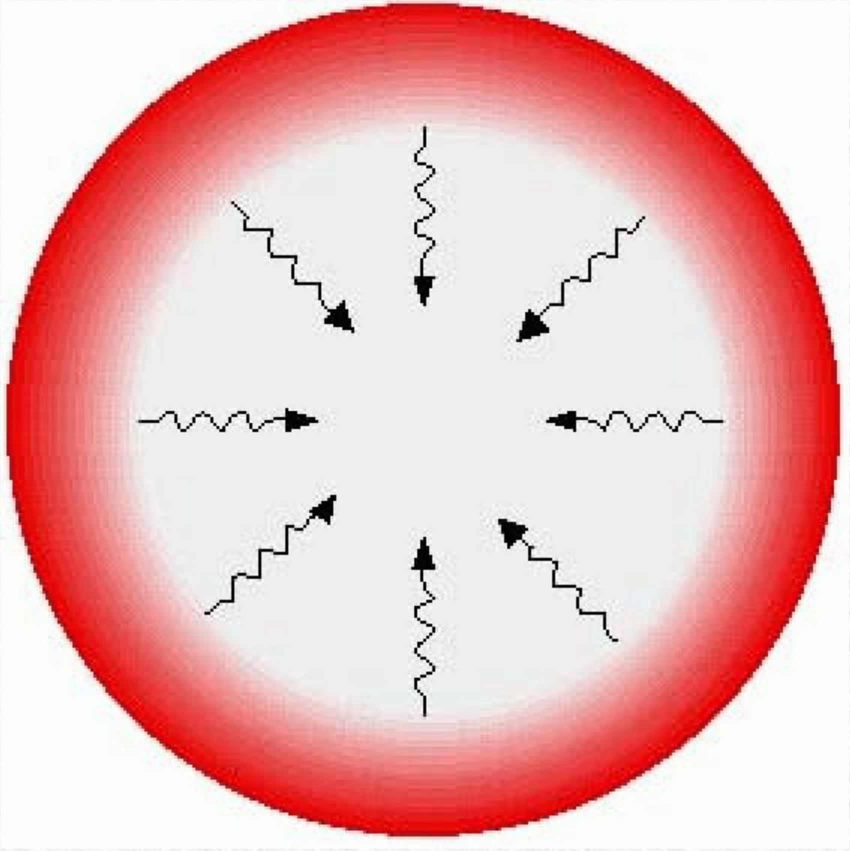
Cosmology Basics

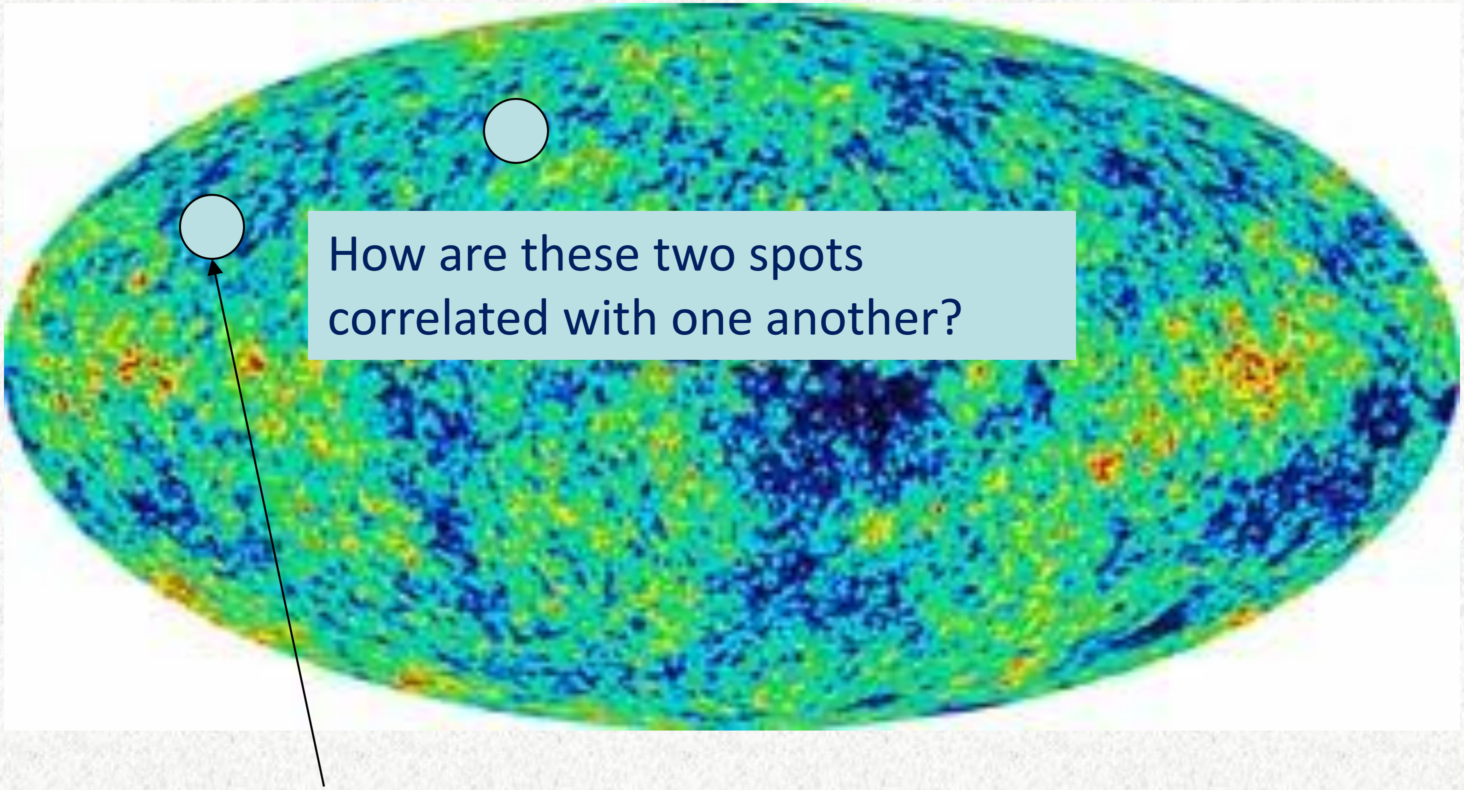
- Friedmann-Robertson-Walker (FRW) Metric and Expansion
- Constituents of the Universe
- Evolution, including ***Dark Energy***
- Thermal History: Recombination, BBN, e^+-e^- annihilation
- ***Neutrinos***
- Thermal History: Neutrino Abundance
- ***Leptogenesis***
- Thermal History: ***Weakly Interacting Massive Particles***
- ***Inflation***
- CMB Anisotropies
- Structure Formation
- Alternatives

We see photons today from last scattering surface when the universe was just 400,000 years old

The temperature of the cosmic microwave background (CMB) is very nearly the same in all directions.

