BSM Higgs Decoupling Scenarios In Light Of Unitarity Constraints

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(ongoing work with H. E. Logan)
Motivation

- If future measurements of the higgs couplings do not show significant deviations and no new particles are discovered we might be in the decoupling limit of a SM extension.

- Unitarity Constraints provide a model independent link between higgs coupling deviations and the scale below which New Physics must appear.

- In a specific model and it isn’t necessary that the higgs couplings approach SM values as slowly as allowed by unitarity (faster decoupling implies NP shows up earlier).

- Studying scalar, fermion and gauge boson extensions can help us find features that are directly related to NP appearing much earlier vs closer to the bound due to Unitarity.
Motivation

• In addition we can identify models in which a higgs coupling is always enhanced or suppressed or allows for both enhancement and suppression.

• An understanding of these aspects of SM extensions should aid in distinguishing between potential NP models.
Parametrization of couplings

\[ \mathcal{L} \supset k V M^2 V^* V^\mu \left[ 1 + a_V \frac{2h}{v_{SM}} + b_V \frac{h^2}{v_{SM}^2} \right] - m_f \bar{f} f \left[ 1 + c_f \frac{h}{v_{SM}} \right] \]

\[ - \frac{1}{2} M_h h^2 \left[ 1 + d_3 \frac{h}{v_{SM}} + d_4 \frac{h^2}{4v_{SM}^2} \right] \]

\[ a, b, c : \text{multiplicative factors by which the SM couplings are modified} \]
Unitarity Limits

- $2 \rightarrow 2$ scattering involving longitudinal $V$ (W or Z), $f$, $h$

\[ V_L V_L \rightarrow V_L V_L : |1 - a^2| < 8\pi \frac{v^2}{s} \]

\[ V_L V_L \rightarrow hh : |b - a^2| < \frac{16\pi}{\sqrt{3}} \frac{v^2}{s} \]

\[ f \bar{f} \rightarrow V_L V_L : |1 - ac| < \frac{16\pi}{3} \frac{v}{m_f} \frac{v}{\sqrt{s}} \]

- In the above $v = 246$ GeV
- these inequalities determine the scale below which EFT is valid
- and the scale below which NP must appear
Scale below which NP enters

\[ V_L V_L \rightarrow V_L V_L : |1 - a^2| < 8\pi \frac{v^2}{s} \]

\[ V_L V_L \rightarrow hh : |b - a^2| < \frac{16\pi v^2}{\sqrt{3}} \frac{v}{s} \]

\[ f \bar{f} \rightarrow V_L V_L : |1 - ac| < \frac{16\pi v}{3} \frac{v}{m_f \sqrt{s}} \]

<table>
<thead>
<tr>
<th>\sqrt{s} \ (\text{TeV})</th>
<th>0.1</th>
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<tbody>
<tr>
<td></td>
<td>1 - a^2</td>
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<td>1 - ac</td>
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<td>b - a^2</td>
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- large deviations lower the NP scale upper bound
- \( a, b \) are more sensitive to the NP scale upper bound
Unitarity Limits

\[ V_L V_L \rightarrow V_L V_L : |1 - a^2| < 8\pi \frac{v^2}{s} \]

\[ V_L V_L \rightarrow hh : |b - a^2| < \frac{16\pi}{\sqrt{3}} \frac{v^2}{s} \]

\[ f \bar{f} \rightarrow V_L V_L : |1 - ac| < \frac{16\pi}{3} \frac{v}{m_f} \frac{v}{\sqrt{s}} \]

Model Independent Power laws for deviations in couplings

\[ hVV : |1 - a| \approx \mathcal{O}(v^2/s) \]

\[ hhVV : |1 - b| \approx \mathcal{O}(v^2/s) \]

\[ h\bar{f}f : |1 - c| \approx \mathcal{O}(v/\sqrt{s}) \]
“Saturation” of Power Laws

- In models with a decoupling limit we can express coupling deviations in powers of $v/M_{\text{new}}$

