Axions: Theory

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2. Possible Solutions
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4. Astrophysical Constraints on Axions
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Our present understanding of the Strong Interactions

QCD: very simple lagrangian, governed by symmetries and particle content:

$$\mathcal{L} = -\frac{1}{4g^2}(G^a_{\mu\nu})^2 + \sum_f \bar{Q}_f(i\not{D} - m_f)Q_f \quad (1)$$

Here $G_{\mu\nu}$ is the field strength of QCD (we will use $F_{\mu\nu}$ for QED), Q_f the various quark flavors.

Extremely successful.

- At very high energies, detailed, precise predictions, well verified at Tevatron and LHC (dramatically in the context of Higgs discovery.
- At low energies, precision studies with lattice gauge theory.

Strong CP Problem

Only one puzzle: can add to QCD lagrangian

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2} G\tilde{G}. \tag{2}$$

Here $\tilde{G}$ ("G-dual") is

$$\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{\mu\nu}; \quad G\tilde{G} = 2\vec{E} \cdot \vec{B}. \tag{3}$$

This term is a total derivative (easy to see in QED), and it is tempting to ignore it. But it turns out to have physical effects, and it violates *parity*, and thus CP.

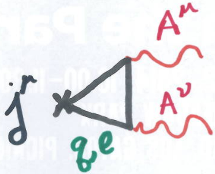
Connection to Anomalies

The possibility of a non-zero θ is related to another possible source of *CP* violation: γ_5 terms in the quark lagrangian (in the language of four component fermions; in terms of two component fermions, this corresponds to complex masses). For a single quark, the issue is a term of the form:

$$\mathcal{L}_m = m(\cos(\theta)\bar{q}q) + i\sin\theta\bar{q}\gamma_5 q). \quad (4)$$

The use of θ here is not an accident; a transformation $q \rightarrow e^{-i\frac{\theta}{2}\gamma_5}q$ gets rid of the would-be CP-violating mass term, but at the price – *as a result of the so-called anomaly* – of inducing a $\theta G\tilde{G}$ term of the type described above.

Anomaly in the current $j^\mu = \bar{q}\gamma^\mu\gamma^5 q$



Neutron Electric Dipole Moment

One can show, using the properties of the chiral lagrangian (current algebra) that θ leads to an electric dipole moment for the neutron:

$$d_n = g_{\pi NN} \frac{\theta m_u m_d}{f_\pi (m_u + m_d)} \langle N_f | \bar{q} \tau^a q | N_f \rangle \ln(m_p/m_\pi) \frac{1}{4\pi^2 m_p} \tag{5}$$

$$= 5.2 \times 10^{-16} \theta cm$$

(this is calculated in an approximation which becomes more and more reliable as the masses of the light quarks become smaller).

From the experimental limit, $d_n < 3 \times 10^{-26}\ \mathrm{e\ cm}$, one has $\theta < 10^{-10}$.

This is a puzzle. Why such a small dimensionless number?

$\theta \to 0$: strong interactions preserve CP. If not for the fact that the rest of the SM violates CP, would be *natural*.

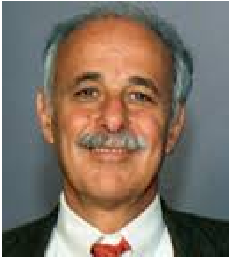
Possible Resolutions

1. $m_u = 0$ If true, $u \to e^{-i\frac{\theta}{2}\gamma_5} u$ eliminates θ from the lagrangian. An *effective* m_u might be generated from non-perturbative effects in the theory (Georgi, McArthur; Kaplan, Manohar) Could result as an accident of discrete flavor symmetries (Banks, Nir, Seiberg), or a result of “anomalous" discrete symmetries as in string theory (M.D.)
2. CP exact microscopically, $\theta = 0$; spontaneous breaking gives the CKM phase but leads, under suitable conditions, to small effective θ (Nelson, Barr). In critical string theories, CP is an exact (gauge) symmetry, spontaneously broken at generic points in typical moduli spaces. A plausible framework.