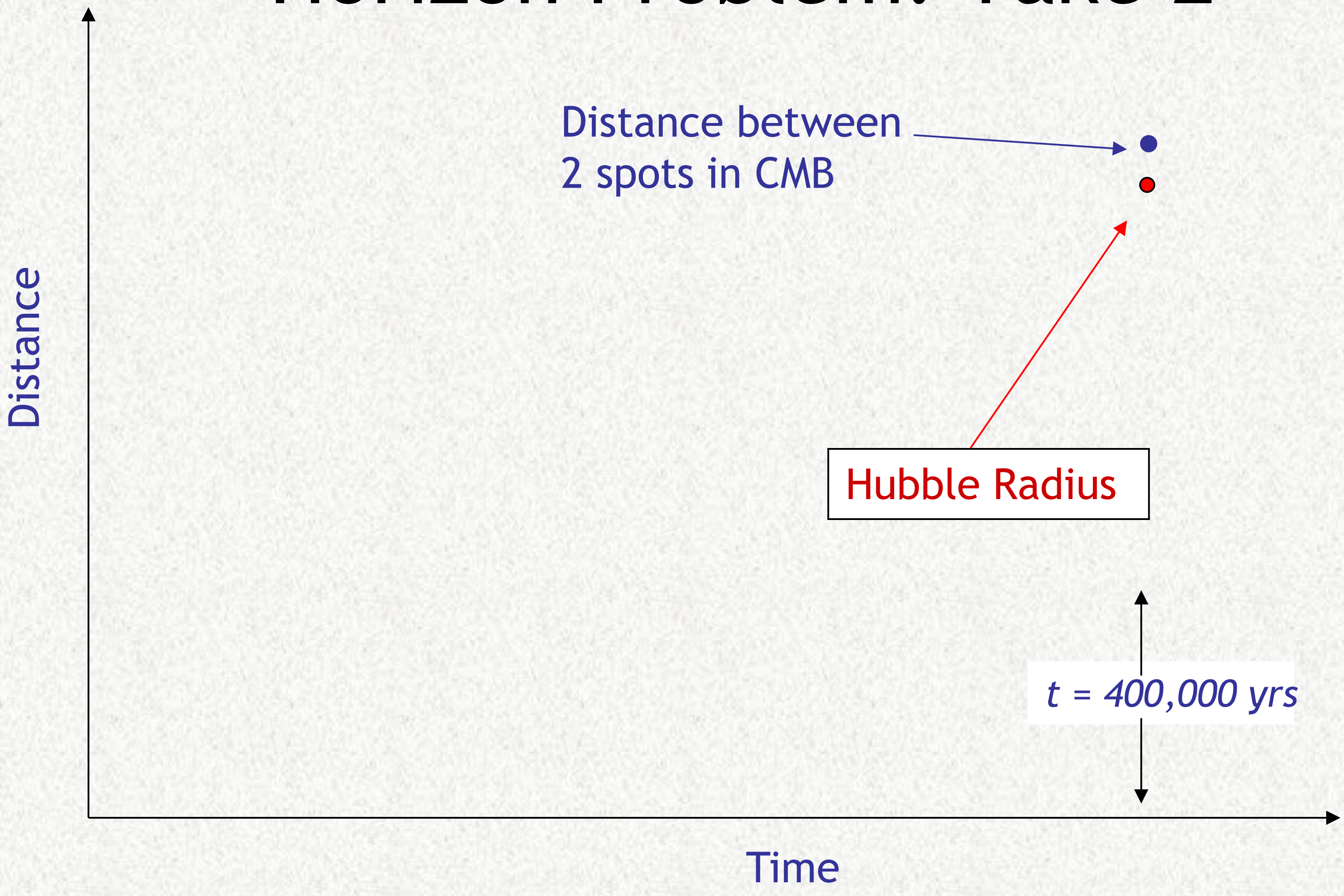


Horizon Problem

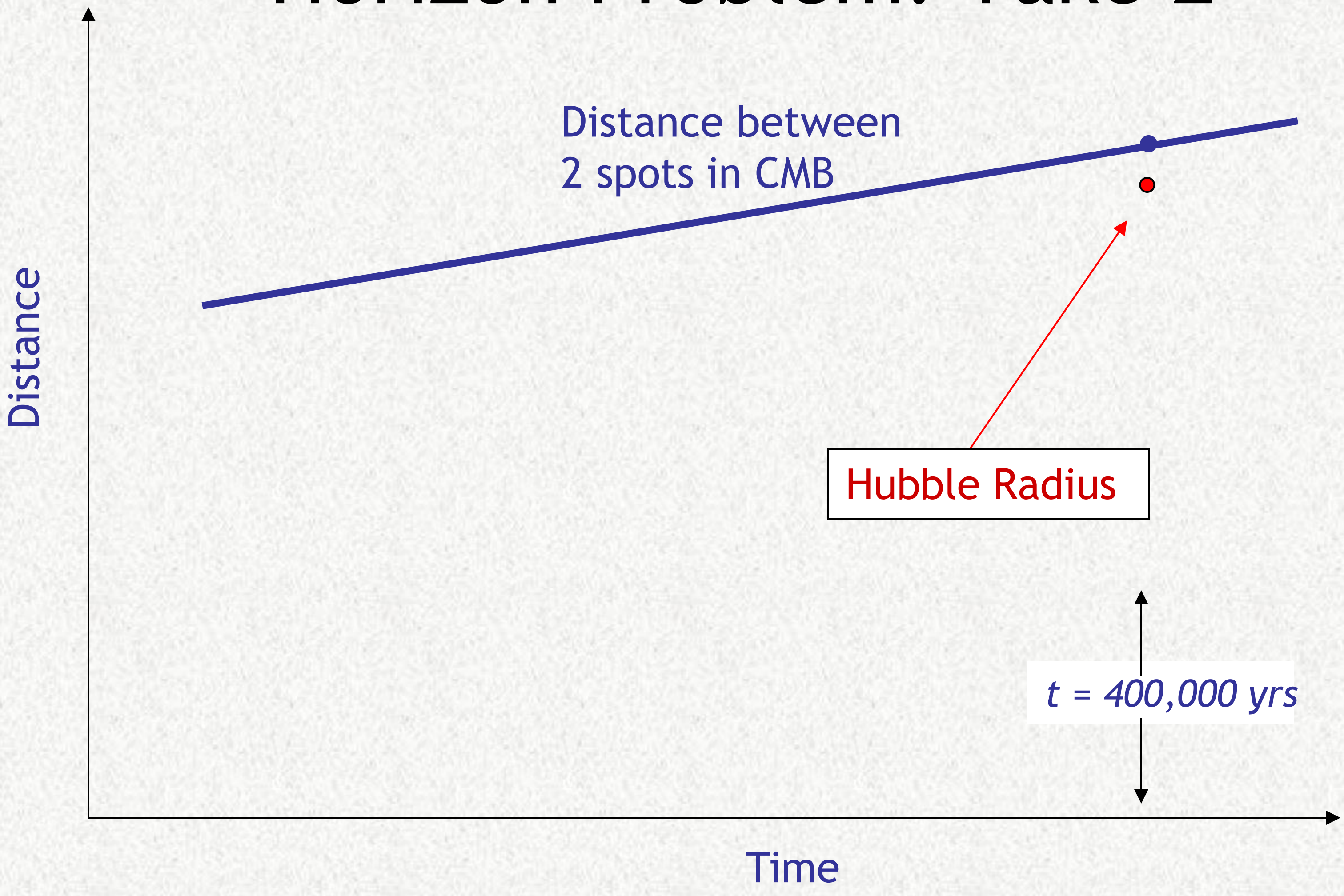


Hubble Radius =c/H (Distance light travels as the Universe doubles in size) at *t=400,000* years

