The decoupling limit of the Georgi-Machacek model

Katy Hartling

Department of Physics
Carleton University
Ottawa, Ontario

khally@physics.carleton.ca

Phenomenology Symposium, May 5-7, 2014

Katy Hartling, Kunal Kumar, and Heather E. Logan.
arXiv:1404.2640v2 [hep-ph]

The Georgi-Machacek model

- The Georgi-Machacek model extends the Standard Model (SM) Higgs sector by two scalar isopspin triplets.[1]
- Provides an interesting and valuable benchmark for:
  - **BSM scalar searches**:
    Contains a doubly charged scalar state.
  - **Studies of Higgs properties**:
    Allows for hVV couplings that are larger than those of the SM, unlike many other Higgs-sector extensions.
- Has been further extended with scalar doublets and incorporated into Little Higgs and SUSY.

[1] H. Georgi and M. Machacek, Nucl. Phys. B **262**, 463 (1985)

The Georgi-Machacek model

- The Georgi-Machacek model contains one doublet and two triplets,

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^{+} \\ -\phi^{+*} & \phi^{0} \end{pmatrix} \qquad \chi = \begin{pmatrix} \chi^{0*} & \xi^{+} & \chi^{++} \\ -\chi^{+*} & \xi^{0} & \chi^{+} \\ \chi^{++*} & -\xi^{+*} & \chi^{0} \end{pmatrix}$$

  with $SU(2)_L \times SU(2)_R$ symmetry, and where $s^{Q*} = (-1)^Q s^{-Q}$.

- $\langle \Phi \rangle = v_\phi/\sqrt{2}$ and $\langle X \rangle = v_\chi$, where $v_{\mathrm{SM}}^2 = v_\phi^2 + 8 v_\chi^2 = (246\,\mathrm{GeV})^2$.

- Scalar mixing yields a physical fiveplet, triplet, and two singlets:

$$\begin{pmatrix} H_5^{++} & H_5^{+} & H_5^{0} & H_5^{-} & H_5^{--} \end{pmatrix}^T, \begin{pmatrix} H_3^{+} & H_3^{0} & H_3^{-} \end{pmatrix}^T, H, h$$

  - Multiplets have identical masses m_5, m_3, m_H, and m_h.
  - Mixing angle α controls how much h comes from the triplets.
  - Ratio v_χ/v controls how much H_3, G come from the triplets.

The scalar potential

$$V(\Phi, X) = \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X)$$
$$+ \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b)$$

Original potential[2]

- Imposed additional Z_2 symmetry for simplicity.
- All scalar masses can be written as $m_i^2 = \lambda_i v^2$.
- Unitarity bounds require $m_i < 700$ GeV $\Rightarrow$ No decoupling limit![3]

Most general potential

- Two additional terms parametrized by M_1, M_2.

[2] M. S. Chanowitz and M. Golden, Phys. Lett. B **165**, 105 (1985).
[3] M. Aoki and S. Kanemura, Phys. Rev. D **77**, 095009 (2008) [arXiv: 0712.4053]

The scalar potential

$$
\begin{aligned}
V(\Phi, X) = & \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X) \\
& + \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b) \\
& - M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}
\end{aligned}
$$

Original potential[2]

- Imposed additional Z_2 symmetry for simplicity.
- All scalar masses can be written as $m_i^2 = \lambda_i v^2$.
- Unitarity bounds require $m_i < 700$ GeV $\Rightarrow$ No decoupling limit![3]

Most general potential

- Two additional terms parametrized by M_1, M_2.

[2] M. S. Chanowitz and M. Golden, Phys. Lett. B **165**, 105 (1985).

[3] M. Aoki and S. Kanemura, Phys. Rev. D **77**, 095009 (2008) [arXiv: 0712.4053]

The scalar potential

$$
\begin{aligned}
V(\Phi, X) = & \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X) \\
& + \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b) \\
& - M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}
\end{aligned}
$$

Original potential

- Imposed additional Z_2 symmetry for simplicity.
- All scalar masses can be written as $m_i^2 = \lambda_i v^2$.
- Unitarity bounds require $m_i < 700$ GeV $\Rightarrow$ No decoupling limit!

Most general potential

- Two additional terms parametrized by M_1, M_2.

Motivation

What effect does this have on the decoupling limit?

