Lattice study of semileptonic B decays into D mesons

in the SM and beyond

Mariam Atoui

based on work with D. Bečirević V. Morénas F. Sanfilippo [arXiv:1312.2914]

Phenomenology 2014 symposium
University of Pittsburgh

Université Blaise Pascal
Clermont Fd-France

5-7 may 2014

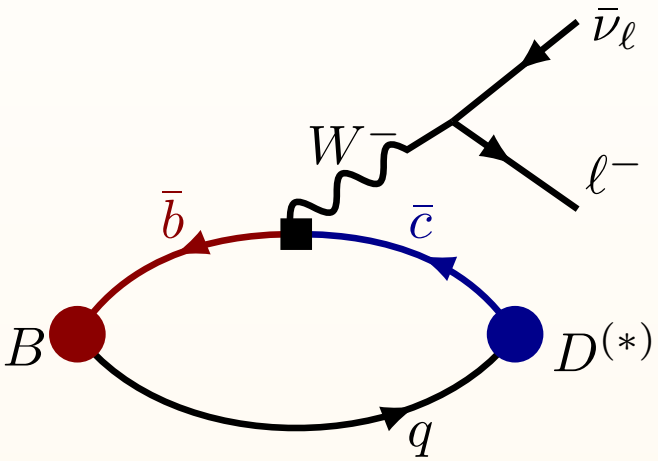
Outline

1. Semileptonic B decays
2. Lattice calculation of form factors for $B_{(s)} \to D_{(s)} \ell \bar{\nu}_\ell$
3. Conclusions

Outline

1. Semileptonic B decays
2. Lattice calculation of form factors for $B_{(s)} \to D_{(s)} \ell \bar{\nu}_\ell$
3. Conclusions

Why semileptonic B decays? (I)



Difference between the inclusive and the exclusive determination of the CKM matrix element V_{cb} ($\simeq 1.5\sigma$)

Inclusive decays : $|V_{cb}|_{\text{inclusive}} = 41.90(70) \times 10^{-3}$

Exclusive decays : decay rates $\propto$ CKM matrix element $\times$ $\underbrace{\text{form factor}}_{\text{theoretical uncertainty}}$

- $B \to D^* \ell \bar{\nu}_\ell$ $\quad \mathscr{F}(1) = 0.906(4)(12)$ $\quad (\mathscr{F}(1) \cdot |V_{cb}| = 35.90(45) \times 10^{-3})$

  Baiely *et al.* [arXiv :1403.0635] (2014)

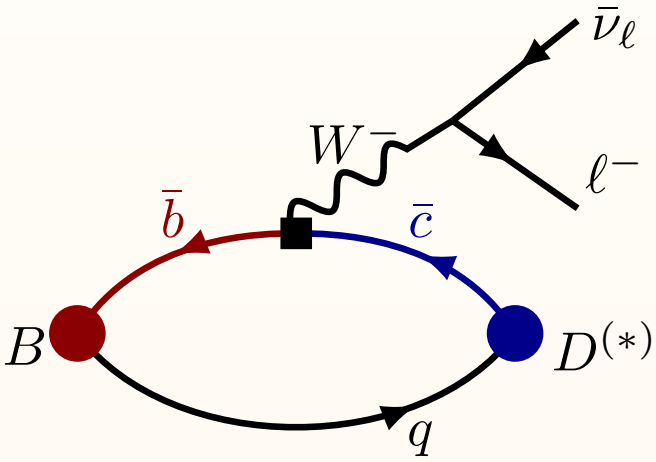
- $B \to D \ell \bar{\nu}_\ell$ $\quad \mathscr{G}(1) = 1.074(24)$ $\quad (\mathscr{G}(1) \cdot |V_{cb}| = 42.6(1.5) \times 10^{-3})$

  Okamoto *et al.* [Nucl. Phys. Proc. Suppl.]

$$\Longrightarrow |V_{cb}|_{\text{exclusive}} = 38.56(89) \times 10^{-3}$$

$\Longrightarrow$ Improve lattice precision and use other lattice approaches

Why semileptonic B decays? (II)



Difference between experimental measurements and theoretical predictions in "$b \to c$" data?

$$R(D) = \frac{\mathscr{B}(B \to D\,\tau\,\bar{\nu}_\tau)}{\mathscr{B}(B \to D\,\ell\,\bar{\nu}_\ell)}$$

Babar Collaboration $\rightsquigarrow$ 0.440 ± 0.072

BaBar Collaboration [Phys. Rev. lett] (2008)

Standard Model $\rightsquigarrow$ 0.31 ± 0.02

Becirevic *et al.* [Phys. lett. B 716] (2013)

around 2σ tension between measurements and SM predictions

Possible New Physics effects in semileptonic B decays?

Proposal : Lattice calculations of $B_s \to D_s \, \ell \, \bar{\nu}$ channels in and beyond the SM.

Why $B_s \to D_s$?

- Experimentally :
  - Could be studied at LHCb and especially at Super-Belle
  - No averaging between neutral and charged modes
  - Soft photon pollution smaller Becirevic *et al.* [Acta Phys. Polon. Supp.] (2010)

$$\frac{\mathscr{B}(B_s \to D_s \, \ell \, \nu \gamma_{\rm soft})}{\mathscr{B}(B_s \to D_s \ell \nu)} < \frac{\mathscr{B}(B \to D^0 \ell \nu \gamma_{\rm soft})}{\mathscr{B}(B \to D^0 \, \ell \, \nu)}$$

- Lattice point of view :
  - much more affordable numerically

Proposal : Lattice calculations of $B_s \to D_s\, \ell\, \bar{\nu}$ channels in and beyond the SM.

