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# Beam-based diagnostic of octupolar component of the fundamental mode in CLIC accelerating structures

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## Conclusions

The damping waveguides of the accelerating structures of CLIC give rise to a four-fold symmetry that allows for a non-zero octupolar component of the fundamental accelerating mode. The effect of this octupolar component has been previously observed and simulated. Here we present some results on measuring the strength of the octupolar component by measuring the position shifts due to octupolar kicks during a vertical scan of the beam. The resulted integrated octupolar strength can be found by third order fits and the measurement resulted in an octupolar strength of  $-14.8 \text{ kTm/m}^3$  and  $-18.5 \text{ kTm/m}^3$ , which is off by about a factor 2 compared to RF simulations. More work is needed on the analysis and simulations before we have a full consistent picture.

## Introduction

Following the LHC experiments there is need for precision measurements of the results found. A multi-TeV lepton collider would be a suitable machine for such a task. One proposed future electron-positron collider is the compact linear collider (CLIC) developed at CERN.

CLIC is based on X-band normal conducting technology and uses a novel two-beam accelerating scheme to accelerating electrons and positrons before collision. Some key parameters:

- Collision energy 3 TeV
- Luminosity  $2 \times 10^{-34} \text{ cm}^{-2} \text{ s}^{-1}$
- X-band technology 12 GHz
- Accelerating gradient 100 MV/m
- ~140,000 accelerating structures
- Two-beam acceleration scheme
- Accelerating gradient 100 MV/m

The large number of accelerating structures makes it crucial to understand the behavior of these structures and how the beam is affected. In this experiment we have attempted to measure the strength of an octupolar component of the fundamental accelerating mode in a CLIC accelerating structure installed and operated in the CLIC test facility 3 at CERN.

## Octupolar component

In the CLIC accelerating structures, there are four radial wave guides in each cell which are designed for damping higher order modes, see figure 4. The four-fold symmetry allows for a non-zero octupolar component of the fundamental accelerating mode. For an ultra-relativistic beam the effect of the transverse electric and magnetic fields can be combined and the total effect of the octupolar kick can be expressed in magnetic units. The strength of the octupolar component is proportional to the accelerating electric field but the RF octupole phase has 90 degree shift with respect to the accelerating mode, so that the RF octupole is at zero-crossing when accelerating mode is on crest and vice versa.

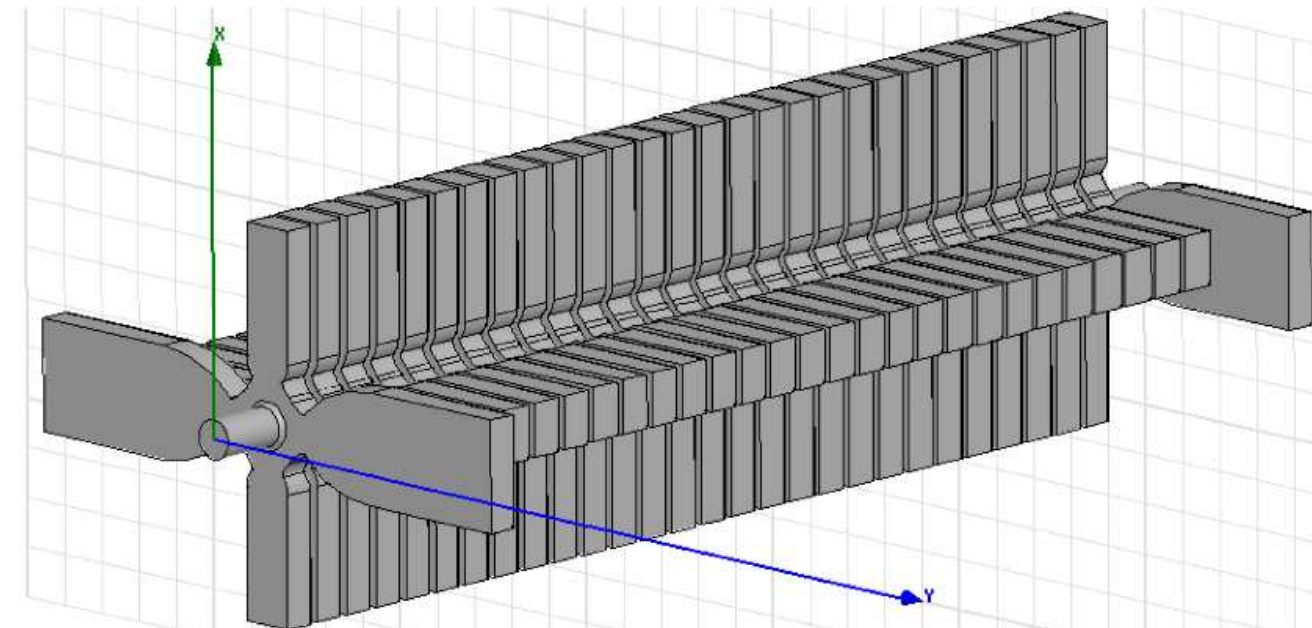


Figure 4. CLIC accelerating structure with damping wave guides.

