Introduzione agli acceleratori di particelle

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Synchrotron

- Frequency modulated but also *B*-field increased **synchronously** to match energy and keep revolution radius constant.
- Magnetic field produced by several bending magnets increases with momentum. For high energies:

$$p = eBr \text{ and } p[\frac{GeV}{c}] \cong 0.3B[T]r[m]$$

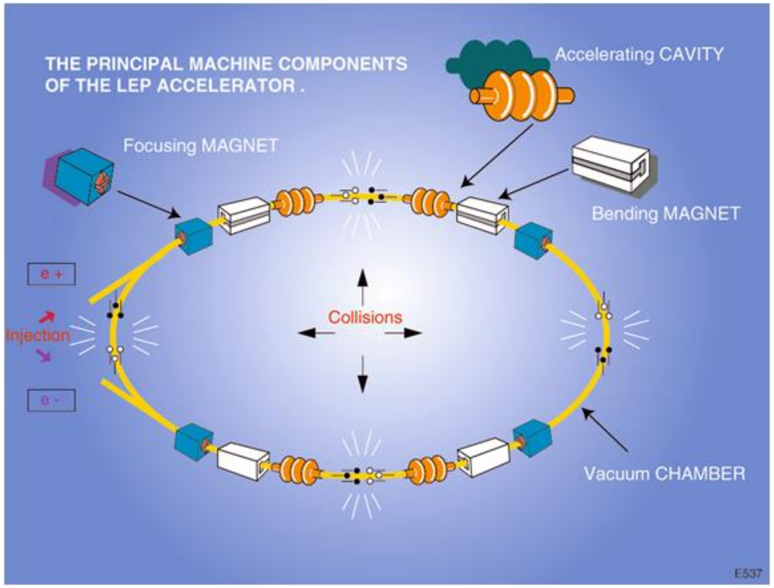
- Practical limitations for magnetic fields => high energies only at large radius
- Need large bending radius and **super-conducting magnets to reach very high energies as in the LHC:** 8.3 T and bending radius of **2.9km** to reach 7 TeV

Summary of Circular Machines

Machine	RF frequency f	Magnetic Field B	Orbit Radius ρ	Comment
Cyclotron	constant	constant	increases with energy	Particles out of synch with RF; low energy beam or heavy ions
Iso-Cyclotron	constant	varies	increases with energy	Particles in synch, but difficult to create stable orbits
Synchro-cyclotron	varies	constant	increases with energy	Stable oscillations
Synchrotron	varies	varies	constant	Flexible machine, high energies possible

Main Components of an Accelerator

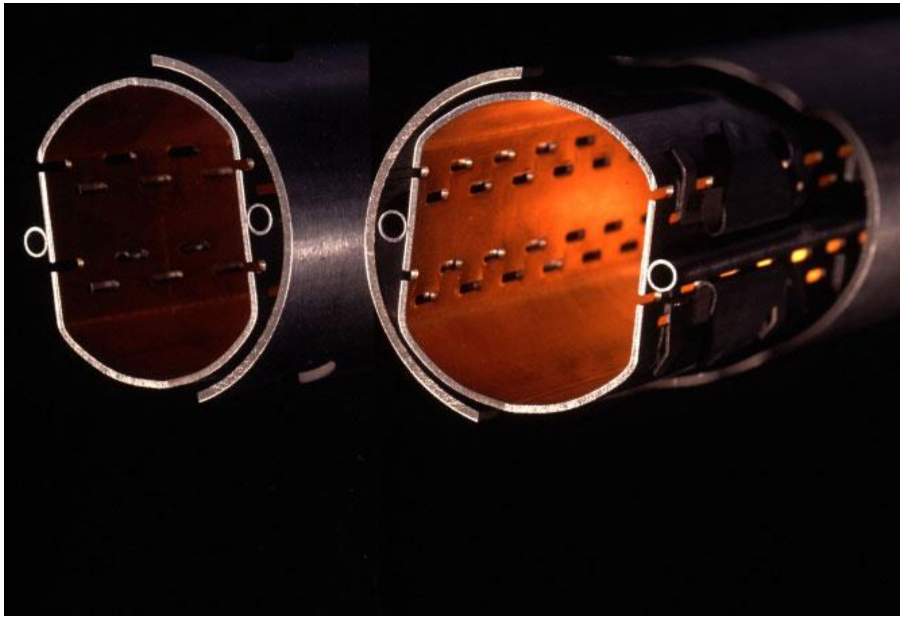
- Employs a vacuum chamber in which the particles travel
- Employs electric fields to impart energy to (accelerate) the particles
- Employs magnetic fields to steer and focus the beam
- Makes collisions either against a fixed target, or between two beams of particles.



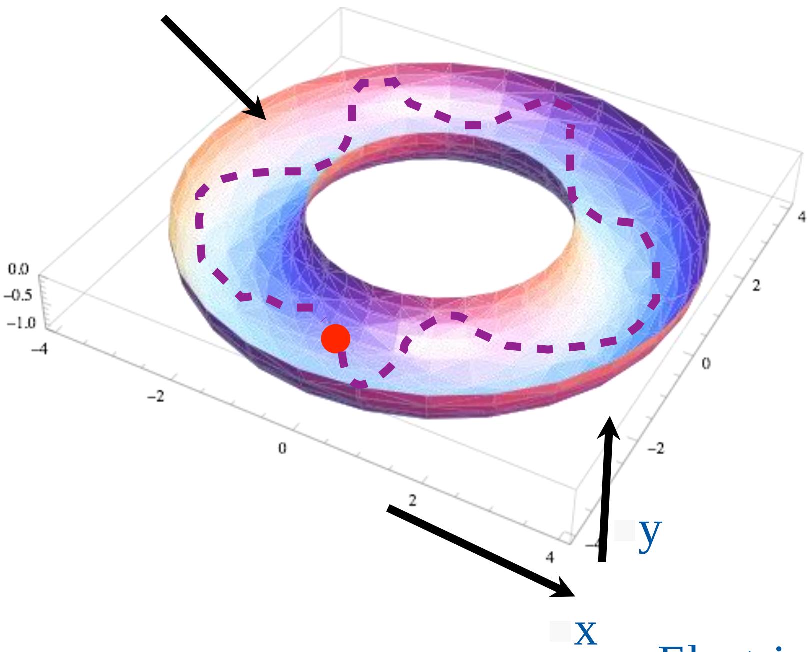
Vacuum System

- Beam particles travel inside a metal pipe (also known as the beam pipe). This is kept at an ultrahigh vacuum to minimise the amount of gas present to avoid collisions between gas molecules and the particles in the beam.

Ultrahigh vacuum <10^{-10} Torr ~10^{-13} atm



How an accelerator works ?



- ***Goal:*** *keep enough* ***CHARGED*** *particles confined in a well defined volume to accelerate them for a sufficiently long time* ***(ms - hours)***
- *How ? Lorentz Force!*

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Electric field accelerates particles

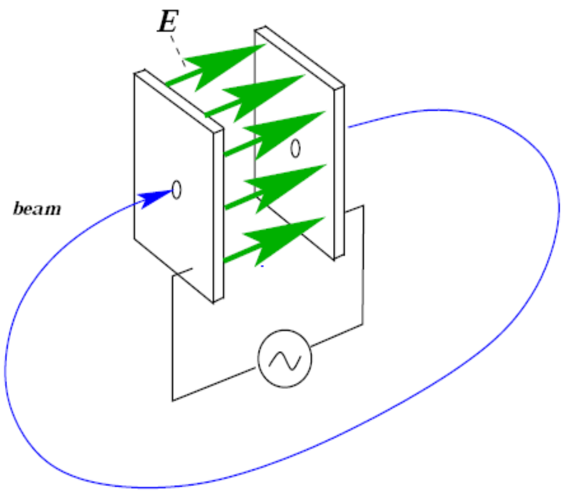
Particles of different energy (speed) behave differently