Why $B_s \to D_s$?

- Experimentally :
  - Could be studied at LHCb and especially at Super-Belle
  - No averaging between neutral and charged modes
  - Soft photon pollution smaller Becirevic *et al.* [Acta Phys. Polon. Supp.] (2010)

$$\frac{\mathscr{B}(B_s \to D_s\, \ell\, \nu \gamma_{\rm soft})}{\mathscr{B}(B_s \to D_s \ell \nu)} < \frac{\mathscr{B}(B \to D^0 \ell \nu \gamma_{\rm soft})}{\mathscr{B}(B \to D^0\, \ell\, \nu)}$$

- Lattice point of view :
  - much more affordable numerically

New Physics in "$b \to c$" ?

- Revisit the SM prediction
- Consider additional tensor and scalar operators in the effective Hamiltonian

$$\boxed{\mathscr{H}_{\rm eff} = \mathscr{H}_{\rm eff}^{\rm SM} + \mathscr{H}_{\rm eff}^{\rm NP}}$$

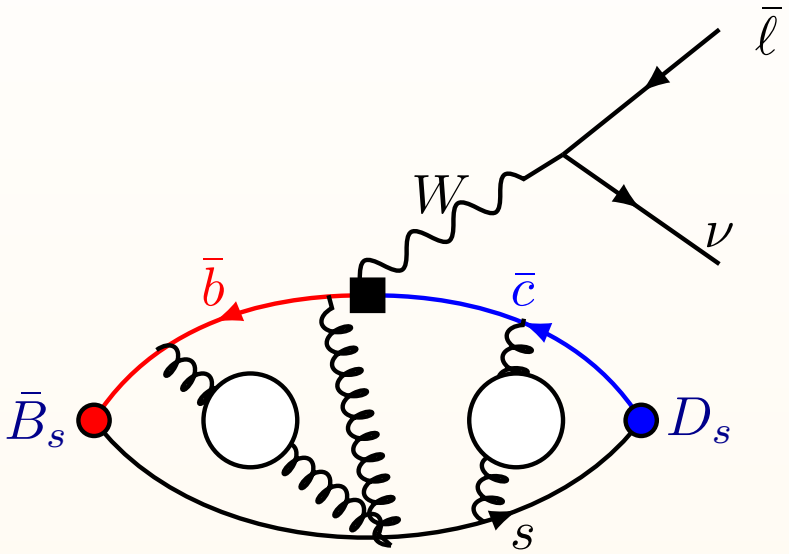
Outline

1. Semileptonic B decays
2. Lattice calculation of form factors for $B_{(s)} \to D_{(s)} \ell \bar{\nu}_\ell$
3. Conclusions

$b \to c\, \ell \bar{\nu}_\ell$ effective Hamiltonian

In Standard Model (SM) :

$$\mathscr{H}_{\text{eff}}^{\text{SM}} \propto \bar{c}\, \gamma_\mu\, (1-\gamma_5)\, b\, \bar{\ell}\, \gamma^\mu\, (1-\gamma_5)\, \nu_\ell$$

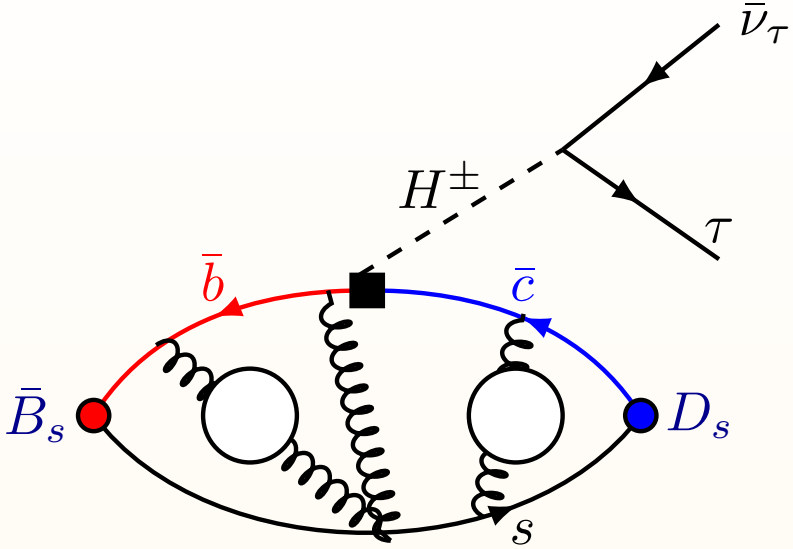


Hadronic matrix elements

$$\langle D_s(p_{D_s}) \,|\, V_\mu \,|\, \bar{B}_s(p_{B_s}) \rangle = F_+(q^2)\, (p_{B_s} + p_{D_s})_\mu$$

$$+ (p_{B_s} - p_{D_s})_\mu \, \left[F_0(q^2) - F_+(q^2)\right] \left(\frac{m_{B_s}^2 - m_{D_s}^2}{q^2} \right) \qquad [0 < q^2 \leq (m_B - m_D)^2]$$

$$\langle D_s(p_{D_s}) \,|\, A_\mu \,|\, \bar{B}_s(p_{B_s}) \rangle = 0$$



New Physics scenario

$$\mathscr{H}_{\text{eff}}^{\text{NP}} \propto g_T \; (\bar{c}\, \sigma_{\mu\nu} b) \; \left(\bar{\ell}\, \sigma^{\mu\nu} \nu\right) + g_S \; (\bar{c}_R\, b_L) \; \left(\bar{\ell}_R\, \nu_L\right)$$

g_T : Coupling of the new tensor term

g_S : Coupling of the scalar term

$$\left\langle D_s(p_{D_s}) \,\middle|\, \bar{c}\, \sigma_{\mu\nu}\, b \,\middle|\, B_s(p_{B_s}) \right\rangle =$$

$$-\, i \left(p_{B_s\,\mu}\, p_{D_s\,\nu} - p_{D_s\,\mu}\, p_{B_s\,\nu} \right) \frac{2\, F_T(q^2)}{m_{B_s} + m_{D_s}}$$