The effect of the octupolar field was observed by sending a large beam through the structure, see figure 5.

## Position shifts

The position shift due to an octupolar field can be expressed as:

$$\hat{Y} - Y = -\frac{k_3 l}{(B\rho)} L(Y^3 + 3Y(\sigma_y^2 - \sigma_x^2 - X^2))$$

$$\hat{X} - X = -\frac{k_3 l}{(B\rho)} L(X^3 + 3X(\sigma_x^2 - \sigma_y^2 - Y^2))$$

where capital letters indicate center-of-mass of the beam,  $X = \langle x \rangle$  and  $Y = \langle y \rangle$ . Hats indicate we have the beam with the octupolar field present. The difference between the two gives the position shift. Finally,  $\sigma_x$  and  $\sigma_y$  denotes the transverse beam size.

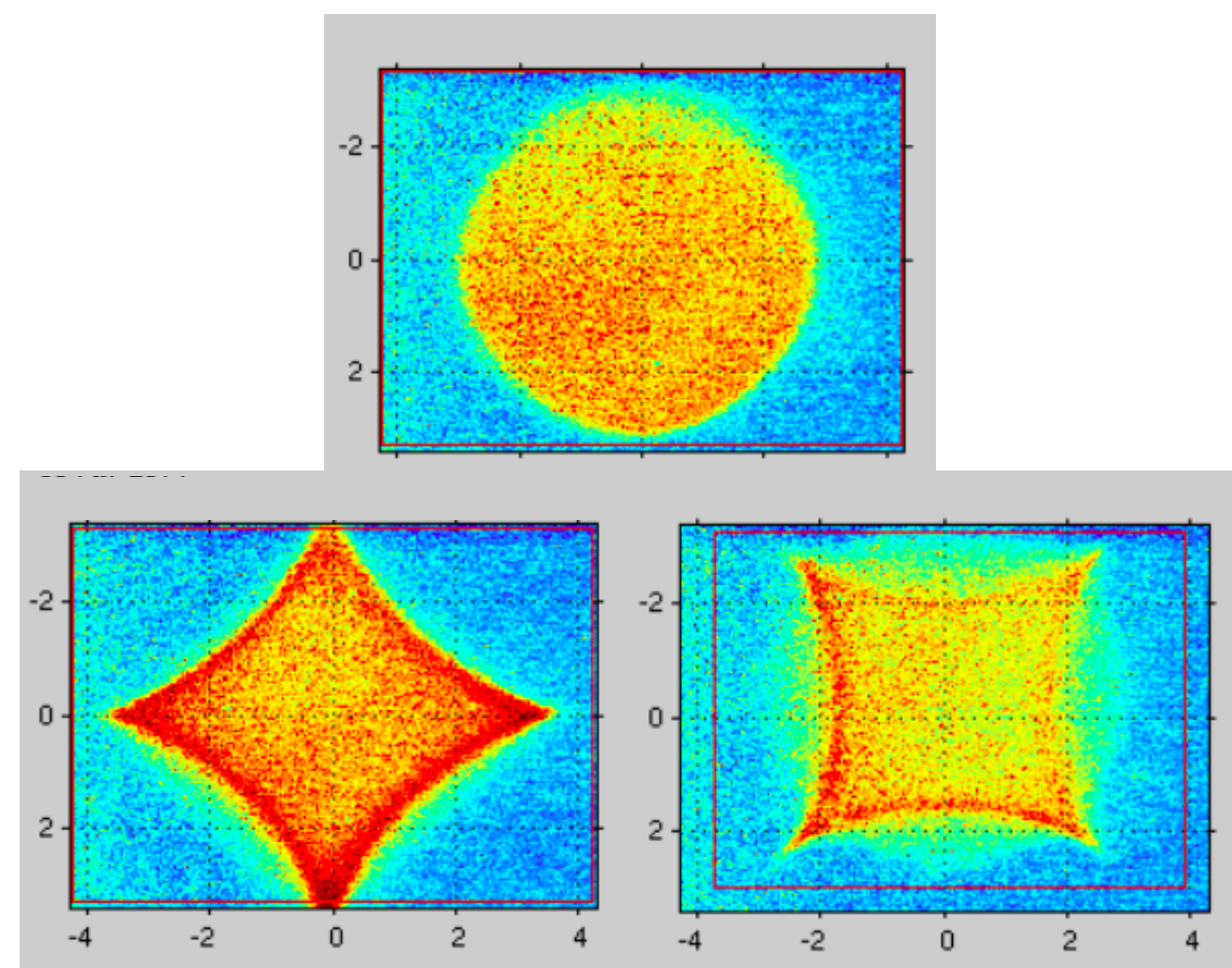


Figure 5. Top: large beam send through the structure, no RF. Bottom right: at zero-crossing towards decelerating phase. Bottom left: at zero-crossing towards accelerating phase. Courtesy of Wilfrid Farabolini, CERN.