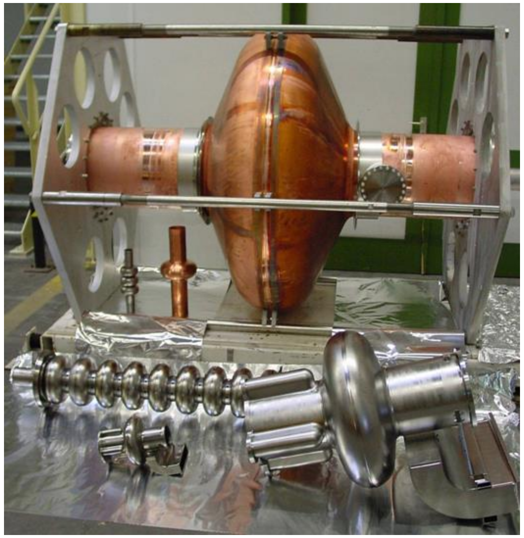
Magnetic field confines particles on a given trajectory

- An **accelerator** is formed by a sequence (called **lattice**) of:
  a) **Magnets** → Magnetic Field
  b) **Accelerating Cavity** → Electric Field

Acceleration (RF) System

- Radiofrequency (RF) electric fields provide acceleration to a beam of particles. RF cavities are located intermittently along the beam pipe. Each time a beam passes the electric field in an RF cavity, some of the energy from the radio wave is transferred to the particles.

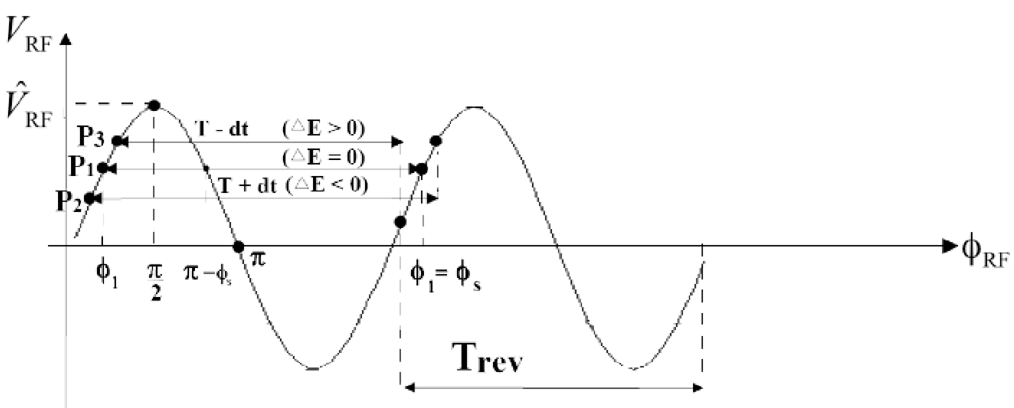


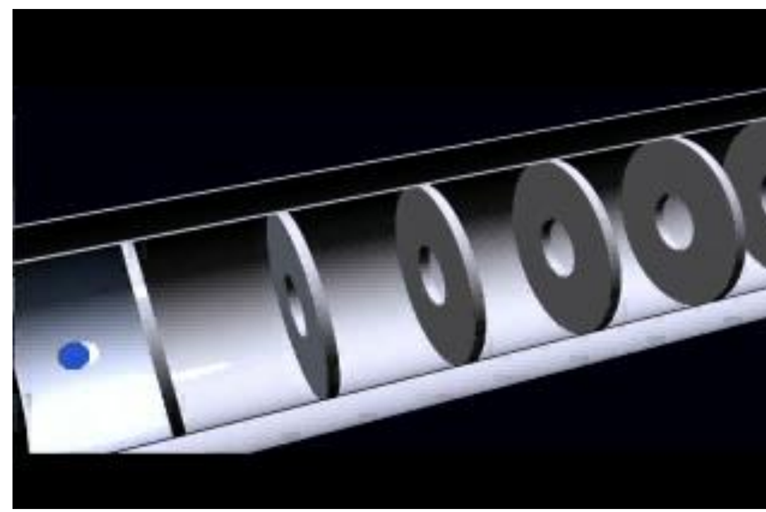


Acceleration

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

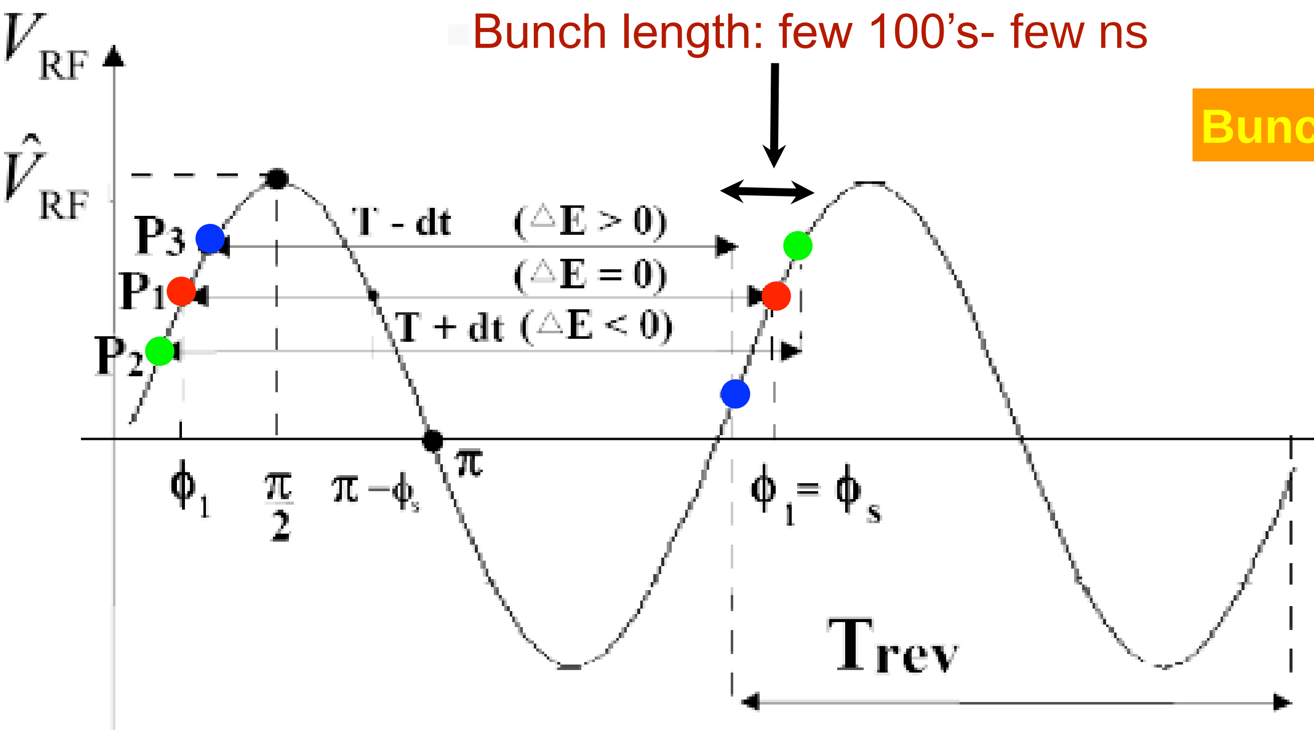
- Particles are accelerated by an RF (radio frequency) electric field which is confined in cavities.
- The electric field varies in time as a sinus wave in such a way, that at each revolution, the particle comes back at the RF to see the acceleration ➔ $f_{RF} = h\ f_{rev}$

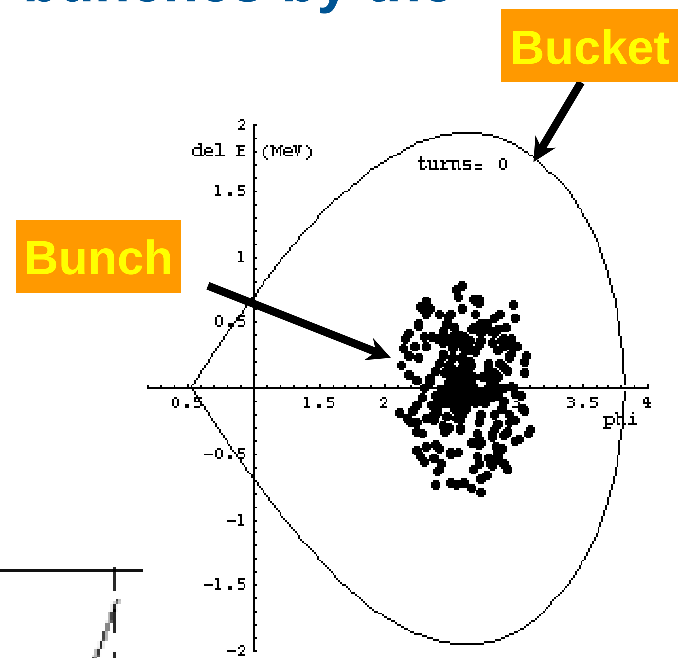




Longitudinal focusing, a pendulum ...