The scalar potential

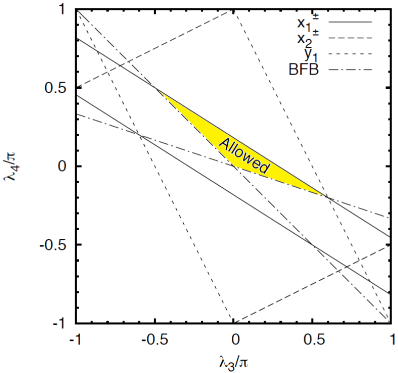
$$V(\Phi, X) = \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X)$$
$$+ \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b)$$
$$- M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}$$

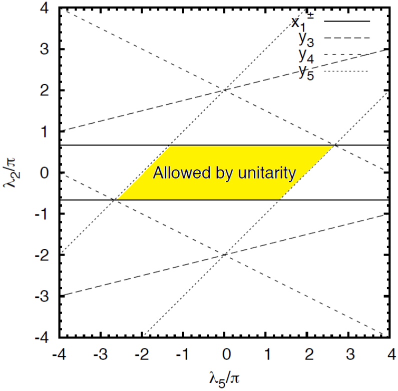
Constraint	Sets parameter
$v = 246$ GeV	v_ϕ
$\partial V/\partial v_\chi = 0$	v_χ
$\partial V/\partial v_\phi = 0$	μ_2
$m_h = 125$ GeV	λ_1

Free parameters: λ_2, λ_3, λ_4, λ_5, M_1, M_2 and mass scale μ_3.

Theoretical constraints

Requiring unitarity and V bounded-from-below constrains λ_i:





Also considered constraints to avoid alternative minima.

- Desired vacuum is a global minimum if λ_3, λ_5, M_1 and M_2 are > 0.
- Otherwise must be checked numerically.

The decoupling limit

Decoupling limit

Does the GM model approach an effective SM-like theory as $\mu_3 \to \infty$?

- Derive expansions of masses and couplings to leading-order in μ_3^{-1}.
- M_i can scale with μ_3^n, but unitarity bound on λ_1 constrains $n \leq 1$.

$$\lambda_1 \approx \frac{1}{8}\left[\frac{m_h^2}{v^2} + \frac{3}{4}\frac{M_1^2}{\mu_3^2}\left(1 - 3(2\lambda_2 - \lambda_5)\frac{v^2}{\mu_3^2} + \frac{3M_1M_2v^2}{\mu_3^4} + \frac{5m_h^2}{3\mu_3^2}\right)\right],$$

- Plot masses and couplings using two test cases A and B:

Case	λ_i	M_i
A	0.1	100 GeV
B	0.1	$\mu_3/3$

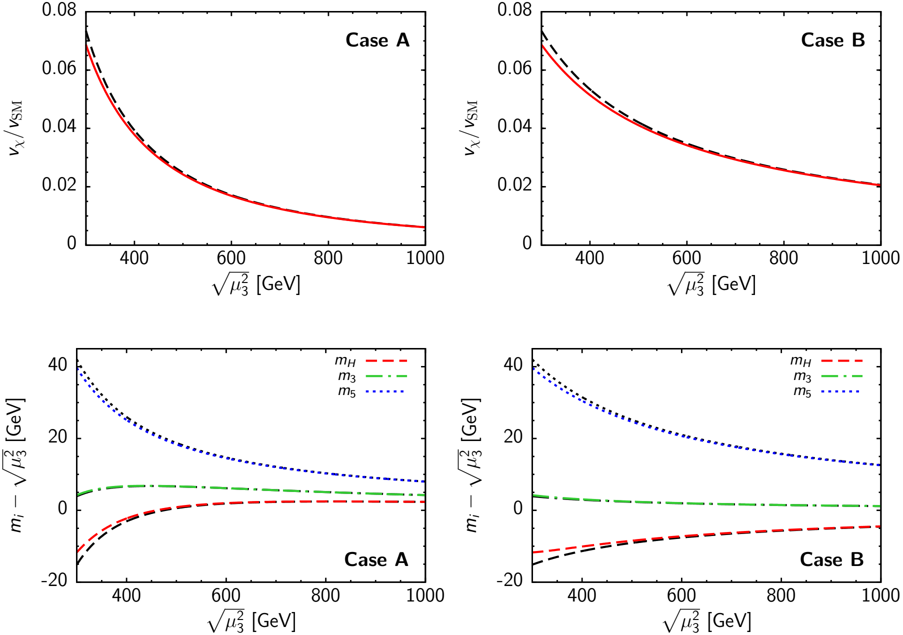
The decoupling limit: vevs, masses, and mixing angles

Decoupling limit

Yes! The most general GM does have a decoupling limit.