**Model X**

$hVV : \ |1 - a| \approx \mathcal{O}(v^2/M_{\text{new}}^2)$

![Graph for Model X](image)

**Model Y**

$hVV : \ |1 - a| \approx \mathcal{O}(v^4/M_{\text{new}}^4)$

![Graph for Model Y](image)

**Unitarity Limit**

$hVV : \ |1 - a| \approx \mathcal{O}(v^2/s)$
Power Laws in Specific Models

• Scalar Sector Extensions
• Fermion Sector Extensions
• Gauge Boson Sector Extensions
Extended Scalar Sector - Type II 2HDM

\[ \nu_{\text{gen}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[ \lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right] (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right\}. \]

J.F. Gunion, H.E. Haber [PRD 67, 075019]

\[ a \approx 1 - \frac{\hat{\lambda}^2 v^4}{2 m_A^4}, \]
\[ b = 1, \]
\[ c \approx 1 + \frac{\hat{\lambda} v^2}{m_A^2} \times \begin{cases} \cot \beta & \text{for up type fermions} \\ -\tan \beta & \text{for down type fermions} \end{cases} \]

- \( a, b \) and \( c \) do not saturate unitarity limit power law

- Are there scalar sector extensions for which this is possible?
Extended Scalar Sector - SM + singlet

- Simplest extension of Higgs sector with interesting phenomenology when mixing is allowed or forbidden

\[ V = \lambda \left( \Phi^+ \Phi - \frac{v^2}{2} \right)^2 + \frac{1}{2} \mu^2 s^2 + \lambda_1 s^4 + \lambda_2 s^2 \left( \Phi^+ \Phi - \frac{v^2}{2} \right) + M_1 s \left( \Phi^+ \Phi - \frac{v^2}{2} \right) + M_2 s^3 \]

- \( s \) does not get a non-zero vev because \( \mu^2 > 0 \)

- Discovered higgs is a mixture of SM doublet higgs field \( \phi \) and a real singlet scalar

\[ h = \phi \cos \theta - s \sin \theta \]
\[ H = \phi \sin \theta + s \cos \theta \]

- Solving for mass eigenstates

\[ M^2_{h,H} = \lambda v^2 + \frac{1}{2} \mu^2 \pm \sqrt{\left( \lambda v^2 - \frac{1}{2} \mu^2 \right)^2 + M_1^2 v^2} \]
Extended Scalar Sector - SM + singlet

- Power laws for coupling modifications
  \[ a = c = \cos \theta \approx 1 - \frac{M_1^2 v^2}{2 \mu^4} \]
  \[ b = \cos^2 \theta \approx 1 - \frac{M_1^2 v^2}{\mu^4} \]

- Decoupling limit where only \( \mu \) is taken large does not saturate \( a, b \) or \( c \)

- Alternatively we could scale \( \mu \) and \( M_1 \)

- \( M_1 \) can scale at most linearly with \( \mu \)
  \[ |M_1|/\sqrt{\mu^2} \lesssim 5.78 \]

- To see this note that
  \[ \lambda = \frac{m_h^2}{v^2} + \frac{M_1^2}{2(\mu^2 - 2m_h^2)} \]

- And it has an upper bound from Unitarity and BFB constraints
  \[ \lambda \in \left( 0, \frac{16\pi}{3} \right) \]
  \[ \lambda_1 \in \left( 0, \frac{4\pi}{3} \right) \]
  \[ \lambda_2 \in \left( -\frac{8\pi}{3}, 8\pi \right) \]
Extended Scalar Sector - SM + singlet

\[ V = \lambda \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 + \frac{1}{2} \mu^2 s^2 + \lambda_1 s^4 + \lambda_2 s^2 \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right) + M_1 s \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right) + M_2 s^3 \]