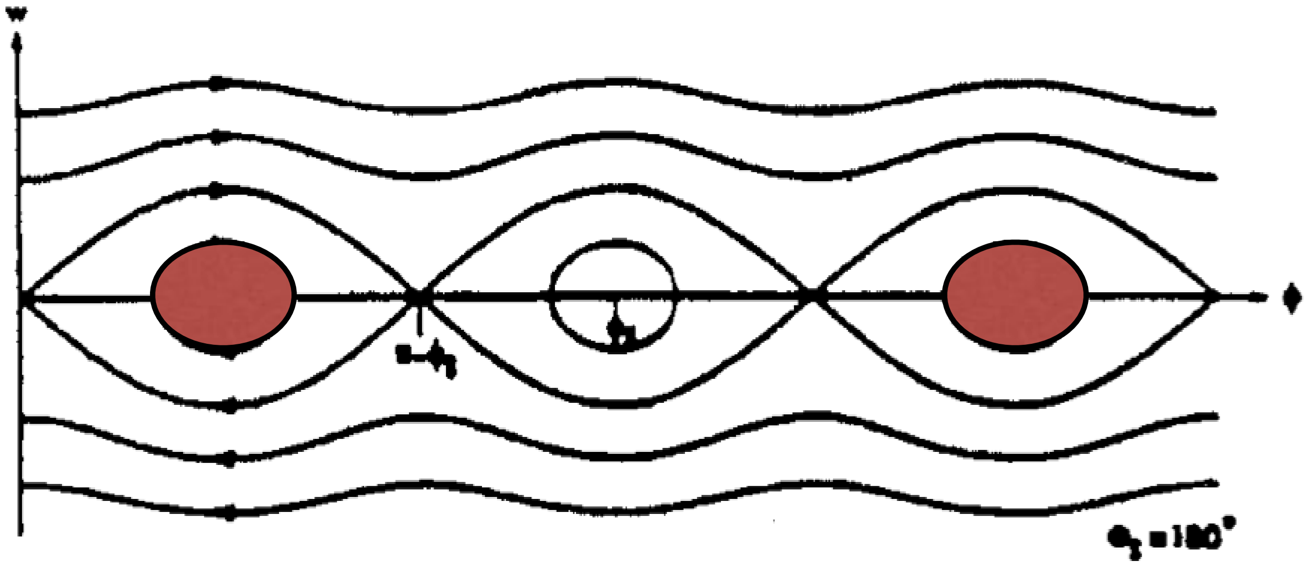
- Particles are confined within a range in phase and energy called **BUCKET** and are grouped into **bunches by the electric field**.





One machine turn ~ some (hundred)microseconds

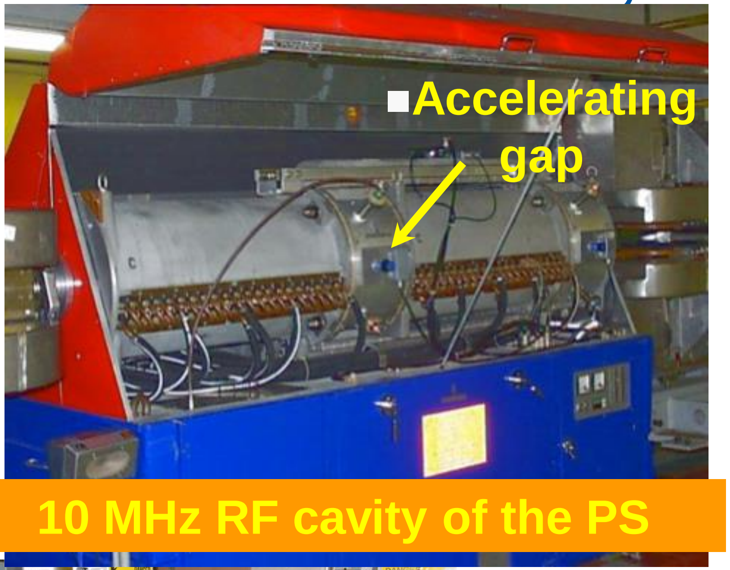
A chain of buckets



- Number of buckets:
- *possible positions along the machine circumference where* ***there could be a bunch.***
- In the example: 3 buckets and 2 bunches ➔ h=3

RF cavities in the CERN PS

The dimension of the cavity changes with the RF wave length



10 MHz RF cavity of the PS

Six 200 MHz Cavities

13 MHz cavity

40 MHz cavity

80 MHz cavity

RF system LHC

- 16 MV/beam
  - ACCELERATION TAKES TIME (20 mins)
  - What is the energy gain per turn?
    - Injection at 450 GeV
    - Ramp to 7000 GeV
    - Circumference 27 km
    - Ramp time = ~20 mins

0.52 MV

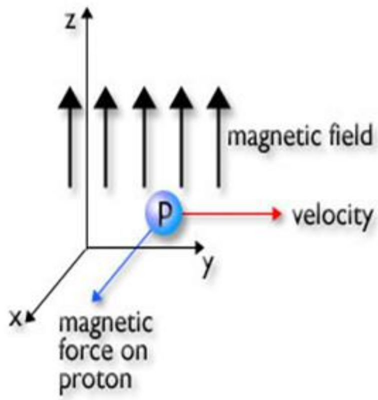
- How long is a wave?
  - f_{RF}= 400 MHz

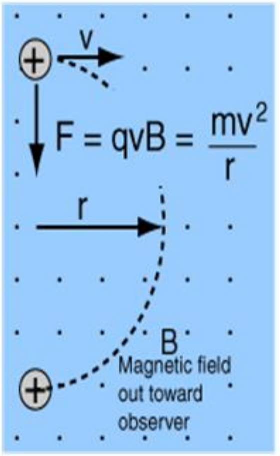
75 cm

RF acceleration (LHC)

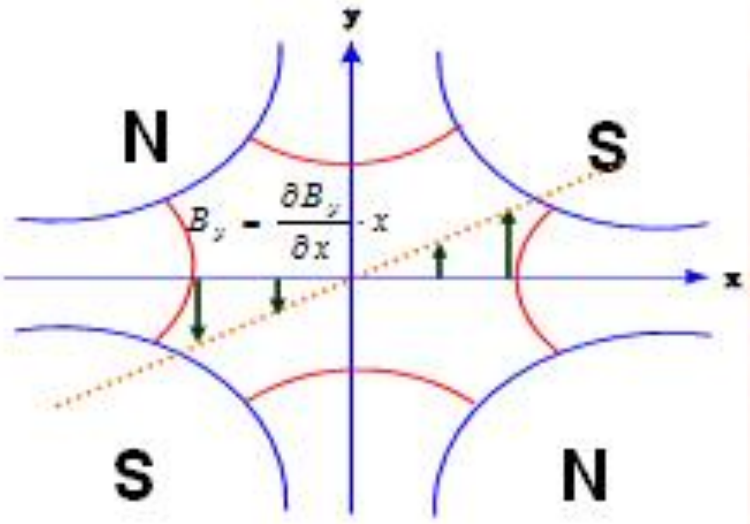
Magnets

- Two main types of magnets:
  - Dipole magnets to bend the path of a beam of particles that would otherwise travel in a straight line. The more energy a particle has, the greater the magnetic field needed to bend its path ➔ larger energy implies larger circumferences or stronger magnetic fields





  - Quadrupole magnets are used to focus a beam, gathering all the particles closer together (like lenses are used to focus a beam of light).