Quantity	Case A	Case B
$\frac{m_{H,3,5}}{\mu_3} - 1$	${\mu_3}^{-2}$	${\mu_3}^{-2}$
v_χ	${\mu_3}^{-2}$	${\mu_3}^{-1}$
$\sin\alpha$	${\mu_3}^{-2}$	${\mu_3}^{-1}$

- BSM scalar masses become degenerate and approach $\mu_3 \approx M_{\mathrm{heavy}}$ with ${\mu_3}^{-2}$, regardless of case.
- Triplet vev $v_\chi \to 0$ implies $G \to G_{\mathrm{SM}}$ and H_3 are all triplet.
- Mixing angle α controls the composition of the light Higgs, therefore $\sin\alpha \to 0$ implies $h \to h_{\mathrm{SM}}$.


Case A
Case B
v_χ / v_{SM}
$\sqrt{\mu_3^2}$ [GeV]
$m_i - \sqrt{\mu_3^2}$ [GeV]
m_H
m_3
m_5
0.08
0.06
0.04
0.02
0
400
600
800
1000
40
20
-20

The decoupling limit: Higgs couplings

Coupling	Quantity	Case A	Case B	2HDM[4]
hVV	$\kappa_V - 1$	μ_3^{-4}	μ_3^{-2}	M_A^{-4}
$hf\bar{f}$	$\kappa_f - 1$	μ_3^{-4}	μ_3^{-2}	M_A^{-2}
hhh	$g_{hhh}/g_{hhh}^{\rm SM} - 1$	μ_3^{-4}	μ_3^{-2}	M_A^{-2}
$h\gamma\gamma$	$\Delta\kappa_\gamma$	μ_3^{-2}	μ_3^{-2}	M_A^{-2}
$hZ\gamma$	$\Delta\kappa_{Z\gamma}$	μ_3^{-2}	μ_3^{-2}	M_A^{-2}

- All couplings may deviate from the SM by $\mathcal{O}(v^2/M_{\rm new}^2)$, up to 10%.

$$\kappa_V \simeq 1 + \frac{3}{8}\frac{M_1^2 v^2}{\mu_3^4} \qquad \kappa_f \approx 1 - \frac{1}{8}\frac{M_1^2 v^2}{\mu_3^4}$$

- hVV may be larger than in the SM, unlike the 2HDM.

[4] J. F. Gunion and H. E. Haber, Phys. Rev. D **67** 075019 (2003) [hep-ph/0207010]

The decoupling limit: Higgs couplings

Coupling	Quantity	Case A	Case B	2HDM[5]
hVV	$\kappa_V - 1$	μ_3^{-4}	μ_3^{-2}	M_A^{-4}
$hf\bar{f}$	$\kappa_f - 1$	μ_3^{-4}	μ_3^{-2}	M_A^{-2}
hhh	$g_{hhh}/g_{hhh}^{\rm SM} - 1$	μ_3^{-4}	μ_3^{-2}	M_A^{-2}
$h\gamma\gamma$	$\Delta\kappa_\gamma$	μ_3^{-2}	μ_3^{-2}	M_A^{-2}
$hZ\gamma$	$\Delta\kappa_{Z\gamma}$	μ_3^{-2}	μ_3^{-2}	M_A^{-2}

- All couplings may deviate from the SM by $\mathcal{O}(v^2/M_{\rm new}^2)$, up to 10%.

$$\kappa_V \simeq 1 + \frac{3}{8}\frac{M_1^2 v^2}{\mu_3^4} \qquad \kappa_f \approx 1 - \frac{1}{8}\frac{M_1^2 v^2}{\mu_3^4}$$

- hVV may be larger than in the SM, unlike the 2HDM.