## Discussion

This is a work in progress. There is some still inconsistency between the two fits and between the fit parameters and their physical interpretation. Due to the relative large number of parameters (4, small data set) we get a high sensitivity in the fit, i.e. a small change will change the parameter and the physical interpretation greatly. However, the most interesting parameter  $a$  in the vertical position shift determines the  $y^3$  and seems stable. Numerical RF simulations of the CLIC structure done by Alexej Grudiev at CERN shows an octupolar strength of  $-73.4 \text{ kTm/m}^3$  at  $V_{\text{acc}} = 22.8 \text{ MV}$ . During our measurements we had  $V_{\text{acc}} = 12 \text{ MV}$ , simulations (octupolar strength has linear dependence on  $V_{\text{acc}}$ ) would then suggest an octupolar component of  $-38 \text{ kTm/m}^3$ , about a factor 2 larger than the measurements.

## Appendix

Here we show in more detail the derivation of the equation for position shift due to octupole field. A single electron traveling in the negative z-direction through a transverse magnetic field gets a kick that can be calculated with the Lorentz force:

$$\vec{F} = q\vec{v} \times \vec{B} = ev\hat{z} \times (B_x\hat{x} + B_y\hat{y}) = -evB_y\hat{x} + evB_x\hat{y}.$$

If the total longitudinal length is  $l$  we get the following kicks

$$\Delta x' = \frac{p_x}{p_z} = \frac{-evB_y l}{p_z v} = -\frac{B_y l}{(B\rho)}$$