- When M1 is scaled linearly with \( \mu \): a and b saturate power law
- \( a, b \) and \( c \) are all suppressed compared to SM
- \( c \) does not saturate UL power law

\[ a = c = \cos \theta \approx 1 - \frac{M_1^2 v^2}{2 \mu^4} \]

\[ b = \cos^2 \theta \approx 1 - \frac{M_1^2 v^2}{\mu^4} \]

- Are there scalar sector extensions where \( a \) and \( b \) are not necessarily suppressed or where \( c \) saturates UL power law?
Extended Scalar Sector - Georgi-Machacek

- Proposed in 1985 as a possible scenario for EWSB
- As we will see this model can generate $a, b > 1$
- Contains the SM doublet along with a complex triplet ($Y=2$) and a real triplet ($Y=0$) arranged so as to preserve custodial SU(2)

$$\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ -\phi^{++} & \phi^0 \end{pmatrix} \quad X = \begin{pmatrix} \chi^0 & \xi^+ & \chi^{++} \\ -\chi^{++} & \xi^0 & \chi^+ \\ \chi^{+++} & -\xi^{++} & \chi^0 \end{pmatrix}$$

$$\langle \Phi \rangle = \frac{v_\phi}{\sqrt{2}} 1_{2\times2} \quad \langle X \rangle = v_\chi 1_{3\times3}$$

- The scalar vevs are constrained from the $W$ boson mass

$$v^2 = v_\phi^2 + 8v_\chi^2$$
Extended Scalar Sector - Georgi-Machacek

- Most general custodial SU(2) preserving potential

\[
V(\Phi, X) = \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) \\
+ \lambda_3 \text{Tr}(X^\dagger XX^\dagger X) + \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b) \\
- M_1 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) (UXU^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t^a X t^b) (UXU^\dagger)_{ab}.
\]

Hartling, KK, Logan[arXiv: 1404.2640]; Aoki, Kanemura [PRD 77,095009]; Chiang, Yagyu [JHEP 1301, 026]

- Has a decoupling limit as \( \mu_3 \) is made large
- All new scalar masses get large with \( \mu_3 \), \( v_\phi \) approaches SM vev, \( v_\chi \) tends to zero

Figs from: Hartling, KK, Logan[arXiv: 1404.2640]
Extended Scalar Sector - Georgi-Machacek

- $a,b$ and $c$ do not saturate power laws
- If $M_1$ is scaled linearly with $\mu_3$, $a$ and $b$ saturate power laws
- Reason for $M_1$ increasing at most linearly is similar to SM + singlet case
- $\lambda_1$ increases with increase in $M_1/\mu_3$ and is bounded from unitarity

$$\lambda_1 \approx \frac{m_h^2}{8v^2} + \frac{3}{32} \frac{M_1^2}{\mu_3^2} \quad \lambda_1 \in \left( -\frac{1}{3}\pi, \frac{1}{3}\pi \right) \approx (-1.05, 1.05)$$

- Therefore $|M_1|/\sqrt{\mu_3^2} \lesssim 3.3$

\[ a = \cos \theta \frac{v_\phi}{v} - \frac{8}{\sqrt{3}} \sin \theta \frac{v_\chi}{v} \approx 1 + \frac{3}{8} \frac{M_1^2 v^2}{\mu_3^4} \]

\[ b = \cos^2 \theta + \frac{8}{3} \sin^2 \theta \approx \left( 1 + \frac{5}{4} \frac{M_1^2 v^2}{\mu_3^4} \right) \]

\[ c = \cos \theta \frac{v}{v_\phi} \approx 1 - \frac{1}{8} \frac{M_1^2 v^2}{\mu_3^4} \]
Extended Scalar Sector - Georgi-Machacek

