EXCLUSIVE W DECAY AND EFFECTIVE FIELD THEORY

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INTRODUCTION

- **We study the radiative exclusive decay mode** $W^{\pm} \rightarrow \pi^{\pm} + \gamma$
  - The W is a massive charged vector boson which mediates weak interactions
  - The pion is a light pseudoscalar meson which is the bound state of an up and down quark

- **Intermediate vector bosons, like W, are produced in high energy accelerators**
  - W bosons are predicted to have mass in the Standard Weinberg-Salam model for weak interactions. $M_W \sim 80.385 \pm 0.015$ GeV

- **Good theoretical calculation of these rare decay modes provide useful insight into electroweak and strong interaction dynamics**
  - $W^{\pm} \rightarrow P^{\pm} + \gamma$, where P is a pseudoscalar meson (π, D_S) have been studied using particular models

    - *L. Arnellos, W. J. Marciano, and Z. Parsa , Nucl. Phys. B 196, 378 (1982)* [BR ~ 3×10^{-9}]

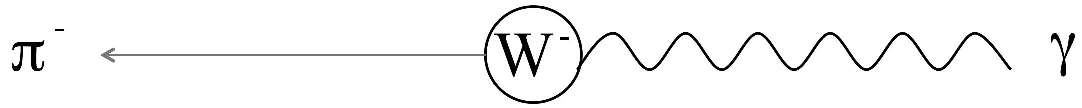
    -*Y. Y. Keum and X. Y. Pham, Mod. Phys. Lett. A, Vol. 9, No. 17 (1994) 1545-1556* [BR ~ 10^{-6}]

- **We employ a model independent approach using effective field theory, and wish to also include 1 loop contributions and resummation of logarithmic terms.**

INTRODUCTION

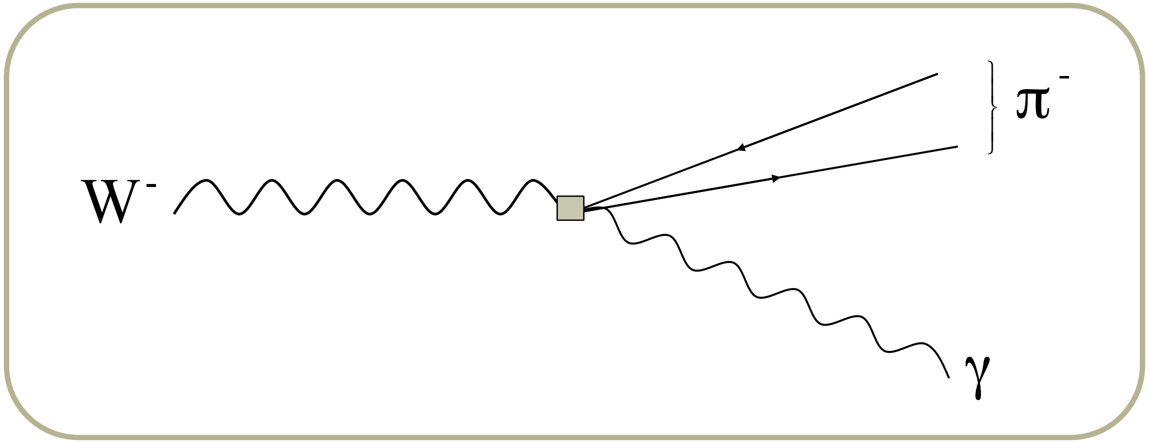
- **Interest coming from measurements of W decay to Psuedoscalar and vector at experiments like CDF**
  - (including Prof. Robert Harr's experimental group from WSU)
  - current bounds from PDG puts branching fraction at < 8×10^{-5}

- **Detection of this radiative 2-body decay mode along with the measurement of the photon energy can provide precise determination of W mass if the decay rate is large enough**



- **In the rest frame of the W, the final products are back to back**
  - The photon is a real, hard (energetic) photon
  - The pion is a collinear bound state of quarks
  - Transverse components become negligible
  - We neglect pion mass so that essentially $m_\pi \rightarrow 0$