[5] J. F. Gunion and H. E. Haber, Phys. Rev. D **67** 075019 (2003) [hep-ph/0207010]

The decoupling limit: Higgs couplings

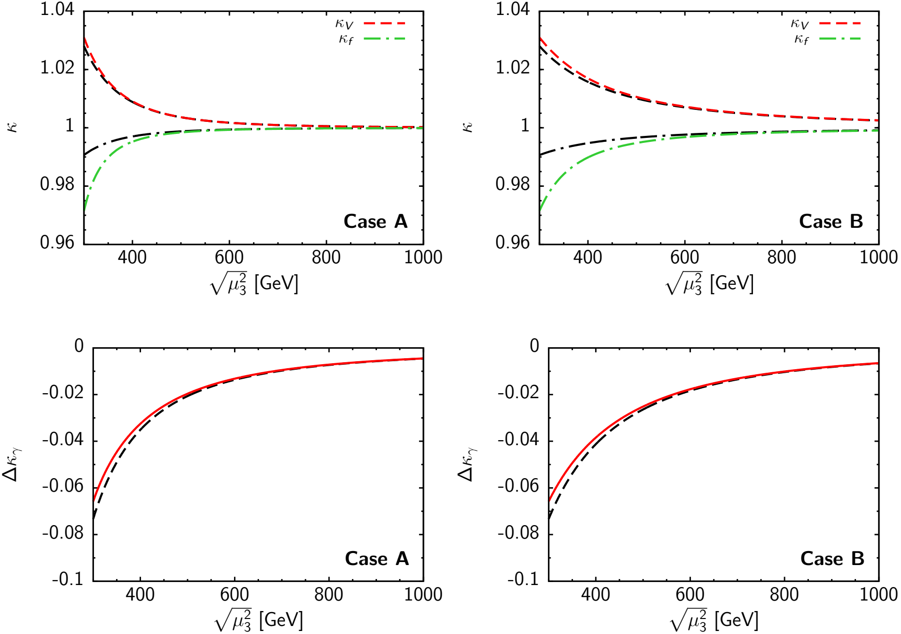
Coupling	Quantity	Case A	Case B	2HDM[4]
hVV	$\kappa_v - 1$	μ_3^{-4}	μ_3^{-2}	M_A^{-4}
$hf\bar{f}$	$\kappa_f - 1$	μ_3^{-4}	μ_3^{-2}	M_A^{-2}
hhh	$g_{hhh}/g_{hhh}^{\mathrm{SM}} - 1$	μ_3^{-4}	μ_3^{-2}	M_A^{-2}
$h\gamma\gamma$	$\Delta\kappa_\gamma$	μ_3^{-2}	μ_3^{-2}	M_A^{-2}
$hZ\gamma$	$\Delta\kappa_{Z\gamma}$	μ_3^{-2}	μ_3^{-2}	M_A^{-2}

- All couplings may deviate from the SM by $\mathcal{O}(v^2/M_{\mathrm{new}}^2)$, up to 10%.

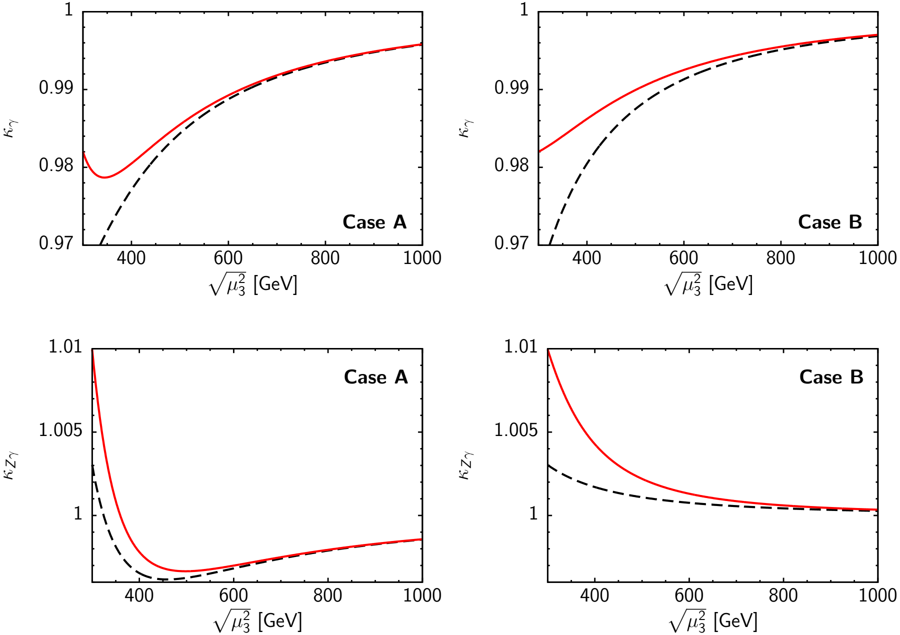
$$\kappa_V \simeq 1 + \frac{3}{8}\frac{M_1^2 v^2}{\mu_3^4} \qquad \kappa_V^{\mathrm{2HDM}} \simeq 1 - \frac{\hat{\lambda}^2 v^4}{2\, m_A^4}$$

- hVV may be larger than in the SM, unlike the 2HDM.