Problems with each of these solutions:

1. $m_u = 0$. Lattice computations seem to rule out (the required non-perturbative effects do not seem to be large enough).
2. Spontaneous CP: special properties required to avoid large θ once CP is spontaneously broken. What would single out such theories?
3. Axions: Our focus today. We will see promise and limitations.

Axions (Peccei-Quinn; Weinberg, Wilczek)



The Peccei-Quinn Symmetry

In a somewhat streamlined language, the Peccei-Quinn proposal was to replace θ by a dynamical field: $\theta \to \frac{a(x)}{f_a}$

It is assumed that $a \to a + \omega f_a$ is a good symmetry of the theory, *violated only by effects of QCD*. Without QCD, θ can take any value.

In QCD *by itself*, the energy is necessarily stationary when

$$\theta_{eff} = \langle \frac{a}{f_a} \rangle = 0. \quad (6)$$

This is simply because *CP* is a good symmetry of QCD if $\theta = 0$, so the vacuum energy (potential) must be an odd function of θ.

One can do better, calculating, again using what we know about chiral symmetry in QCD, the axion potential:

$$V(a) = -m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d} \cos(a/f_a) \tag{7}$$

This gives, for the axion mass:

$$m_a = 0.6 \text{ meV} \left(\frac{10^{10} \text{ GeV}}{f_a} \right). \tag{8}$$

Peccei and Quinn actually constructed a model for this phenomenon, which was a modest extension of the Standard Model with an extra Higgs doublet. They didn't phrase the problem in quite the way I did above, and didn't appreciate that their model had a light, pseudoscalar particle, a. This was quickly recognized by Weinberg and Wilczek, who calculated its mass and the properties of its interactions. It quickly become clear that the original axion idea was not experimentally viable.

The Invisible Axion

But in the more general picture described above, the problems with the axion are easily resolved. The strength of the axion's interactions are proportional to $1/f_a$. This is because of the Peccei-Quinn symmetry. The symmetry requires that axion interactions appear only with derivatives of the axion field; on dimensional grounds, these come with powers of $\frac{\partial_\mu}{f_a}$ (momenta – q^μ/f_a). QCD terms which break the symmetry also come with powers of $1/f_a$. So if f_a is large enough, the axion will be hard to detect (it becomes "harmless" or "invisible").

The scale, f_a, might be associated with some high scale of physics (M_{gut}? M_p? – more later).

Sample couplings

1. Axion to two photons (notation of PDG):

$$\mathcal{L}_{\gamma\gamma} = \frac{1}{4} G_{a\gamma\gamma}\, a\, F\tilde{F} \tag{9}$$

where now F is the *electromagnetic* field strength.

$$G_{a\gamma\gamma} = \frac{\alpha}{2\pi}\left(\frac{E}{N} - \frac{4}{3}\frac{4+z}{1+z}\right)\frac{1+z}{\sqrt{z}}\frac{m_a}{m_\pi f_\pi} \quad z = \frac{m_u}{m_d} \tag{10}$$

E, N are the electromagnetic and QCD anomalies.

2. Axion to quarks, leptons:

$$\mathcal{L}_{aff} = \frac{C_f}{2f_a}\bar{\psi}_f \gamma^\mu \gamma_5 \psi_f\, \partial_\mu a. \tag{11}$$

The detailed coefficients depend on the model.

DFSZ

Add to the Standard Model an additional Higgs doublet (e.g. as in supersymmetry), i.e. two doublets, H_u, H_d, plus a singlet, ϕ. Impose the Peccei-Quinn symmetry:

$$\phi \rightarrow e^{i\alpha}\phi;\ \ H_u \rightarrow e^{-i\frac{\alpha}{2}} H_u;\ \ H_d \rightarrow e^{-i\frac{\alpha}{2}} H_d \tag{12}$$

Require potential such that

$$\langle \phi \rangle = \frac{f_a}{\sqrt{2}}. \tag{13}$$

This breaks the PQ symmetry spontaneously. (Pseudo-)Goldstone boson:

$$\mathrm{Im}\ \phi = \frac{a}{\sqrt{2}}.$$

a couples to $G\tilde{G}$, $F\tilde{F}$. Also couples to leptons, quarks.

$$\frac{E}{N} = 8/3; \quad C_e = \frac{\cos^2\beta}{3} \qquad \tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}. \tag{14}$$

As expected, as f_a becomes large, the axion's interactions with other particles become weaker. Once $f_a >> \frac{\mathrm{GeV}}{G_F}$, unobservable in accelerator experiments.