THE DECAY

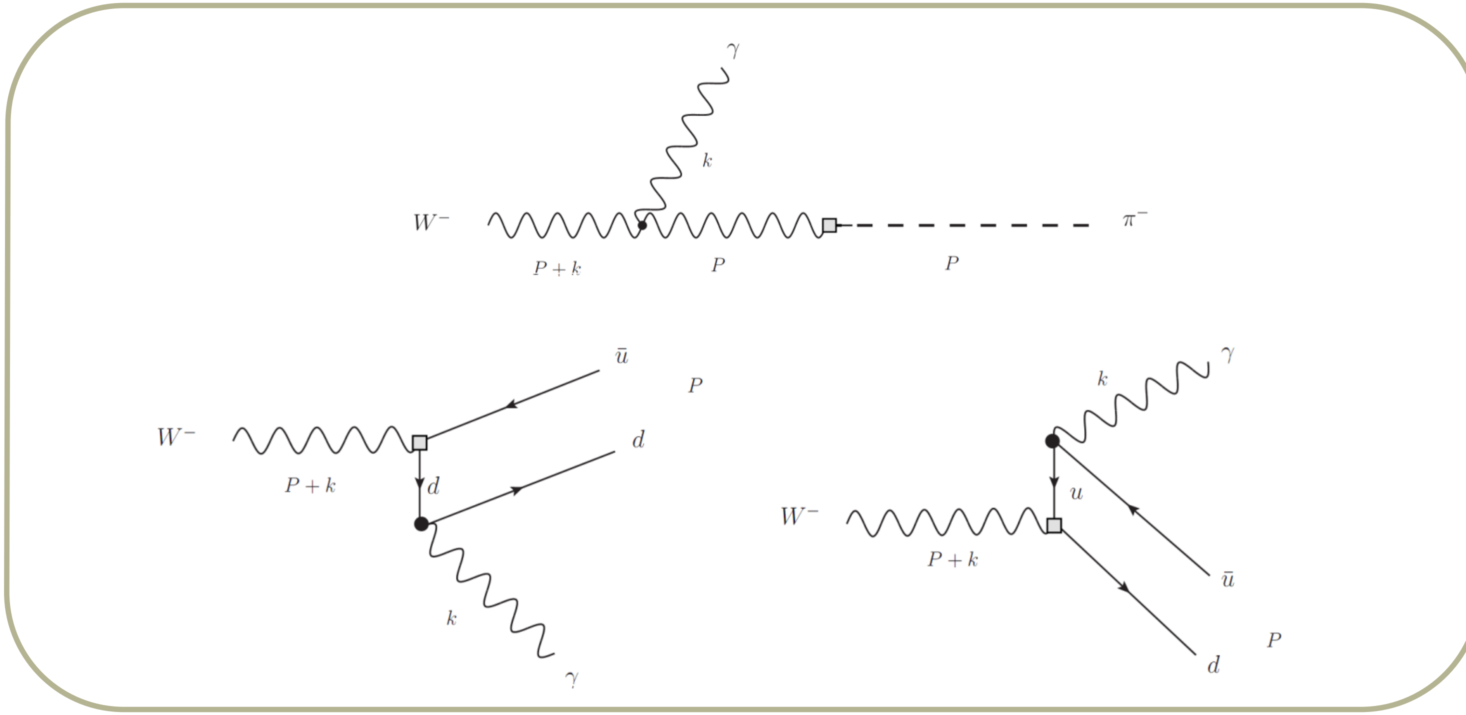


- **General Structure of the amplitude**
  - Required by Lorentz invariance, Gauge invariance, equations of motion {orthogonality conditions of polarizations $\epsilon(k)\cdot(k) = \epsilon(P+k)\cdot(P+k) = 0$ }

$$\mathcal{M} = -\frac{ge}{2\sqrt{2}}|V_{ud}|[H_2(P\cdot k g^{\mu\lambda} - P_\mu k_\lambda) + iH_6\epsilon^{\alpha\beta\mu\lambda}k_\alpha P_\beta]\epsilon^*_\mu(k)\epsilon_\lambda(P+k)$$

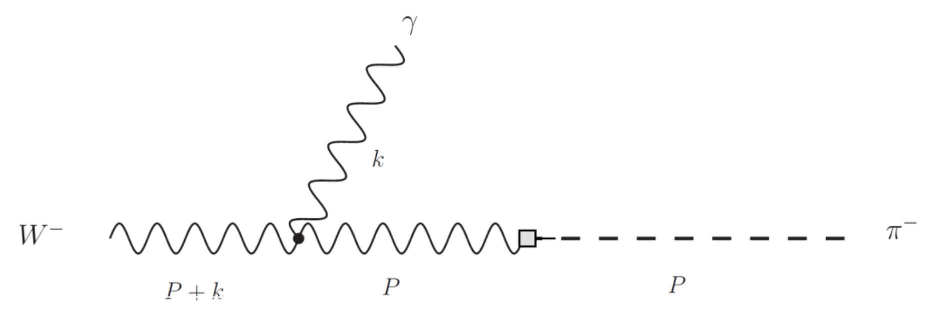
  - H_2 and H_6 are form factors corresponding to axial-vector and vector component of the weak current.
  - The W boson carries momentum of P+k, where P is the Pion momentum and k is the Photon momentum