[4] J. F. Gunion and H. E. Haber, Phys. Rev. D **67** 075019 (2003) [hep-ph/0207010]



κ_V
κ_f
κ
1.04
1.02
1
0.98
0.96
400
600
800
1000
$\sqrt{\mu_3^2}$ [GeV]
Case A
Case B
$\Delta\kappa_\gamma$
0
-0.02
-0.04
-0.06
-0.08
-0.1



1
0.99
0.98
0.97
κ_γ
400
600
800
1000
$\sqrt{\mu_3^2}$ [GeV]
Case A
Case B
1.01
1.005
$\kappa_{Z\gamma}$

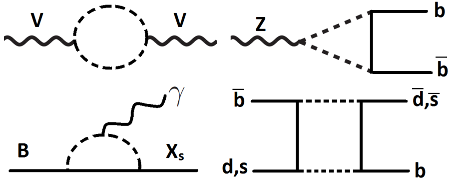
Summary and future work

Summary:

- The most general Georgi-Machacek model has a decoupling limit.
- In general the decoupling is faster in Case A than Case B.
- Couplings may deviate from the SM by $\mathcal{O}(v^2/M^2_{\rm new})$, up to 10%.
- A precision measurement of the hVV coupling may distinguish between the GM and 2HDM models.

Future work: Consider constraints from experimental measurements:

- Oblique parameters
- $R_b = \dfrac{\mathrm{BR(Z \to b\bar{b})}}{\mathrm{BR(Z \to hadrons)}}$
- $B \to X_s\gamma$
- $\bar{B}^0 - B^0$ meson mixing



Thank you!

Thanks also to my supervisor, Heather Logan, and to Kunal Kumar.

This work has been supported by Carleton University, NSERC, and OGS.

Extra slides

Scalar potential

$$
\begin{aligned}
V(\Phi, X) =& \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X) \\
&+ \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b) \\
&- M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}
\end{aligned}
$$

Here $\tau^a = \sigma^a/2$ with σ^a being the Pauli matrices, and

$$
t^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad t^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},
$$

The matrix U rotates X into the Cartesian basis, and is given by

$$
U = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}.
$$

Particle mixing

Goldstone boson, triplet and fiveplet states ($c_H \equiv \cos\theta_H = \frac{v_\phi}{v}$):

$$G^+ = c_H \phi^+ + s_H \frac{(\chi^+ + \xi^+)}{\sqrt{2}}$$

$$G^0 = c_H \phi^{0,i} + s_H \chi^{0,i}$$

$$H_3^+ = -s_H \phi^+ + c_H \frac{(\chi^+ + \xi^+)}{\sqrt{2}}$$

$$H_3^0 = -s_H \phi^{0,i} + c_H \chi^{0,i}$$

$$H_5^{++} = \chi^{++}$$

$$H_5^+ = \frac{(\chi^+ - \xi^+)}{\sqrt{2}}$$

$$H_5^0 = \sqrt{\frac{2}{3}}\xi^0 - \sqrt{\frac{1}{3}}\chi^{0,r}$$

The two custodial SU(2) singlets:

$$H_1^0 = \phi^{0,r} \qquad H_1^{0\prime} = \sqrt{\frac{1}{3}}\xi^0 + \sqrt{\frac{2}{3}}\chi^{0,r}.$$

These states mix by an angle α to form the singlets h and H:

$$h = \cos\alpha\, H_1^0 - \sin\alpha\, H_1^{0\prime} \qquad H = \sin\alpha\, H_1^0 + \cos\alpha\, H_1^{0\prime}$$

