INTRODUCTION

• We study the radiative exclusive decay mode \( W^\pm \to \pi^\pm + \gamma \)
  • The W is a massive charged vector boson which mediates weak interactions
  • The pion is a light pseudoscalar meson which is the bound state of an up and down quark

• Intermediate vector bosons, like W, are produced in high energy accelerators
  • W bosons are predicted to have mass in the Standard Weinberg-Salam model for weak interactions. \( M_W \sim 80.385 \pm 0.015 \text{ GeV} \)

• Good theoretical calculation of these rare decay modes provide useful insight into electroweak and strong interaction dynamics
  • \( W^\pm \to P^\pm + \gamma \), where P is a pseudoscalar meson (\( \pi, D_S \)) have been studied using particular models

• We employ a model independent approach using effective field theory, and wish to also include 1 loop contributions and resummation of logarithmic terms.
INTRODUCTION

• Interest coming from measurements of W decay to Psuedoscalar and vector at experiments like CDF
  • (including Prof. Robert Harr’s experimental group from WSU)
  • current bounds from PDG puts branching fraction at $< 8 \times 10^{-5}$

• Detection of this radiative 2-body decay mode along with the measurement of the photon energy can provide precise determination of W mass if the decay rate is large enough

$$\pi^- \quad W^- \quad \gamma$$

• In the rest frame of the W, the final products are back to back
  • The photon is a real, hard (energetic) photon
  • The pion is a collinear bound state of quarks
  • Transverse components become negligible
  • We neglect pion mass so that essentially $m_{\pi} \rightarrow 0$
• General Structure of the amplitude
  
  - Required by Lorentz invariance, Gauge invariance, equations of motion  
  \{orthogonality conditions of polarizations \( \epsilon(k) \cdot (k) = \epsilon(P + k) \cdot (P + k) = 0 \}\}

  \[ \mathcal{M} = - \frac{g e}{2\sqrt{2}} |V_{ud}| [H_2(P \cdot k g^{\mu\lambda} - P_\mu k_\lambda) + i H_6 \epsilon^{\alpha\beta\mu\lambda} k_\alpha P_{\beta}] \epsilon^{*}_\mu(k) \epsilon_\lambda(P + k) \]

  - \(H_2\) and \(H_6\) are form factors corresponding to axial-vector and vector component of the weak current.

  - The \(W\) boson carries momentum of \(P+k\), where \(P\) is the Pion momentum and \(k\) is the Photon momentum.
THE DECAY

\[ W^\pm \rightarrow \pi^\pm + \gamma \]

- 3 feynman diagrams contribute to the decay width at tree level
  - Trilinear non abelian weak coupling from standard model
    - \(-ie[(2P + k)_{\mu}g_{\alpha\lambda} - (P + 2k)_{\lambda}g_{\mu\lambda} - (P - k)_{\lambda}g_{\mu\lambda}]\)
  - Weak vertex describing axial current interacting with the W
    - \(-i\frac{g}{2\sqrt{2}} \gamma^\lambda(1 - \gamma_5)\)
  - QED vertex describing vector current interacting with the photon
    - \(-ieQ \gamma^\mu\)
TREE LEVEL DIAGRAMS

- **W pole diagram**

\[ M_W = -\frac{eg}{2\sqrt{2}} |V_{ud}| f_\pi \epsilon_\mu^*(k) \epsilon_\lambda (P + k) \left( g^{\alpha\beta} - \frac{P^\alpha P^\beta}{P^2 - m_W^2} \right) \]

\[ \times \left[ (2P)_\mu g_{\alpha\lambda} - (P + 2k)_\alpha g_{\mu\lambda} - 2P_\lambda g_{\mu\alpha} \right] P_\beta, \]

- Pion decay constant defined by

\[ \langle \pi | \bar{d} \gamma_\beta (1 - \gamma_5) u | 0 \rangle = i f_\pi P_\beta \]

- Using orthogonality conditions it simplifies and we see contribution only to the axial vector form factor

\[ M_W = -\frac{eg}{2\sqrt{2}} |V_{ud}| f_\pi \epsilon_\mu^*(k) \epsilon_\lambda (P + k) g_{\mu\lambda} \]
TREE LEVEL DIAGRAMS