THE DECAY $W^{\pm} \rightarrow \pi^{\pm} + \gamma$



- **3 feynman diagrams contribute to the decay width at tree level**
  - Trilinear non abelian weak coupling from standard model
    - ● $-ie[(2P+k)_{\mu}g_{\alpha\lambda} - (P+2k)_{\alpha}g_{\mu\lambda} - (P-k)_{\lambda}g_{\mu\alpha}]$
  - Weak vertex describing axial current interacting with the W
    - ◻ $-i\frac{g}{2\sqrt{2}}\ \gamma^{\lambda}(1-\gamma_5)$
  - QED vertex describing vector current interacting with the photon
    - ● $-ie\,Q\,\gamma^{\mu}$

TREE LEVEL DIAGRAMS



- **W pole diagram**

$$\mathcal{M}_W = -\frac{eg}{2\sqrt{2}}|V_{ud}|f_\pi\epsilon^*_\mu(k)\epsilon_\lambda(P+k)\frac{(g^{\alpha\beta}-P^\alpha P^\beta/m_W^2)}{P^2-m_W^2}$$
$$\times[(2P)_\mu g_{\alpha\lambda}-(P+2k)_\alpha g_{\mu\lambda}-2P_\lambda g_{\mu\alpha}]P_\beta,$$

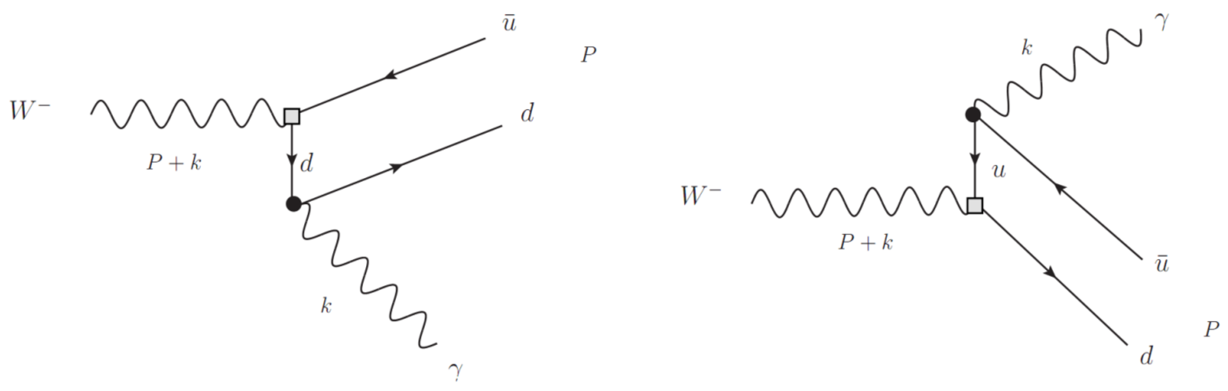
- Pion decay constant defined by

$$\langle\pi|\bar{d}\gamma_\beta(1-\gamma_5)u|0\rangle = if_\pi P_\beta$$

- Using orthogonality conditions it simplifies and we see contribution only to the axial vector form factor

$$\mathcal{M}_W = -\frac{eg}{2\sqrt{2}}|V_{ud}|f_\pi\epsilon^*_\mu(k)\epsilon_\lambda(P+k)g_{\mu\lambda}$$

TREE LEVEL DIAGRAMS



- **Structure dependent diagrams**

$$\mathcal{M}_d = \frac{-ige}{2\sqrt{2}} Q_d \langle \pi | \bar{d} \gamma^\mu \frac{(\not{P}_d + \not{k})}{(P_d + k)^2} \gamma^\lambda (1 - \gamma_5) u | 0 \rangle \epsilon^*_\mu(k) \epsilon_\lambda(P + k)$$

$$\mathcal{M}_u = \frac{-ige}{2\sqrt{2}} Q_u \langle \pi | \bar{d} \gamma^\lambda (1 - \gamma_5) \frac{(-\not{P}_u - \not{k})}{(P_u + k)^2} \gamma^\mu u | 0 \rangle \epsilon^*_\mu(k) \epsilon_\lambda(P + k)$$

