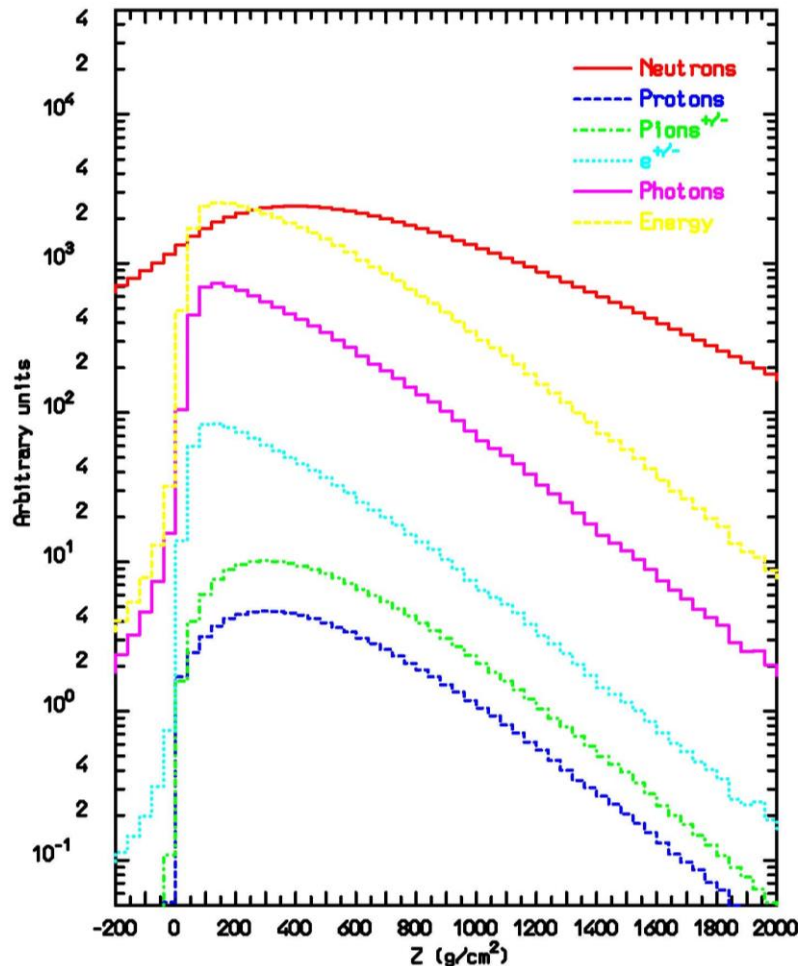# Hadron-Nucleus Interactions
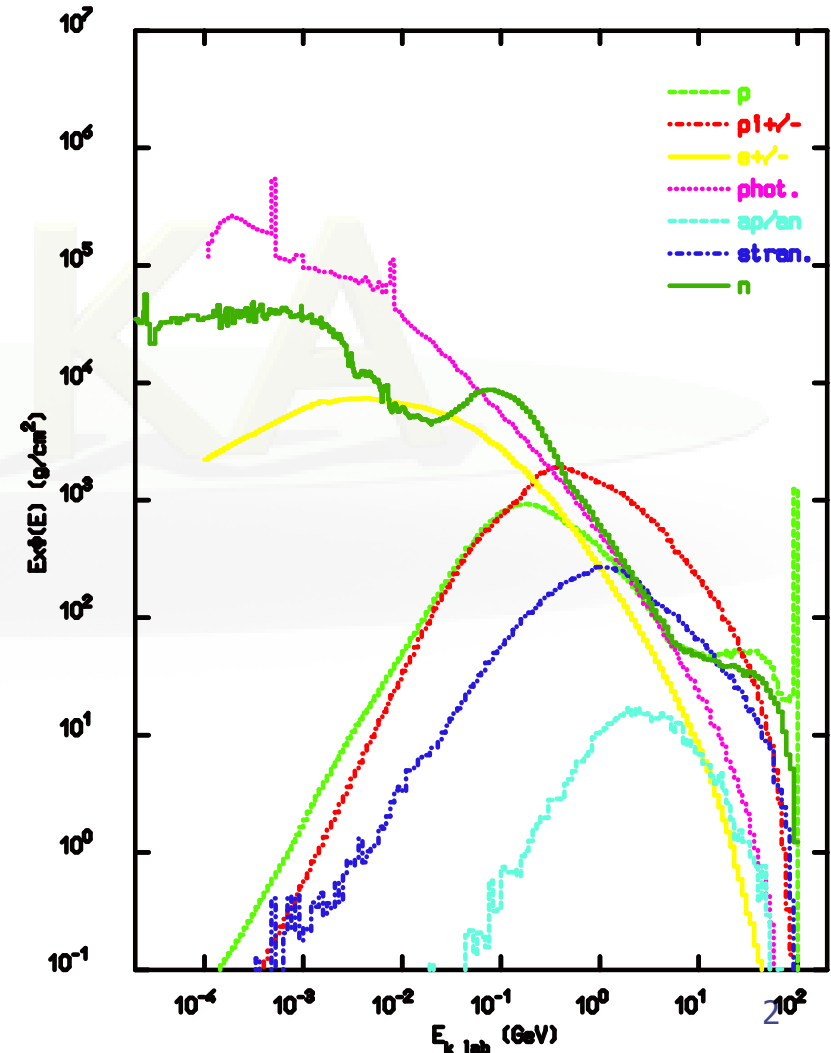
Beginners' FLUKA Course

# Hadronic showers: many particle species, wide energy range

**100 GeV p on Pb**
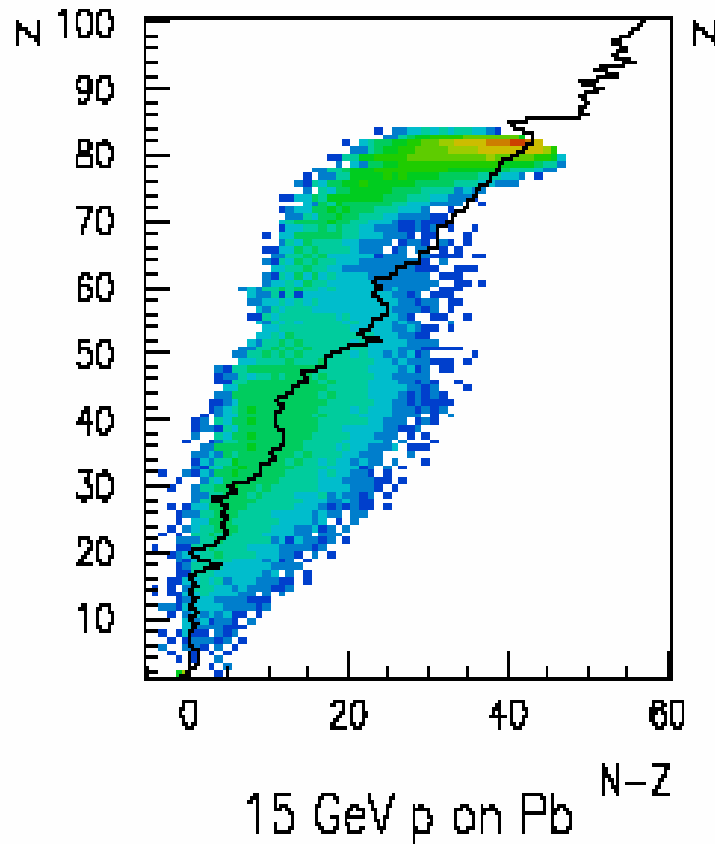**shower longitudinal development**

**100 GeV p in a Fe absorber:**
**space-averaged particle spectra**

# Hadronic showers: we want to simulate also what is left !

**Residual nuclei distribbution Following 15 GeV proton interactions on Lead**



15 GeV p on Pb

- The MC should be able to reproduce this complexity, maintaining correlations and cross-talks among the shower components
- Hadronic interactions have to be simulated over a wide energy range with an high degree of accuracy
- Nuclear effects are essential
- => need detailed nuclear interaction models
- => need checks on simple data (thin target)
- A summary of the hadronic interaction models used in FLUKA is given in the following

# The FLUKA hadronic Models

Hadron-nucleus: PEANUT

**Elastic,exchange**
Phase shifts
data, eikonal

**P<3-5GeV/c**
Resonance prod
and decay

Sophisticated
G-Intranuclear Cascade

Gradual onset of
Glauber-Gribov multiple
interactions
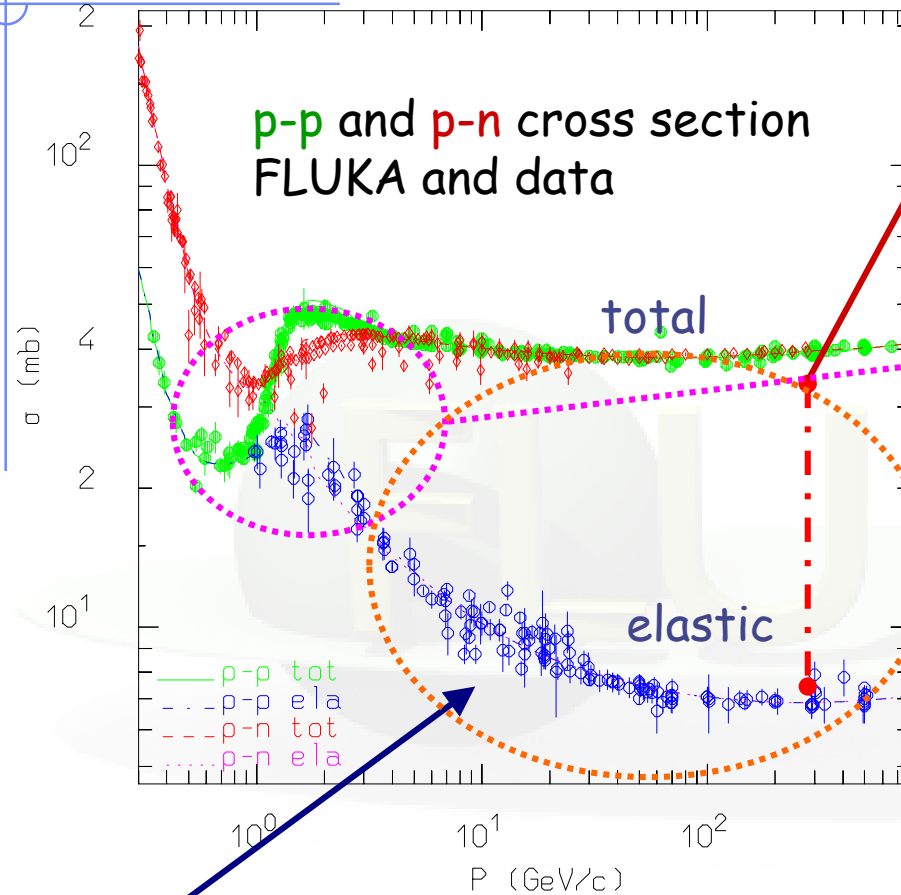
hadron

hadron

Preequilibrium

Coalescence

**low E**
**π,K**
Special

**High Energy**
DPM
hadronization

Evaporation/Fission/Fermi break-up
γ deexcitation

# Hadron-nucleon interaction models

p-p and p-n cross section
FLUKA and data

total

elastic

σ (mb)

P (GeV/c)

p-p tot
p-p ela
p-n tot
p-n ela

Particle production interactions: two kinds of models

Those based on "resonance" production and decays, cover the energy range up to 3–5 GeV