$\Longrightarrow$ 3 form factors : $\underbrace{\underbrace{F_+,\; F_0}_{\text{In SM}} \quad \text{and} \quad F_T}_{\text{NP}}$

Going to the Lattice

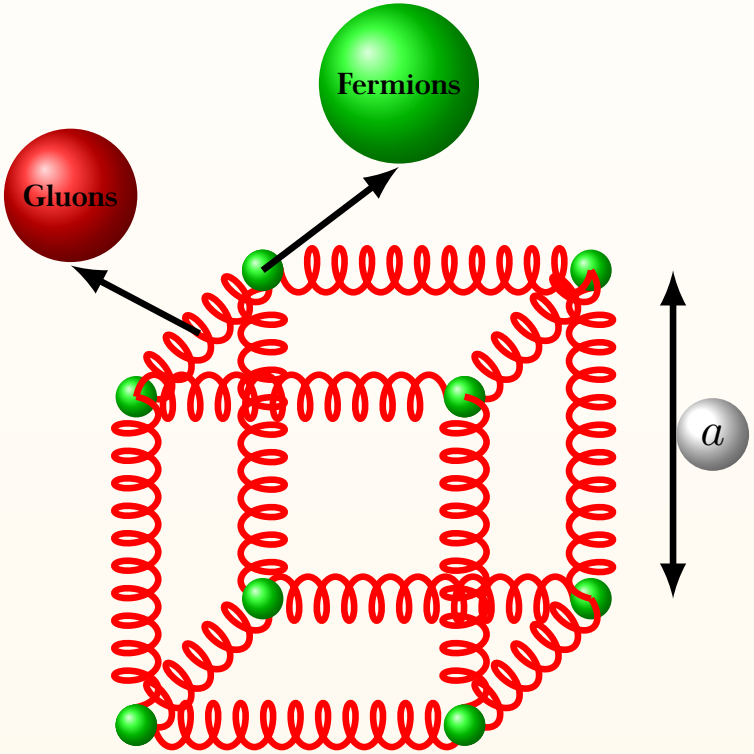
WHY

The only known way to systematically and rigorously solve the quantum theory of strong interactions

INTEREST

No uncontrolled hypothesis, pure QCD!!

Going to the lattice



- Space time discretization (K. Wilson 1974)
- Sufficiently small lattice spacing a

  $a \simeq 0.05 \cdots 0.10$ fm $\Rightarrow$ "continuum physics"
- Hypercubic lattice $L^3 \times T$

  $L \simeq 2.0 \cdots 4.0$ fm (sufficiently large)

  $\Rightarrow$ "no finite size effects"

EXPECTED PROBLEM $(B_s \to D_s \ell \bar{\nu}_\ell)$

Computation of quantities involving heavy quarks (cut-off effects $am_b \gg 1$)

Approach : Interpolation to the physical b mass from the charm region (ratio method)

Atoui *et al.* [arXiv :1310.5238] (2013), P. Dimopoulos*et al.* [ETM Collaboration] [JHEP 1201, 046] (2012)

Simulation setup

- Wilson twisted mass Dirac operator with two degenerate flavors ($N_f = 2$)

$$Q^{(\chi)} = \gamma_\mu D_\mu + m + i\mu\gamma_5 - \frac{a}{2}\Box \quad , \quad m + 4 = \frac{1}{2\kappa}$$

- "Tree-level Symanzik" improved Gauge-action
- 4 lattice spacings $\Rightarrow$ extrapolation in the continuum
- $m_\pi \in [280, 500]$ MeV
- Ten heavy quark masses $m_c = m_h^{(0)}$, $m_b = m_h^{(9)}$

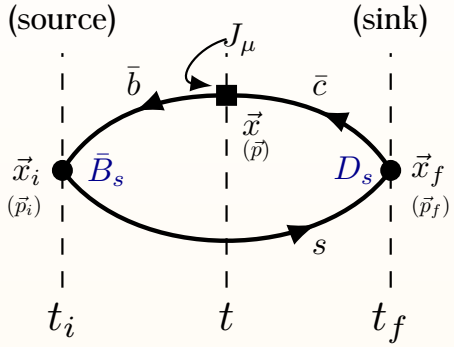
[ETM Collaboration, arXiv :1010.3659, arXiv : 1107.1441]

- Renormalization constants computed non-perturbatively in RI-MOM scheme

Constantinou *et al.* [ETM Collaboration, arXiv :1201.5025]

- Smearing parameters : $\kappa_s = 4$, $N = 30$, $\alpha_{\rm APE} = 0.5$ and $N_{\rm APE} = 20$

Matrix elements extraction



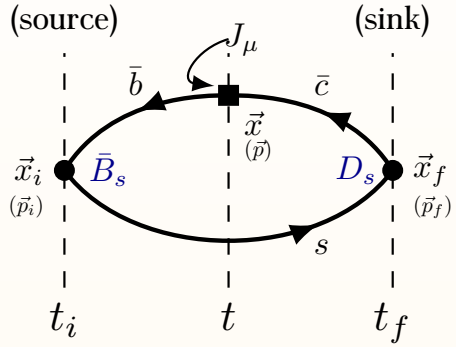
- Kinematics : rest frame of B_s

  $p_{B_s} = (m_{B_s}, \vec{0})$ $\quad p_{D_s} = (E_{D_s}, \vec{p}_{D_s})$ with $\vec{p}_{D_s} = \dfrac{\pi \vec{\theta}_0}{L}$