Beam guidance (dipoles)

- Consider only a uniform magnetic field **B** in the direction perpendicular to the particle motion over a length **l**. From the ideal trajectory and after considering that the transverse velocities $\mathbf{v_x \ll v_s, v_y \ll v_s}$, we have that the radius ρ and angle α of curvature are

$$\varrho = \frac{pc}{qB} = \frac{\beta E}{qB} \qquad \alpha = \frac{qBl}{pc}$$

- We define the **magnetic rigidity** $B\varrho \equiv \frac{q}{pc}$

- In more practical units $\boxed{\beta E_{tot}[GeV] = 0.2998|B\rho|[Tm]}$

- For ions with charge multiplicity **Z** and atomic number **A**, the energy per nucleon is

$$\beta \bar{E}_{tot}[GeV/u] = 0.2998\frac{Z}{A}|B\rho|[Tm]$$

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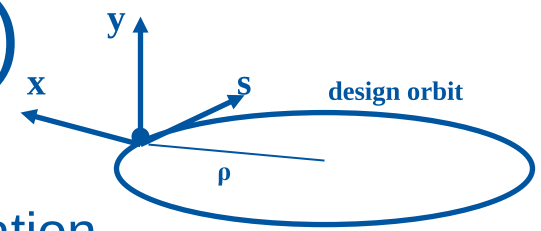
Beam focusing (quadrupoles)

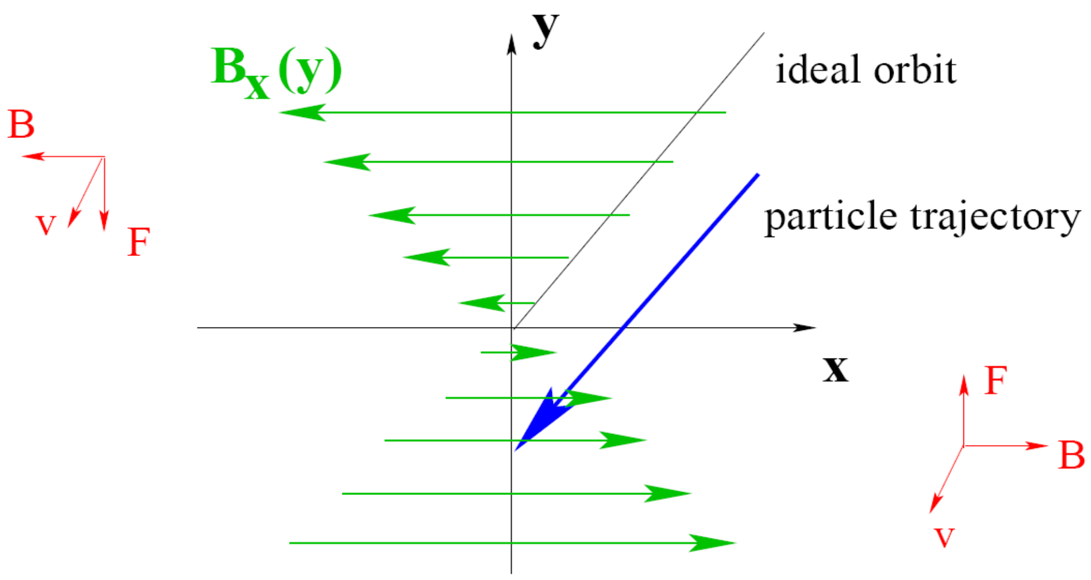


- Consider a particle in the design orbit.
- In the horizontal plane it does an harmonic oscillation

  $x = x_0 \cos(\omega t + \phi)$ with frequency $\omega = \frac{v_s}{\rho}$

- The horizontal acceleration is described by $\frac{d^2x}{ds^2} = \frac{d^2x}{v_s^2 dt^2} = -\frac{1}{\rho^2}x$
- There is a **week focusing** effect in the horizontal plane.
- In the vertical plane, only force is gravitation. The particle will be displaced vertically following the usual law $\Delta y = \frac{1}{2} g \Delta t^2$

- Setting g = 10 m/s^2, the particle will be displaced by **18mm** (LHC dipole aperture) in **60ms** (a few hundreds of turns in LHC)
- Need of **focusing**!



Focusing elements

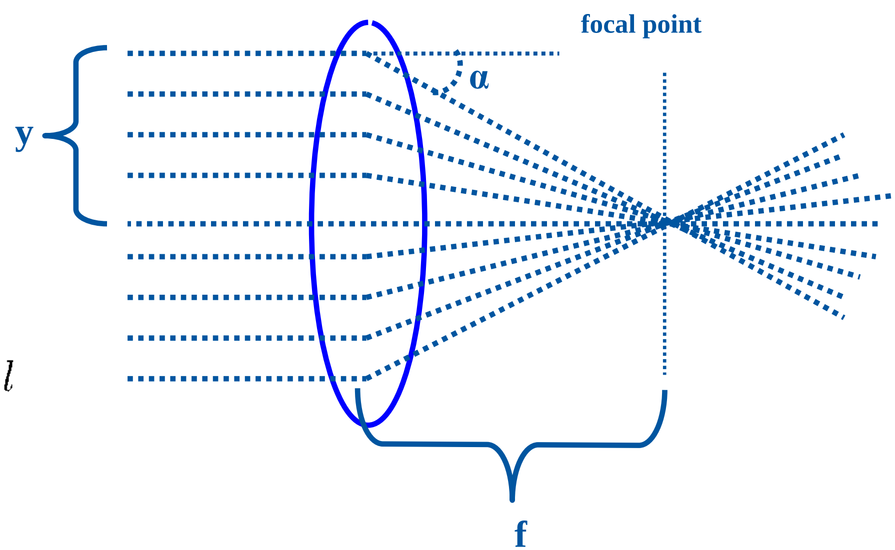
- Magnetic element that deflects the beam by an angle proportional to the distance from its centre (equivalent to ray **optics**) provides focusing.
- For a focal length f the deflection angle is $\alpha = -\frac{y}{f}$
- A magnetic element with length **l** and with a gradient **g** has a field

$B_x = gy$ so that the deflection angle is $\alpha = -\frac{qgyl}{pc} = -\frac{qgyl}{\beta E}$

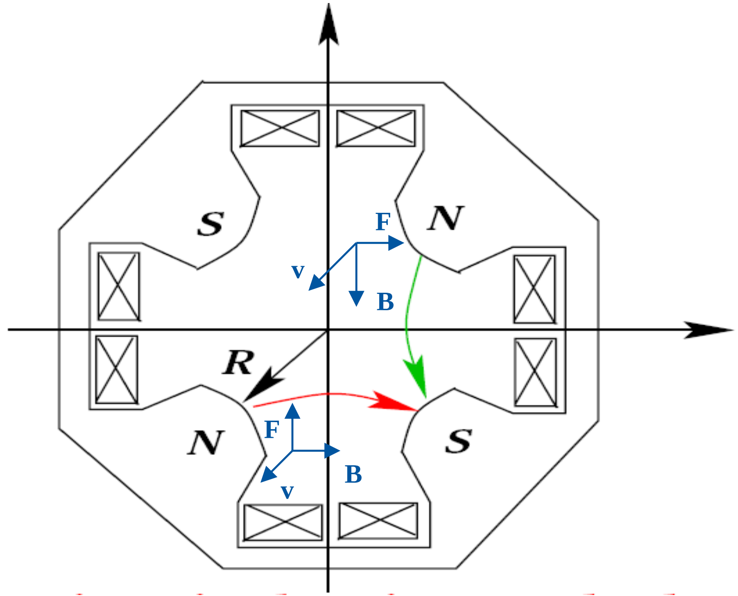
- The normalised focusing strength $k = \frac{qg}{\beta E}$
- In more practical units, for **Z=1**