- \( a \) and \( b \) saturate power laws
- \( a \) and \( b \) are \textit{enhanced} compared to SM
- \( c \) does not saturate power laws

\[
a = \cos \theta \frac{v_\phi}{v} - \frac{8}{\sqrt{3}} \sin \theta \frac{v_\chi}{v} \approx 1 + \frac{3}{8} \frac{M_1^2 v^2}{\mu_3^4}
\]

\[
b = \cos^2 \theta + \frac{8}{3} \sin^2 \theta \approx \left( 1 + \frac{5}{4} \frac{M_1^2 v^2}{\mu_3^4} \right)
\]

\[
c = \cos \theta \frac{v}{v_\phi} \approx 1 - \frac{1}{8} \frac{M_1^2 v^2}{\mu_3^4}
\]

- A feature that leads to saturation of \( a \) and \( b \) in scalar sector extensions is the presence of a dimensionful parameter (e.g. \( M_1 \)) that is the coefficient of a term trilinear in scalar fields

\[
V = \lambda \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 + \frac{1}{2} \mu^2 s^2 + \lambda_1 s^4 + \lambda_2 s^2 \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right) + M_1 s \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right) + M_2 s^3
\]

- Do fermion extensions saturate the power law for \( c \)?
Extended Fermion Sector

- Adding new vector-like top partners: $\chi_{L,R} \sim (3, 1)_{2/3}$

  eg: KK, R. V.-Morales, F. Yu [PRD 86,113002]

- Mass terms for top and top partners:

  $$\mathcal{L} \supset -y_t \tilde{H} \tilde{Q}_L t_R - y_L \tilde{H} \tilde{Q}_L \chi_R - M \tilde{\chi}_L \chi_R + \text{h.c.}$$

- In the $(t, \chi)$ basis the (non-diagonal) mass and interaction matrices are

  $$\hat{M} = \begin{pmatrix} m & \xi_L \\ 0 & M \end{pmatrix} \quad \hat{N}_h = \begin{pmatrix} m & \xi_L \\ 0 & 0 \end{pmatrix}$$

  where $\xi_L = \frac{y_L v}{\sqrt{2}} \quad m = \frac{y_t v}{\sqrt{2}}$
Extended Fermion Sector

- After switching to basis of mass eigenstates $\mathbf{t} \equiv (t_1, t_2)$

\[
\mathcal{L} \supset -\bar{t} \left( \hat{M}_D + \frac{h}{v} \hat{V}_h \right) P_R \mathbf{t} + \text{h.c.}
\]

\[
\hat{M}_D = \hat{L} \hat{M} \hat{R}^\dagger \\
\hat{V}_h = \hat{L} \hat{N}_h \hat{R}^\dagger
\]

- The 1,1 entry of $\hat{V}_h$ gives us the $h tt \bar{t}$ coupling

- This model has a decoupling limit as $M$ is made large

- Assuming $\frac{\xi_L}{M}$, $\frac{m}{M} \ll 1$

\[
c = \frac{m}{m_t} \left( 1 - \frac{3}{2} \frac{\xi_L^2}{M^2} \right) \approx 1 - \frac{y_L^2}{2} \left( \frac{v^2}{M^2} \right)
\]

- while $a = b = 1$

- Doesn’t saturate power law for $c$ as well
**Extended Gauge Boson Sector**

- For completeness we consider the effect of an extended gauge boson sector

\[
\begin{align*}
SU(2)_1 \times SU(2)_2 \times U(1)_Y & \quad \rightarrow \quad SU(2)_W \times U(1)_Y \\
SU(2)_1 \times SU(2)_2 \times U(1)_Y & \quad \rightarrow \quad U(1)_{em}
\end{align*}
\]

\[\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v_\phi + \phi^0 + i\phi^0) \end{pmatrix}, \quad \Delta = \begin{pmatrix} \eta^0 \chi^+ \\ \eta^- \chi^0 \end{pmatrix} = \langle \Delta \rangle + \begin{pmatrix} \frac{1}{\sqrt{2}}(\eta^0_r + i\eta^0_i) \quad \chi^+ \\ \eta^- \quad \frac{1}{\sqrt{2}}(\chi^0_r + i\chi^0_i) \end{pmatrix}, \quad \frac{v_\Delta}{2} \text{ diag } (1, 1).\]