  (twisted boundary conditions)

  $\longrightarrow w = v_{B_s} \cdot v_{D_s} \in \{1, 1.004, 1.02, 1.04, 1.06\}$

Matrix elements extraction



- Kinematics : rest frame of B_s

$$p_{B_s} = (m_{B_s}, \vec{0}) \quad p_{D_s} = (E_{D_s}, \vec{p}_{D_s}) \text{ with } \vec{p}_{D_s} = \frac{\pi \vec{\theta}_0}{L}$$

(twisted boundary conditions)

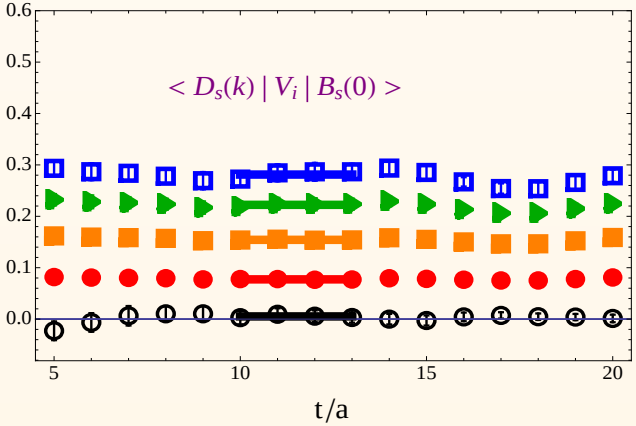
$$\longrightarrow w = v_{B_s} \cdot v_{D_s} \in \{1, 1.004, 1.02, 1.04, 1.06\}$$

- Two- and three- point correlators

$$\boxed{\text{Ratio } \mathscr{R}(t)} = \frac{\mathscr{C}^{(3)}(t, t_i, t_f, \vec{p}_i, \vec{p}_f)}{\mathscr{C}^{(2)}_{(B_s)}(t - t_i, \vec{p}_f) \cdot \mathscr{C}^{(2)}_{(D_s)}(t_f - t, \vec{p}_i)} \cdot \sqrt{\mathscr{Z}_{B_s}} \cdot \sqrt{\mathscr{Z}_{D_s}}$$

$$\xrightarrow[t - t_i \to \infty]{t_f - t \to \infty} \langle D_s(\vec{p}_f) | J_\mu | B_s(\vec{p}_i) \rangle$$

$$\mathscr{Z}_H = |\langle 0 | \mathscr{O}_H | H \rangle|^2 \quad \text{obtained from} \quad \mathscr{C}^{(2)}_H(t) \xrightarrow[t \gg 0]{} \frac{\mathscr{Z}_H}{2M_H} \left[\exp(-M_H\, t) + \exp(M_H\, t)\right]$$



At large time, we observe the stable signal (plateau) which is the desired hadronic matrix element

Extraction of $\mathscr{G}_s(1)$

HQET decomposition

$$\frac{1}{\sqrt{m_{B_s} m_{D_s}}} \left\langle D_s(p_{D_s}) \,\middle|\, V_\mu \,\middle|\, \bar{B}_s(p_{B_s}) \right\rangle = h_+(w)\,(v_{B_s} + v_{D_s})_\mu + (v_{B_s} - v_{D_s})_\mu \, h_-(w)$$

$$\text{with } F_+(q^2) = \frac{m_B + m_D}{\sqrt{4 m_B m_D}} \cdot h_+(w) \left[1 - \frac{m_B - m_D}{m_B + m_D} \frac{h_-(w)}{h_+(w)} \right],$$

$$\text{and } F_0(q^2) = \frac{\sqrt{m_B m_D}}{m_B + m_D} (w+1) \cdot h_+(w) \left[1 - \frac{m_B + m_D}{m_B - m_D} \frac{w-1}{w+1} \frac{h_-(w)}{h_+(w)} \right]$$

- The decay rate depends on $\mathscr{G}_s(w)^{B_s \to D_s}$ (in the limit of vanishing lepton masses)

$$\boxed{\mathscr{G}_s(w) = h_+(w) - \frac{m_{D_s} - m_{B_s}}{m_{D_s} + m_{B_s}} h_-(w)}$$

needed to extract V_{cb}

  - Experiments : $\mathscr{G}_s(w)|V_{cb}|$
  - Lattice QCD provides normalization : the zero recoil point (Isgur-Wise point) $\mathscr{G}_s(1)$