$$k[m^{-2}] = 0.2998 \frac{g[T/m]}{\beta E[GeV]}$$

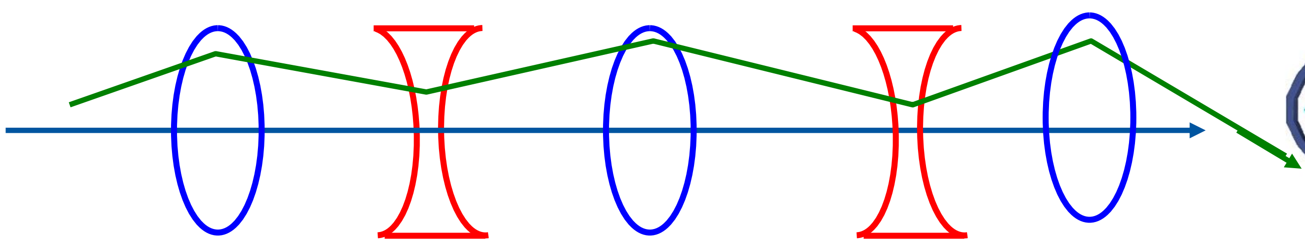
- The focal length becomes $f^{-1} = k\ l$ and the deflection angle is $\alpha = -k\ y\ l$

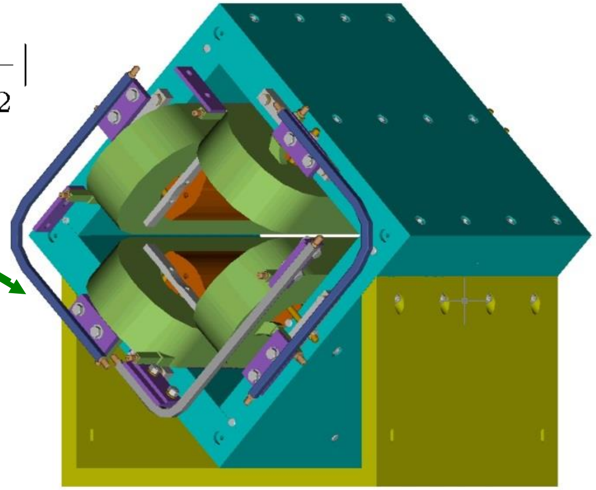


Quadrupoles

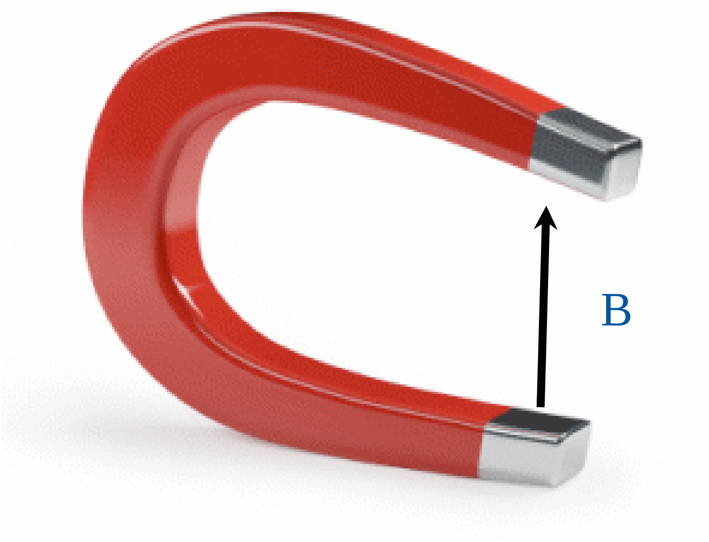


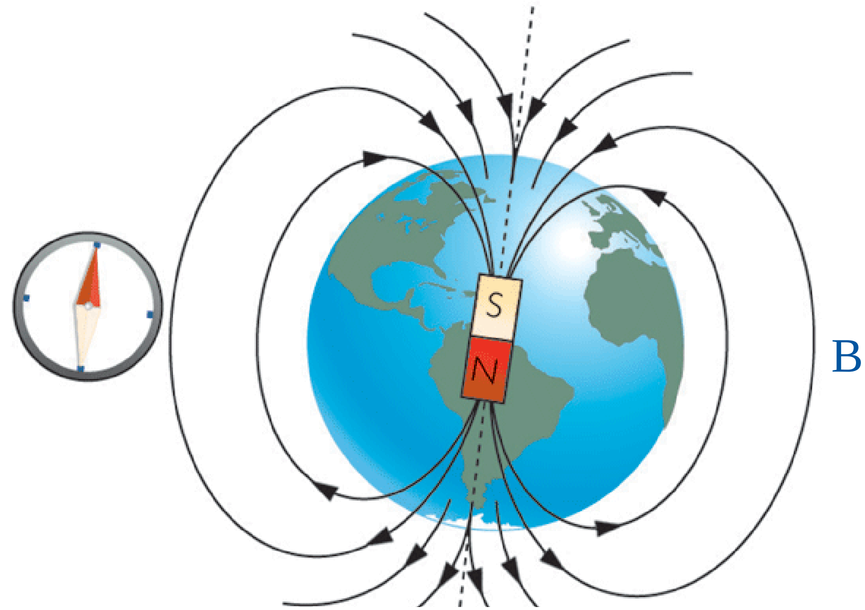
- Quadrupoles are focusing in one plane and defocusing in the other
- The field is $(B_x, B_y) = g(y, x)$
- The resulting force $(F_x, F_y) = k(y, -x)$
- Need to alternate focusing and defocusing to control the beam (**alternating gradient focusing)**
- From optics we know that a combination of two lenses with focal lengths **f1** and **f2** separated by a distance **d**
  $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \ f_2}$$
- If $f_1 = -f_2$, there is a net focusing effect, i.e. $\frac{1}{f} = |\frac{d}{f_1 \ f_2}|$





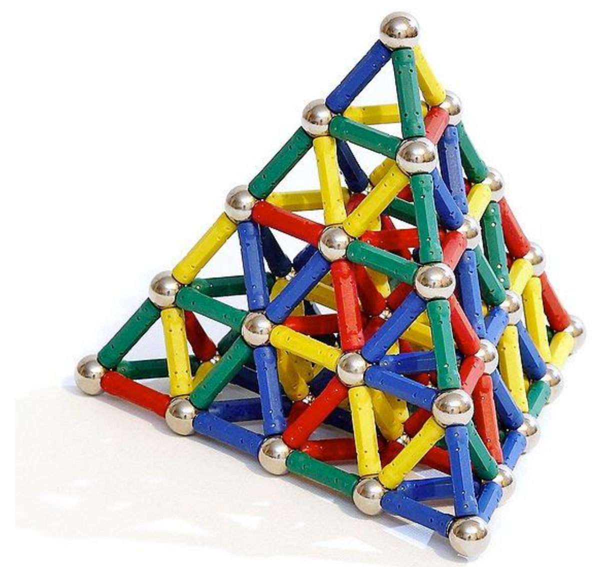
Two dipoles and magnets you should know very well





- **Earth Magnetic Field : ~ 0.6 Gauss**

- **Typical SPS dipole field: ~ 20000 Gauss (2Tesla)**



Magnets

- Normal conducting magnets:
  - field dominated by the magnetization of the iron yoke
  - Maximum fields limited to less than 2 Tesla (~40000 the terrestrial magnetic field)

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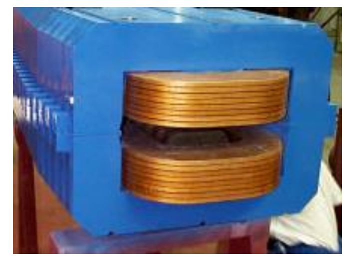
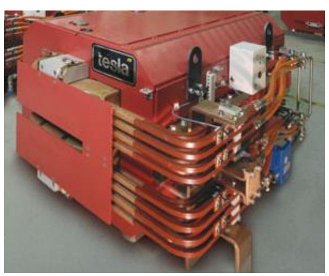
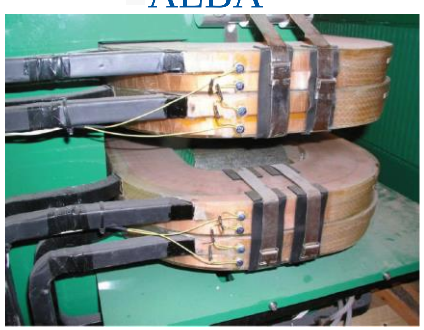