• Structure dependent diagrams

\[
M_d = \frac{-ige}{2\sqrt{2}} Q_d \langle \pi | \bar{d} \gamma^\mu \frac{(P_d + k)}{(P_d + k)^2} \gamma^\lambda (1 - \gamma_5) u | 0 \rangle \epsilon^*_\mu(k) \epsilon_\lambda(P + k) \\
M_u = \frac{-ige}{2\sqrt{2}} Q_u \langle \pi | \bar{d} \gamma^\lambda (1 - \gamma_5) \frac{(P_u - k)}{(P_u + k)^2} \gamma^\mu u | 0 \rangle \epsilon^*_\mu(k) \epsilon_\lambda(P + k)
\]
PION WAVE FUNCTION

• In order to evaluate the matrix elements we need to construct the Pion wave function

• Need to describe the distribution of quark momentums within the pion
  • Assign longitudinal momentum fractions to the quarks with respect to pion momentum
    \[ P_d = xP \]
    \[ P_u = \bar{x}P, \]

• Employ light cone distribution amplitude for light pseudoscalar mesons (from Brodsky and Lepage)
  \[
  \langle \pi(P)|\bar{d}(z_2)u(z_1)|0\rangle = \frac{i f_\pi}{4} (P\gamma_5) \int_0^1 dx e^{i(xP\cdot z_2 + \bar{x}P\cdot z_1)} \phi_\pi(x)
  \]
  • Where \( \phi_\pi(x) = 6x(1-x) \) is the leading twist distribution amplitude for the light meson in the asymptotic limit

• We then just have to integrate over the momentum fraction \( x \)
AMPLITUDE

- Total tree level Amplitude corresponding to the decay

\[
\mathcal{M}_{W^+d} = \frac{g_e}{2\sqrt{2}} |V_{ud}| \frac{f_\pi}{m_W^2} [P \cdot k g^{\mu\lambda} - P_\mu P_\lambda + i \epsilon^{\alpha\beta\mu\lambda} k_\alpha P_\beta] \epsilon_\mu^*(k) \epsilon_\lambda(P + k)
\]

- Compare to the general expression to obtain the form factors

\[
H_6 = H_2 = -\frac{f_\pi}{m_W^2}
\]

- Decay Width obtained from squaring the amplitude and integrating over the 2-body phase space

\[
\Gamma = \frac{\alpha G_F}{24\sqrt{2}} |V_{ud}|^2 m_W^3 \left(1 - \frac{m_\pi^2}{m_W^2}\right)^3 \left(\frac{2f_\pi^2}{m_W^2}\right)
\]

- Using numerical values from the Particle Data Group

\[
BR(W^\pm \rightarrow \pi^\pm + \gamma) = 3.2 \times 10^{-9}
\]
The mass of the decaying W gives us a hard scale \( M_W \)

In the W boson rest frame the decay produces back to back -
- A real, hard photon
- A light pion in the opposite direction
- Both carrying about half the rest mass energy (in the limit of a massless pion)
- The quarks constituting the pion are therefore expected to be collinear

We can choose the momentum of the photon along the light cone direction, and work in the light cone basis

- Parametrize the light cone direction along \( z \) so that we define for a collinear particle

\[
\begin{align*}
n^\mu &= (1, 0, 0, 1) \\
\bar{n}^\mu &= (1, 0, 0, -1)
\end{align*}
\]

\( n^2 = \bar{n}^2 = 0 \) and \( n \cdot \bar{n} = 2 \)

We can set the momentum of the particles in this basis to be

\[
\begin{align*}
P^\mu &= \frac{M_W}{2} n^\mu \\
k^\mu &= \frac{M_W}{2} \bar{n}^\mu
\end{align*}
\]
\[ W^\pm \rightarrow \pi^\pm + \gamma \] IN SCET

- The general expression for the amplitude can be written in the SCET basis

\[ \mathcal{M} = -\frac{ge}{2\sqrt{2}} |V_{ud}| M_W^2 \left[ H_2(2g^{\mu\lambda} - n_\mu \bar{n}_\lambda) + iH_6 \epsilon^{\mu\lambda\alpha\beta} \bar{n}_\alpha n_\beta \epsilon^*_\mu(k) \epsilon_\lambda(P + k) \right] \]