$\mathscr{G}_s(1)$ in the continuum

- Heavy quark masses $(a \cdot m_h \ll 1)$ m_h^i $i \in \{0, \cdots, 9\}$

$$m_c \leq m_h^i \leq m_b \qquad \frac{m_h^{i+1}}{m_h^i} = \lambda = 1.17$$

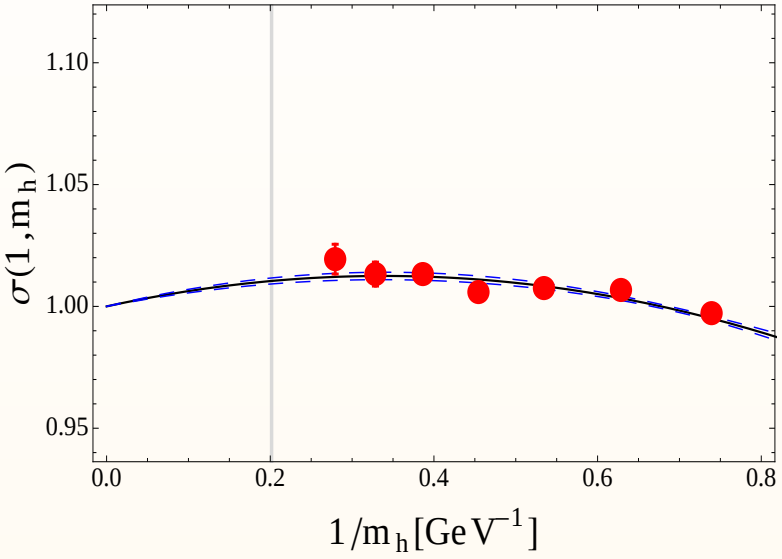
- Determine for each m_h^i the ratio

$$\Sigma_i(a^2, m_l) = \frac{\mathscr{G}_s(1, m_h^{(i+1)})}{\mathscr{G}_s(1, m_h^{(i)})}, \text{ for each}$$

  - gauge ensemble
  - light quark mass

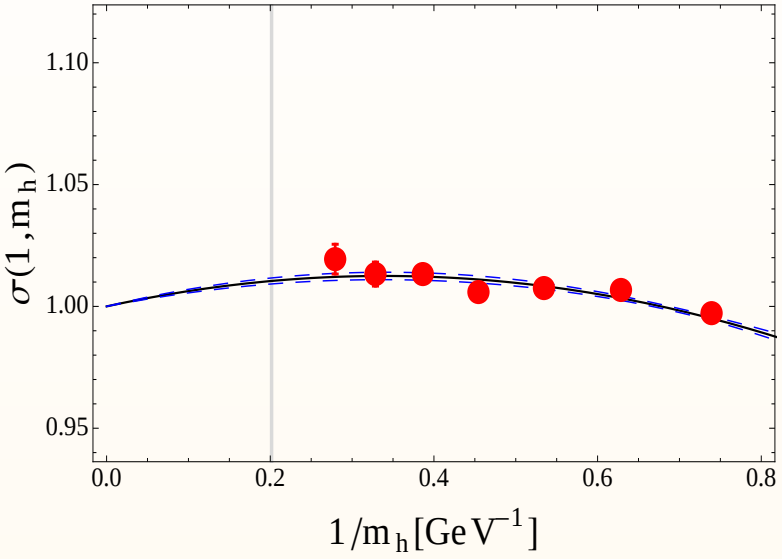
$\Rightarrow$ Extrapolate Σ_i to the continuum $\sigma_i = \lim\limits_{\substack{m_l^{\text{sea}} \to m_{ud} \\ a \to 0}} \Sigma_i$

$$\Sigma_i = \left[\alpha^s + \beta^s \frac{m_l^{\text{sea}}}{m_s} + \gamma^s \frac{a^2}{a_{3.9}^2}\right]$$



$\mathscr{G}_s(1) \xrightarrow{m_h \to \infty}$ constant $\Rightarrow$ fit with

$$\boxed{\sigma(1, m_h) = 1 + \frac{s_1}{m_h} + \frac{s_2}{m_h^2}}$$



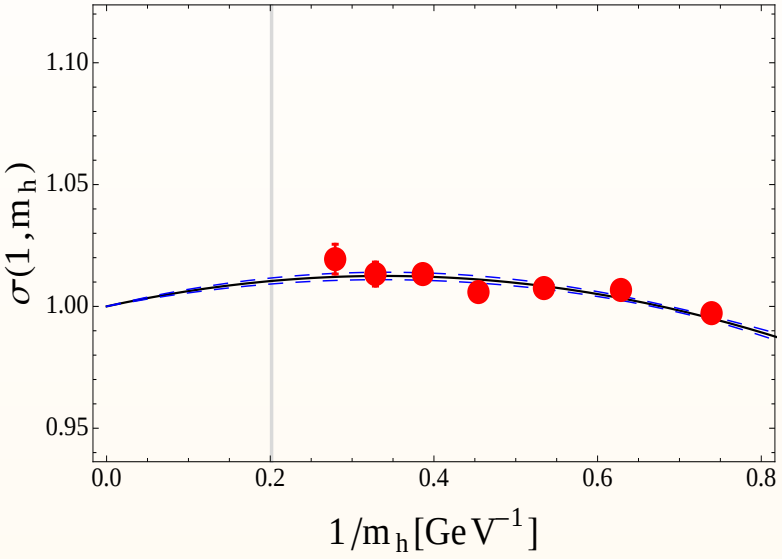
$$\mathscr{G}_s(1) \xrightarrow{m_h \to \infty} \text{constant} \Rightarrow \text{fit with}$$

$$\boxed{\sigma(1, m_h) = 1 + \frac{s_1}{m_h} + \frac{s_2}{m_h^2}}$$

Interpolate to the physical b quark mass

$$\mathscr{G}_s(1, m_b) = \mathscr{G}_s(1, m_h = m_c)\, \sigma_0\, \sigma_1 \cdots \sigma_8 \sigma_9$$

with $[\sigma_i = \sigma(1, m_h^{(i)})]$ and $\mathscr{G}_s(1, m_h = m_c) = 1$ (elastic form factor fixed by ward indendity)