SLS

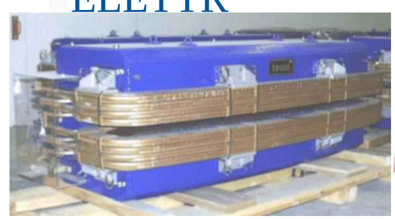
SPRING-8



SOLEIL

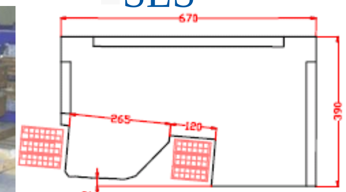


DIAMOND



CLS

Gap=45 mm B= 1.35 T G= 3.8T/m



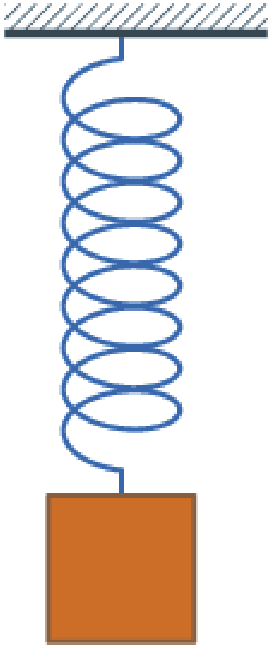
ASP

SPEAR3

Gap = 50 mm B= 1.4 T G= 3.6 T/m

Classical mechanics.... spring with a mass

$$F = ma = m\frac{d^2x}{dt^2} = -kx$$



with *k* the spring constant and *m* the mass

Solution of the equation of motion is a periodic function:

$$x(t) = A\cos(2\pi f\, t + \phi)$$

with 1/period equals to

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Equation of motion (transverse)

- Equation of motion of a particle in a circular accelerator composed by a sequence of elements, each one eventually with a ***k*** at a position ***s*** of the ring, repeated at every ***C***

$$\frac{d^2x}{ds} + K(s)x = 0$$

Hill's Equation

$$x(s) = a\sqrt{\beta(s)}\cos(\phi(s) + \phi_0)$$

$$\phi(s) = \int \frac{1}{\beta(s)} ds$$

Local force at a position ***s*** in the ***ring***

$$K(s) = 1/\rho^2 + k(s)$$

$$K(s) = K(s + C)$$

Dipoles

Quadrupoles

This imply some periodicity of the solution

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Envelope of the betatronic motion (Twiss parameters)

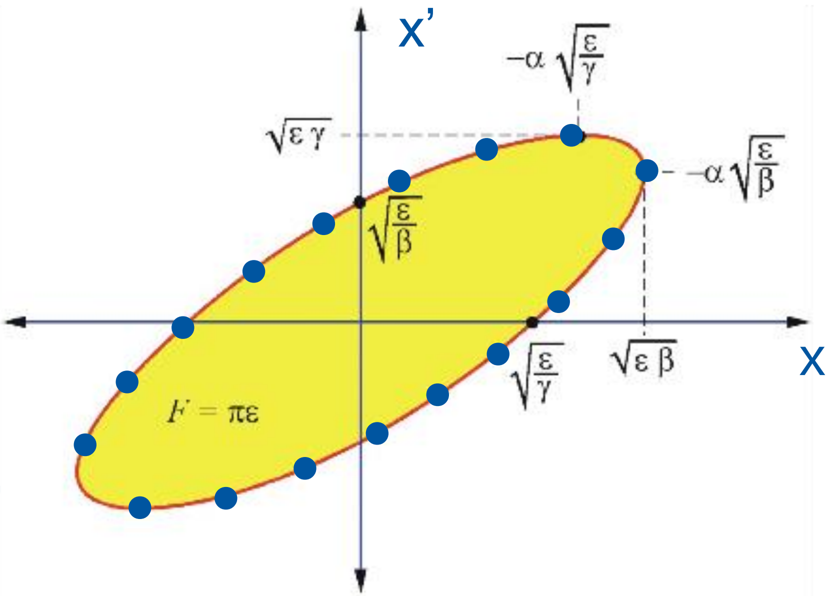
Defining $x'(s) = \frac{dx}{ds}$ (beam divergence) it can be demonstrated that

$\gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s) = \varepsilon$

where:

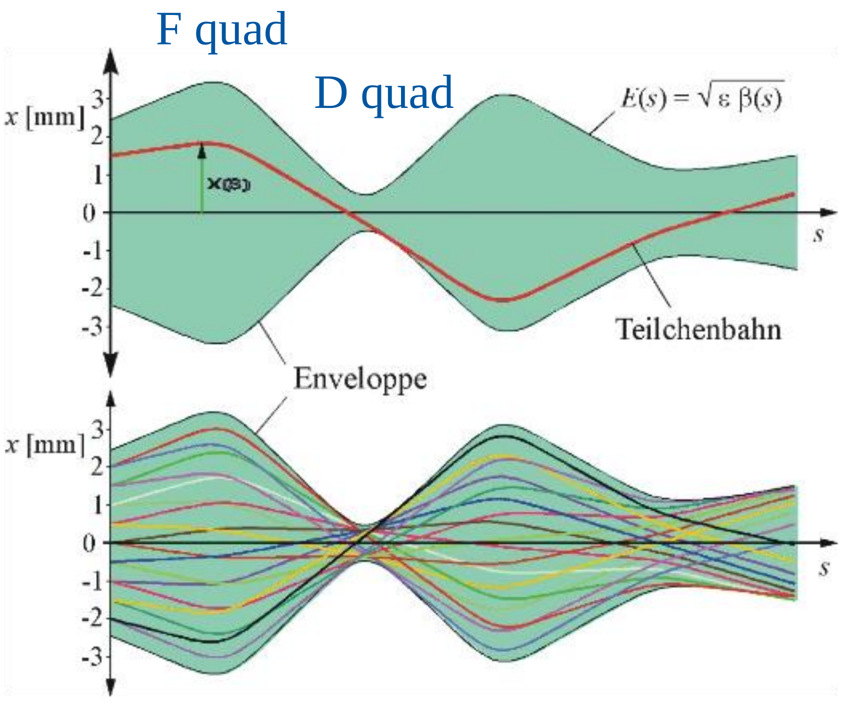
- $\alpha(s) \equiv -\frac{1}{2}\frac{d\beta}{ds}$
- $\gamma(s) \equiv \frac{1+\alpha^2(s)}{\beta(s)}$
- $\varepsilon \equiv a^2$

Twiss parameters (α, β, γ) at a given position and emittance ε in the ring provide a complete description of the motion of the particles and of the beam envelope



Definition of envelope

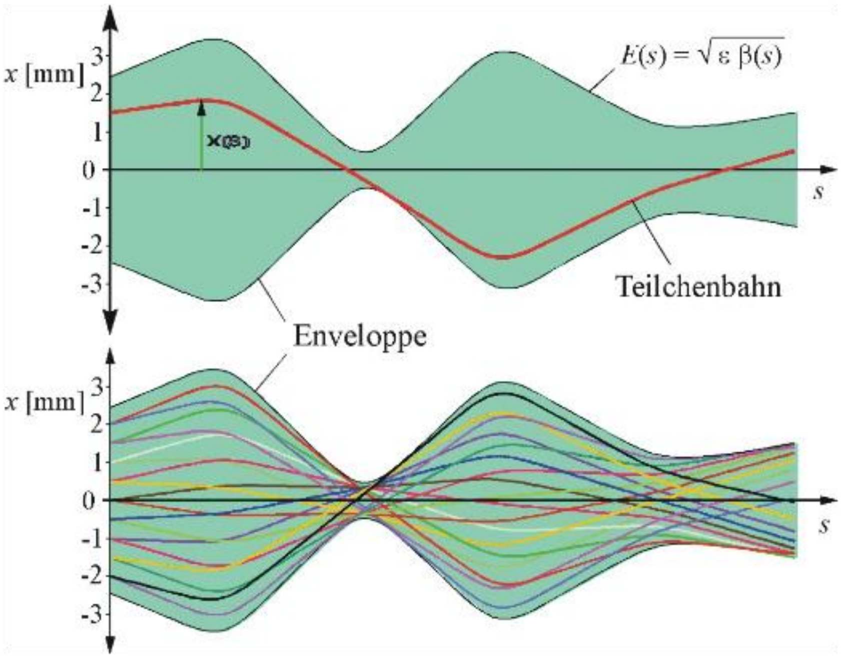
- The beam envelope can be defined for example as the maximum amplitude for which the particle remains in the machine vacuum chamber.