Those based on quark/parton string models, which provide reliable results up to several tens of TeV

Elastic, charge exchange and strangeness exchange reactions:
• Available phase-shift analysis and/or fits of experimental differential data
• At high energies, standard eikonal approximations are used

# Nonelastic hN interactions at intermediate energies

- $N_1 + N_2 \rightarrow N'_1 + N'_2 + \pi$    threshold at 290 MeV, important above 700 MeV,
- $\pi + N \rightarrow \pi' + \pi'' + N'$     opens at 170 MeV.
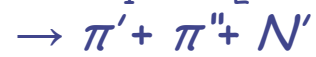
Anti-nucleon -nucleon open at rest !
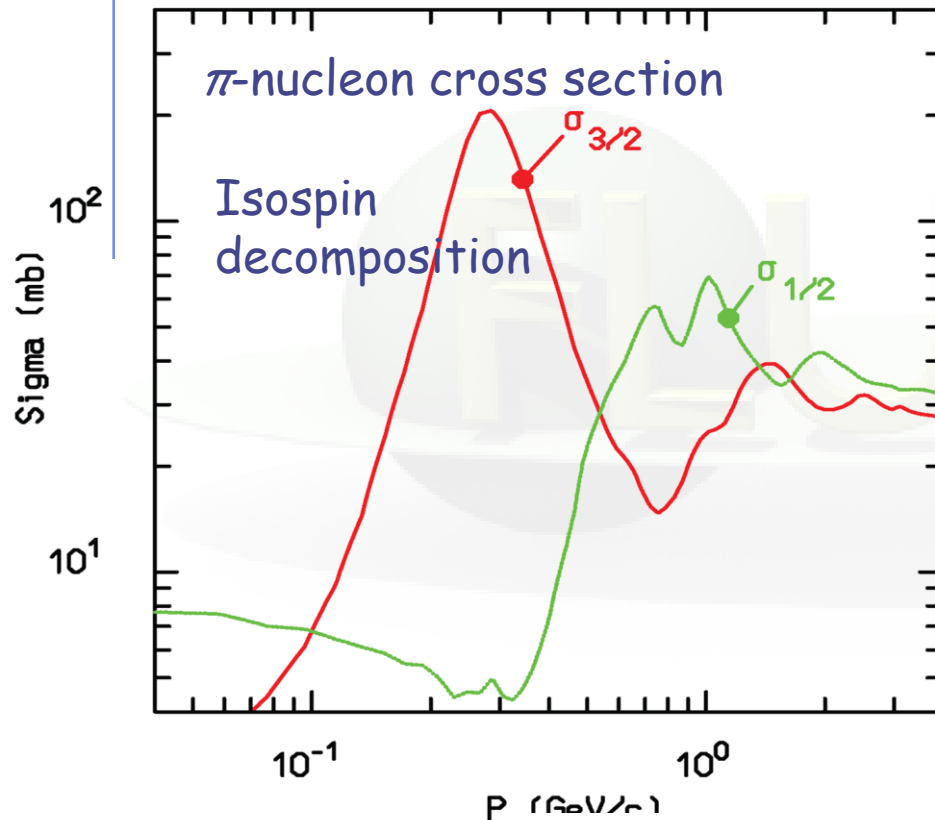
Dominance of the $\Delta$ resonance and of the $N*$ resonances

$\rightarrow$ isobar model

$\rightarrow$ all reactions proceed through an intermediate state containing at least one resonance.

*FLUKA: $\approx$ 60 resonances, and $\approx$ 100 channels*

$\pi$-nucleon cross section

Isospin decomposition

$\sigma_{3/2}$

$\sigma_{1/2}$

Sigma (mb)

$10^2$

$10^1$

$10^{-1}$     $10^0$

P (GeV/c)

Resonance energies, widths, cross sections, branching ratios from data and conservation laws, whenever possible. Inferred from inclusive cross sections when needed

$N_1 + N_2 \rightarrow N''_1 + \Delta(1232)$     $\rightarrow N'_1 + N'_2 + \pi$

$\pi + N \rightarrow \Delta(1600) \rightarrow \pi + \Delta(1232) \rightarrow \pi' + \pi'' + N'$

$N_1 + N_2 \rightarrow \Delta_1(1232) + \Delta_2(1232)$     $\rightarrow N'_1 + \pi_1 + N'_2 + \pi_2$

# Inelastic hN at high energies: (DPM, QGSM, ...)

- Problem: "soft" interactions → QCD perturbation theory cannot be applied.
- Interacting strings (quarks held together by the gluon-gluon interaction into the form of a string)
- Interactions treated in the Reggeon-Pomeron framework
- each of the two hadrons splits into 2 colored partons → combination into 2 colourless chains → 2 back-to-back jets
- each jet is then hadronized into physical hadrons

# Hadron-hadron collisions: chain examples
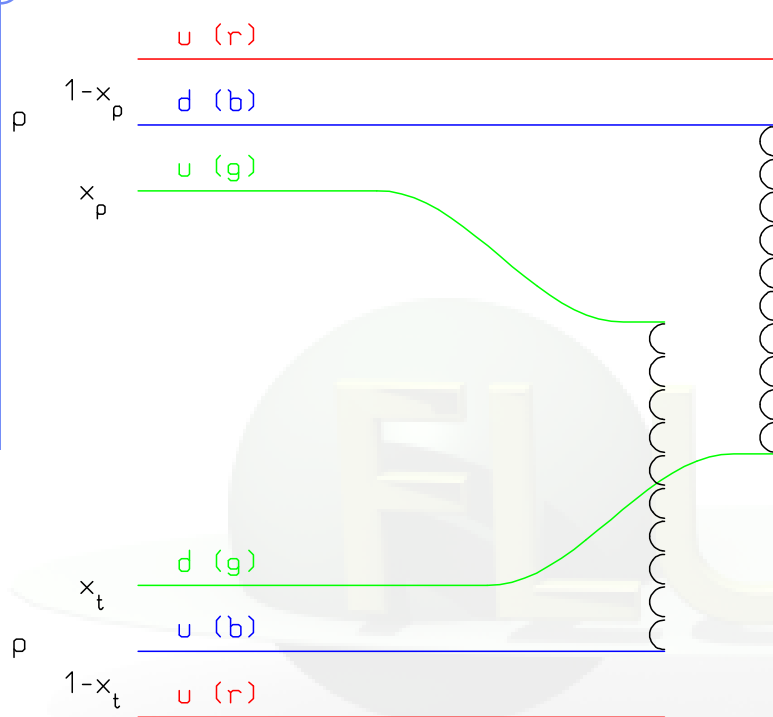


Leading two-chain diagram in DPM for p-p scattering. The color (red, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities
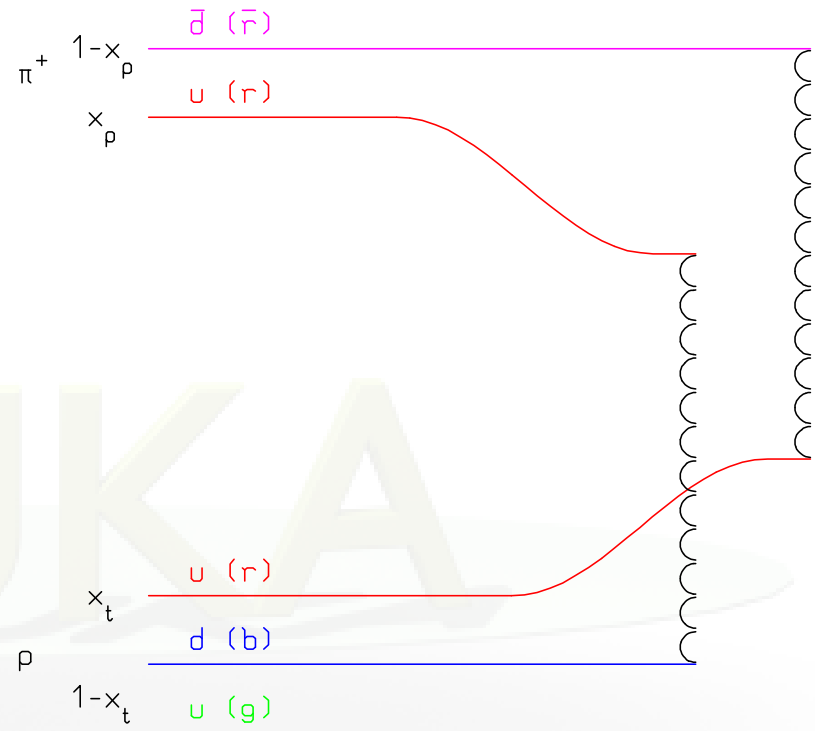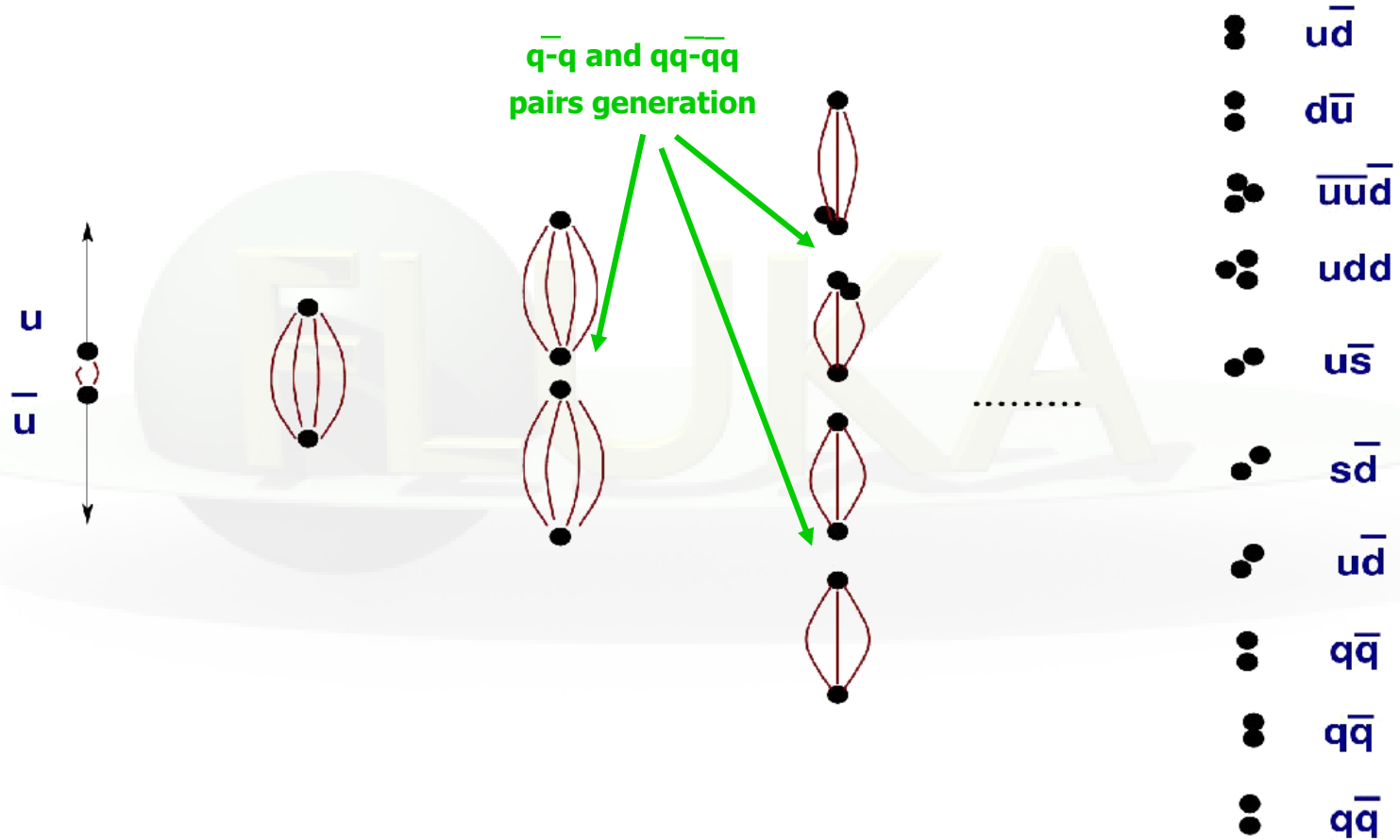
Leading two-chain diagram in DPM for $\pi^+$-p scattering. The color (red, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities

# Hadronization example



$\bar{q}$-q and qq-$\overline{qq}$ pairs generation

$u\bar{d}$
$d\bar{u}$
$\overline{uu}\bar{d}$
udd
$u\bar{s}$
$s\bar{d}$
$u\bar{d}$
$q\bar{q}$
$q\bar{q}$
$q\bar{q}$

9

# DPM and hadronization

### from DPM:

- Number of chains
- Chain composition
- Chain energies and momenta
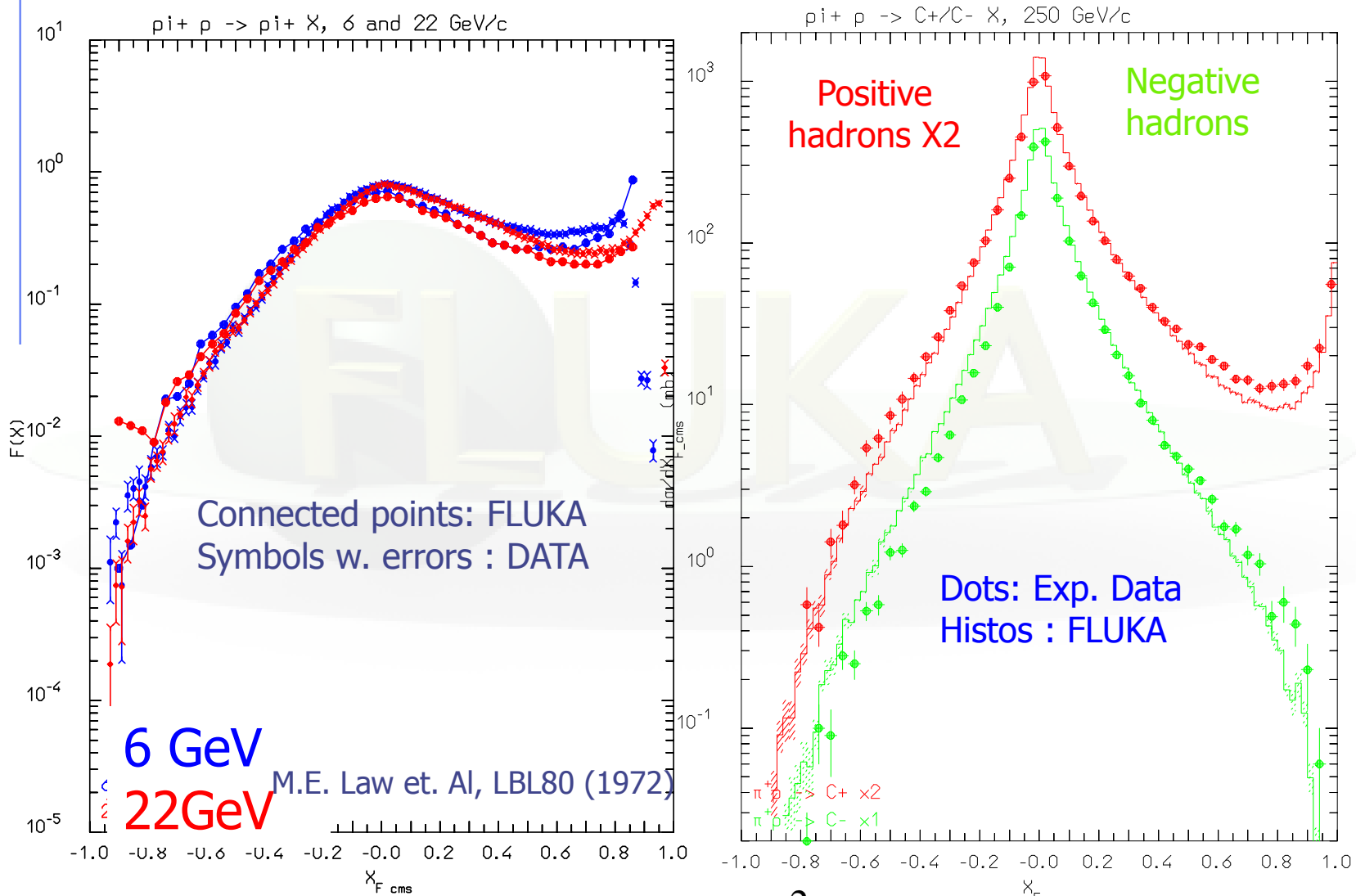- Diffractive events

**Almost No Freedom**

### Chain hadronization

- Assumes chain universality
- Fragmentation functions from hard processes and $e+e-$ scattering
- Transverse momentum from uncertainty considerations
- Mass effects at low energies

**The same functions and (few) parameters for all reactions and energies**

# Inelastic hN interactions: examples

$\pi^+ + p \rightarrow \pi^+ + X$ (6 & 22 GeV/c)    $\pi^+ + p \rightarrow$ Ch$^+$/Ch$^-$ + X (250 GeV/c)



Connected points: FLUKA
Symbols w. errors : DATA

6 GeV
22GeV
M.E. Law et. Al, LBL80 (1972)

Positive hadrons X2

Negative hadrons

Dots: Exp. Data
Histos : FLUKA

$$x_F = \frac{p_L}{p_{L\max}} = \frac{2p_L}{\sqrt{s}}$$

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# Nuclear interactions in PEANUT:

**Target nucleus description (density, Fermi motion, etc)**

↓

**Glauber-Gribov cascade with formation zone**

↓

**Generalized IntraNuclear cascade**

↓

**Preequilibrium stage with current exciton configuration and excitation energy**
(all non-nucleons emitted/decayed + all nucleons below 30-100 MeV)

↓

**Evaporation/Fragmentation/Fission model**

↓

**γ deexcitation**

t (s)

$10^{-23}$

$10^{-22}$

$10^{-20}$

$10^{-16}$

# Nucleon Fermi Motion

- Fermi gas model: Nucleons = Non-interacting Constrained Fermions

  Momentum distribution $\propto \dfrac{dN}{dk} = \dfrac{|k|^2}{2\pi^2}$

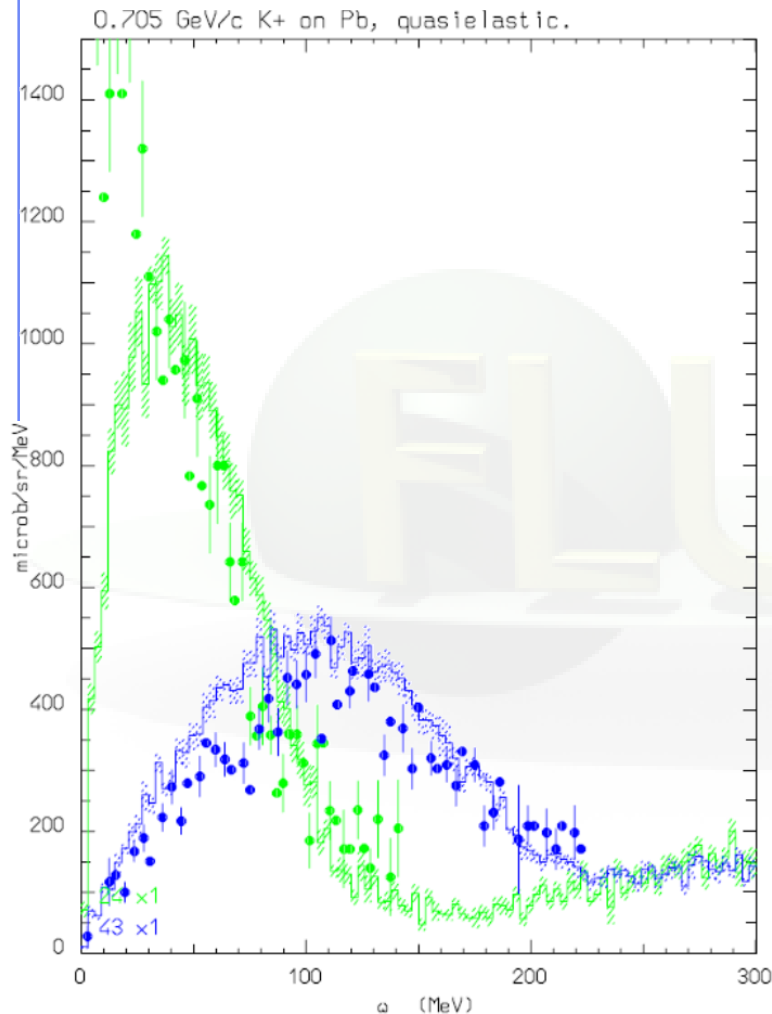  for *k* up to a (local) Fermi momentum *$k_F(r)$* given by

$$k_F(r) = \left[3\pi^2 \rho_N(r)\right]^{\frac{1}{3}}$$

The Fermi energy ($k_F \approx 1.36$ fm, $P_F \approx 260$ MeV/c, $E_F \approx 35$ MeV, at nuclear max. density) is customarily used in building a self-consistent Nuclear Potential

Depth of the potential well $\equiv$ Fermi Energy + Nuclear Binding Energy

# Positive kaons as a probe of Fermi motion



0.705 GeV/c K+ on Pb, quasielastic.

$K^+$    $K^0$

No low mass S=1 baryons $\rightarrow$
weak $K^+$N interaction
only elastic and ch. exch. up to
$\approx 800$ MeV/c

$(K^+, K^{+\prime})$ on Pb vs residual excita-
tion, 705 MeV/c, at $24°$ and $43°$.
Histo: FLUKA, dots: data (Phys:
Rev. C51, 669 (1995))

On free nucleon: recoil energy :
43 MeV at $24°$ , 117 MeV at $43°$.