$\mathscr{G}_s(1) \xrightarrow{m_h \to \infty}$ constant $\Rightarrow$ fit with

$$\sigma(1, m_h) = 1 + \frac{s_1}{m_h} + \frac{s_2}{m_h^2}$$

Interpolate to the physical b quark mass

$$\mathscr{G}_s(1, m_b) = \mathscr{G}_s(1, m_h = m_c)\, \sigma_0\, \sigma_1 \cdots \sigma_8 \sigma_9$$

with $[\sigma_i = \sigma(1, m_h^{(i)})]$ and $\mathscr{G}_s(1, m_h = m_c) = 1$ (elastic form factor fixed by ward indendity)

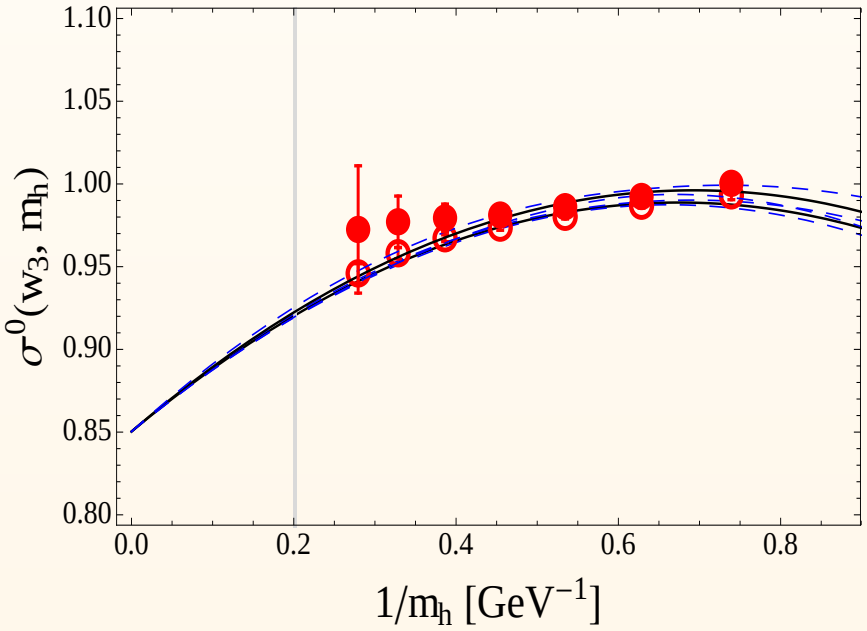
- $\mathscr{G}_s(1) = 1.052(46)$ $\Rightarrow$ *should be used to extract V_{cb} from $B_s \to D_s$ data*
- $\mathscr{G}(1) = 1.079(29)$ $\Rightarrow$ $V_{cb} = (39.48 \pm 1.06_{\text{th.}} \pm 1.39_{\text{exp}}) \times 10^{-3}$ *(from $B \to D$ data)*

$\mathscr{R}_0(q^2) = F_0(q^2)/F_+(q^2)$ *near zero recoil*

Importance

- Discuss the recent discrepancy between the experimentally measured $\mathscr{B}(B \to D\tau\nu_\tau)/\mathscr{B}(B \to D\ell\nu_\ell)$ and its SM prediction
- More significant for the case of the τ-lepton

Strategy : compute the ratio of $\mathscr{R}_0$ at successive m_h^i :



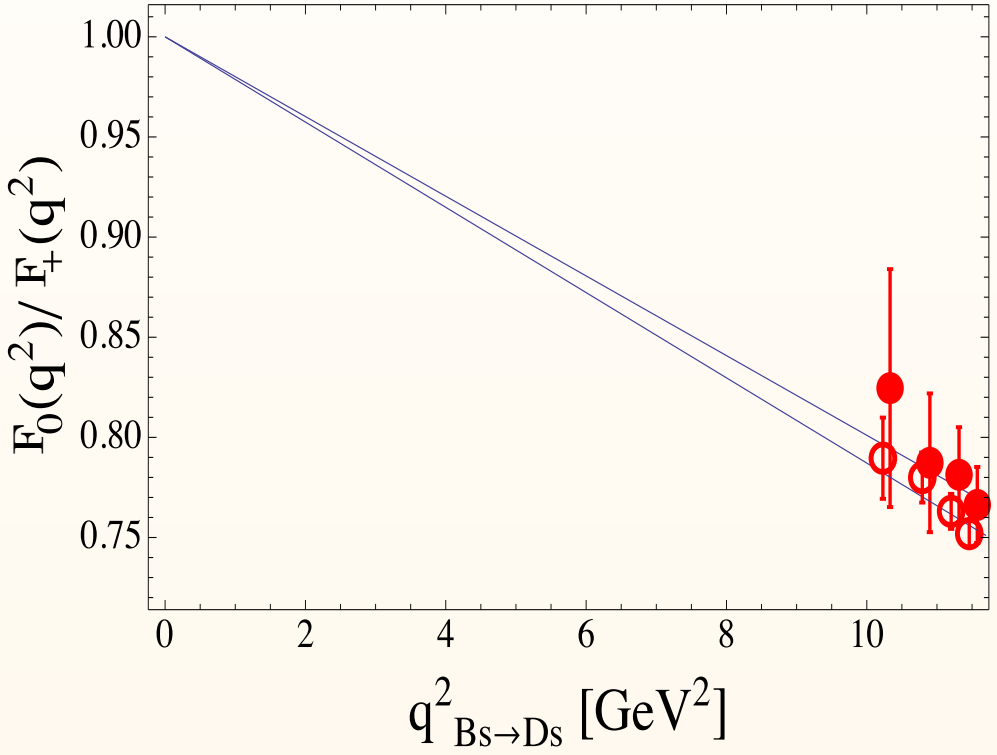
$$\sigma^0(w, m_h) = \frac{1}{\lambda} + \frac{s_1''}{m_h} + \frac{s_2''}{m_h^2}$$

