Obtaining a nonzero θ_{13} in lepton models based on $SO(3) \to A_4$

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Outline

- Tri-bimaximal mixing found and lost; how do A_4 models cope?
- Basics of $SO(3) \rightarrow A_4$ model
- Nonzero θ_{13} in $SO(3) \rightarrow A_4$ model
- Summary

Tri-bimaximal mixing and A_4 models

- For long time, PMNS matrix thought to be consistent with TBM

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\leftarrow \sin(\theta_{13}) e^{i\delta_{13}}$

- Highly specific pattern… discrete lepton flavor symmetries?

Tri-bimaximal mixing and A_4 models

- Simplest A_4 lepton models
  - Components
    - SM Higgs and leptons
    - Right handed neutrinos
    - Scalar flavons ϕ, ϕ'

    } Representations of A_4
  - Features
    - Additional Z_2 symmetry so flavons sectorized
    - Flavons gain VEV $\langle\phi\rangle = (v, v, v)$, $\langle\phi'\rangle = (v', 0, 0)$

$\Rightarrow$ TBM!

Tri-bimaximal mixing… lost

- Recently, θ_{13} found to be much larger!
- Daya Bay (2012):

$$\sin^2(2\theta_{13}) = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$$

- RENO (2012):

$$\sin^2(2\theta_{13}) = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst})$$

- TBM ruled out!

How do A_4 models cope?

- Actually many A_4 models already allow this!
- Approaches:
  - Higher dimension operators, e.g. [Altarelli and Meloni (2009)]
  - More flavons, e.g. [Chen et al (2013)]
  - Perturb flavon alignments, e.g. [King (2011)]
  - Radiative corrections, e.g. [Antusch et al (2003)]

Continuous symmetry $\rightarrow A_4$

- Class of models based on continuous symmetry $\rightarrow A_4$.
- Originally motivated to explain origin of A_4.
- Specific example: $SO(3) \rightarrow A_4$ [Berger and Grossman (2009)]
- Turns out θ_{13} already nonzero at tree level!
- Even better: θ_{13} related to $\frac{\cancel{A4}\ \text{scale}}{\cancel{SO(3)}\ \text{scale}}$

Basics of SO(3) → A_4 model

Field	$SU(2)_L$	$U(1)_Y$	$SO(3)_F$	Z_2	
ψ_l	$\mathbf{2}$	$-\frac{1}{2}$	$\mathbf{3}$	$-$	Left-handed leptons
ψ_f	$\mathbf{1}$	-1	$\mathbf{3}$	$-$	
ψ_e	$\mathbf{1}$	-1	$\mathbf{1}$	$+$	Right-handed leptons
ψ_m	$\mathbf{1}$	-1	$\mathbf{5}$	$+$	
ψ_n	$\mathbf{1}$	0	$\mathbf{3}$	$-$	
H	$\mathbf{2}$	$\frac{1}{2}$	$\mathbf{1}$	$+$	Scalars
ϕ	$\mathbf{1}$	0	$\mathbf{3}$	$-$	
ϕ'	$\mathbf{1}$	0	$\mathbf{3}$	$+$	
ϕ_5	$\mathbf{1}$	0	$\mathbf{5}$	$-$	
T	$\mathbf{1}$	0	$\mathbf{7}$	$-$	

Basics of SO(3) $\rightarrow$ A_4 model

- Most general Lagrangian for leptons

$$L_l = -y_e \overline{\psi_l^a} \frac{H}{\Lambda} \phi^a \psi_e - y_m \overline{\psi_l^a} \frac{H}{\Lambda} \phi^b \psi_m^{ab} - y_m^T \overline{\psi_l^a} \frac{H}{\Lambda} T^{abc} \psi_m^{bc} - y_m^5 \grave{o}^{abc} \overline{\psi_l^a} \frac{H}{\Lambda} \phi_5^{bd} \psi_m^{cd}$$

$$-y_e' \overline{\psi_f^a} \phi^a \psi_e - y_m' \overline{\psi_f^a} \phi^b \psi_m^{ab} - y_m^{T'} \overline{\psi_f^a} T^{abc} \psi_m^{bc} - y_m^{5'} \grave{o}^{abc} \overline{\psi_f^a} \phi_5^{bd} \psi_m^{cd}$$

$$L_v = -M \overline{\psi_n^{ca}} \psi_n^a - \frac{x_v}{\Lambda} \overline{\psi_n^{ca}} \psi_n^b \phi'^c T^{abc} - y_v \overline{\psi_l^a} H \psi_n^a$$

- Flavons gain VEV

$$\langle T \rangle = v_T x^{(a} y^b z^{c)}, \qquad \langle \phi \rangle = \begin{pmatrix} v \\ v \\ v \end{pmatrix}, \qquad \langle \phi_5 \rangle = \begin{pmatrix} 0 & v_5 & v_5 \\ v_5 & 0 & v_5 \\ v_5 & v_5 & 0 \end{pmatrix}, \qquad \langle \phi' \rangle = \begin{pmatrix} v' \\ 0 \\ 0 \end{pmatrix}$$