- Most general CP conserving potential with the Z\(_2\) symmetry \(\Delta \leftrightarrow \tilde{\Delta}\)

\[
V = m^2_\Phi \Phi^\dagger \Phi + \frac{\lambda_\Phi}{2} (\Phi^\dagger \Phi)^2 + (m^2_\Delta + \lambda_0 \Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \frac{\lambda_\Delta}{2} [\text{Tr}(\Delta^\dagger \Delta)]^2 - \frac{\tilde{\lambda}}{2} \left|\text{Tr}(\Delta^\dagger \tilde{\Delta})\right|^2 \\
- \left[\frac{\tilde{\lambda}'}{4} \left(\text{Tr}(\Delta^\dagger \Delta)\right)^2 + \text{h.c.}\right]
\]
Extended Gauge Boson Sector

- Of the 12 fields in the bi-doublet and doublet: 6 are “eaten” by gauge bosons
- Of the remaining 6 there are 4 odd under $Z_2$: $H^0, A^0, H^\pm$
- And 2 are even under $Z_2$: $h^0, H'^0$
- Decoupling limit corresponds to making $v_\Delta$ large
- In terms of the original doublet and bi-doublet fields

  \[ h^0 = \phi_r^0 \cos \alpha_h - \frac{1}{\sqrt{2}} (\chi_r^0 + \eta_r^0) \sin \alpha_h \]

  \[ H'^0 = \phi_r^0 \sin \alpha_h + \frac{1}{\sqrt{2}} (\chi_r^0 + \eta_r^0) \cos \alpha_h \]

- In general there is an effect on $a,b,c$ from mixing between the two $Z_2$ even scalars
- Since we want to isolate the effect of the additional bosons we choose this mixing angle $\alpha_h$ to be zero
### Extended Gauge Boson Sector

**Calculating** $a_W, a_Z, b_W, b_Z$

- Generated by the scalar kinetic terms: $(D_\mu \Phi)^\dagger D_\mu \Phi + \text{Tr} [(D_\mu \Delta)^\dagger D_\mu \Delta]$
- where $D_\mu = \partial_\mu - ig_Y Y B_\mu - ig_1 \vec{T}_1 \cdot \vec{W}_{1\mu} - ig_2 \vec{T}_2 \cdot \vec{W}_{2\mu}$
- Defining $t_{\theta_0} = g_1/g_2$

<table>
<thead>
<tr>
<th>$a_W$</th>
<th>$a_Z$</th>
<th>$b_W$</th>
<th>$b_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 + \frac{v^2}{v_\Delta^2} s_{\theta_0}^2$</td>
<td>$1 + \frac{v^2}{v_\Delta^2} s_{\theta_0}^2 (1 + t_{\theta_0}^2 c_{\theta_0}^2 (1 - 2t_{\theta_0} s_{\theta_0}^2))$</td>
<td>$1$</td>
<td>$1 + \frac{v^2}{v_\Delta^2} t_{\theta_0}^2 s_{\theta_0}^2 c_{\theta_0}^2 (1 - 2t_{\theta_0} s_{\theta_0}^2)$</td>
</tr>
</tbody>
</table>

- $a_W$ can be enhanced like in GM model while couplings to Z may be enhanced or suppressed depending on $t_{\theta_0}$
- The fermion coupling modification again does not saturate unitarity power law

$$c = \frac{v}{v_\phi} = 1 - \frac{v^2}{v_\Delta^2} s_{\theta_0}^2$$

- Always suppressed
Results

• Extended Scalar Sectors (SM+s, GM) can saturate $a$ and $b$ power laws when there is a trilinear term whose dimensionful coefficient can be made large
• Further discrimination is possible based on whether $a$ and $b$ can be enhanced in a model (e.g. Georgi-Machacek)
• None of the extensions (to our knowledge) saturate the power the law for $c$
• Gauge Boson Sector extensions that enhance $a$ and $b$ are possible as well