- We can perform a matching at the required order to the operator in the effective theory and thus obtain the relevant form factors in SCET

- Since the pion consists of collinear quarks the full theory matches onto operators of the form

\[ \mathcal{O} = [\bar{\xi}_{n,P_d} W] \Gamma C'(\bar{n} \cdot \mu) [W^\dagger \xi_{n,P_u}] \]

  - C is the hard matching coefficient that describes short distance interactions
  - W’s are Wilson lines built out of collinear gluon fields that ensures that the operator is gauge invariant under collinear gauge transformations
  - Wilson lines are required to connect the fields since they are non local
\( W^{\pm} \to \pi^{\pm} + \gamma \) \text{ IN SCET}

- **Tree level amplitude in SCET**

\[
\mathcal{M}_d = \frac{-i e^g}{2\sqrt{2}} Q_d \langle \pi | [\bar{\xi}_{n,P_d} W] \gamma_\perp \frac{x \not{n} + \not{\bar{\nu}}}{2xM_w} \gamma^\lambda (1 - \gamma_5) [W^\dagger \xi_{n,P_u}] | 0 \rangle \epsilon_\mu^*(k) \epsilon_\lambda (P + k)
\]

\[
\mathcal{M}_u = \frac{-i e^g}{2\sqrt{2}} Q_u \langle \pi | [\bar{\xi}_{n,P_d} W] \gamma^\lambda (1 - \gamma_5) \frac{-\not{x} \not{n} - \not{\bar{\nu}}}{2xM_w} \gamma_\perp^\mu [W^\dagger \xi_{n,P_u}] | 0 \rangle \epsilon_\mu^*(k) \epsilon_\lambda (P + k).
\]

- only the transverse component of the electromagnetic vertex is relevant since from orthogonality of the photon polarization to the momentum along the light cone \( \gamma_\perp^\mu = \gamma^\mu - \frac{\not{n} \gamma_5}{2} - \frac{\not{\bar{\nu}} \gamma_5}{2} \)

- Using equations of motion for collinear fields \( \not{n} \xi_{n,P} = 0 \), \( \not{\bar{\nu}} \xi_{n,P} = \xi_{n,P} \) we can simplify and obtain the amplitude and match onto the general expression to obtain the form factors

\[
H_2 = \frac{i}{M_W^3} \left( \frac{Q_d}{x} - \frac{Q_u}{\bar{x}} \right) \langle \pi | [\bar{\xi}_{n,P_d} W] \not{n} (1 - \gamma_5) [W^\dagger \xi_{n,P_u}] | 0 \rangle - \frac{4f_\pi}{M_W^2}
\]

\[
H_6 = \frac{i}{M_W^3} \left( \frac{Q_d}{x} + \frac{Q_u}{\bar{x}} \right) \langle \pi | [\bar{\xi}_{n,P_d} W] \not{n} (1 - \gamma_5) [W^\dagger \xi_{n,P_u}] | 0 \rangle.
\]

- We can boost the expression involving the pion wavefunction defined by Brodsky-Lepage in the SCET frame to collinear fields

- We can therefore obtain a factorization

\[
C_{2,6} \langle \pi | [\bar{\xi}_{n,P_d} W] \Gamma [W^\dagger \xi_{n,P_u}] | 0 \rangle = -if_\pi \not{n} \cdot P_\pi \int_0^1 C_{2,6} \phi_\pi (x) dx.
\]
1 LOOP CONTRIBUTIONS

- Full Theory vs Effective theory
SUMMARY & OBJECTIVES

• Studied the radiative exclusive decay of W to a Pion and an energetic photon

• Obtained the relevant form factors and branching ratio at tree level in QCD and SCET

• No enhancement was found at tree level

• Showed factorization into hard matching coefficient and light cone distribution function at tree level in SCET

• Calculate one-loop order $\alpha_s$ correction in the effective theory

• Extend Calculation to the case of $W \rightarrow D \gamma$