KSVZ Model

Here one has a field, ϕ, and a new quark, q and $\bar{q}$, which will be very heavy. q and $\bar{q}$ are assumed to carry color but to be $SU(2) \times U(1)$ singlets. In two component language, the Peccei-Quinn symmetry is assumed to be

$$\phi \rightarrow e^{i\alpha}\phi \quad q \rightarrow e^{-i\frac{\alpha}{2}} q \, '\bar{q} \rightarrow e^{-i\frac{\alpha}{2}} \bar{q}. \tag{15}$$

ϕ is assumed to have an expectation value:

$$\langle \phi \rangle = \frac{f_a}{\sqrt{2}}. \tag{16}$$

The imaginary part of ϕ is the axion:

$$\phi = \frac{1}{\sqrt{2}} f_a + ia. \tag{17}$$

But these are just two of a wealth of possible modes, characterized by the coefficients above. These two, however, are often used as benchmarks to characterize the capabilities of different experimental detection schemes, as well as to illustrate the range of possible astrophysical phenomena.

Astrophysical Constraints

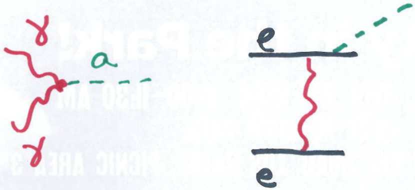
Axion interactions are "semi weak", in the sense that cross sections go as $1/f_a^2$, as opposed to weak interactions which behave as $1/v^4$. So even for large f_a, reaction rates can be comparable to those for neutrinos. This raises a worry about stars, where various processes can produce axions. If interaction rates are large compared to those for neutrinos, excessive amounts of energy will be carried off by axions. More detailed studies in particular astrophysical environments place lower limits on f_a.

Sources of Astrophysical Constraints

Partial list:

1. The sun
2. Red Giants, Globular Clusters
3. SN 1987a
4. White dwarfs

Primakoff process, axion bremstrahlung.



Axion Luminosity

In sun:

$$L_a = G^2_{a\gamma\gamma} \times 1.85 \times 10^{17} L_\odot \tag{18}$$

so

$$G_{a\gamma\gamma} < 7 \times 10^{-10}. \tag{19}$$

Stronger constraint from globular clusters, $7 \rightarrow 1$.

Axions as Dark Matter

Paradoxically, because the axion is so weakly interacting, it can play a significant role in the early universe.

In an FRW universe:

$$\ddot{a} + 3\,H\dot{a} + m_a^2\, a = 0 \tag{20}$$

a is overdamped for $H > m_a$; oscillates for $H < m_a$.

Because there is nothing special about the point $a = 0$, initially the axion might be a homogeneous field, with non-zero $a = a_0$ ($\theta = \theta_0$) (more about this assumption later).

m_a is small; e.g. for

$$f_a = 10^{16}\mathrm{GeV}, m_a = 10^{-9}\ \mathrm{eV}; \quad f_a = 10^{11}\mathrm{GeV} \quad m_a = 6 \times 10^{-4}\ \mathrm{eV}. \qquad (21)$$

$H = 10^{-9},\ 10^{-4}$ eV when the temperature of the universe is about $1,\ 10^2$ GeV. This is late compared to, e.g., the likely times of inflation.

Solving the axion equation of motion

$$\ddot{\theta} + 3\,H\dot{\theta} + m_a^2\,\theta = 0 \tag{22}$$

Seek a solution of form, for large t:

$$\theta(t) = \theta_0 f(t)\cos(m_a t) \tag{23}$$

with $f(t)$ slowly varying. For radiation/matter dominated eras:

$$H = \frac{1}{2t}; f = \left(\frac{t_0}{t}\right)^{3/4} \quad H = \frac{2}{3t}; f = \left(\frac{t_0}{t}\right) \tag{24}$$

Each of these solutions falls off as $1/R(t)^3$. In other words, the system behaves like pressure-less dust, a collection of zero momentum axions.