# (Generalized) IntraNuclear Cascade

- Primary and secondary particles moving in the nuclear medium
- Target nucleons motion and nuclear well according to the Fermi gas model
- Interaction probability
  $$\sigma_{free} + \text{Fermi motion} \times \rho(r) + \text{exceptions (ex. } \pi)$$
- Glauber cascade at higher energies
- Classical trajectories (+) nuclear mean potential (resonant for $\pi$)
- Curvature from nuclear potential $\rightarrow$ refraction and reflection
- Interactions are incoherent and uncorrelated
- Interactions in projectile-target nucleon CMS $\rightarrow$ Lorentz boosts
- Multibody absorption for $\pi$, $\mu^-$, $K^-$
- Quantum effects (Pauli, formation zone, correlations...)
- Exact conservation of energy, momenta and all addititive quantum numbers, including nuclear recoil

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# hA at high energies: Glauber-Gribov cascade with formation zone

- ## Glauber cascade

  - Quantum mechanical method to compute Elastic, Quasi-elastic and Absorption hA cross sections from Free hadron-nucleon scattering + nuclear ground state
  - Multiple Collision expansion of the scattering amplitude

- ## Glauber-Gribov

  - Field theory formulation of Glauber model
  - Multiple collisions $\leftrightarrow$ Feynman diagrams
  - High energies: exchange of one or more Pomerons with one or more target nucleons (a closed string exchange)

- ## Formation zone (=materialization time)

# Glauber Cascade

Quantum mechanical method to compute all relevant hadron-nucleus cross sections from hadron-nucleon scattering:

$$S_{hN}(\vec{b},s) = e^{i\chi_{hN}(\vec{b},s)} = \eta_{hN}(\vec{b},s)e^{2i\delta_{hN}(\vec{b},s)}$$

and nuclear ground state wave function $\Psi_i$

**Total**
$$\sigma_{hAT}(s) = 2\int d^2\vec{b}\int d^3\vec{u}\,\left|\Psi_i(\vec{u})\right|^2\left[1 - \prod_{j=1}^{A}\mathrm{Re}\,S_{hN}(\vec{b}-\vec{r}_{j\perp},s)\right]$$

**Elastic**
$$\sigma_{hA\,el}(s) = \int d^2\vec{b}\left|\int d^3\vec{u}\,\left|\Psi_i(\vec{u})\right|^2\left[1 - \prod_{j=1}^{A}S_{hN}(\vec{b}-\vec{r}_{j\perp},s)\right]\right|^2$$

**Scattering**
$$\sigma_{hA\,\Sigma f}(s) \equiv \sum_{f}\sigma_{hA\,fi}(s) = \int d^2\vec{b}\int d^3\vec{u}\,\left|\Psi_i(\vec{u})\right|^2\left|\left|1 - \prod_{j=1}^{A}S_{hN}(\vec{b}-\vec{r}_{j\perp},s)\right|\right|^2$$

**Absorption (particle prod.)**

$$\sigma_{hA\,abs}(s) \equiv \sigma_{hA\,T}(s) - \sigma_{hA\,\Sigma f}(s)$$

$$= \int d^2\vec{b}\int d^3\vec{u}\,\left|\Psi_i(\vec{u})\right|^2\left\{1 - \left\{\prod_{j=1}^{A}1 - \left[1 - \left|S_{hN}(\vec{b}-\vec{r}_{j\perp},s)\right|^2\right]\right\}\right\}$$

Absorption probability over a given *b* and nucleon configuration

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# Gribov interpretation of Glauber multiple collisions

Therefore the absorption cross section is just the integral in the impact parameter plane of the probability of getting at least one non-elastic hadron-nucleon collision

and the overall average number of collision is given by

$$\langle \nu \rangle = \frac{Z\sigma_{hpr} + N\sigma_{hnr}}{\sigma_{hAabs}}$$

- Glauber-Gribov model = Field theory formulation of Glauber model
- Multiple collision terms $\Rightarrow$ Feynman graphs
- At high energies : exchange of one or more pomerons with one or more target nucleons

- In the Dual Parton Model language: (neglecting higher order diagrams):
  Interaction with **n** target nucleons $\Rightarrow$ **2n** chains
  - Two chains from projectile valence quarks + valence quarks of one target nucleon $\Rightarrow$ valence-valence chains
  - 2(n-1) chains from sea quarks of the projectile + valence quarks of target nucleons $\Rightarrow$ 2(n-1) sea-valence chains

# Chain examples
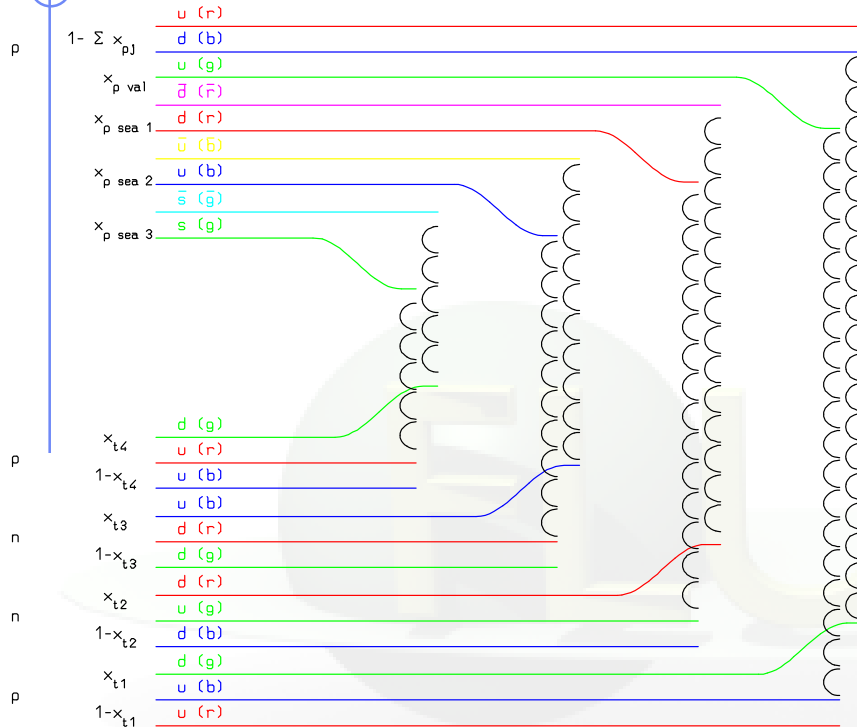


Leading two-chain diagram in DPM for **p-A Glauber scattering** with 4 collisions

The color (red, antired, blue, antiblue, green, and antigreen) and quark combination shown in the figure is just one of the allowed possibilities
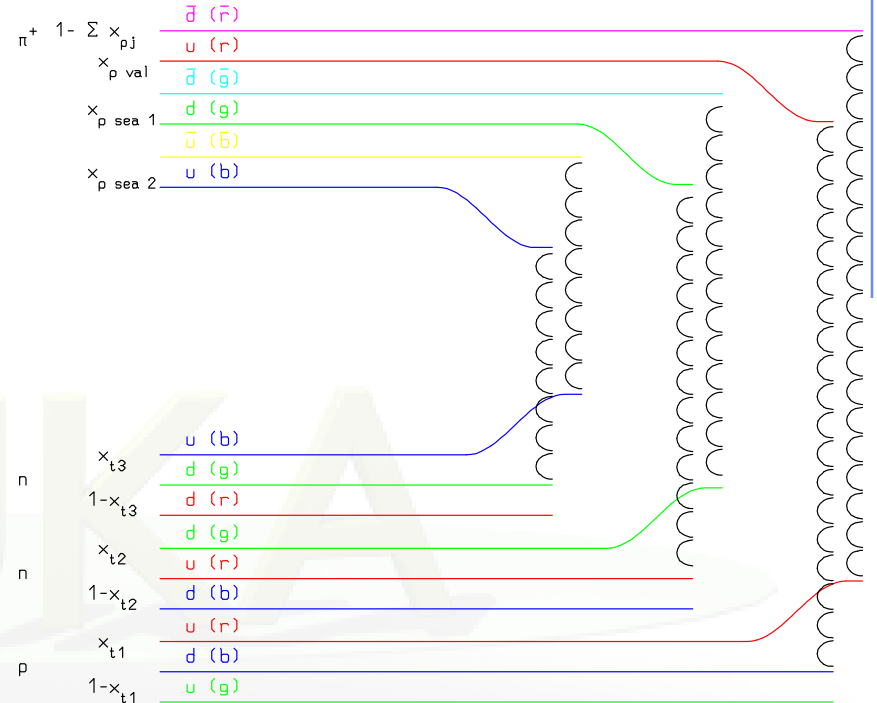
Leading two-chain diagram in DPM for $\pi^+$**-A Glauber scattering** with 3 collisions

The color (red, antired, blue, antiblue, green, and antigreen) and quark combination shown in the figure is just one of the allowed possibilities

# Formation zone

Naively: "materialization" time (originally proposed by Stodolski).
Qualitative estimate:

In the frame where *p// =0*

$$\bar{t} = \Delta t \approx \frac{\hbar}{E_T} = \frac{\hbar}{\sqrt{p_T^2 + M^2}}$$

Particle proper time

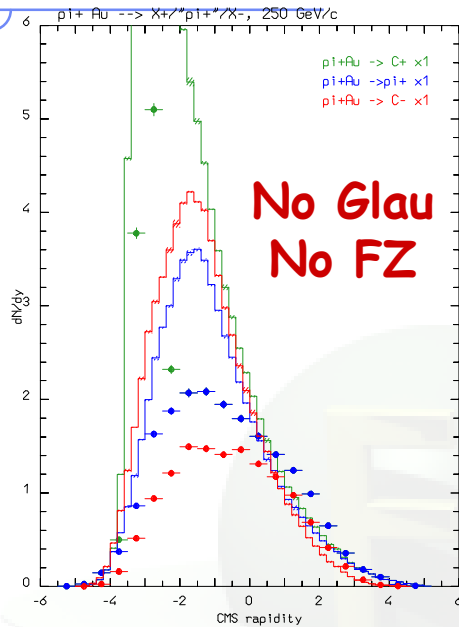$$\tau = \frac{M}{E_T}\bar{t} = \frac{\hbar M}{p_T^2 + M^2}$$

Going to the nucleus system

$$\Delta x_{for} \equiv \beta\, c \cdot t_{lab} \approx \frac{p_{lab}}{E_T}\bar{t} \approx \frac{p_{lab}}{M}\tau = k_{for}\frac{\hbar p_{lab}}{p_T^2 + M^2}$$

Condition for possible reinteraction inside a nucleus:
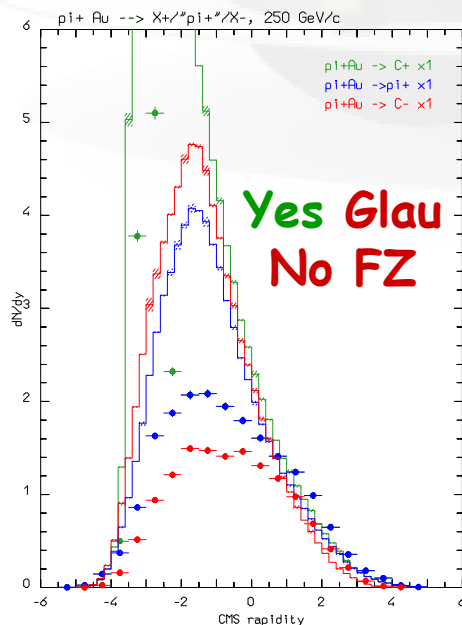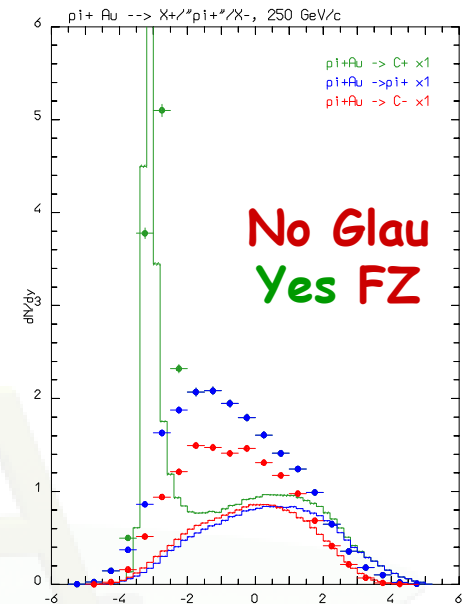
$$\Delta x_{for} \leq R_A \approx r_0 A^{\frac{1}{3}}$$
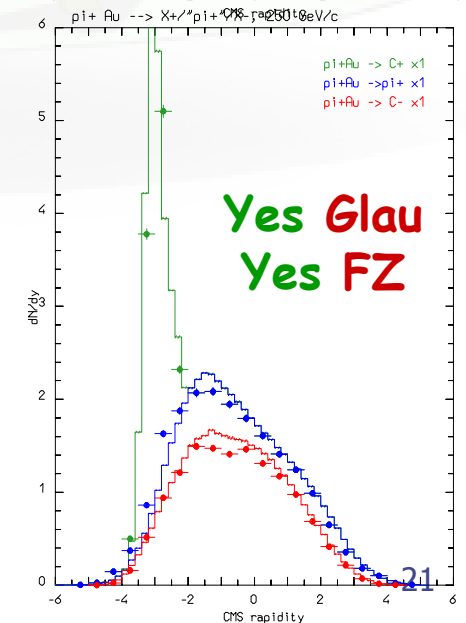
# Effect of Glauber and Formation Zone

Rapidity distribution of charged particles produced in 250 GeV $\pi^+$ collisions on Gold

Points: exp. data (Agababyan et al., ZPC50, 361 (1991)).

