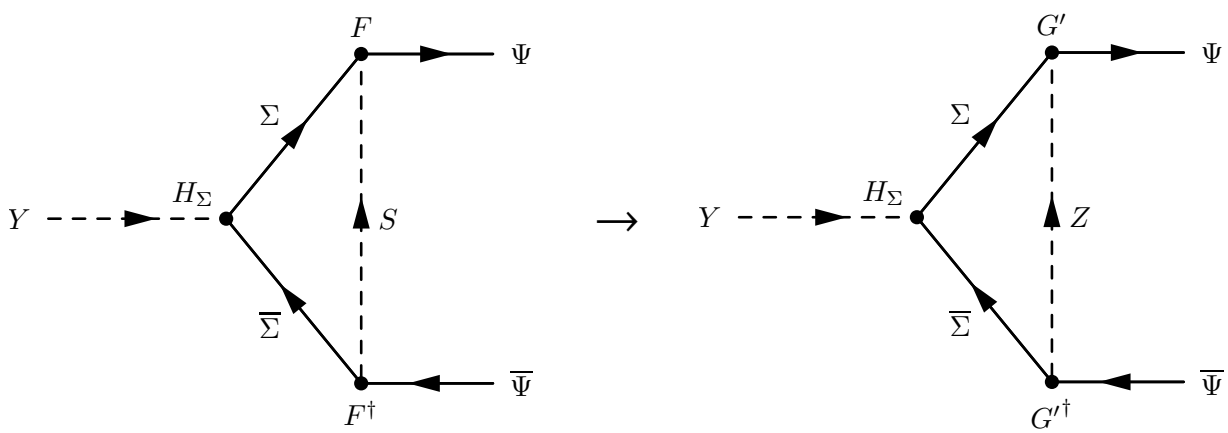
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

☞ replace $S \sim \mathbf{1}_0$ by $Z \sim \mathbf{1}_8$ ↷ interaction

$$\mathscr{L}^{Z}_{\text{toy}} = g' \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi}\Sigma\right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij} \, Z \, \overline{\Psi}_i \Sigma_j + \text{h.c.}$$

$$G' = g' \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}$$

and leads to new interference diagram



CP Conservation vs Symmetry Enhancement

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➡ different contribution to decay asymmetry: $\varepsilon^{S}_{Y\to\overline{\Psi}\Psi} \to \varepsilon^{Z}_{Y\to\overline{\Psi}\Psi}$

☞ total CP asymmetry of the Y decay vanishes if
$$\begin{cases} \text{(i)} & M_Z = M_X \\ \text{(ii)} & |g| = |g'| \\ \text{(iii)} & \varphi = 0 \end{cases}$$

☞ relations (i)—(iii) can be due to an outer automorphism

$$X \overset{u_3}{\longleftrightarrow} Z\,, \quad Y \overset{u_3}{\longrightarrow} Y\,, \quad \Psi \overset{u_3}{\longrightarrow} U_{u_3}\,\Sigma^C \quad \& \quad \Sigma \overset{u_3}{\longrightarrow} U_{u_3}\,\Psi^C$$

requires $q_\Sigma = -q_\Psi$

$$U_{u_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

... BUT this enlarges $\Delta(27) \to \text{SG}(54,5) \simeq \Delta(27) \rtimes \mathbb{Z}_2^{u_3}$

SG(54, 5): group name from GAP library

Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

field	X	Y	Z	Ψ	Σ	ϕ
$\Delta(27)$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$
U(1)	$2q_\Psi$	0	$2q_\Psi$	q_Ψ	$-q_\Psi$	0

- SG$(54,5)$: $\begin{cases} (X,Z) & : \text{ doublet} \\ (\Psi,\Sigma^C) & : \text{ hexaplet} \\ \phi & : \text{ non–trivial 1–dim. representation} \end{cases}$
- non–trivial $\langle\phi\rangle$ breaks SG$(54,5) \to \Delta(27)$
- allowed coupling leads to mass splitting $\mathscr{L}^{\phi}_{\text{toy}} \supset M^2\left(|X|^2+|Z|^2\right) + \left[\frac{\mu}{\sqrt{2}}\langle\phi\rangle\left(|X|^2-|Z|^2\right) + \text{h.c.}\right]$

CG coefficient of SG$(54,5)$

- ➥ CP asymmetry with calculable phases

$$\varepsilon_{Y\to\overline{\Psi}\Psi} \propto |g|^2\,|h_\Psi|^2\,\text{Im}\left[\,\omega\,\right]\left(\text{Im}\left[I_X\right]-\text{Im}\left[I_Z\right]\right)$$

phase predicted by group theory

Group theoretical origin of CP violation!