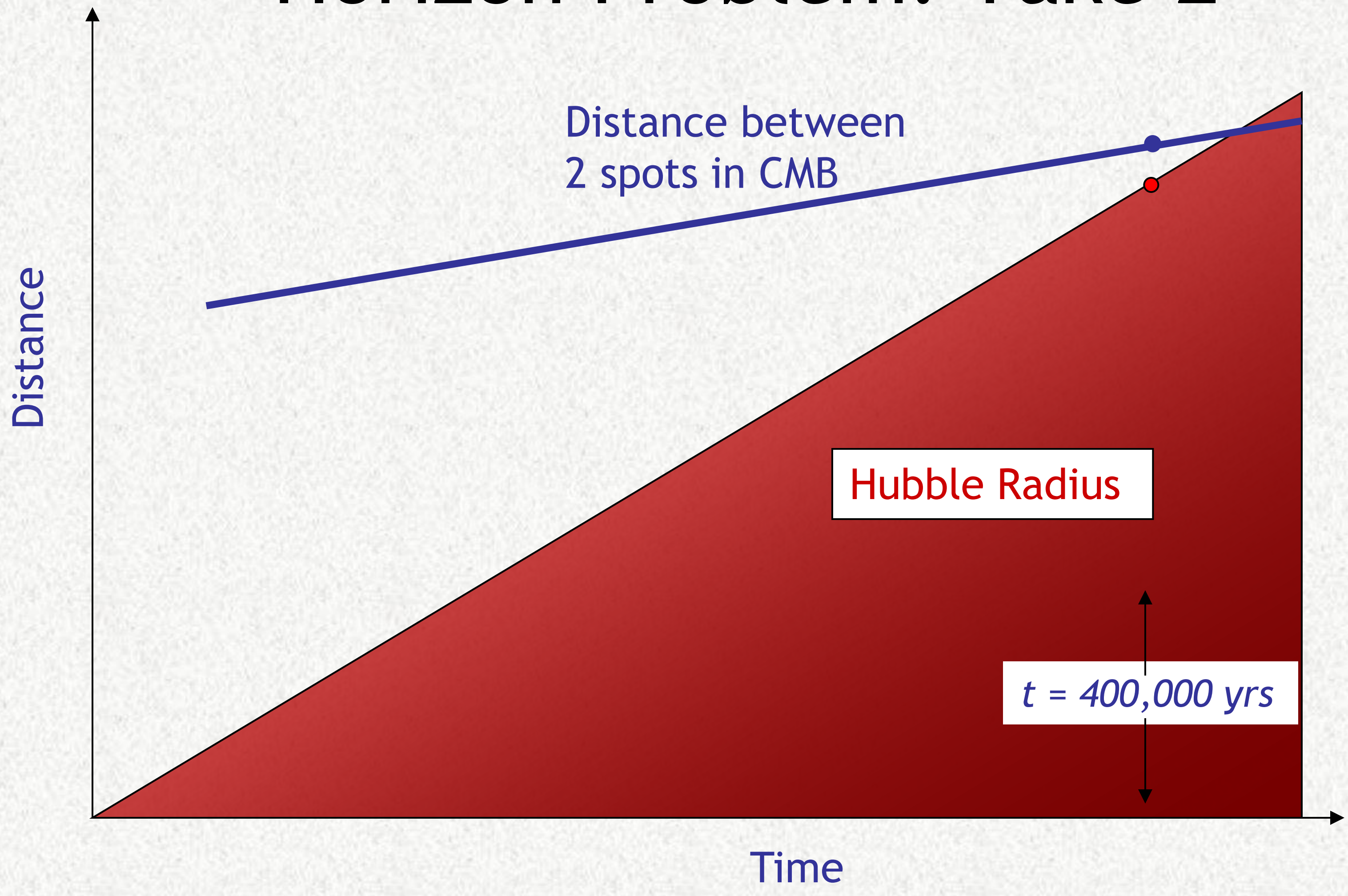
Horizon Problem: Take 2



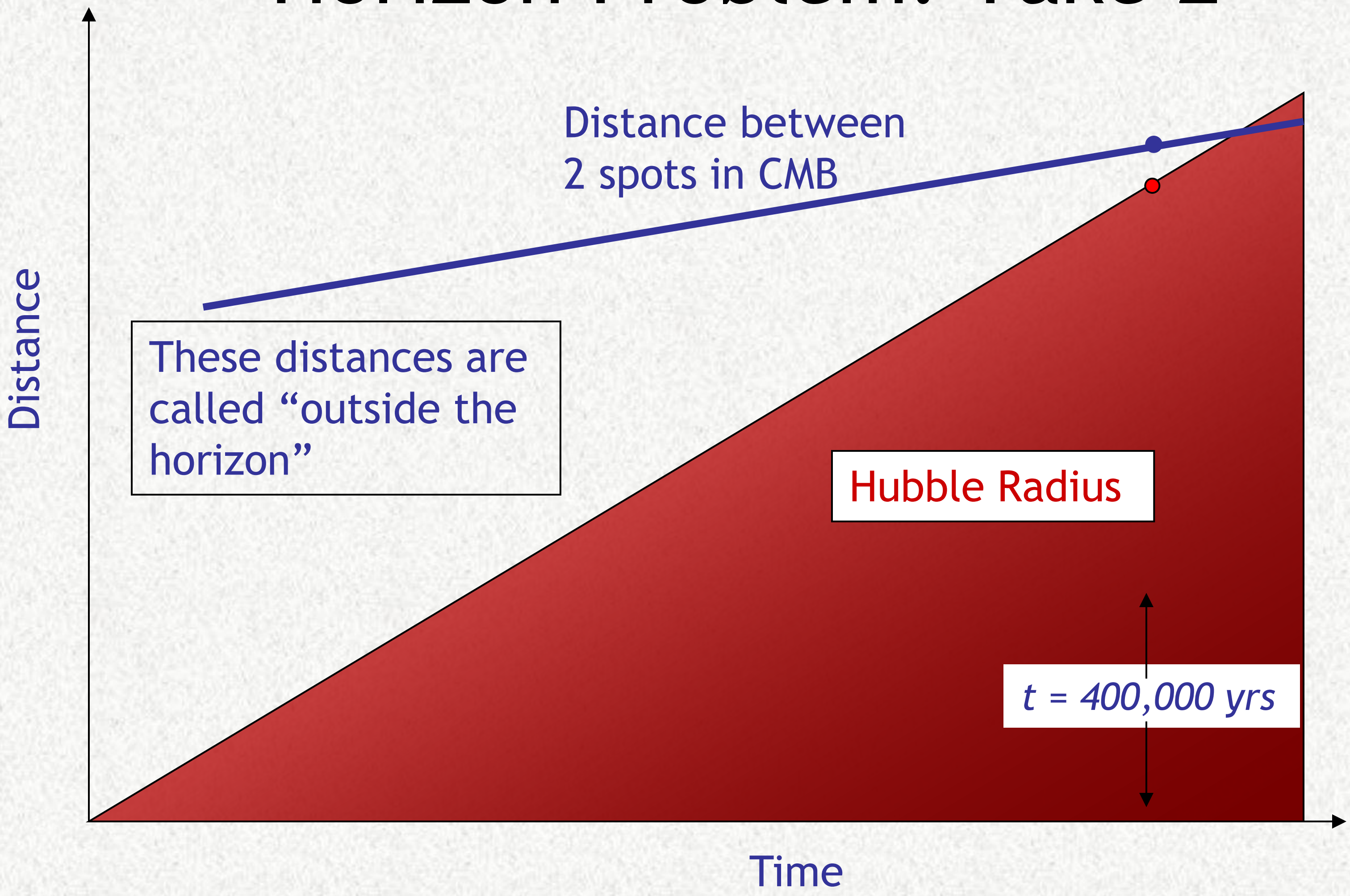
Horizon Problem: Take 2



Horizon Problem: Take 2



Horizon Problem: Take 2



Comoving Horizon

For particles which move at the speed of light *$ds^2=0$*, so *$dx=dt/a$*. Integrating up to time *t* gives the total comoving distance traveled by light since the beginning of expansion.