( rapidity $\approx$ -ln(tg($\theta$/2)) )

Large Effects on:
Multiplicity
Energy distribution
Angular distribution

**No Glau No FZ**

**No Glau Yes FZ**

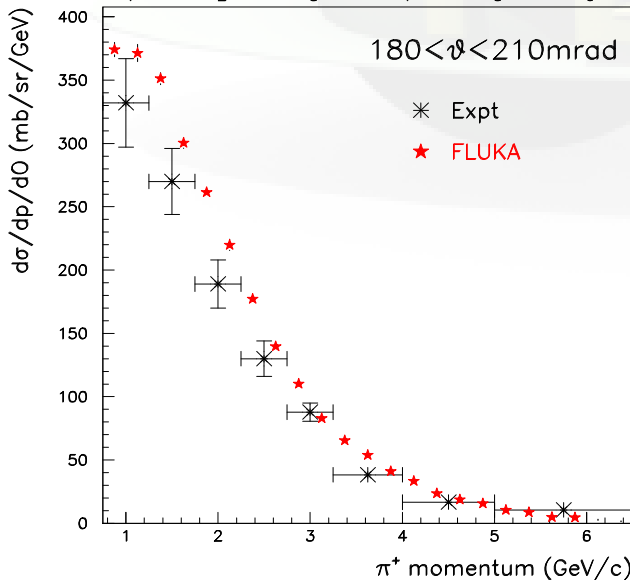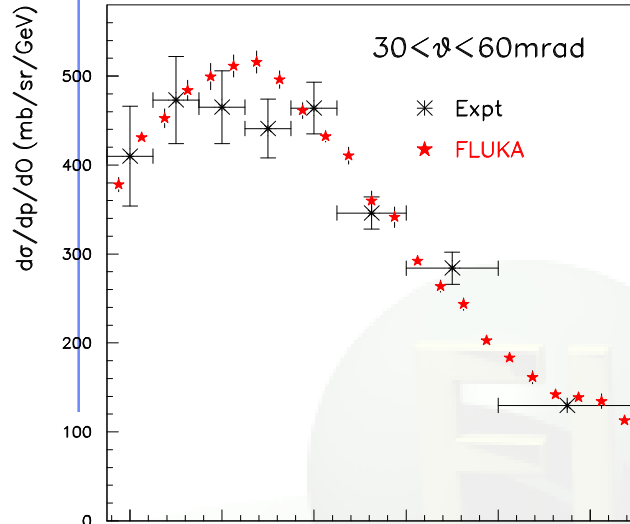**Yes Glau No FZ**

**Yes Glau Yes FZ**

# WARNING

- PEANUT has been extended to cover the whole energy range in late 2006

- A less refined GINC model is available for projectile momenta above about 5 GeV/c, and was the only choice until recently

- However: the extended peanut is NOT yet the default, mainly because of some ambiguity in the definition of quasi-elastic cross sections.

- Whenever activation is a concern or, "precise" particle production calculations in the energy range above 5 GeV are required, the PEANUT extended model should be switched on

- To activate PEANUT at all energies:

```
PHYSICS 1000.  1000.  1000.  1000.  1000.  1000.  PEATHRESH
```
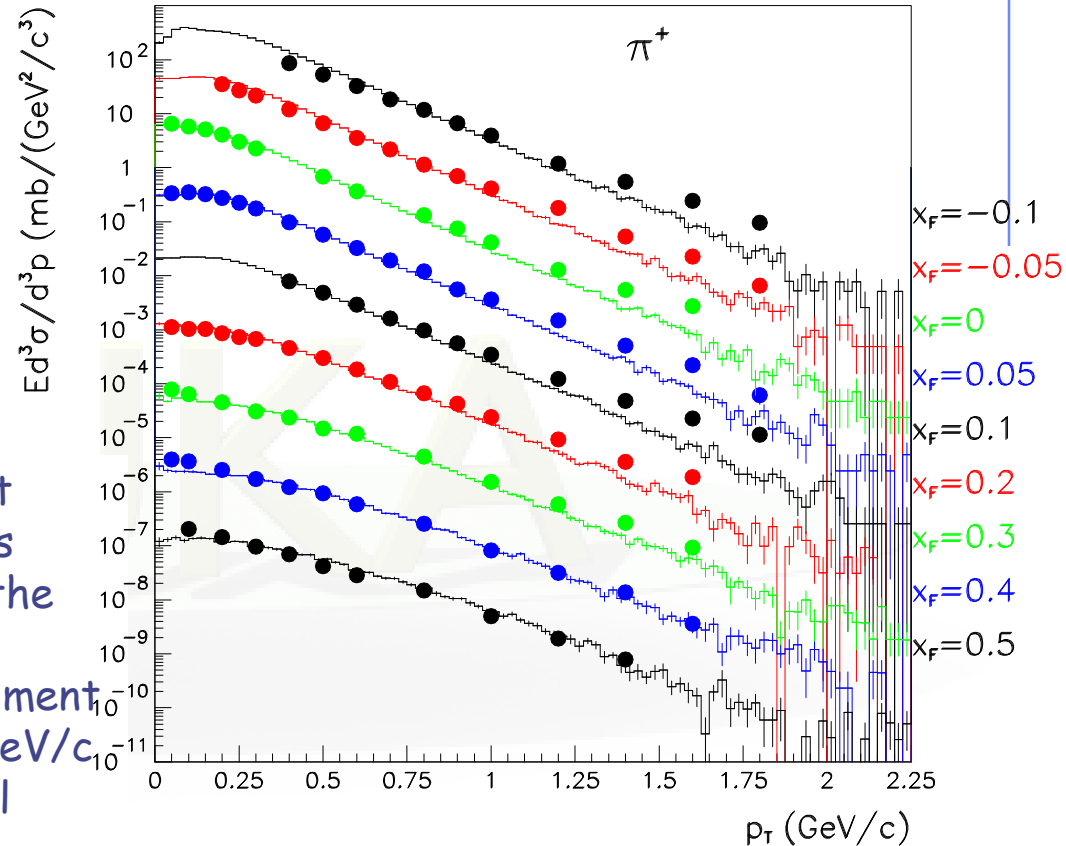
# Nonelastic hA interactions at high energies: examples

Harp expt. , 12.9 GeV/c p on Al



$30 < \vartheta < 60$ mrad

* Expt
* FLUKA

$d\sigma/dp/d\Omega$ (mb/sr/GeV)

$180 < \vartheta < 210$ mrad

* Expt
* FLUKA

$\pi^+$ momentum (GeV/c)

NA49 expt. , 158 GeV/c p on C



$\pi^+$

$E d^3\sigma/d^3p$ (mb/(GeV$^2$/c$^3$))

$x_F = -0.1$
$x_F = -0.05$
$x_F = 0$
$x_F = 0.05$
$x_F = 0.1$
$x_F = 0.2$
$x_F = 0.3$
$x_F = 0.4$
$x_F = 0.5$

$p_T$ (GeV/c)

Recent results from the HARP experiment 12.9 GeV/c p on Al $\pi^+$ production at different angles

Double differential $\pi$+ production for p C interactions at 158 GeV/c, as measured by NA49 (symbols) and predicted by FLUKA (histograms)

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# Double differential pion production