PION WAVE FUNCTION

- **In order to evaluate the matrix elements we need to construct the Pion wave function**

- **Need to describe the distribution of quark momentums within the pion**
  - Assign longitudinal momentum fractions to the quarks with respect to pion momentum

$$P_d = xP$$

$$P_u = \bar{x}P,$$

- **Employ light cone distribution amplitude for light pseudoscalar mesons (from Brodsky and Lepage)**

$$\langle \pi(P)|\bar{d}(z_2)u(z_1)|0\rangle = \frac{if_\pi}{4}(\not{P}\gamma_5)\int_0^1 dx e^{i(xp\cdot z_2+\bar{x}p\cdot z_1)}\phi_\pi(x)$$

  - Where $\phi_\pi(x) = 6x(1-x)$ is the leading twist distribution amplitude for the light meson in the asymptotic limit

- **We then just have to integrate over the momentum fraction x**

AMPLITUDE

- Total tree level Amplitude corresponding to the decay

$$\mathcal{M}_{W+u+d} = \frac{ge}{2\sqrt{2}}|V_{ud}|\frac{f_\pi}{m_W^2}[P\cdot k g^{\mu\lambda} - P_\mu k_\lambda + i\epsilon^{\alpha\beta\mu\lambda}k_\alpha P_\beta]\epsilon^*_\mu(k)\epsilon_\lambda(P+k)$$

- Compare to the general expression to obtain the form factors

$$H_6 = H_2 = \frac{-f_\pi}{m_W^2}$$

- Decay Width obtained from squaring the amplitude and integrating over the 2-body phase space

$$\Gamma = \frac{\alpha G_F}{24\sqrt{2}}|V_{ud}|^2 m_W^3 \left(1 - \frac{m_\pi^2}{m_W^2}\right)^3 \left(\frac{2f_\pi^2}{m_W^2}\right)$$

- Using numerical values from the Particle Data Group

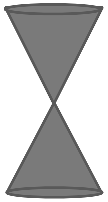
$$BR[W^\pm \to \pi^\pm + \gamma] = 3.2 \times 10^{-9}$$

SOFT COLLINEAR EFFECTIVE THEORY

- **The mass of the decaying W gives us a hard scale M_W**
- **In the W boson rest frame the decay produces back to back -**
  - A real, hard photon
  - A light pion in the opposite direction
  - Both carrying about half the rest mass energy (in the limit of a massless pion)
  - The quarks constituting the pion are therefore expected to be collinear
- **We can choose the momentum of the photon along the light cone direction, and work in the light cone basis**

- **Parametrize the light cone direction along z so that we define for a collinear particle**