$\mathscr{R}_0(q^2) \propto 1/m_h$ (HQET)

● dependence on m_l^{sea}
○ independent on m_l^{sea}

At defined w and at m_b :

$$\mathscr{R}_0(w) = \mathscr{R}_0(w, m_h^{(i)})\, \sigma^0_{(i)}\, \sigma^0_{(i+1)} \cdots \sigma^0_{(8)} \sigma^0_{(9)}$$



● dependence on m_l^{sea}
○ independent on m_l^{sea}

$F_0(0) = F_+(0) \;\Rightarrow$ fit with $\mathscr{R}_0(q^2) = 1 - \alpha q^2$

$\alpha = 0.021(1)$ consistent with existing results

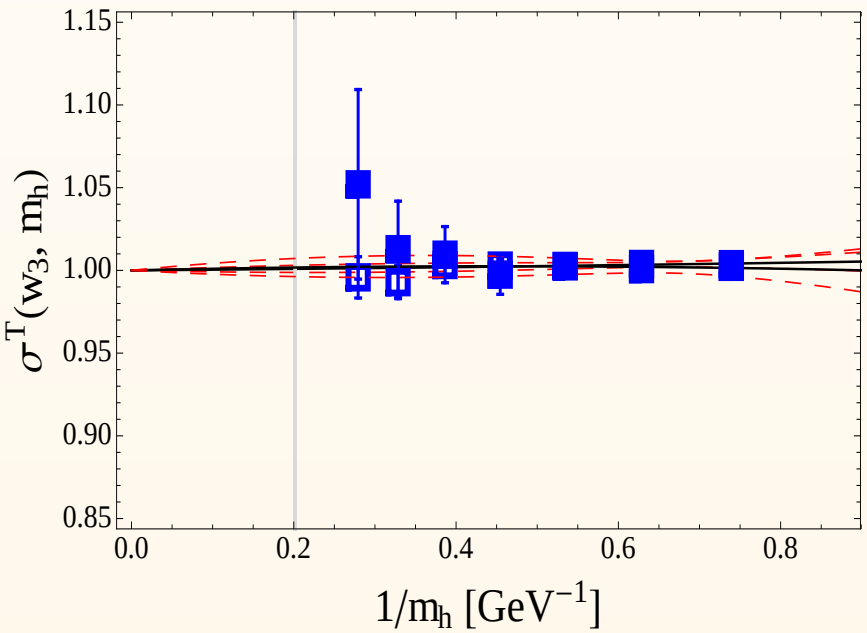
Bailey *et al.* [Phys. Rev. D 85] (2012) de Divitiis *et al.* [Phys. lett. B 655] (2007)

The tensor form factor F_T

Goal : *determine the ratio* $\mathscr{R}_T = \dfrac{F_T}{F_+}$ *near zero recoil*

(F_T is obtained in the $\overline{\text{MS}}$ scheme and at $\mu = m_b$)

Strategy : *compute the ratio of* $\mathscr{R}_T$ *at successive* m_h^i :



$$\sigma^T(w, m_h) = 1 + \frac{s_1'}{m_h} + \frac{s_2'}{m_h^2}$$

$\mathscr{R}_T(q^2) \propto$ constant (HQET)

■ (dependence on m_l^{sea})

□ (not dependent on m_l^{sea})

After the continuum extrapolation, we use $\sigma^T(w)$ to determine :

$$\mathscr{R}_T(w, m_b) = \mathscr{R}_T(w, m_h^{(i)})\, \sigma^T_{(i)}\, \sigma^T_{(i+1)} \cdots \sigma^T_{(8)} \sigma^T_{(9)}$$

w	1.004	1.016	1.036	1.062
$\mathscr{R}_T$	1.076(68)	1.062 (76)	0.975(94)	0.920 (111)

- Quark model predictions : $\mathscr{R}_T(q^2) = 1.03(1)$ (Melikhov *et al.*, Phys. Rev. D **62**, (2000))
- Our LQCD data : Error increases when going to higher w
  - unable to check the flatness of $\mathscr{R}_T(q^2)$
  - $F_T(q^2) \neq 0$ is a signature of NP effects

Outline

1. Semileptonic B decays
2. Lattice calculation of form factors for $B_{(s)} \to D_{(s)} \ell \bar{\nu}_\ell$
3. Conclusions