Be(p,pi+x) @ 17.5 GeV/c

Au(p,pi+x) @ 17.5 GeV/c

0.95 ×16
0.85 ×8
0.75 ×4
0.65 ×2
0.55 ×1
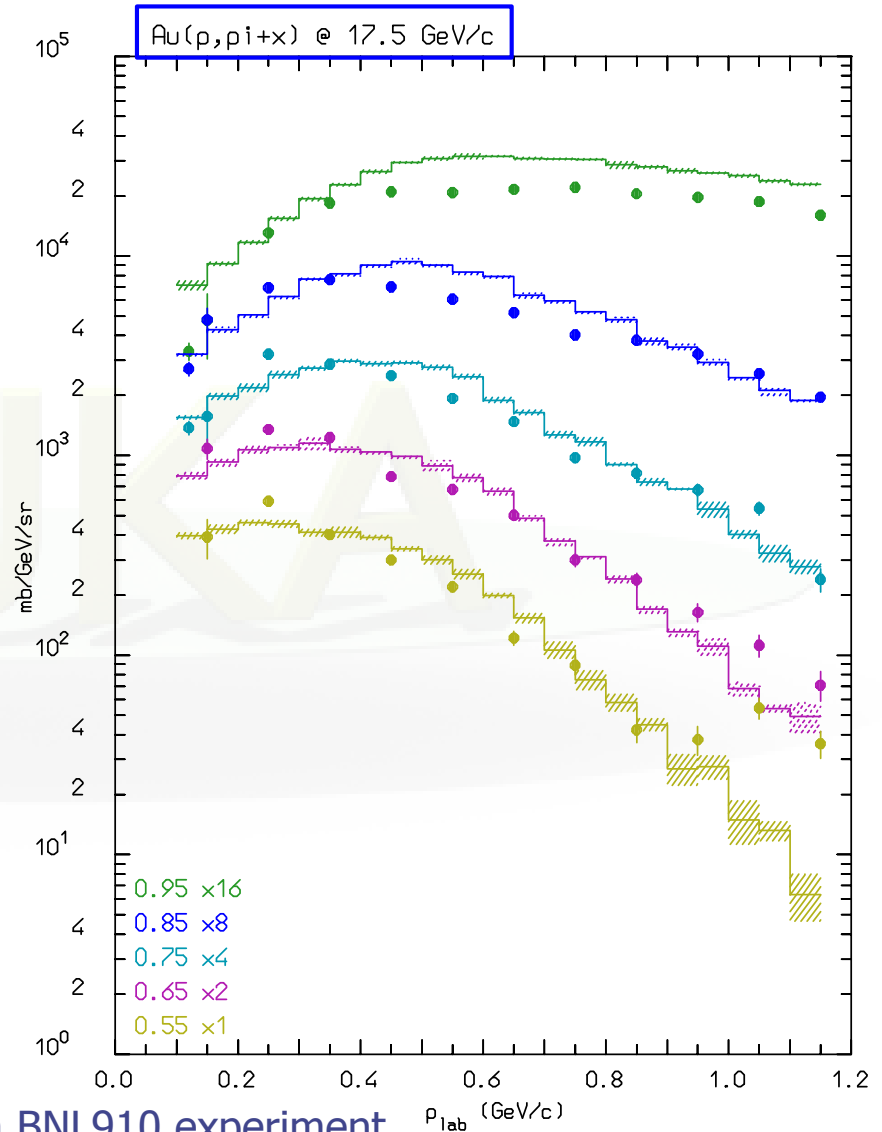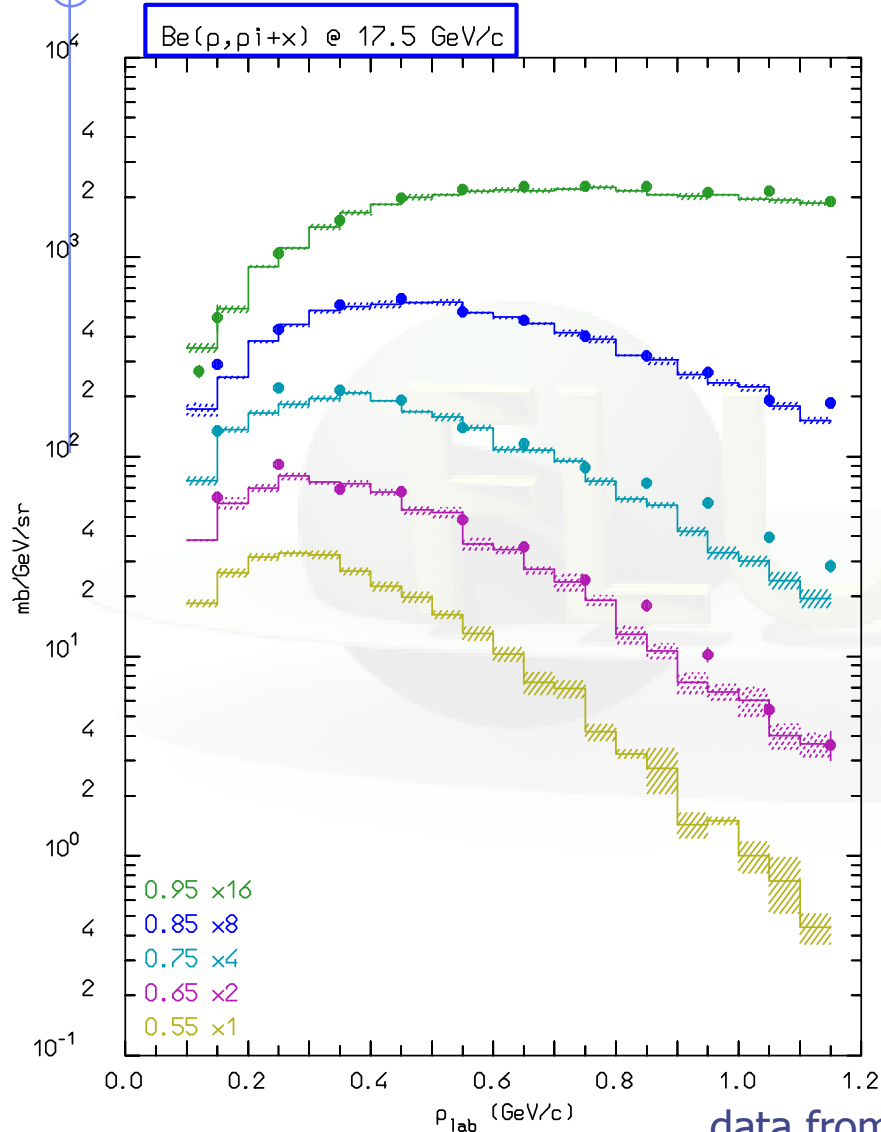
data from BNL910 experiment

# Pions: nuclear medium effects

Free π N interactions ⟹

⟹ Non resonant channel

⟹ P-wave resonant Δ production

Δ in nuclear medium ⟹ decay ⟹ elastic scattering, charge exchange

⟹ reinteraction ⟹ Multibody pion absorption

**Assuming for the free resonant σ a Breit-Wigner form with width Γ_F**

$$\sigma_{res}^{Free} = \frac{8\pi}{p_{cms}^2} \frac{M_\Delta^2 \Gamma_F^2(p_{cms})}{\left(s - M_\Delta^2\right)^2 + M_\Delta^2 \Gamma_F^2(p_{cms})}$$

**An ``in medium'' resonant σ ($\sigma_{res}^A$) can be obtained adding to Γ_F the imaginary part of the (extra) width arising from nuclear medium**

$$\tfrac{1}{2}\Gamma_T = \tfrac{1}{2}\Gamma_F - \mathrm{Im}\,\Sigma_\Delta \qquad \Sigma_\Delta = \Sigma_{qe} + \Sigma_2 + \Sigma_3$$

**(Oset et al., NPA 468, 631)**

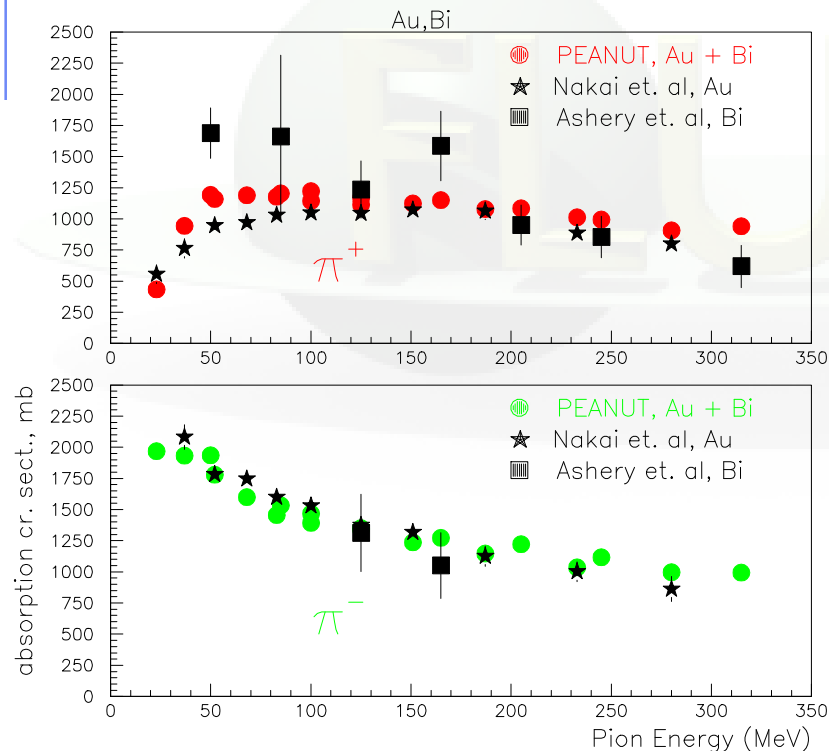**quasielastic scattering, two and three body absorption**

**The in-nucleus $\sigma_t^A$ takes also into account a two-body s-wave absorption $\sigma_s^A$ derived from the optical model**

$$\sigma_t^A = \sigma_{res}^A + \sigma_t^{Free} - \sigma_{res}^{Free} + \sigma_s^A \qquad \sigma_s^A(\omega) = \tfrac{4\pi}{p}\left(1 + \tfrac{\omega}{2m}\right)\mathrm{Im}\,B_0(\omega)\rho$$

# Pion absorption

Pion absorption cross section on Gold and Bismuth in the $\Delta$ resonance region (multibody absorption in PEANUT)
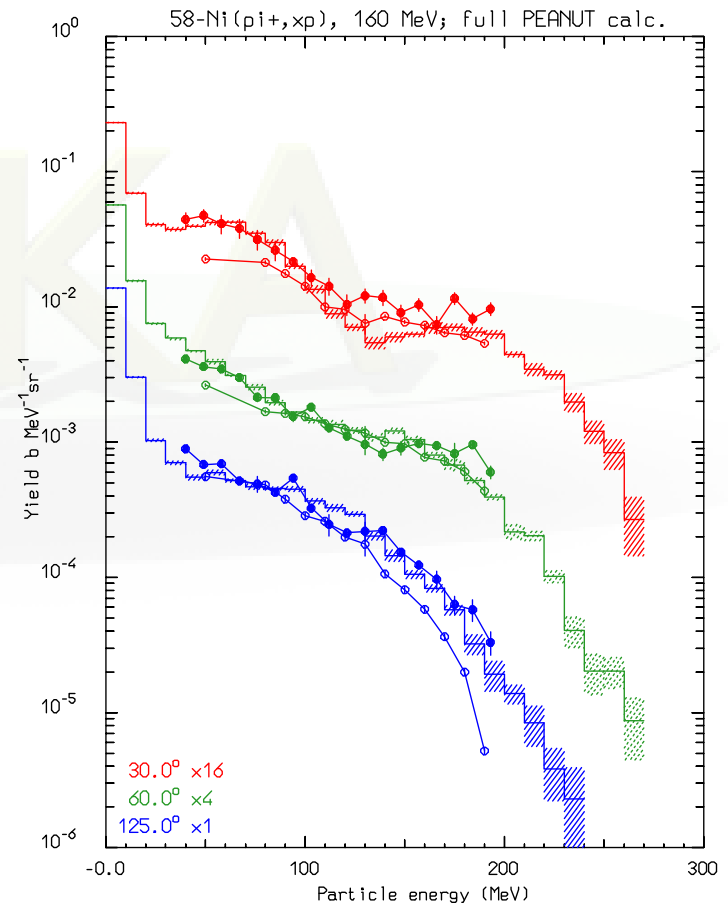
# Preequilibrium emission

For $E > \pi$ production threshold $\rightarrow$ only (G)INC models
At lower energies a variety of preequilibrium models

## Two leading approaches

The quantum-mechanical multistep model:
Very good theoretical background
Complex, difficulties for multiple emissions

The semiclassical exciton model
Statistical assumptions
Simple and fast
Suitable for MC

**Statistical assumption:**
any partition of the excitation energy $E^*$ among N, $N = N_h + N_p$, excitons has the same probability to occur
Step: nucleon-nucleon collision with $N_{n+1} = N_n + 2$ ("never come back approximation)
Chain end = equilibrium = $N_n$ sufficiently high or excitation energy below threshold