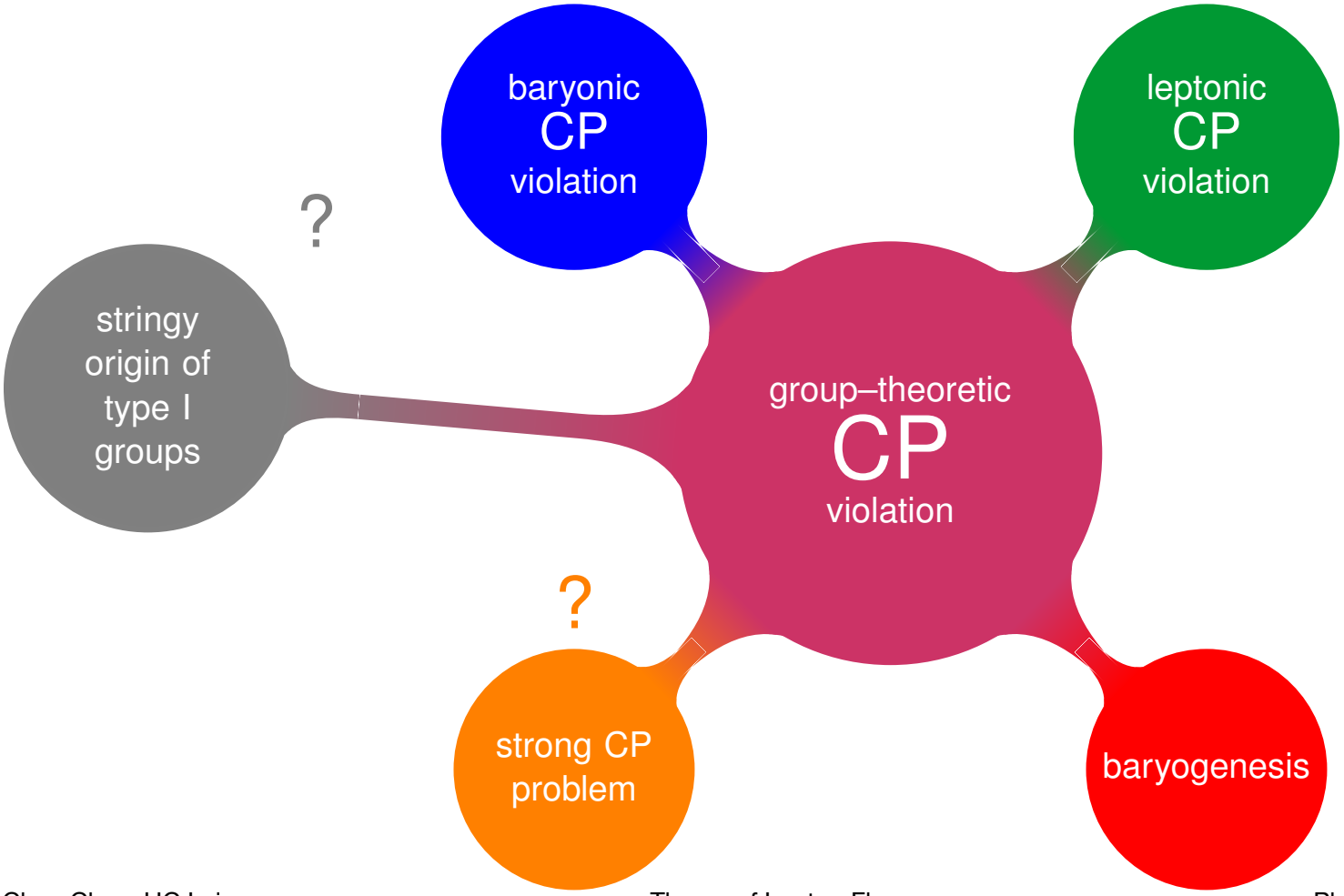
M.-C.C., K.T. Mahanthappa (2009)

Conclusions

- Discrete family symmetries: correlations among observables
- Kähler corrections induced (and determined) by flavon VEVs (with order parameter ~ θ_c)
  - similar in structure to RG corrections, but can be along different directions
  - size of Kähler corrections generically dominate RG corrections (no loop suppression, contributions from copious heavy states)
  - non-zero CP phases can be induced
  - additional parameters (Kähler coefficients) introduced
- robustness of model predictions diminished given the presence of these potentially sizable corrections and new parameters
- theoretical understanding of Kähler corrections crucial for achieving precision compatible with experimental accuracy