and

$$\Delta y' = \frac{p_y}{p_z} = \frac{evB_x l}{p_z v} = \frac{B_x l}{(B\rho)}.$$

A general multipole field can be written as

$$B_y + iB_x = C_n(x + iy)^{n-1}$$

## Compact Linear Collider (CLIC)

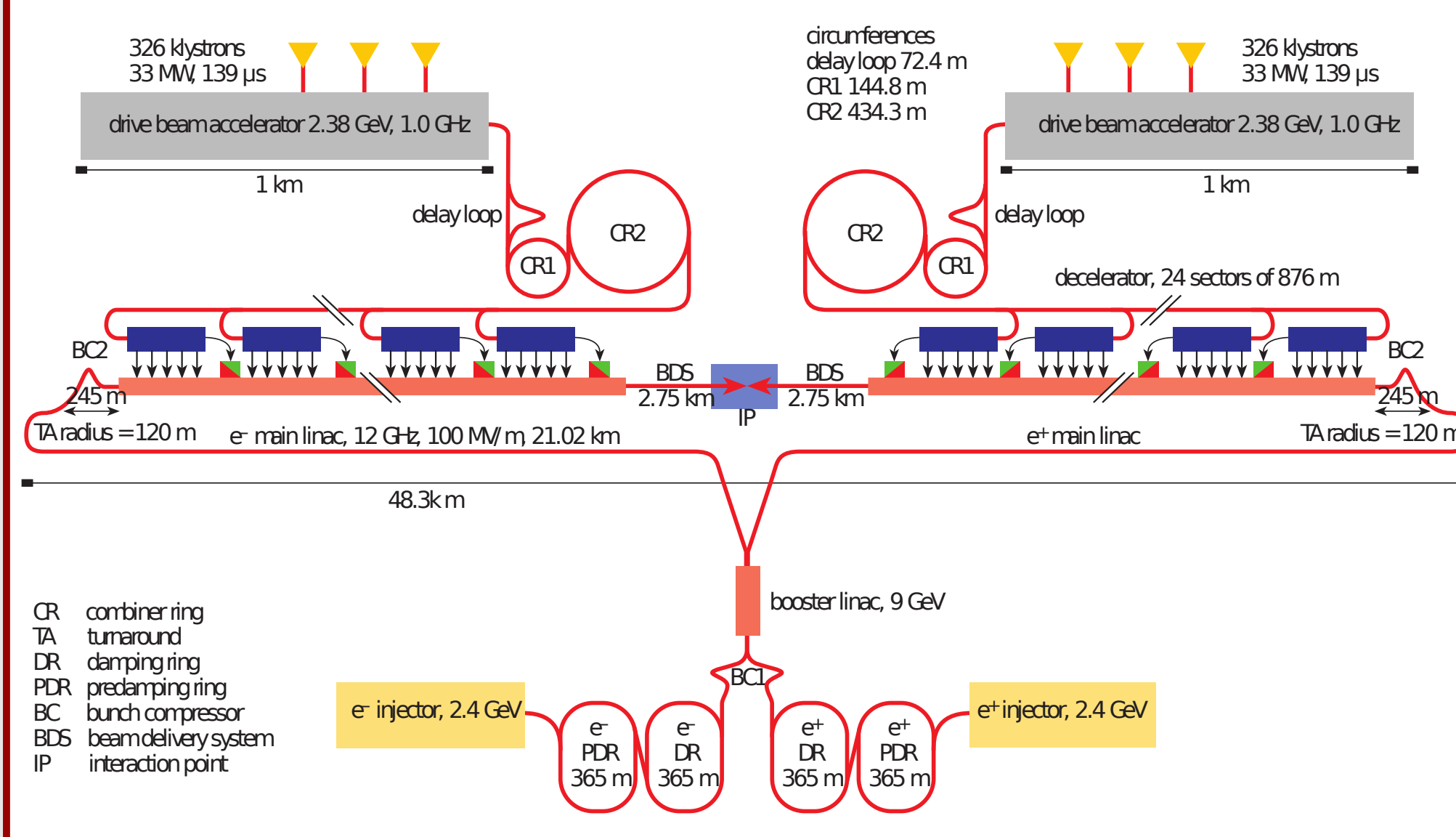


Figure 1. The Compact Linear Collider (CLIC). Upper part of the figure shows the drive beam complex. Two high intensity low energy electron beams are decelerated through periodic structures (blue boxes) in order to generate RF power for accelerating the main beam (lower part of the figure).