$$\eta = \int dx = \int_0^t \frac{dt'}{a(t')}$$

□ is called the *comoving horizon.*

Exercise: Compute □ in a matter dominated and radiation dominated universe

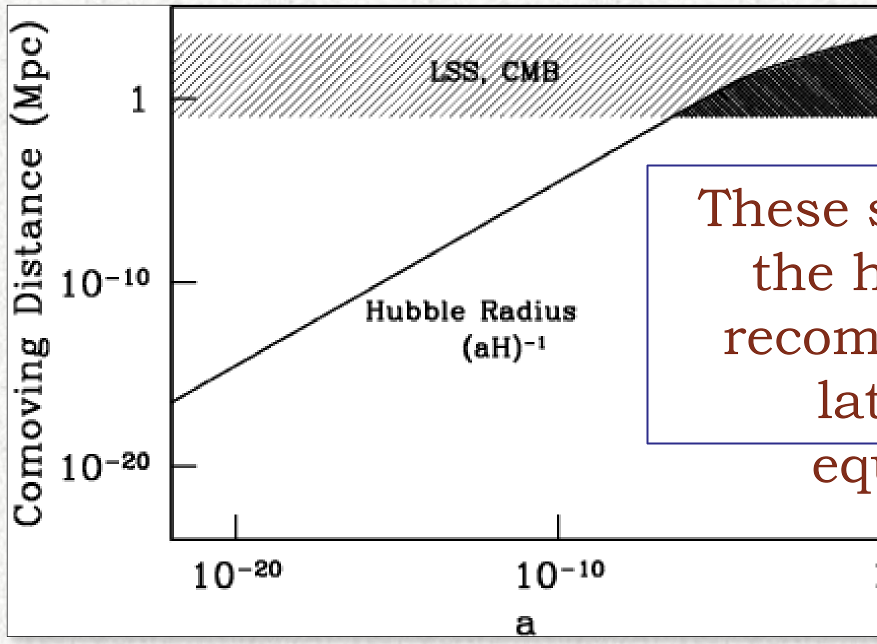
Hubble Radius

Can rewrite as integral over **comoving Hubble radius** *$(aH)^{-1}$*, roughly the comoving distance light can travel as the universe expands by a factor of 2.

$$\int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{da'/dt'}\frac{1}{a'} = \int_0^a \frac{da'}{a'}\frac{1}{a'H(a')}$$

Horizon Problem: Take 3

Compare comoving Cosmological Scales with comoving Hubble Radius



These scales entered the horizon after recombination, too late to have equilibrated