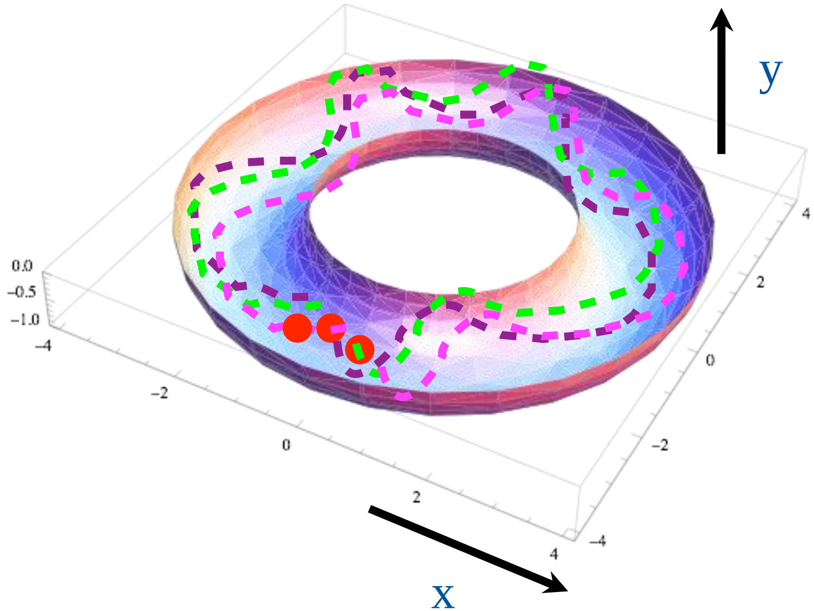


Beam physical dimension

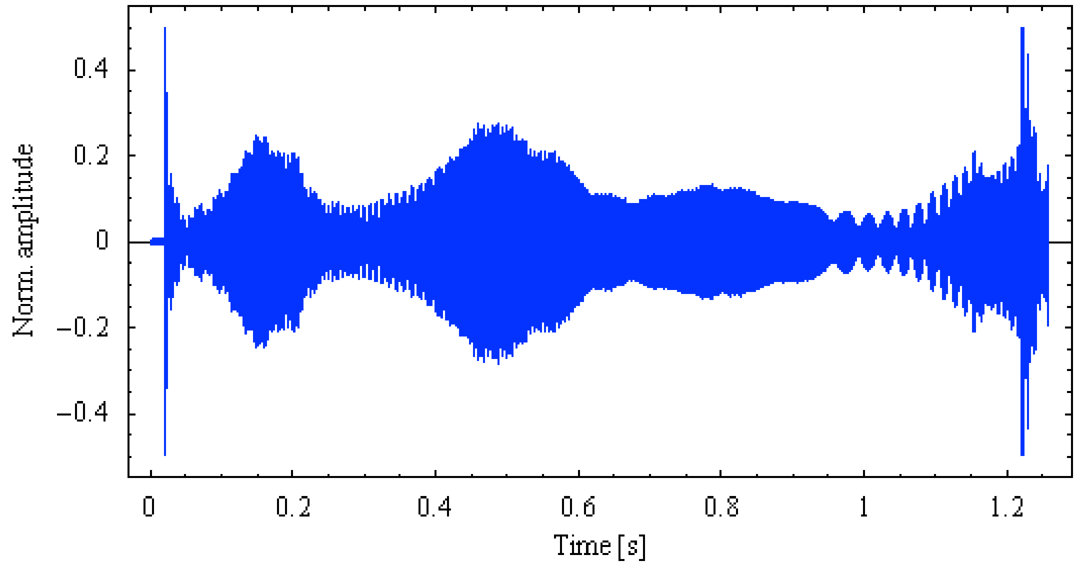
Tune

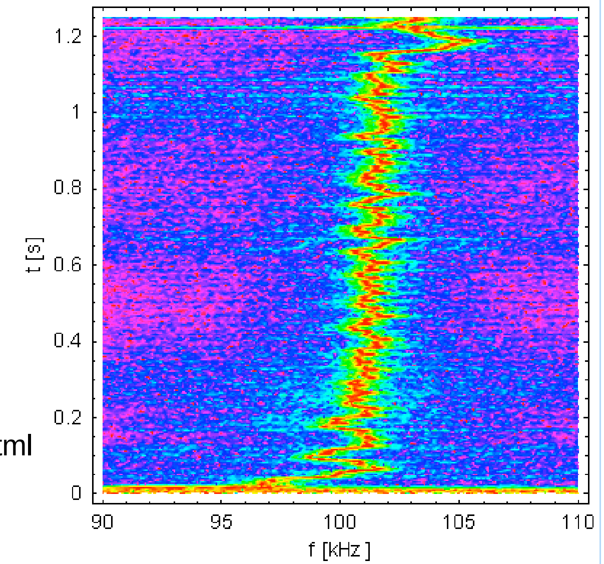
- Tune Q: number of oscillations (called betatronic) in the xx‘ plane a particle does in one machine turn.
- $$Q = \frac{\phi(s+C) - \phi(s)}{2\pi}$$
- The tune depends on the quadrupoles settings