Conclusion & Outlook

(Type I) Discrete groups afford a new origin of CP violation:



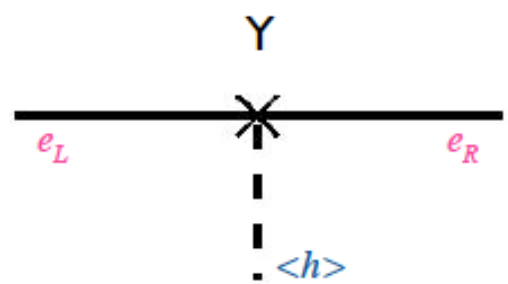
BACK-UP SLIDES

Origin of CP Violation

- CP violation ⇔ complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>



- A New Origin of CP Violation in discrete groups:
  - complex CG coefficients

M.-C. C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)
M.-C. C., K.T. Mahanthappa (2009)

- Canonical CP transformation
  - for a scalar field: $\phi(x) \xmapsto{\mathcal{CP}} \eta_{\mathcal{CP}}\ \phi^*(\mathcal{P}x)$

freedom of re–phasing fields

Generalized CP Transformation

- ☞ setting w/ discrete symmetry G
- ☞ generalized CP transformation
- ☞ invariant contraction/coupling in A_4 or T'

Holthausen, Lindner, and Schmidt (2013)

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi \left(x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3\right)$$

$\omega = \mathrm{e}^{2\pi \mathrm{i}/3}$

- ☞ canonical CP transformation maps A_4/T' invariant contraction to something non–invariant

➡ need generalized CP transformation $\widetilde{\mathcal{CP}}$: $\phi \xmapsto{\widetilde{\mathcal{CP}}} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

How (Not) to Generalize CP

proper CP transformations

- ☞ map field operators to *their own* Hermitean conjugates
- ☞ violation of physical CP is prerequisite for a non–trivial

$$\varepsilon_{i\to f} = \frac{\left|\Gamma\left(i\to f\right)\right|^2 - \left|\Gamma\left(\bar{\imath}\to \bar{f}\right)\right|^2}{\left|\Gamma\left(i\to f\right)\right|^2 + \left|\Gamma\left(\bar{\imath}\to \bar{f}\right)\right|^2}$$

- ➥ connection to observed ~~CP~~, baryogenesis & . . .

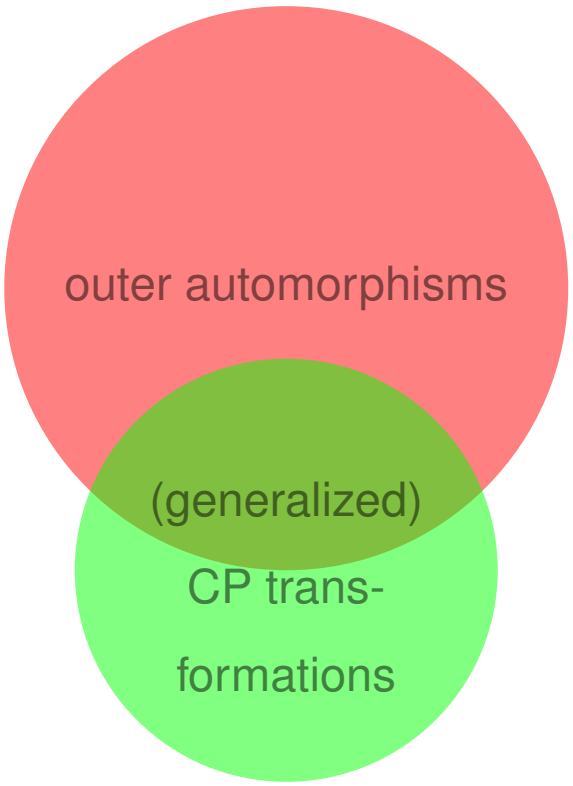
CP–like transformations

- ☞ map some field operators to some other operators
- ☞ such transformations have sometimes been called "generalized CP transformations" in the literature
- ☞ however, imposing CP–like transformations does **not** imply physical CP conservation
- ➥ **NO** connection to observed ~~CP~~, baryogenesis & . . .