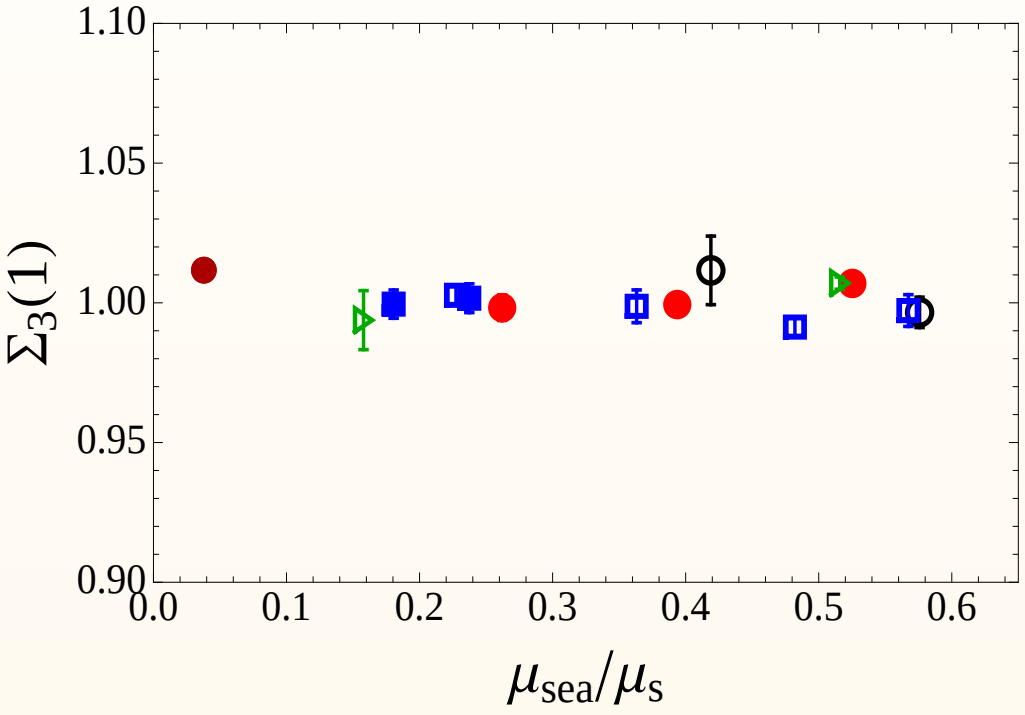
- Dynamical lattice computation of the $\bar{B}_s \to D_s \ell \bar{\nu}$ form factors

  (unquenched QCD + non perturbative renormalization)

  - Computation of $\boxed{\mathscr{G}_s(1) : 1.059(47)}$
  - Study of $\boxed{F_0/F_+}$ and $\boxed{F_T/F_+}$ near the zero recoil limit :
    - First LQCD determination of F_T/F_+
    - Discussion of physics beyond the SM
  - Interpolation to the bottom quark region using ratios of physical quantities
- Follow the same strategy to compute $\mathscr{F}(1)$ from $B_{(s)} \to D^*_{(s)}$ data

Thanks

$$\Sigma_3(1) = \mathscr{G}_s(1, m_h^{(4)})/\mathscr{G}_s(1, m_h^{(3)})$$



○ for $\beta = 3.80$
□ for $\beta = 3.90$ (24^3)
■ for $\beta = 3.90$ (32^3)
● for $\beta = 4.05$
▷ for $\beta = 4.20$
● for $\beta = 4.05$

β	$(L/a,\ T/a)$	$a\mu_\ell$	$a\mu_s$	$a\mu_h$
3.80	(24, 48)	0.0080, 0.0110	0.0194	0.2331, 0.2742, 0.3225, 0.3793, 0.4461, 0.5246, 0.6170, 0.7257, 0.8536, 1.0040
3.90	(32, 64)	0.0030, 0.0040	0.0177	0.2150, 0.2529, 0.2974, 0.3498, 0.4114, 0.4839, 0.5691, 0.6694, 0.7873, 0.9260
	(24, 48)	0.0040, 0.0064 0.0085, 0.0100		
4.05	(32, 64)	0.0030, 0.0080	0.0154	0.1849, 0.2175, 0.2558, 0.3008, 0.3538, 0.4162, 0.4895, 0.5757, 0.6771,0.7963
4.20	(48, 96)	0.0020	0.0129	0.1566, 0.1842, 0.2166, 0.2548, 0.2997, 0.3525, 0.4145, 0.4876, 0.5734,0.6745
	(32, 64)	0.0065		

β	3.80	3.90	4.05	4.20
Z_V	0.5816(2)	0.6103(3)	0.6451(3)	0.686(1)
Z_A	0.746(11)	0.746(6)	0.772(6)	0.780(6)
Z_T	0.73(2)	0.750(9)	0.798(7)	0.822(4)
a (fm)	0.098(3)	0.085(3)	0.067(2)	0.054(1)

$w\ \left(q^2_{B_s\to D_s}\right)$ [GeV^2]	$\mathscr{G}_s(w)_{\beta_s^{(i)}\neq 0}$	$\mathscr{G}_s(w)_{\beta_s^{(i)}=0}$
1. (11.54)	1.052(47)	1.073(17)
1.004 (11.46)	1.052(47)	1.075(16)
1.016 (11.20)	1.029(49)	1.063(15)
1.036 (10.79)	1.044(51)	1.034(17)
1.062 (10.23)	0.986(57)	1.004(20)

$$\frac{\mathscr{G}_s(w)}{\mathscr{G}_s(1)} = \left(\frac{2}{1+w}\right)^{2\rho^2} \qquad \rho^2 = 1.2(8)(\beta_s^i \neq 0) \quad 1.1(3)(\beta_s^i = 0)$$

$$f_T(q^2,\mu) = \frac{c_T(\mu)}{c_T(\mu_0)} f_T(q^2,\mu_0)$$

where,

$$c_T^{\rm lo}(\mu) = a_s(\mu)^{\gamma_0/\beta_0}\,,$$

$$c_T^{\rm nlo}(\mu) = c_T^{\rm lo}(\mu)\left(1 + \frac{\gamma_1\beta_0 - \gamma_0\beta_1}{\beta_0^2} a_s(\mu)\right)\,,$$

$$c_T^{\rm nnlo}(\mu) = c_T^{\rm nlo}(\mu) + \frac{1}{2} c_T^{\rm lo}(\mu)\left[\left(\frac{\gamma_1\beta_0 - \gamma_0\beta_1}{\beta_0^2}\right)^2 + \frac{\gamma_2}{\beta_0} + \frac{\beta_1^2\gamma_0}{\beta_0^3} - \frac{\beta_1\gamma_1 + \beta_2\gamma_0}{\beta_0^2}\right] a_s(\mu)^2\,,$$