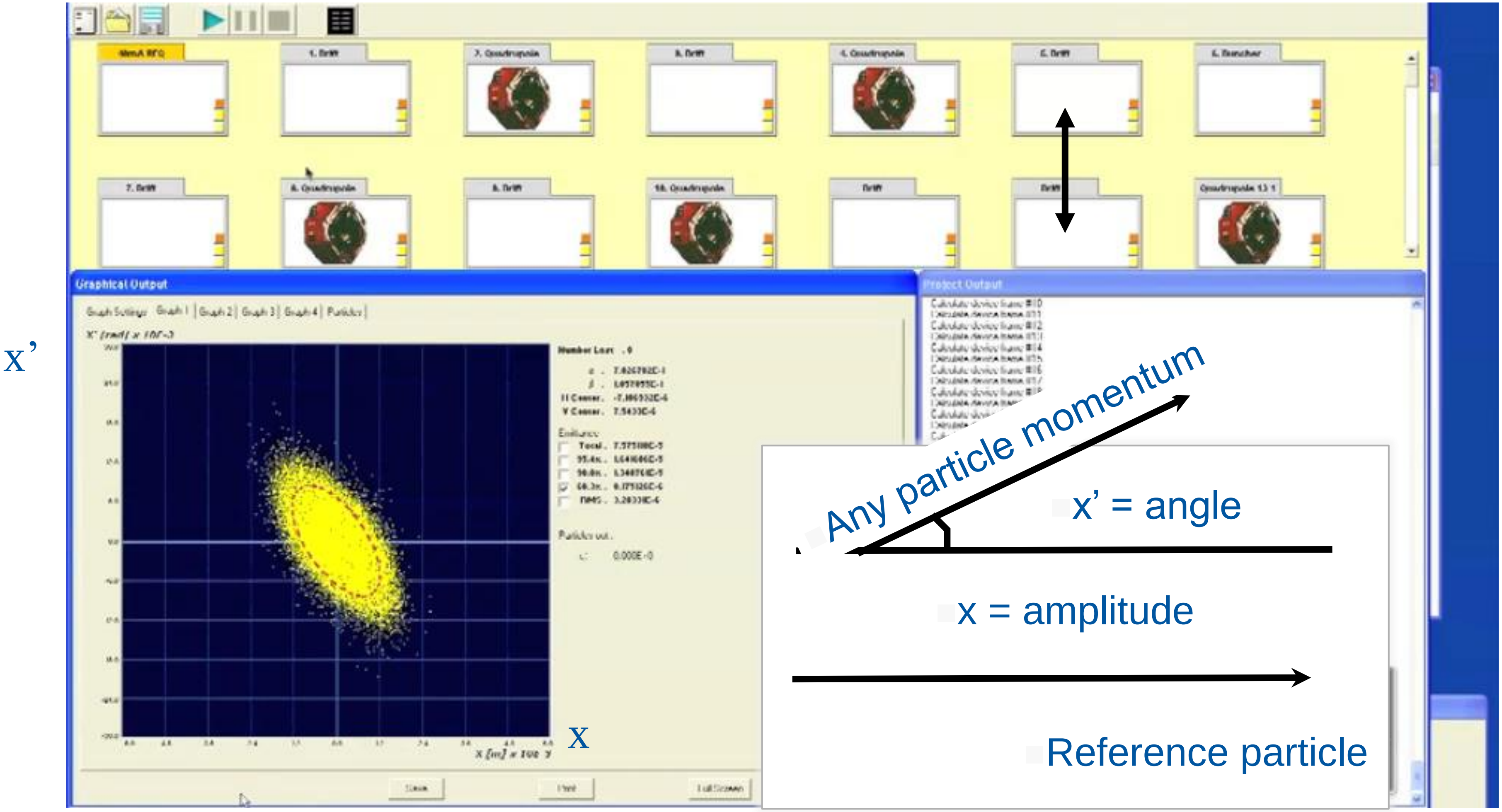
Tune: number of betatron oscillation in the transverse plane



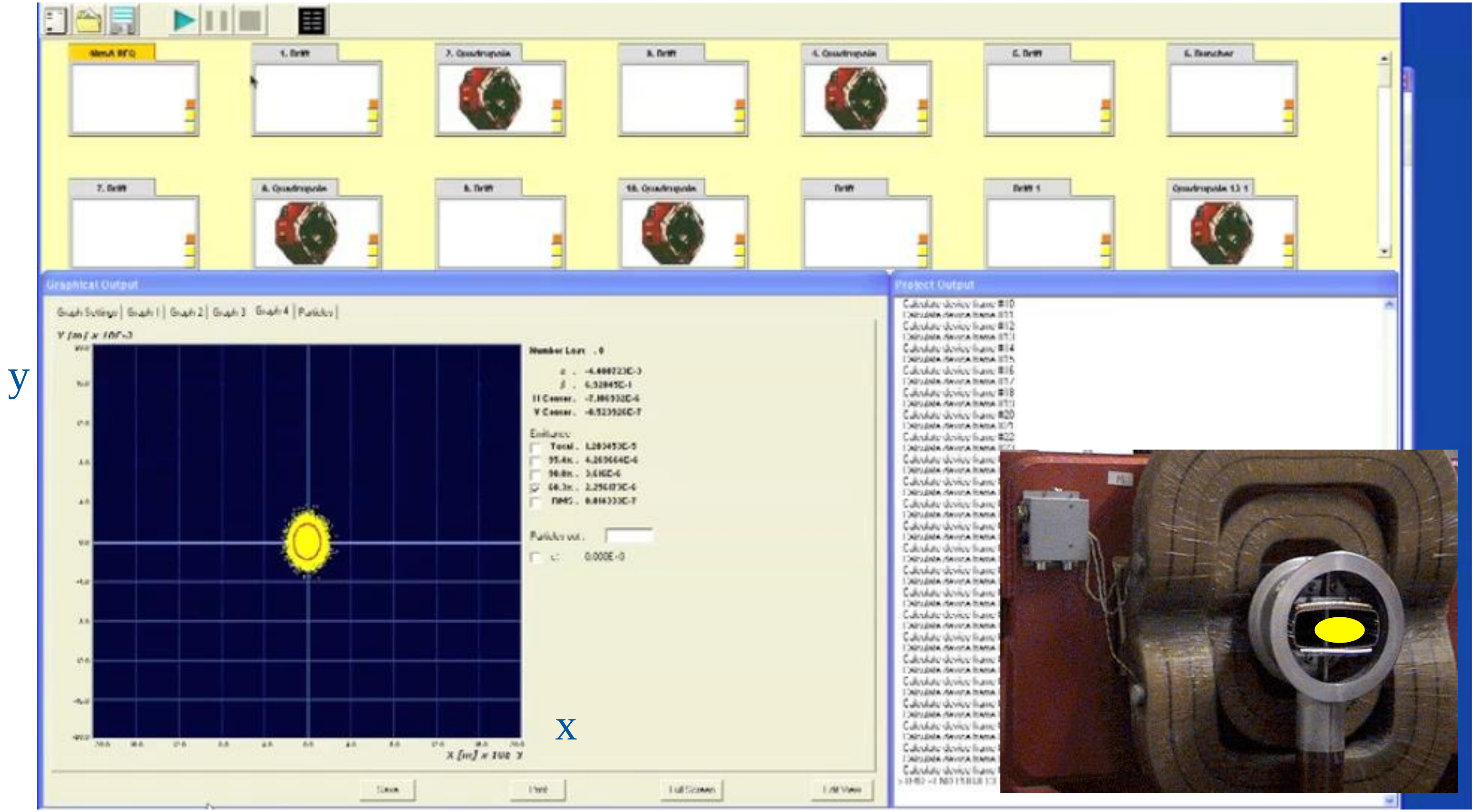


http://mgasior.web.cern.ch/mgasior/pro/3D-BBQ/ps.html

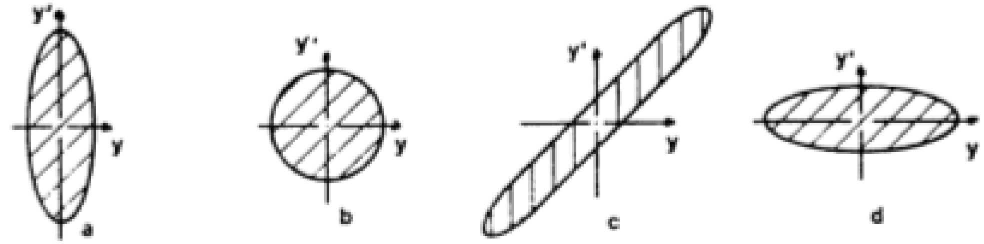
Particle transport in a lattice (phase space)



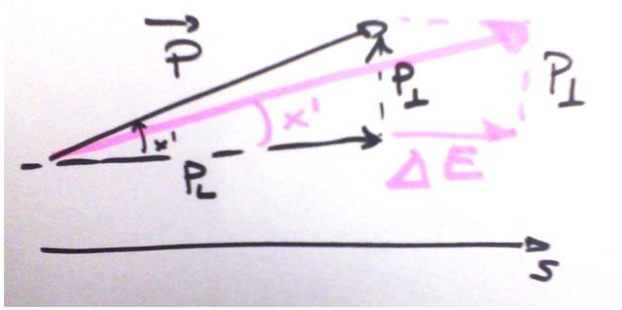
Particle transport in a lattice (real space)



Liouville theorem



- Theorem: In the vicinity of a particle, the particle density in phase space is a constant if the particle move in an external magnetic field or in a general field which the force do not depend upon velocity the beam is like an incompressible fluid in phase space
- Implications:
  a. the emittance is conserved when the beam is transported via a magnetic system. The ellipse is distorted/streched but its surface is conserved.
  b. the emittance is NOT conserved if we accelerate, except if we normalize the emittance wrt to βγ (relativistic). x' is reduced by the acceleration.



A synchrotron at a glance: LEIR