Theoretical constraints

Requiring unitarity and V bounded-from-below constrains λ_i:

$$\begin{aligned}
\lambda_1 &\in \left(0, \frac{1}{3}\pi\right) \simeq (0, 1.05) && \text{(Unitarity, BFB)} \\
\lambda_2 &\in \left(-\frac{2}{3}\pi, \frac{2}{3}\pi\right) \simeq (-2.09, 2.09) && \text{(Unitarity)} \\
\lambda_3 &\in \left(-\frac{1}{2}\pi, \frac{3}{5}\pi\right) \simeq (-1.57, 1.88) && \text{(Unitarity, BFB)} \\
\lambda_4 &\in \left(-\frac{1}{5}\pi, \frac{1}{2}\pi\right) \simeq (-0.628, 1.57) && \text{(Unitarity, BFB)} \\
\lambda_5 &\in \left(-\frac{8}{3}\pi, \frac{8}{3}\pi\right) \simeq (-8.38, 8.38) && \text{(Unitarity, BFB)} \\
|M_1|/\sqrt{\mu_3^2} &\lesssim 3.3 && (\lambda_1 \text{ Unitarity}) \\
|M_2|/\sqrt{\mu_3^2} &\lesssim 1.2 && (8\, v_\chi^2 \leq v^2) \qquad (1)
\end{aligned}$$

Desired vacuum is a global minimum if λ_3, λ_5, M_1 and M_2 are > 0.

Decoupling expansions

Vacuum expectation values:

$$v_\chi \approx \frac{M_1 v^2}{4\mu_3^2}\left(1-(2\lambda_2-\lambda_5)\frac{v^2}{\mu_3^2}+\frac{M_1(3M_2-M_1)v^2}{2\mu_3^4}\right)$$

$$v_\phi \approx v\left(1-\frac{M_1^2 v^2}{4\mu_3^4}\right)$$

Masses:

$$m_H \approx \mu_3\left[1+(2\lambda_2-\lambda_5)\frac{v^2}{2\mu_3^2}+\frac{3M_1(M_1-4M_2)v^2}{8\mu_3^4}\right]$$

$$m_3 \approx \mu_3\left[1+\left(2\lambda_2-\frac{\lambda_5}{2}\right)\frac{v^2}{2\mu_3^2}+\frac{M_1(M_1-3M_2)v^2}{4\mu_3^4}\right]$$

$$m_5 \approx \mu_3\left[1+\left(2\lambda_2+\frac{\lambda_5}{2}\right)\frac{v^2}{2\mu_3^2}+\frac{3M_1M_2v^2}{4\mu_3^4}\right]$$

Mixing angle:

$$\sin\alpha \approx -\frac{\sqrt{3}M_1 v}{2\mu_3^2}\left[1-2(2\lambda_2-\lambda_5)\frac{v^2}{\mu_3^2}+\frac{m_h^2}{\mu_3^2}+\frac{M_1(24M_2-5M_1)v^2}{8\mu_3^4}\right]$$

Decoupling expansions

Tree-level couplings

$$\kappa_V \approx 1 + \frac{3}{8}\frac{M_1^2 v^2}{\mu_3^4}$$

$$\kappa_f \approx 1 - \frac{1}{8}\frac{M_1^2 v^2}{\mu_3^4}$$

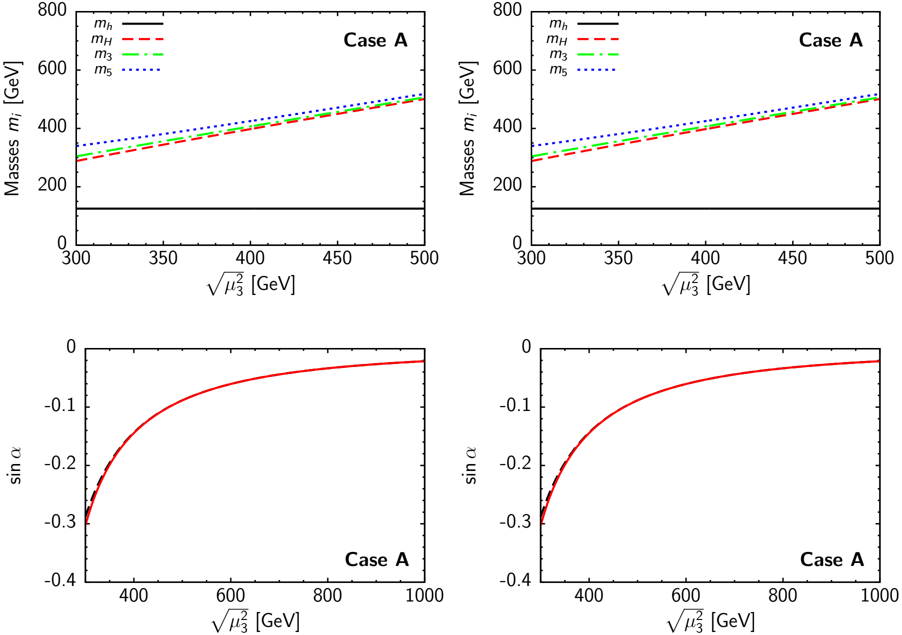
$$g_{hhVV} \approx \frac{2m_V^2}{v^2}\left(1 + \frac{5}{4}\frac{M_1^2 v^2}{\mu_3^4}\right)$$