$$n^\mu = (1, 0, 0, 1)$$



$$\bar{n}^\mu = (1, 0, 0, -1)$$

$$n^2 = \bar{n}^2 = 0 \text{ and } n \cdot \bar{n} = 2$$

- **We can set the momentum of the particles in this basis to be**

$$P^\mu = \frac{M_W}{2} n^\mu$$

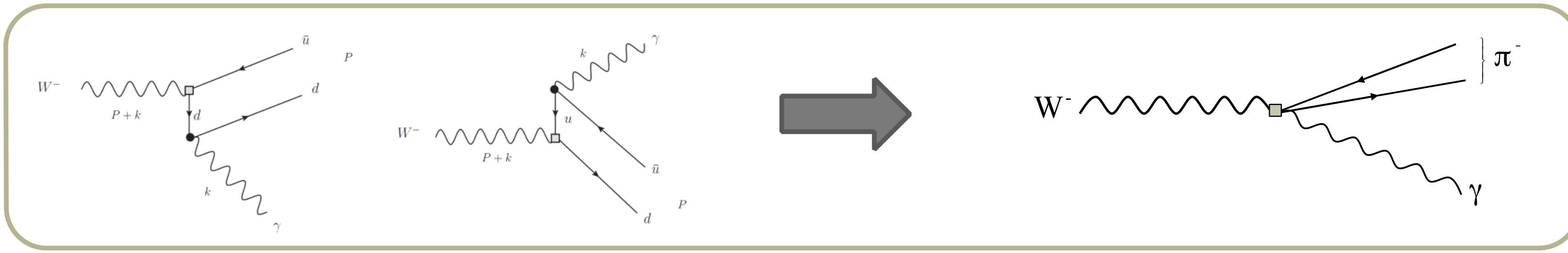
$$k^\mu = \frac{M_W}{2} \bar{n}^\mu$$

$W^{\pm} \to \pi^{\pm} + \gamma$ IN SCET

- **The general expression for the amplitude can be written in the SCET basis**

$$\mathcal{M} = -\frac{ge}{2\sqrt{2}}|V_{ud}|\frac{M_W^2}{4}[H_2(2g^{\mu\lambda} - n_\mu \bar{n}_\lambda) + iH_6\epsilon^{\mu\lambda\alpha\beta}\bar{n}_\alpha n_\beta]\epsilon^*_\mu(k)\epsilon_\lambda(P+k)$$

- **We can perform a matching at the required order to the operator in the effective theory and thus obtain the relevant form factors in SCET**



- **Since the pion consists of collinear quarks the full theory matches onto operators of the form**

$$\mathcal{O} = [\bar{\xi}_{n,P_d} W]\Gamma C(\bar{n}\cdot p, \mu)[W^\dagger \xi_{n,P_u}]$$

  - C is the hard matching coefficient that describes short distance interactions
  - W's are Wilson lines built out of collinear gluon fields that ensures that the operator is gauge invariant under collinear gauge transformations
  - Wilson lines are required to connect the fields since they are non local

$W^{\pm} \to \pi^{\pm} + \gamma$ IN SCET

- **Tree level amplitude in SCET**

$$\mathcal{M}_d = \frac{-ige}{2\sqrt{2}} Q_d \langle \pi | [\bar{\xi}_{n,P_d} W] \gamma_\perp^\mu \frac{(x \not{n} + \not{\bar{n}})}{2x M_w} \gamma^\lambda (1-\gamma_5) [W^\dagger \xi_{n,P_u}] | 0 \rangle \epsilon_\mu^*(k) \epsilon_\lambda(P+k)$$

$$\mathcal{M}_u = \frac{-ige}{2\sqrt{2}} Q_u \langle \pi | [\bar{\xi}_{n,P_d} W] \gamma^\lambda (1-\gamma_5) \frac{(-\bar{x} \not{n} - \not{\bar{n}})}{2\bar{x} M_w} \gamma_\perp^\mu [W^\dagger \xi_{n,P_u}] | 0 \rangle \epsilon_\mu^*(k) \epsilon_\lambda(P+k).$$

  - only the transverse component of the electromagnetic vertex is relevant since from orthogonality of the photon polarization to the momentum along the light cone $\gamma_\perp^\mu = \gamma^\mu - \frac{n^\mu \not{\bar{n}}}{2} - \frac{\bar{n}^\mu \not{n}}{2}$

- **Using equations of motion for collinear fields** $\not{n} \xi_{n,P} = 0$, $\frac{\not{n}\not{\bar{n}}}{4} \xi_{n,P} = \xi_{n,P}$ **we can simplify and obtain the amplitude and match onto the general expression to obtain the form factors**

$$H_2 = \frac{i}{M_W^3} \left(\frac{Q_d}{x} - \frac{Q_u}{\bar{x}} \right) \langle \pi | [\bar{\xi}_{n,P_d} W] \not{\bar{n}} (1-\gamma_5) [W^\dagger \xi_{n,P_u}] | 0 \rangle - \frac{4 f_\pi}{M_W^2}$$

$$H_6 = \frac{i}{M_W^3} \left(\frac{Q_d}{x} + \frac{Q_u}{\bar{x}} \right) \langle \pi | [\bar{\xi}_{n,P_d} W] \not{\bar{n}} (1-\gamma_5) [W^\dagger \xi_{n,P_u}] | 0 \rangle.$$