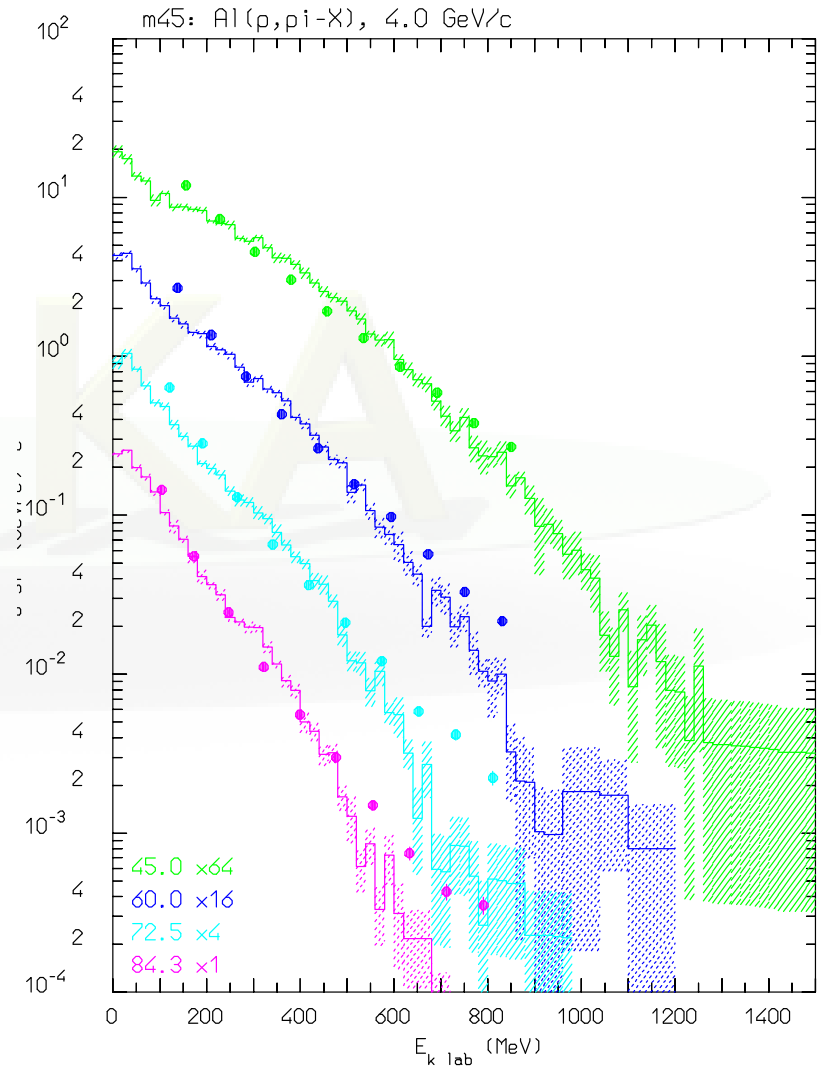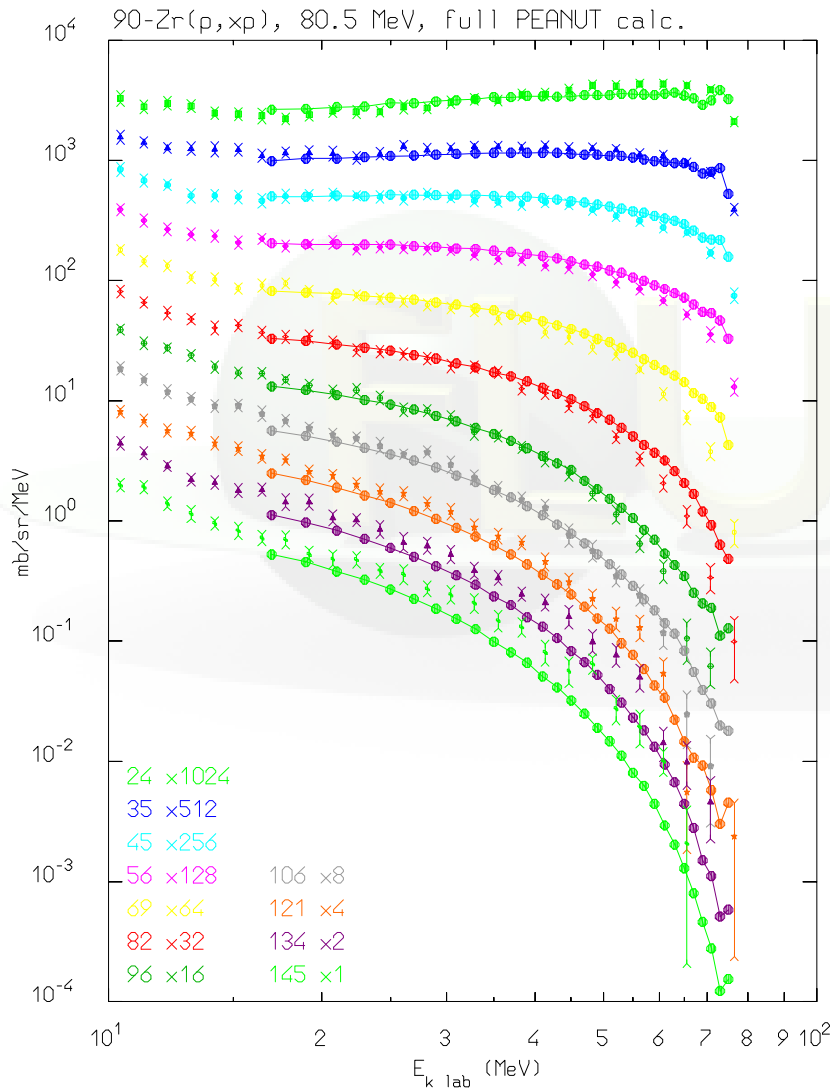*N$_1$ depends on the reaction type and cascade history*

# Thin target example
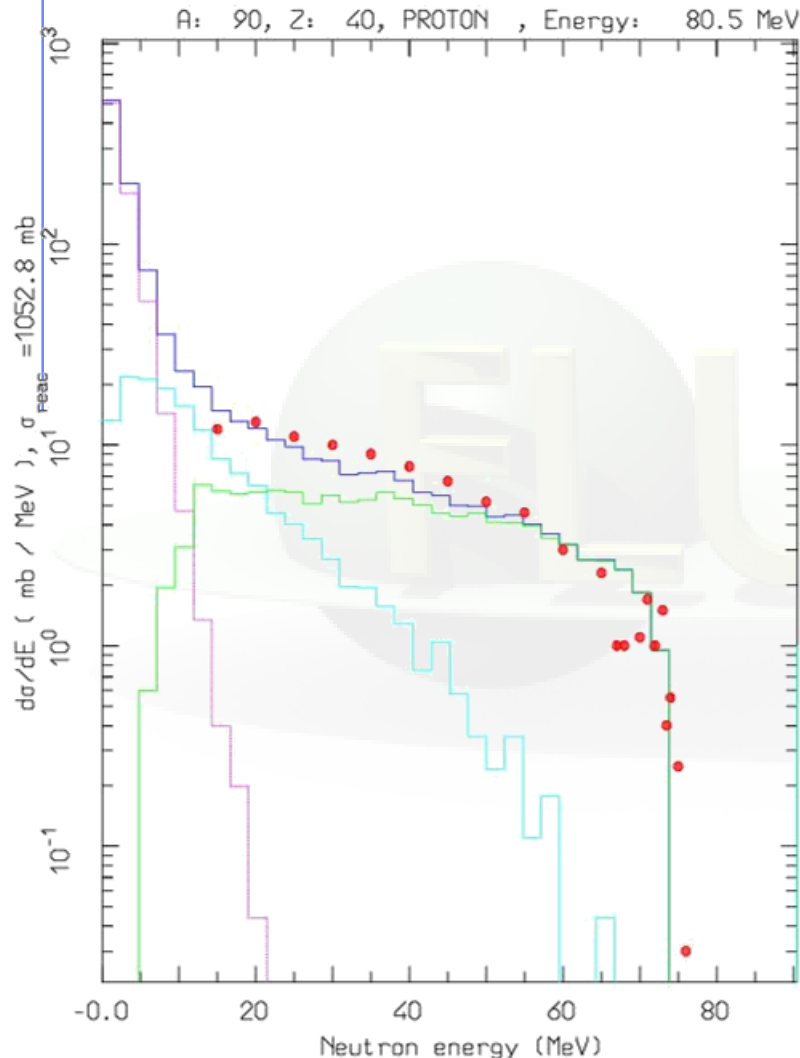


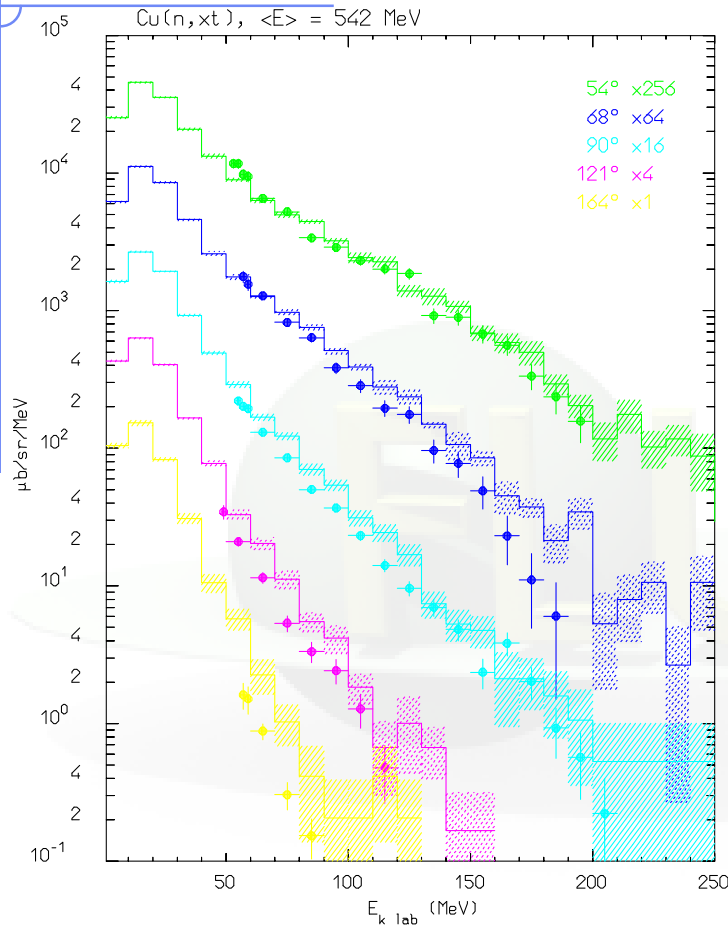Angle-integrated $^{90}$Zr(p,xn) at 80.5 MeV

The various lines show the total, INC, preequilibrium and evaporation contributions

Experimental data from M. Trabandt et al., Phys. Rev. C39, 452 (1989)

# Thin target examples

p + $^{90}$Zr $\rightarrow$ p + X (80 MeV)      p + Al $\rightarrow$ $\pi^-$ + X (4 GeV/c)

# Coalescence



Cu(n,xt), <E> = 542 MeV

54° ×256
68° ×64
90° ×16
121° ×4
164° ×1

μb/sr/MeV

$E_{k\ lab}$ (MeV)

High energy light fragments are emitted through the coalescence mechanism: "put together" emitted nucleons that are near in phase space.