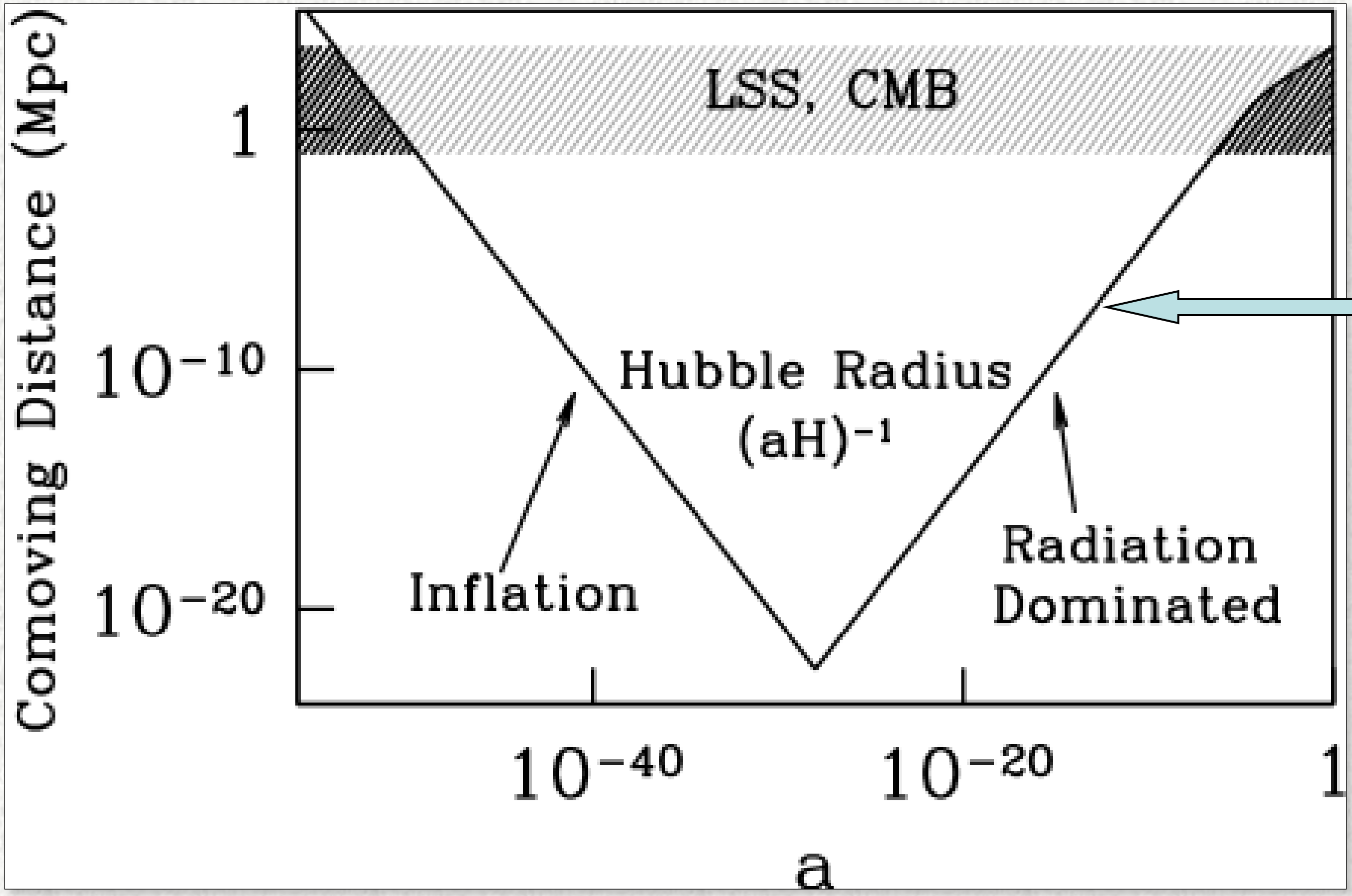
Hubble Radius

Can rewrite □ as integral over **comoving Hubble radius** *(aH)*$^{-1}$, roughly the comoving distance light can travel as the universe expands by a factor of 2.