$$g_{hhh} \approx \frac{3m_h^2}{v}\left\{1 - \frac{M_1^2 v^2}{\mu_3^4}\left[\frac{7}{8} - \frac{3}{2}\frac{v^2}{m_h^2}\left((2\lambda_2 - \lambda_5) + \frac{M_1 M_2}{\mu_3^2}\right)\right]\right\}$$

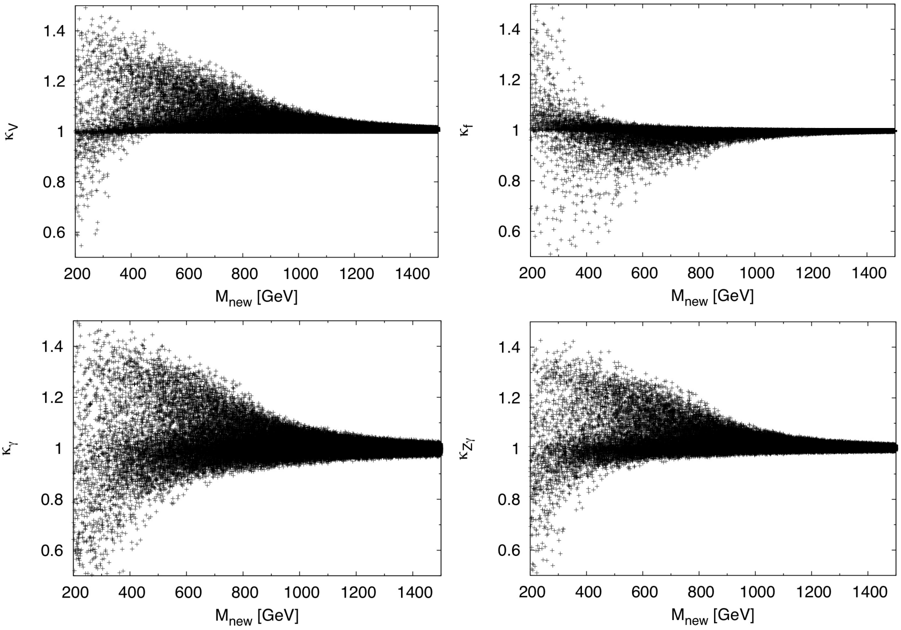
Loop couplings

$$\Delta\kappa_\gamma \simeq -\frac{1}{F_1(M_W) + \frac{4}{3}F_{1/2}(m_t)}\frac{2v^2}{3\mu_3^2}\left[6\lambda_2 + \lambda_5 + \frac{M_1^2 + 12 M_1 M_2}{4\mu_3^2}\right],$$

$$\Delta\kappa_{Z\gamma} \simeq \frac{1}{2(A_W + A_f)}\frac{1 - 2s_W^2}{s_W c_W}\frac{2v^2}{3\mu_3^2}\left[6\lambda_2 + \lambda_5 + \frac{M_1^2 + 12 M_1 M_2}{4\mu_3^2}\right]$$



Case A
m_h
m_H
m_3
m_5
Masses m_i [GeV]
$\sqrt{\mu_3^2}$ [GeV]
Case A
m_h
m_H
m_3
m_5
Masses m_i [GeV]
$\sqrt{\mu_3^2}$ [GeV]
$\sin\alpha$
Case A
$\sqrt{\mu_3^2}$ [GeV]
$\sin\alpha$
Case A
$\sqrt{\mu_3^2}$ [GeV]



1.4
1.2
1
0.8
0.6
κ_V
200
400
600
800
1000
1200
1400
M_{new} [GeV]
1.4
1.2
1
0.8
0.6
κ_f
200
400
600
800
1000
1200
1400
M_{new} [GeV]
1.4
1.2
1
0.8
0.6
κ_γ
200
400
600
800
1000
1200
1400
M_{new} [GeV]
1.4
1.2
1
0.8
0.6
$\kappa_{Z\gamma}$
200
400
600
800
1000
1200
1400
M_{new} [GeV]