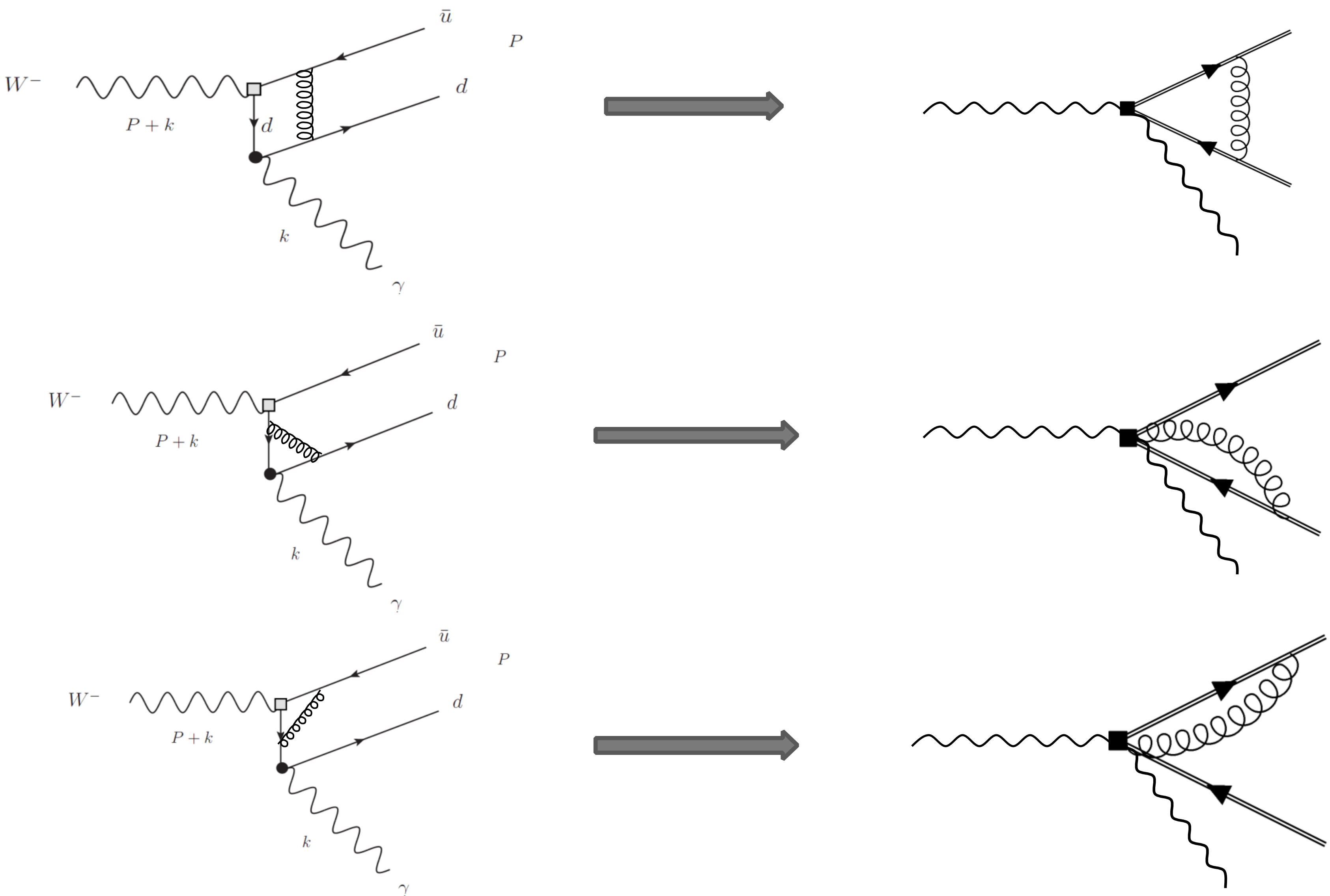
- **We can boost the expression involving the pion wavefunction defined by Brodsky-Lepage in the SCET frame to collinear fields**

- **We can therefore obtain a factorization**

$$C_{2,6} \langle \pi | [\bar{\xi}_{n,P_d} W] \Gamma [W^\dagger \xi_{n,P_u}] | 0 \rangle = -i f_\pi \bar{n} \cdot P_\pi \int_0^1 C_{2,6} \phi_\pi(x) dx.$$

1 LOOP CONTRIBUTIONS

- **Full Theory** **vs** **Effective theory**



SUMMARY & OBJECTIVES

- **Studied the radiative exclusive decay of W to a Pion and an energetic photon**
- **Obtained the relevant form factors and branching ratio at tree level in QCD and SCET**
- **No enhancement was found at tree level**
- **Showed factorization into hard matching coefficient and light cone distribution function at tree level in SCET**
- **Calculate one-loop order α_s correction in the effective theory**
- **Extend Calculation to the case of $W \to D\,\gamma$**