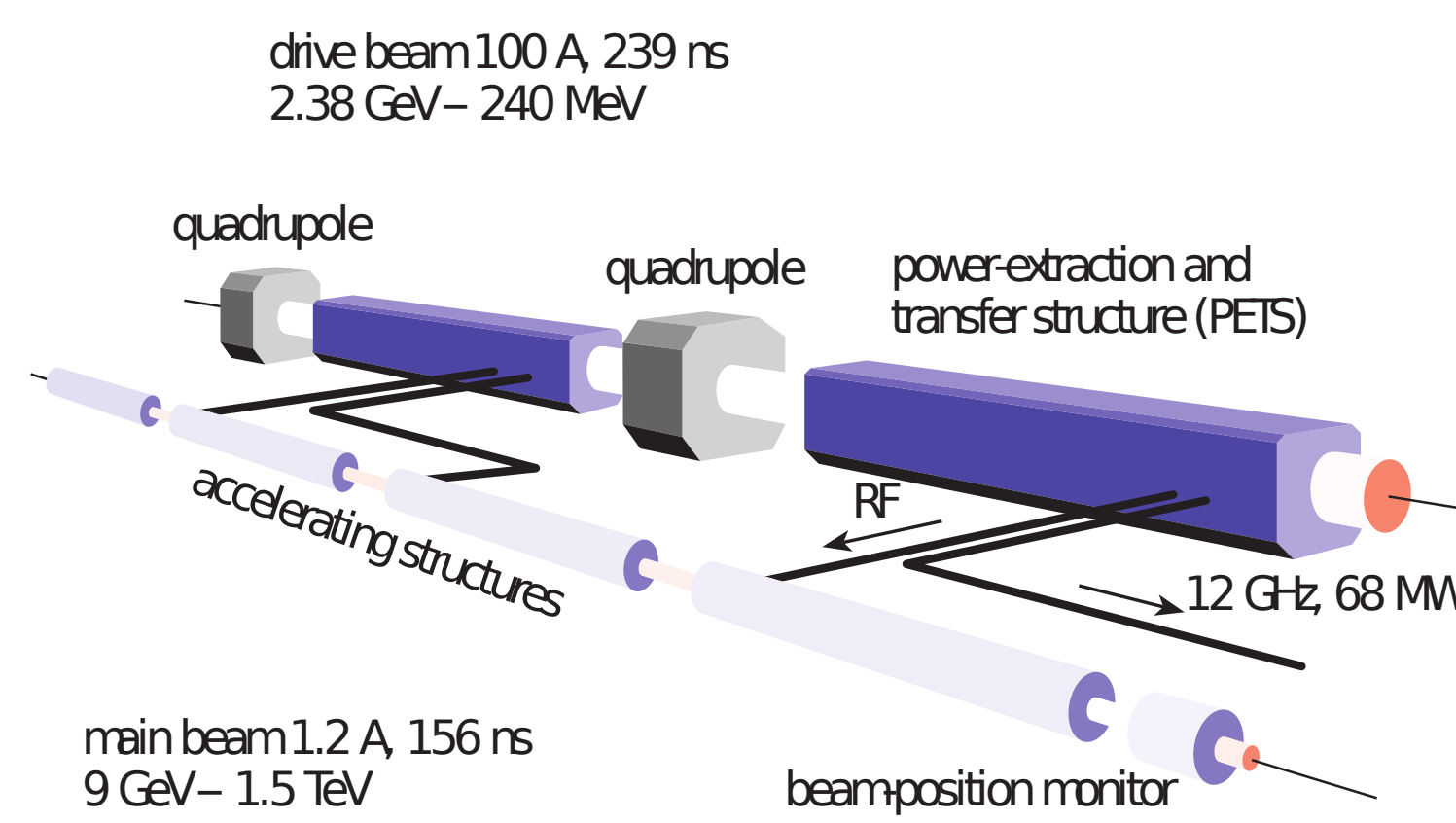


Figure 2. Two-beam acceleration scheme. The drive beam is decelerated in the power-extraction and transfer structures (PETS) and the generated 12 GHz RF power is guided to the accelerating structures where the main beam is accelerated.

## CLIC Test Facility 3 (CTF3)

The CLIC test facility 3 (CTF3) at CERN was constructed to prove the CLIC accelerator concept. This involves generating the drive beam and two-beam acceleration. The drive beam consist of a 150 MeV electron linac with a long macro pulse that is later compressed in a delay loop and a combiner ring in order to create a high-intensity beam.

In the CTF3 experimental area (CLEX) there is a small electron accelerator called CALIFES that generates the probe beam, which simulates the CLIC main beam. At the two-beam test stand (TBTS) the drive beam is passed through a PETS and the generated RF power is guided to a CLIC accelerator structure in the probe beam beam-line.

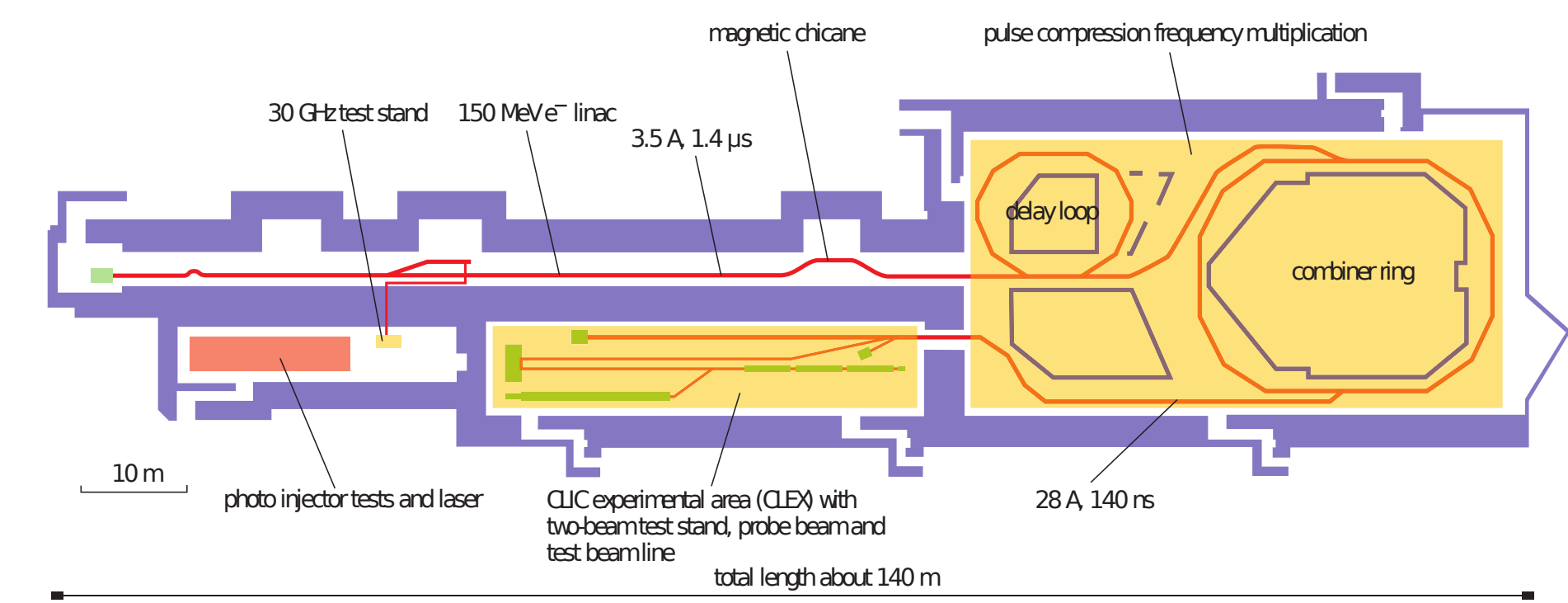


Figure 3. CLIC test facility 3 (CTF3).