When the axion starts to oscillate, it constitutes a fraction of the energy density of order $\theta_0^2 \frac{f_a^2}{M_p^2}$; with $f_a = 10^{11}$ this is about 10^{-14}.

During the radiation dominated era, the energy density in matter (axions) falls off as T^3, as opposed to the radiation, which falls off as T^4. The usual matter-radiation equality occurs for $T \approx$ eV. At this time, the energy density in axions, indeed, is of order the energy density in radiation.

Larger f_a: matter domination too early. Smaller: axions only a fraction of the dark matter.

So indeed for $f_a \approx 10^{11}$ GeV and $\theta_0 \approx 1$, the axions come to dominate the energy density of the universe at the approximate time of matter-radiation equality. More careful calculation takes into account the temperature dependence of the axion mass, and yields:

$$\Omega_a h^2 = 0.11\ \theta_0^2 \left(\frac{f_a}{5 \times 10^{11}\ \mathrm{GeV}}\right)^{1.184}. \quad (25)$$

So from the combination of astrophysical and cosmological considerations, the axion decay constant/mass lies in a rather narrow range. At the high end of this range, the axion constitutes the dark matter.

The Axion and Inflation

So far, we have assumed that the axion, initially, takes the same value throughout the universe. Within the framework of inflationary big-bang cosmology, this only makes sense if the Peccei-Quinn transition (the “turning on" of the expectation value of ϕ) occurred before inflation, so that the initial value of the field is the same everywhere in the observable universe.

If this is not the case, the production of axions leads to similar limits, but involves more complicated processes, including the production of axion strings.

BICEP2, if correct, may constrain the axion further (isocurvature fluctuations).

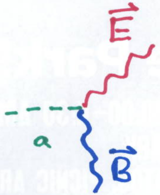
Searching for the Axion

For $f_a > 10^8$ GeV, the axion is extremely weakly interacting. In scattering experiments, it is produced rarely and detection is essentially impossible.

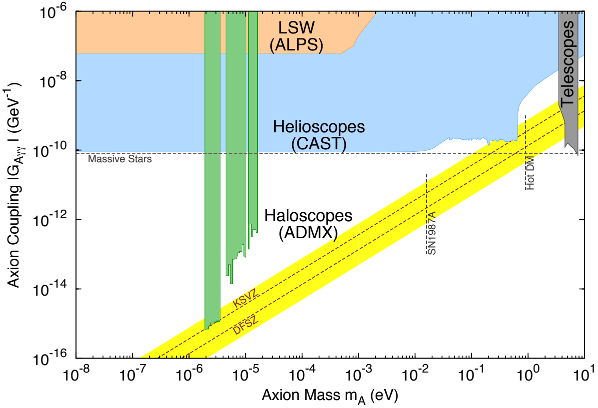
However, if we assume that the axion constitutes the dark matter, we are living in a sea of axions, and we might hope to detect them. The main interaction at our disposal is the interaction with the electromagnetic field characterized by $G_{a\gamma\gamma}$: In particular, in a strong magnetic field, an axion can convert into a photon. If the magnetic field is in a cavity, this means we can hope to produce a cavity excitation.