Example : double differential t production from 542 MeV neutrons on Copper

**Warning**: coalescence is OFF by default
Can be important, ex for . residual nuclei.
To activate it:

```
PHYSICS 1.                                    COALESCE
```

If coalescence is on, switch on Heavy ion transport and interactions (see later)

# Equilibrium particle emission (evaporation, fission and nuclear break-up)

From statistical considerations and the detailed balance principle, the probabilities for emitting a particle of mass $m_j$, spin $S_j \hbar$ and energy E, or of fissioning are given by:

(i, f for initial/final state, Fiss for fission saddle point)

**Probability per unit time of emitting a particle j with energy E**

$$P_J = \frac{(2S_j + 1) m_j c}{\pi^2 \hbar^3} \int_{V_j}^{U_i - Q_j - \Delta_f} \frac{\rho_f(U_f)}{\rho_i(U_i)} \sigma_{inv}(E) E \, dE$$

**Probability per unit time of fissioning**

$$P_{Fiss} = \frac{1}{2\pi\hbar} \int_0^{U_i - B_{Fiss}} \frac{\rho_{Fiss}(U_i - B_{Fiss} - E)}{\rho_i(U_i)} dE$$

- ρ's: nuclear level densities
- U's: excitation energies
- $V_j$'s: possible Coulomb barrier for emitting a particle type j
- $B_{Fiss}$: fission barrier

- $Q_j$'s: reaction Q for emitting a particle type j
- $\sigma_{inv}$: cross section for the inverse process
- Δ's: pairing energies

*Neutron emission is strongly favoured because of the lack of any barrier*
*Heavy nuclei generally reach higher excitations because of more intense cascading*

# Equilibrium particle emission

- Evaporation: Weisskopf-Ewing approach
  - ~600 possible emitted particles/states (A<25) with an extended evaporation/fragmentation formalism
  - Full level density formula with level density parameter A,Z and excitation dependent
  - Inverse cross section with proper sub-barrier
  - Analytic solution for the emission widths (neglecting the level density dependence on U, taken into account by rejection
  - Emission energies from the width expression with no. approx.
- Fission: past, improved version of the Atchison algorithm, now
  - $\Gamma_{fis}$ based of first principles, full competition with evaporation
  - Improved mass and charge widths
  - Myers and Swiatecki fission barriers, with exc. en. Dependent level density enhancement at saddle point
- Fermi Break-up for A<18 nuclei
  - ~ 50000 combinations included with up to 6 ejectiles
- $\gamma$ de-excitation: statistical + rotational + tabulated levels
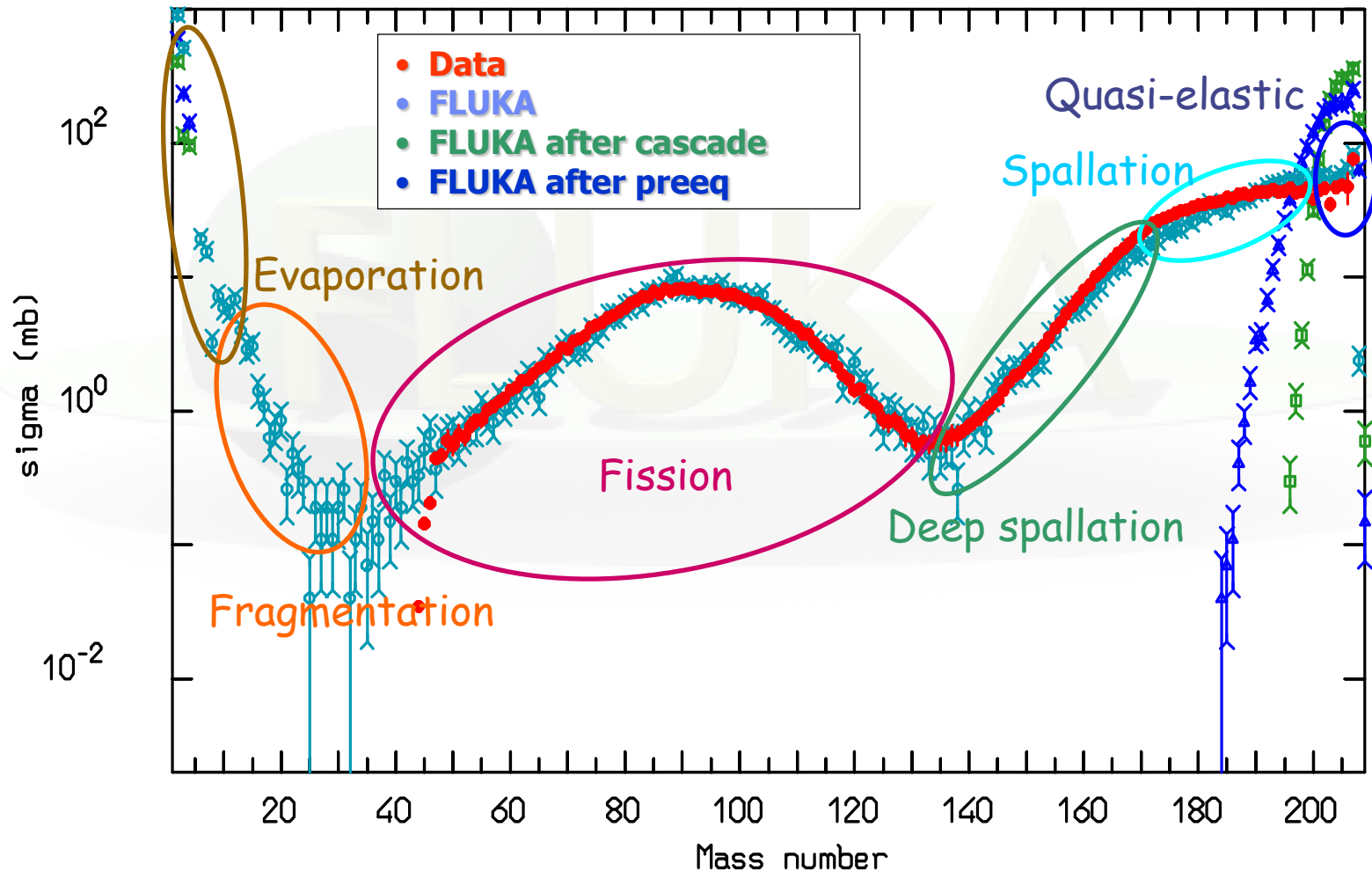
# Residual Nuclei

- The **production of residuals** is the result of the **last step** of the nuclear reaction, thus it is influenced by all the previous stages

- **Residual mass** distributions are **very well reproduced**

- Residuals near to the compound mass are usually well reproduced

- However, the production of **specific isotopes may be influenced by additional problems** which have little or no impact on the emitted particle spectra (Sensitive to details of evaporation, Nuclear structure effects, Lack of spin-parity dependent calculations in most MC models)



$Log_{10}$ N of residual nuclei

1.5 GeV p on U     N-Z

1.5 GeV p on Pb    N-Z

15 GeV p on Pb     N-Z
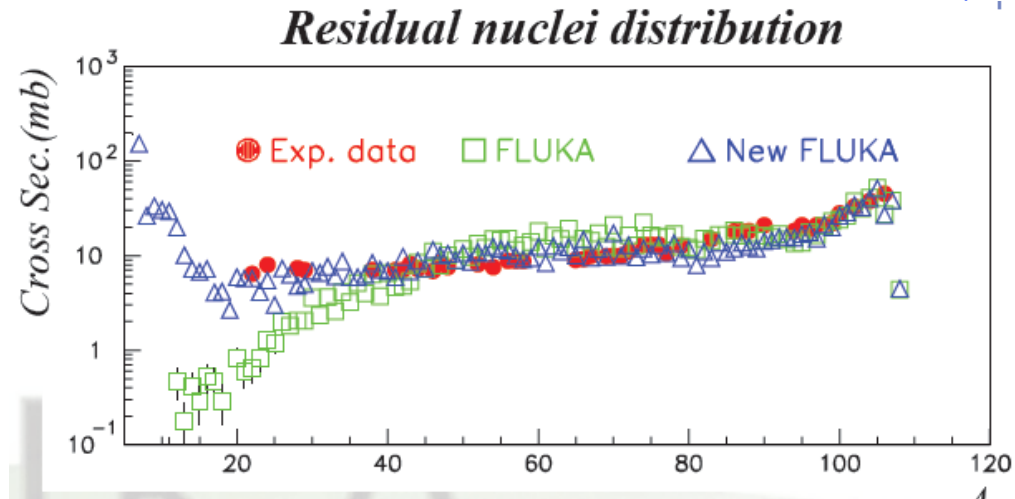
150 GeV p on Pb    N-Z
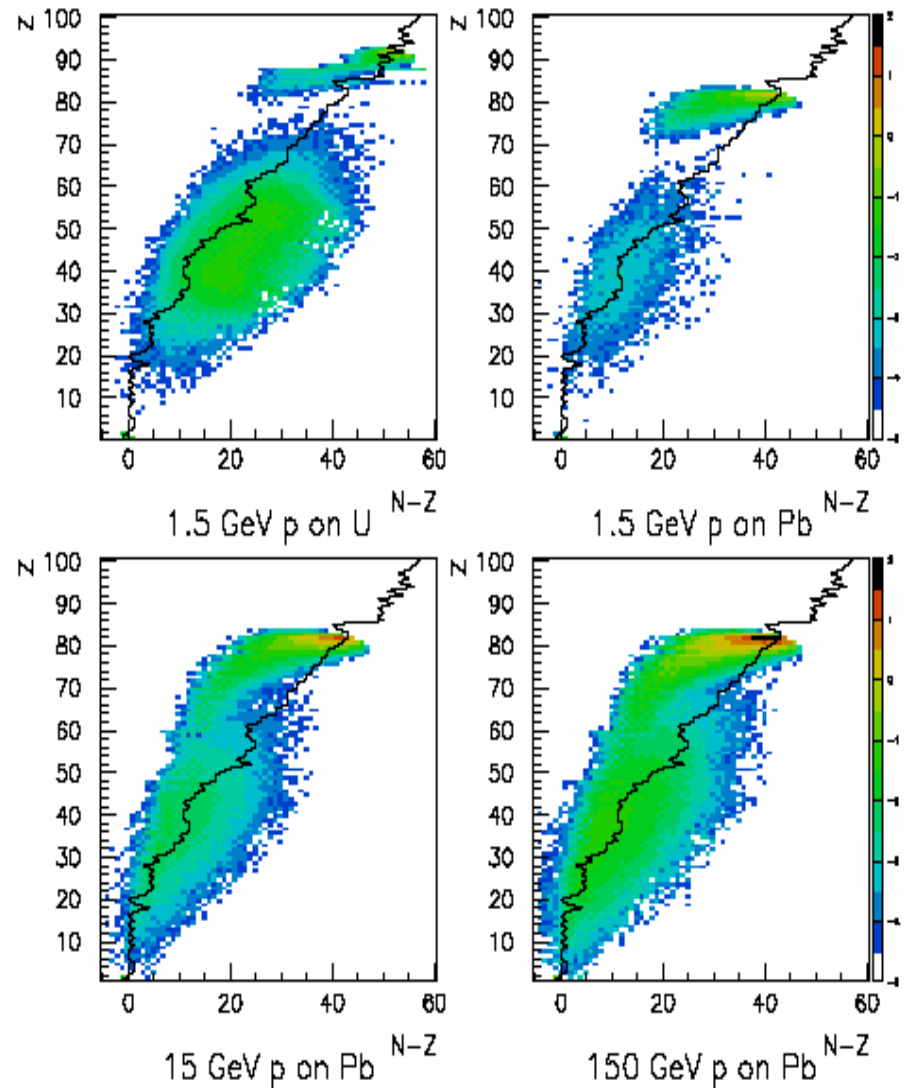
# Example of fission/evaporation

1 A GeV $^{208}$Pb + p reactions Nucl. Phys. A 686 (2001) 481-524

# Residual nuclei

Experimental and computed
residual nuclei mass distribution
for Ag(p,x)X at 300 GeV

Data from
Phys. Rev. C19 2388 (1979) and
Nucl. Phys. A543, 703 (1992)



Residual nuclei distribution

The fragmentation model has  much improved the FLUKA predictions

Also for A-A interactions

Warning: fragmentation is OFF by default, because it is a cpu-eater.
It is NECESSARY to activate it for activation studies:

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PHYSICS 3.                                                          EVAPORAT
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If fragmentation is on, switch on Heavy ion transport and interactions (see later)