## Measurement at the two-beam test stand

We performed a scan of the probe beam for different transverse positions inside the accelerating structure (ACS). We used two correctors upstream from the ACS to scan the beam parallel to the beam line. We recorded images with an in-line screen and a CCD camera. The probe beam was operated at twice the repetition rate (0.8 Hz and 1.6 Hz) compared to the drive beam. This setup makes it possible to record images for the beam with RF present in the ACS simultaneously as the beam with no RF. Prior to the scan we measured the incoming beam energy and the energy after on-crest acceleration, the energy gain will then tell us the accelerating gradient.

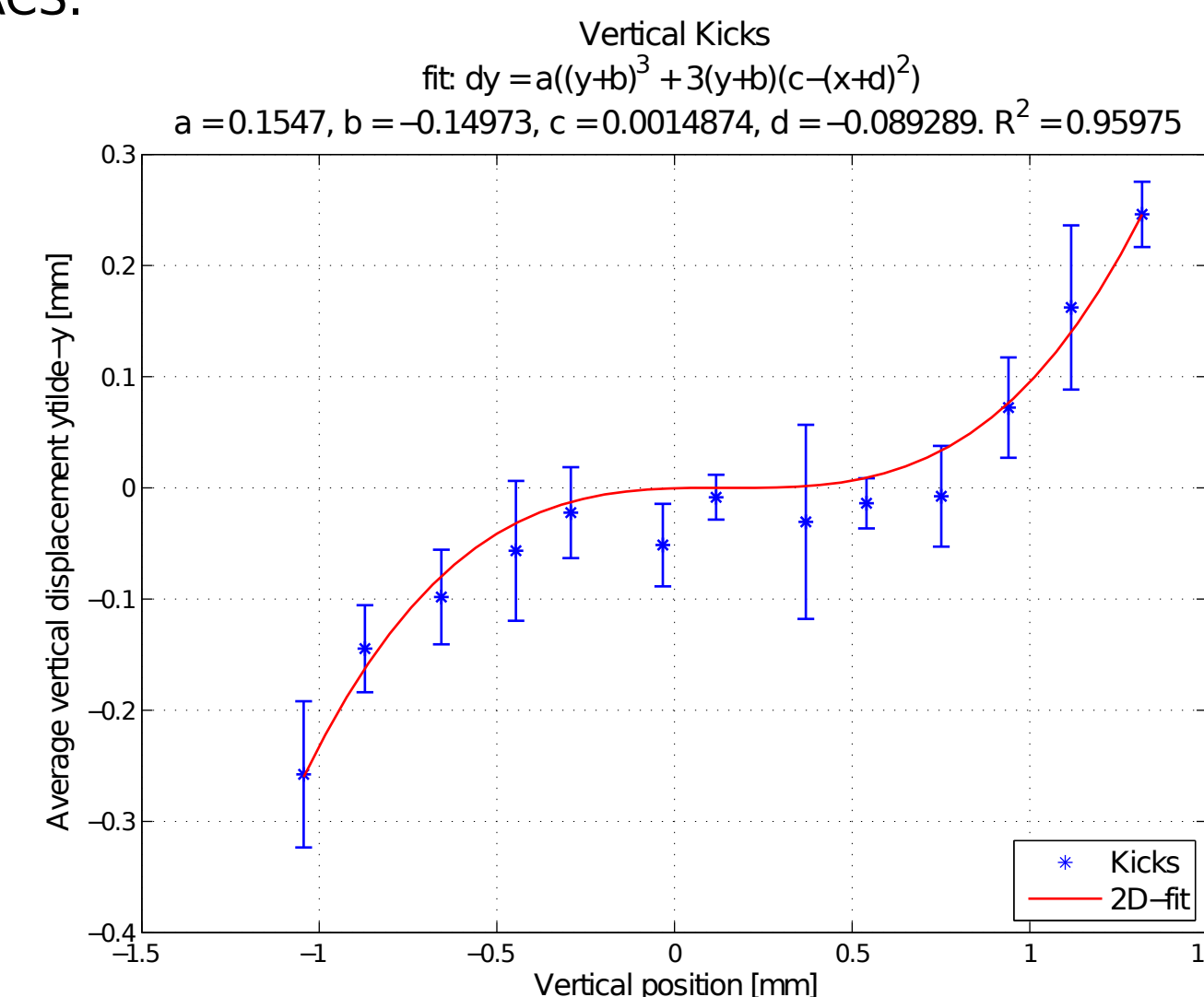
## Scan results

We scanned the beam vertically and measured the position of the center-of-mass of the beam and compared the beam with RF power in the structure to the beam with no RF present. The phase was adjusted to have the beam on zero-crossing of RF, where the octupolar effect is strongest. Below we have plotted the position shifts due to octupolar kicks and made 2D fits on the forms:

$$\Delta y = a[(y+b)^3 + 3(y+b)(c-(x+d)^2)]$$

$$\Delta x = a[(x+d)^3 + 3(x+d)(-c-(y+b)^2)]$$

where  $b$  and  $d$  represents the fact that there is an offset between the center of the screen and the center of the ACS.



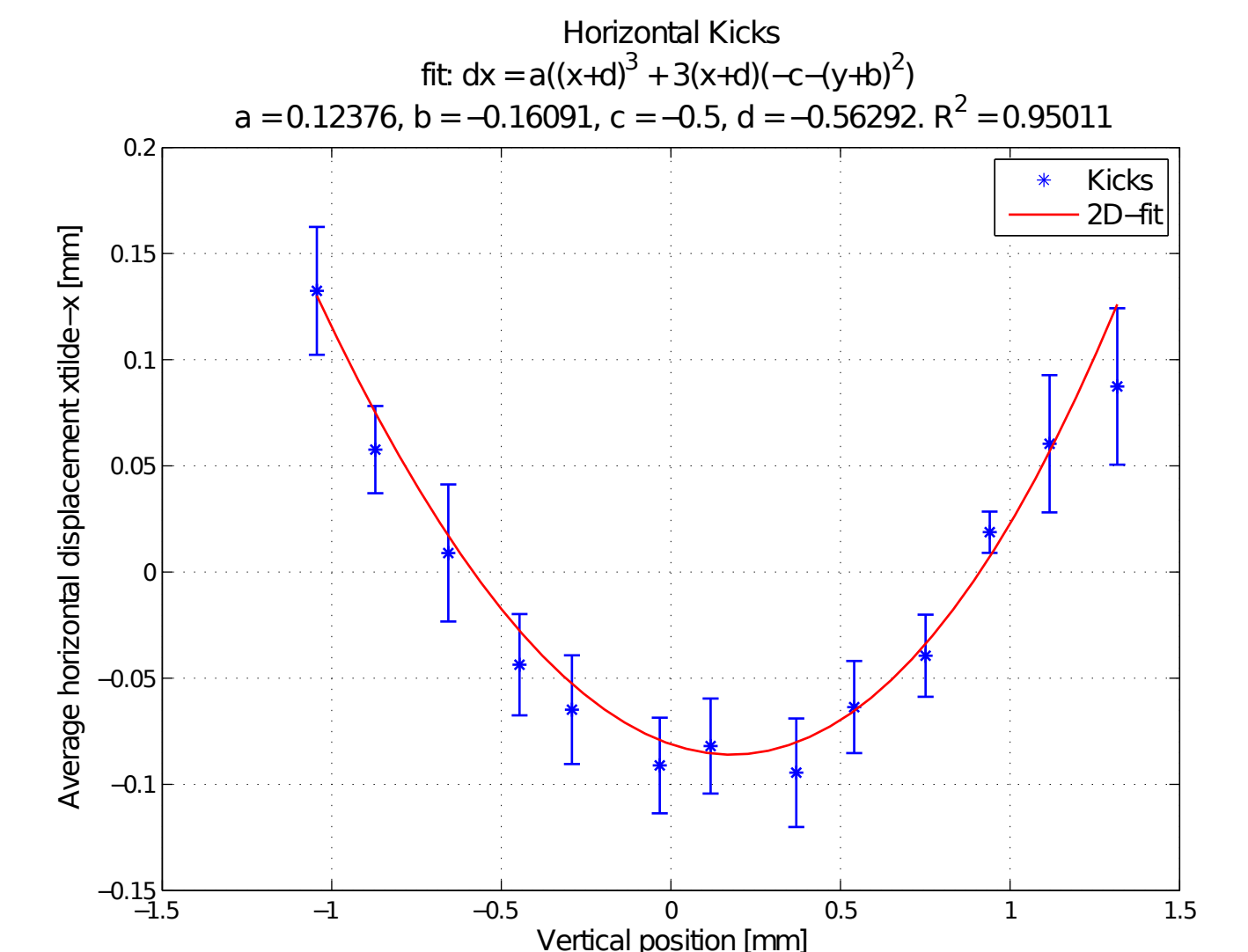
The fit parameter  $a$  is related to the octupolar component strength according to:

$$k_3 l = -\frac{a(B\rho)}{L}.$$

The distance between ACS and screen is  $L = 5.4 \text{ m}$  and the energy of the non-accelerated beam was 194 MeV. This gives us the octupolar strength:

$$k_3 l = -18.5 \text{ kTm/m}^3 \text{ (vertical shift)}$$

$$k_3 l = -14.8 \text{ kTm/m}^3 \text{ (horizontal shift)}$$



if we set  $n = 4$  we get the octupolar field

$$B_x = -k_3(y^3 - 3yx^2)$$

$$B_y = k_3(x^3 - 3xy^2)$$

where we have used the MAD convention  $C_4 = k_3$ . Combining this with the result above we get

$$\Delta x' = -\frac{k_3 l}{(B\rho)}(x^3 - 3xy^2)$$

$$\Delta y' = -\frac{k_3 l}{(B\rho)}(y^3 - 3yx^2).$$

So far we have only considered a single electron. To get the kick of a distribution we must take the expectation value:

$$\Delta Y' = \langle \Delta y' \rangle = \left\langle -\frac{k_3 l}{(B\rho)}(y^3 - 3xy^2) \right\rangle = -\frac{k_3 l}{(B\rho)} [\langle y^3 \rangle - 3 \langle x^2 y \rangle].$$

Let us consider a 2D Gaussian distribution without coupling

$$\Phi(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{(x-X)^2}{2\sigma_x^2} - \frac{(y-Y)^2}{2\sigma_y^2}}.$$

The first expectation value now becomes

$$\langle y^3 \rangle = \iint y^3 \Phi(x, y) dx dy = \frac{1}{\sqrt{2\pi}\sigma_y} \int y^3 e^{-\frac{(y-Y)^2}{2\sigma_y^2}} dy$$

where the limits of all integrals are  $(-\infty, \infty)$ . Variable substitution gives us

$$\langle y^3 \rangle = \frac{1}{\sqrt{2\pi}\sigma_y} \int (\tilde{y} + Y)^3 e^{-\tilde{y}^2/2\sigma_y^2} d\tilde{y} = \frac{1}{\sqrt{2\pi}\sigma_y} \int (\tilde{y}^3 + 3\tilde{y}^2 Y + 3\tilde{y} Y^2 + Y^3) e^{-\tilde{y}^2/2\sigma_y^2} d\tilde{y}$$

The first and third terms are odd function over symmetric interval and integrates to zero. This leaves us with

$$\begin{aligned} \langle y^3 \rangle &= \frac{1}{\sqrt{2\pi}\sigma_y} \int (3\tilde{y}^2 Y + Y^3) e^{-\tilde{y}^2/2\sigma_y^2} d\tilde{y} \\ &= 3Y \frac{1}{\sqrt{2\pi}\sigma_y} \int \tilde{y}^2 e^{-\tilde{y}^2/2\sigma_y^2} d\tilde{y} + Y^3 \frac{1}{\sqrt{2\pi}\sigma_y} \int e^{-\tilde{y}^2/2\sigma_y^2} d\tilde{y} \\ &= 3Y\sigma_y^2 + Y^3. \end{aligned}$$

The second expectation value can be calculated as

$$\begin{aligned} \langle x^2 y \rangle &= \iint x^2 y \Phi(x, y) dx dy \\ &= \left( \frac{1}{\sqrt{2\pi}\sigma_x} \int x^2 e^{-\frac{(x-X)^2}{2\sigma_x^2}} dx \right) \left( \frac{1}{\sqrt{2\pi}\sigma_y} \int y e^{-\frac{(y-Y)^2}{2\sigma_y^2}} dy \right) \\ &= (X^2 + \sigma_x^2) Y. \end{aligned}$$

This gives us

$$\Delta Y' = -\frac{k_3 l}{(B\rho)} [Y^3 + 3Y(\sigma_y^2 - \sigma_x^2 - X^2)].$$

The position on a screen at a distance  $L$  can be written  $\hat{Y} = Y + L\Delta Y'$ .

This yields

$$\hat{Y} = Y - \frac{k_3 l}{(B\rho)} L(Y^3 + 3Y(\sigma_y^2 - \sigma_x^2 - X^2))$$

The position shift in  $X$  is calculated in the same way.

$$\hat{X} = X - \frac{k_3 l}{(B\rho)} L(X^3 + 3X(\sigma_x^2 - \sigma_y^2 - Y^2)).$$

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