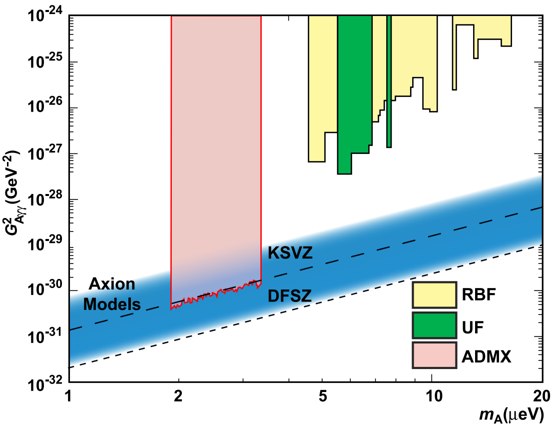
Axion Detection Process

$G_{a\gamma\gamma}\ F\tilde{F} = G_{a\gamma\gamma}\vec{E}\cdot\vec{B}.$

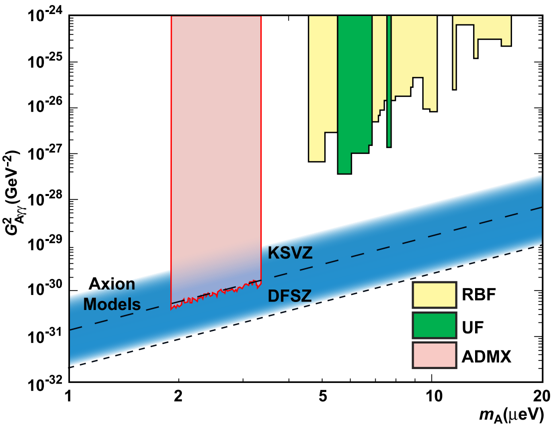


There are many challenges. The axion is quite narrow and we don't know it's mass with anything like precision. So one needs to be able to sweep through many small frequency steps. One needs a cavity of very high quality. The most impressive effort of this type is the ADMX experiment. All of this you will hear about in Professor Rosenberg's lectures.


LSW
(ALPS)
Helioscopes
(CAST)
Telescopes
Massive Stars
Hot DM
SN1987A
Haloscopes
(ADMX)
KSVZ
DFSZ
Axion Coupling |$G_{A\gamma\gamma}$| (GeV^{-1})
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1}
Axion Mass m_A (eV)



10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^{-28}
10^{-29}
10^{-30}
10^{-31}
10^{-32}
$G^2_{A\gamma\gamma}$ (GeV^{-2})
KSVZ
DFSZ
Axion Models
RBF
UF
ADMX
1
2
5
10
20
m_A(μeV)



10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^{-28}
10^{-29}
10^{-30}
10^{-31}
10^{-32}
$G^2_{A\gamma\gamma}$ (GeV^{-2})
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m_A(μeV)
Axion Models
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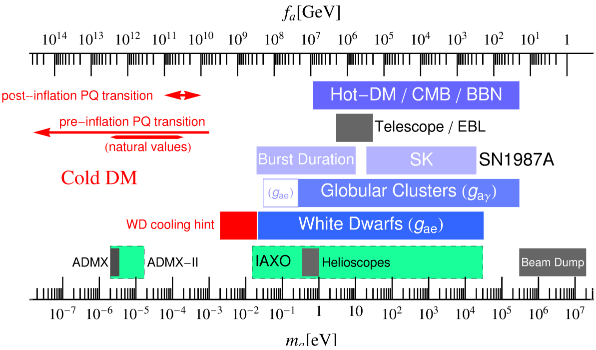

f_a[GeV]
10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^{9} 10^{8} 10^{7} 10^{6} 10^{5} 10^{4} 10^{3} 10^{2} 10^{1} 1
post–inflation PQ transition
pre–inflation PQ transition
(natural values)
Cold DM
Hot–DM / CMB / BBN
Telescope / EBL
Burst Duration
SK
SN1987A
(g_{ae})
Globular Clusters ($g_{a\gamma}$)
WD cooling hint
White Dwarfs (g_{ae})
ADMX
ADMX–II
IAXO
Helioscopes
Beam Dump
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7}
m_a[eV]

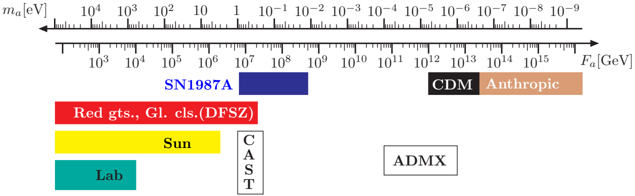

FIG. 15: A cartoon for the F_a bounds.

How Robust are the Cosmological Limits on the Axion

The axion cosmology we have described assumes that the universe was in thermal equilibrium at very early times, times much shorter than the axion mass.