3D coordinate basis vectors

$\Rightarrow$ 6 $\times$ 6 mass matrices

Obtaining U_{PMNS}

- U_{PMNS} characterize charged-current interactions between light eigenstates.
- 6×6 mass matrices:
  - Block-diagonalise 6×6 mass matrices.
  - Then diagonalise 3×3 mass matrices.
- Neutrinos:

$$M_\nu^{3\times 3} = \begin{pmatrix} -\frac{y_\nu^2 v_H^2}{M} & 0 & 0 \\ 0 & -\frac{y_\nu^2 M v_H^2 \Lambda^2}{M^2\Lambda^2 - x_\nu^2 v'^2 v_T^2} & \frac{y_\nu^2 x_\nu v_H^2 v' v_T \Lambda}{M^2\Lambda^2 - x_\nu^2 v'^2 v_T^2} \\ 0 & \frac{y_\nu^2 x_\nu v_H^2 v' v_T \Lambda}{M^2\Lambda^2 - x_\nu^2 v'^2 v_T^2} & -\frac{y_\nu^2 M v_H^2 \Lambda^2}{M^2\Lambda^2 - x_\nu^2 v'^2 v_T^2} \end{pmatrix}, \quad \text{of form} \begin{pmatrix} a & 0 & 0 \\ 0 & b & c \\ 0 & c & b \end{pmatrix}$$

Obtaining U_{PMNS}

- Charged leptons: $M_l^{6\times 6} \equiv \begin{pmatrix} \frac{v_H}{\Lambda}A & \frac{v_H}{\Lambda}B \\ C & D \end{pmatrix}$

$$\omega = e^{i\frac{2\pi}{3}}$$

$$A = \begin{pmatrix} y_e v & \left[y_m v + y_m^5 v_5(\omega^2-\omega)\right] & \left[y_m v + y_m^5 v_5(\omega-\omega^2)\right] \\ y_e v & \left[y_m v + y_m^5 v_5(\omega^2-\omega)\right]\omega & \left[y_m v + y_m^5 v_5(\omega-\omega^2)\right]\omega^2 \\ y_e v & \left[y_m v + y_m^5 v_5(\omega^2-\omega)\right]\omega^2 & \left[y_m v + y_m^5 v_5(\omega-\omega^2)\right]\omega \end{pmatrix}, \quad \text{of form} \begin{pmatrix} a & b & c \\ a & b\omega & c\omega^2 \\ a & b\omega^2 & c\omega \end{pmatrix}$$

$$B = \begin{pmatrix} y_m v + 2y_m^T v_T & y_m v + y_m^5 v_5 & -y_m^5 v_5 \\ y_m v & 2y_m^T v_T & y_m v \\ y_m v + y_m^5 v_5 + y_m^T v_T & y_m v - y_m^5 v_5 & y_m^T v_T \end{pmatrix}$$

$$C = \begin{pmatrix} y_e' v & \left[y_m' v + y_m^{5'} v_5(\omega^2-\omega)\right] & \left[y_m' v + y_m^{5'} v_5(\omega-\omega^2)\right] \\ y_e' v & \left[y_m' v + y_m^{5'} v_5(\omega^2-\omega)\right]\omega & \left[y_m' v + y_m^{5'} v_5(\omega-\omega^2)\right]\omega^2 \\ y_e' v & \left[y_m' v + y_m^{5'} v_5(\omega^2-\omega)\right]\omega^2 & \left[y_m' v + y_m^{5'} v_5(\omega-\omega^2)\right]\omega \end{pmatrix}, \quad \text{of form} \begin{pmatrix} a & b & c \\ a & b\omega & c\omega^2 \\ a & b\omega^2 & c\omega \end{pmatrix}$$

$$D = \begin{pmatrix} y_m' v + 2y_m^{T'} v_T & y_m' v + y_m^{5'} v_5 & -y_m^{5'} v_5 \\ y_m' v & 2y_m^{T'} v_T & y_m' v \\ y_m' v + y_m^{5'} v_5 + y_m^{T'} v_T & y_m' v - y_m^{5'} v_5 & y_m^{T'} v_T \end{pmatrix}$$

Nonzero θ_{13}

- Block diagonalize $M_l^{6\times6}(M_l^{6\times6})^\dagger$

$$M_l^{3\times3}(M_l^{3\times3})^\dagger = \frac{v_H^2}{\Lambda^2}\left[AA^\dagger + BB^\dagger - (AC^\dagger + BD^\dagger)(CC^\dagger + DD^\dagger)^{-1}(CA^\dagger + DB^\dagger)\right]$$

- Useful to define $E \equiv B - (y_m^T / y_m^{T'})D$
- Then $A, C, E \sim O(v, v_5)$, $\quad B, D \sim O(v_T)$
- Expanding in small parameter $\epsilon = O(v/v_T) \sim O(v'/v_T)$
  - LO: $M_l^{3\times3} = \frac{v_H}{\Lambda}\left(A - \frac{y_m^T}{y_m^{T'}}C\right)$

    θ_{13} still 0!