$$\int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{da'/dt'}\frac{1}{a'} = \int_0^a \frac{da'}{a'}\frac{1}{a'H(a')}$$

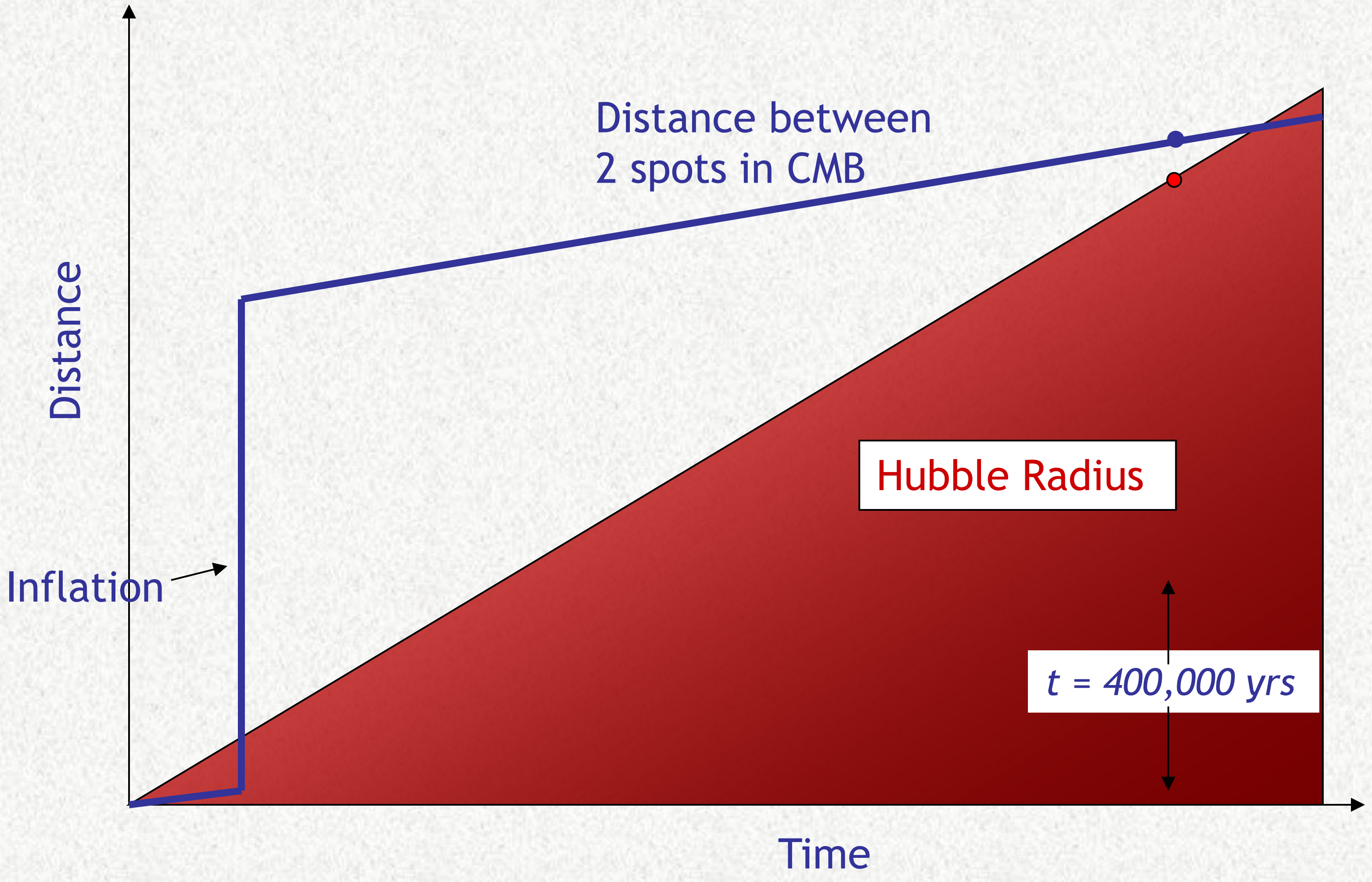
The horizon can be large even if the Hubble radius is small: e.g. if most of the contribution to □ came from early times.