There are reasons to question this assumption. For example, suppose that nature is approximately supersymmetric. Then the axion has a *scalar* superpartner, the saxion. This particle is long lived. If it decays through the two photon interaction (or its superpartners), its lifetime is of order

$$\Gamma = \left(\frac{\alpha}{4\pi}\right)^2 \frac{m_{saxion}^3}{f_a^2} = \left(10^5\ \mathrm{s}\right)^{-1} \frac{m_{saxion}^3}{\mathrm{TeV}^3} \left(\frac{10^{16}}{f_a}\right)^2 \tag{26}$$

where we have taken the grand unified scale as our benchmark axion decay constant. Even at 10^{11} GeV, this is after the axions start to oscillate.

The saxion, when it decays, “reheats" the universe. This temperature should be higher than the temperature at which nucleosynthesis occurs (say 10 MeV).

When the axion starts to oscillate, the universe is dominated by the saxion. It’s energy density is of order $m_a^2 f_a^2$, while the total energy is of order $m_a^2 M_p^2$, so the axion energy fraction is approximately

$$f_a^2/M_p^2 \tag{27}$$

After the saxion decays to radiation, the fractional energy density grows with $1/T$. So between 10 MeV and 1 eV, it grows by 10^7. This gives

$$f_a < 10^{15} \tag{28}$$

a much weaker limit than before. Could be weaker still.

Axion *Quality*

Finally, we turn to a theoretical question: Why are there axions at all? More precisely, why should there be a Peccei-Quinn symmetry, and how good a symmetry does this have to be?

General belief (supported by studies of string theory): *a theory of quantum gravity does not possess (exact) global symmetries.*

Then hopeless? No: symmetry might be an accidental consequence of other symmetries.
Example: discrete symmetries.

$$\phi \rightarrow \phi e^{\frac{2\pi i}{N}}. \tag{29}$$

So leading symmetry breaking terms in potential might take the form:

$$\mathcal{L}_{symm-breaking} = \frac{\phi^N}{M_p^{N-4}} \tag{30}$$

If N is large, these terms would seem very small. But they have to be *extremely* small to insure the smallness of θ. One needs, e.g., the linear term in the a potential

$$V = \frac{1}{2} m_a^2 \, a^2 + \Gamma a + \ldots \tag{31}$$

to be such that

$$\frac{\Gamma}{m_a^2} < 10^{-10} f_a \tag{32}$$

This translates into a requirement that $N > 12$, if $f_a = 10^{11}$; even larger for larger f_a

Why should this be?

Axions in String Theory

We have seen that axions, from the perspective of effective field theory, are surprising. It has long been known that axions are common in string theory, indeed axion-like objects seem ubiquitous. What insight do they give and what expectations do they lead to?

Examples:

1. Heterotic string contains an axion (always) which couples universally to all of the gauge groups.
2. In heterotic and other string theories, antisymmetric tensor fields in higher dimensions become pseudoscalars in four dimensions with axion type couplings.
3. All of these fields exhibit approximate, continuous shift symmetries, $a \to a + \omega f_a$. They typically exhibit *exact* discrete shift symmetries, $a \to a + 2\pi f_a$. The breaking of the continuous symmetries is suppressed, at weak coupling, by $e^{-2\pi/\alpha}$.

Cosmology of String Theoretic Axions

So string theory has axions suitable for solving the strong CP problem. But what about their cosmology?

1. f_a large; in what is imagined a typical string phenomenology, $f_a \geq 10^{15}$ GeV.
2. If approximate supersymmetry, axions accompanied by saxions, other “moduli". Can dilute axions as above. May require surprisingly low f_a.

Detecting string scale axions

Clearly challenging. Recently pursued by P.Graham, S. Rajendran, and others.

Strategies involve noting that if axions are the dark matter, $\theta \propto \cos(m_a t)$, and using (searching for) time varying dipole moments. Some prototype experiments under discussion. Workshop at ICTP last June.

Outlook

Axions are a well-motivated dark matter candidate.

The most straightforward approach suggests they should be detectable in cavity experiments.

Theoretical arguments, however, suggest that in a more complete cosmology, axions might be lighter.

In either case, their detection and study would be an extraordinary development.

This has been a quick tour. If you seek a more leisurely pedagogical introduction, there are many good reviews. A sampling:

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M. Dine, "Supersymmetry and string theory: Beyond the standard model," Cambridge, UK: Cambridge Univ. Pr. (2007) 515 p