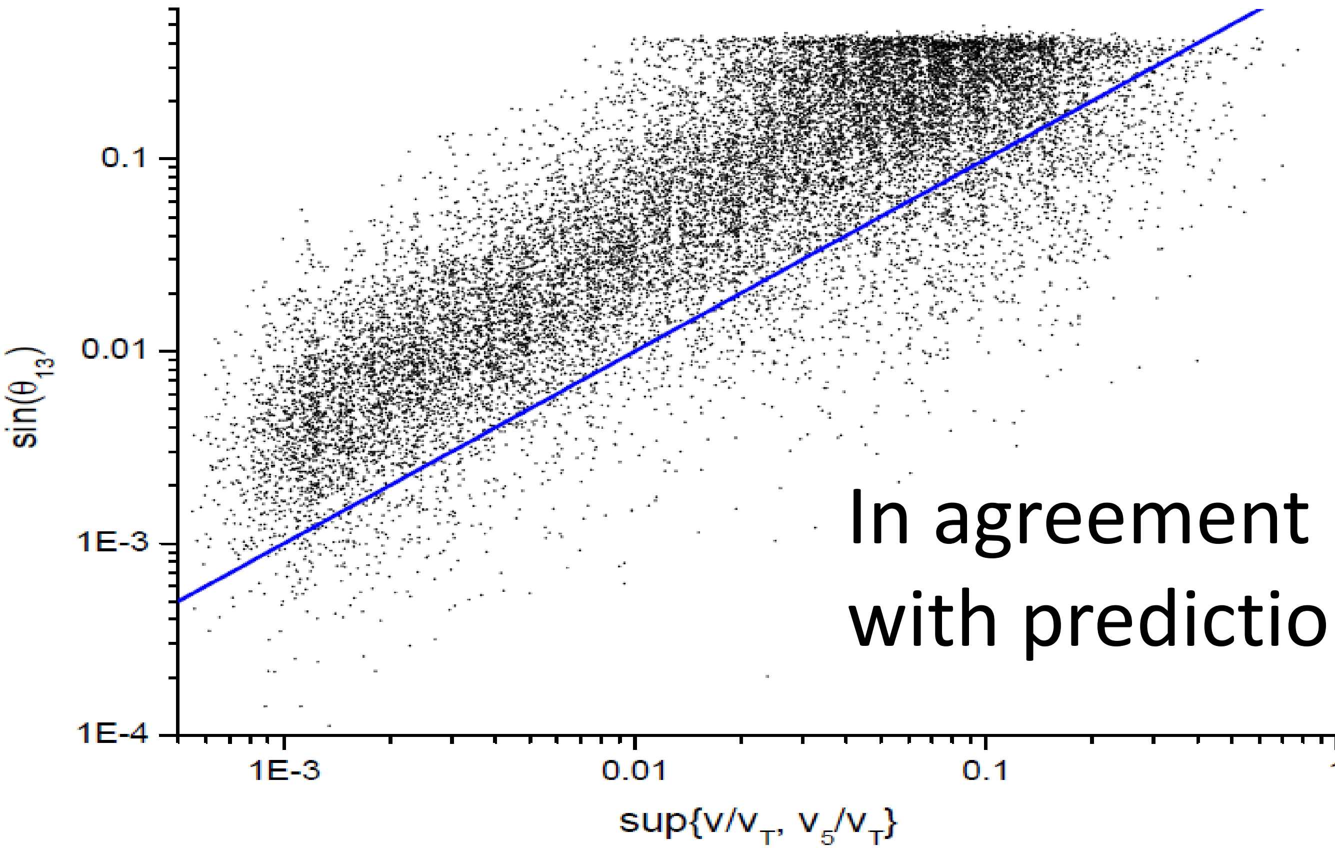
  - NLO: $M_l^{3\times3} = \frac{v_H}{\Lambda}\left(A - \frac{y_m^T}{y_m^{T'}}C - ED^{-1}C\right)$

    Suggests $\theta_{13} \sim \varepsilon$!

    ~~A4~~ scale / ~~SO(3)~~ scale

Nonzero θ_{13}

- Actual contribution to θ_{13} from deviation turns out to be ~ $(m_\tau/m_\mu)\ \varepsilon$ ~ 10ε
- Random simulation results



In agreement with predictions!

Summary and further work

- TBM ruled out, but many A_4 models allow for this scenario
- In particular, SO(3) $\rightarrow$ A_4 model allow nonzero tree-level θ_{13} of size $\sim \frac{m_\tau}{m_\mu} \frac{\cancel{\text{A4}}\ \text{scale}}{\cancel{\text{SO(3)}}\ \text{scale}}$
- % level separation of A_4 and SO(3) breaking scales

Thank you!