Physical CP Transformation

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

☞ Not every outer automorphism defines a physical CP transformation!



proper CP transformations:

class–inverting automorphisms of G

Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:

$$\mathrm{FS}(\boldsymbol{r}_i) \;:=\; \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g^2) \;=\; \frac{1}{|G|}\sum_{g\in G}\mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{r}_i) \;=\; \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.} \end{cases}$$

- Twisted Frobenius indicator

Bickerstaff, Damhus (1985); Kawanaka, Matsuyama (1990)

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;=\; \frac{1}{|G|}\sum_{g\in G}\left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta}\left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;=\; \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall\, i, & \text{if } u \text{ is class–inverting and involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\epsilon_{Y\to\overline{\Phi}\Phi} = \frac{\Gamma(Y\to\overline{\Phi}\Phi) - \Gamma(Y^*\to\overline{\Phi}\Phi)}{\Gamma(Y\to\overline{\Phi}\Phi) + \Gamma(Y^*\to\overline{\Phi}\Phi)}$$

$$\propto \ \mathrm{Im}\,[I_S]\ \mathrm{Im}\left[\mathrm{tr}\left(F^\dagger\, H_\Psi\, F\, H_\Sigma^\dagger\right)\right] + \mathrm{Im}\,[I_X]\ \mathrm{Im}\left[\mathrm{tr}\left(G^\dagger\, H_\Psi\, G\, H_\Sigma^\dagger\right)\right]$$

$$= \ |f|^2\ \mathrm{Im}\,[I_S]\ \mathrm{Im}\,[h_\Psi\, h_\Sigma^*] + |g|^2\ \mathrm{Im}\,[I_X]\ \mathrm{Im}\,[\omega\, h_\Psi\, h_\Sigma^*]\ .$$

one-loop integral $I_S = I(M_S, M_Y)$

one-loop integral $I_X = I(M_X, M_Y)$

- properties of ε
  - invariant under rephasing of fields
  - independent of phases of f and g
  - basis independent

Some Outer Automorphisms of Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- sample outer automorphisms of Δ(27)

$$
\begin{aligned}
u_1 &: \mathbf{1}_1 \leftrightarrow \mathbf{1}_2\,,\ \mathbf{1}_4 \leftrightarrow \mathbf{1}_5\,,\ \mathbf{1}_7 \leftrightarrow \mathbf{1}_8\,,\ \mathbf{3} \to U_{u_1}\,\mathbf{3}^* \\
u_2 &: \mathbf{1}_1 \leftrightarrow \mathbf{1}_4\,,\ \mathbf{1}_2 \leftrightarrow \mathbf{1}_8\,,\ \mathbf{1}_3 \leftrightarrow \mathbf{1}_6\,,\ \mathbf{3} \to U_{u_2}\,\mathbf{3}^* \\
u_3 &: \mathbf{1}_1 \leftrightarrow \mathbf{1}_8\,,\ \mathbf{1}_2 \leftrightarrow \mathbf{1}_4\,,\ \mathbf{1}_5 \leftrightarrow \mathbf{1}_7\,,\ \mathbf{3} \to U_{u_3}\,\mathbf{3}^* \\
u_4 &: \mathbf{1}_1 \leftrightarrow \mathbf{1}_7\,,\ \mathbf{1}_2 \leftrightarrow \mathbf{1}_5\,,\ \mathbf{1}_3 \leftrightarrow \mathbf{1}_6\,,\ \mathbf{3} \to U_{u_4}\,\mathbf{3}^* \\
u_5 &: \mathbf{1}_i \leftrightarrow \mathbf{1}_i{}^*\,,\ \mathbf{3} \to U_{u_5}\,\mathbf{3}
\end{aligned}
$$

- twisted Frobenius-Schur indicators

$\boldsymbol{R}$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{1}_4$	$\mathbf{1}_5$	$\mathbf{1}_6$	$\mathbf{1}_7$	$\mathbf{1}_8$	$\mathbf{3}$	$\overline{\mathbf{3}}$
$\mathrm{FS}_{u_1}(\boldsymbol{R})$	1	1	1	0	0	0	0	0	0	1	1
$\mathrm{FS}_{u_2}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_3}(\boldsymbol{R})$	1	0	0	0	0	1	0	1	0	1	1
$\mathrm{FS}_{u_4}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_5}(\boldsymbol{R})$	1	1	1	1	1	1	1	1	1	0	0

  - none of the u_i maps all representations to their conjugates
  - however, it is possible to impose CP in (non-generic) models, where only a subset of representations are present, e.g. $\{\boldsymbol{r}_i\} \subset \{\mathbf{1}_0, \mathbf{1}_5, \mathbf{1}_7, \mathbf{3}, \overline{\mathbf{3}}\}$