Inflation in terms of Comoving Distances



Cosmological scales *outside the horizon*

Inflation in terms of Physical Distances



Early Dark Energy

Inflation correspond to an epoch in which the comoving Hubble radius decreases.

$$\frac{d}{dt}(aH)^{-1} = \frac{d}{dt}(\dot{a})^{-1} = \frac{-\ddot{a}}{\dot{a}^2} < 0$$

$$\Rightarrow \ddot{a} > 0$$

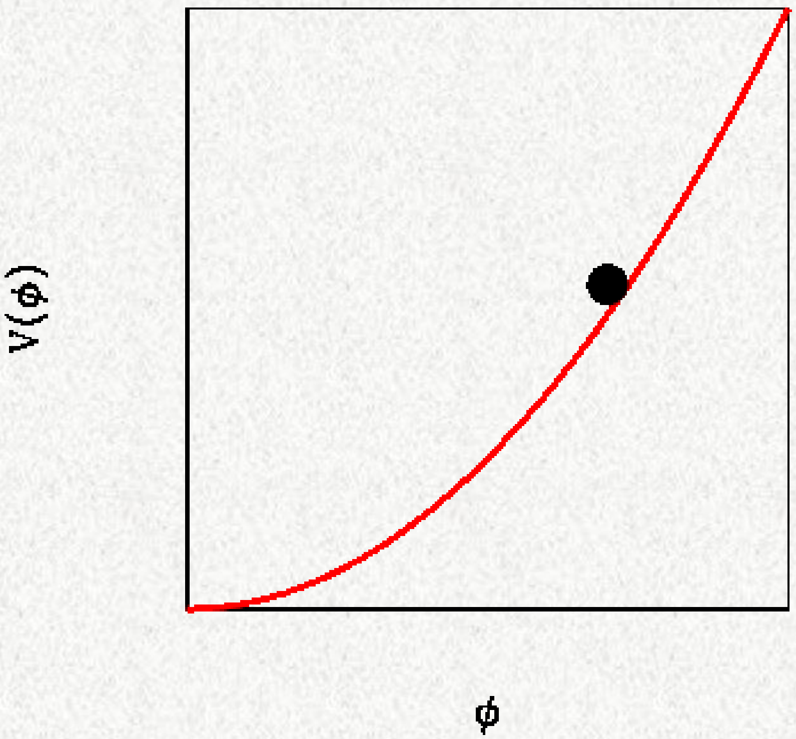
Inflation is an epoch early in which dark energy dominated the universe. This early dark energy has a density ~10^{100} times larger than late dark energy.

Typically model inflation with scalar field

Require:

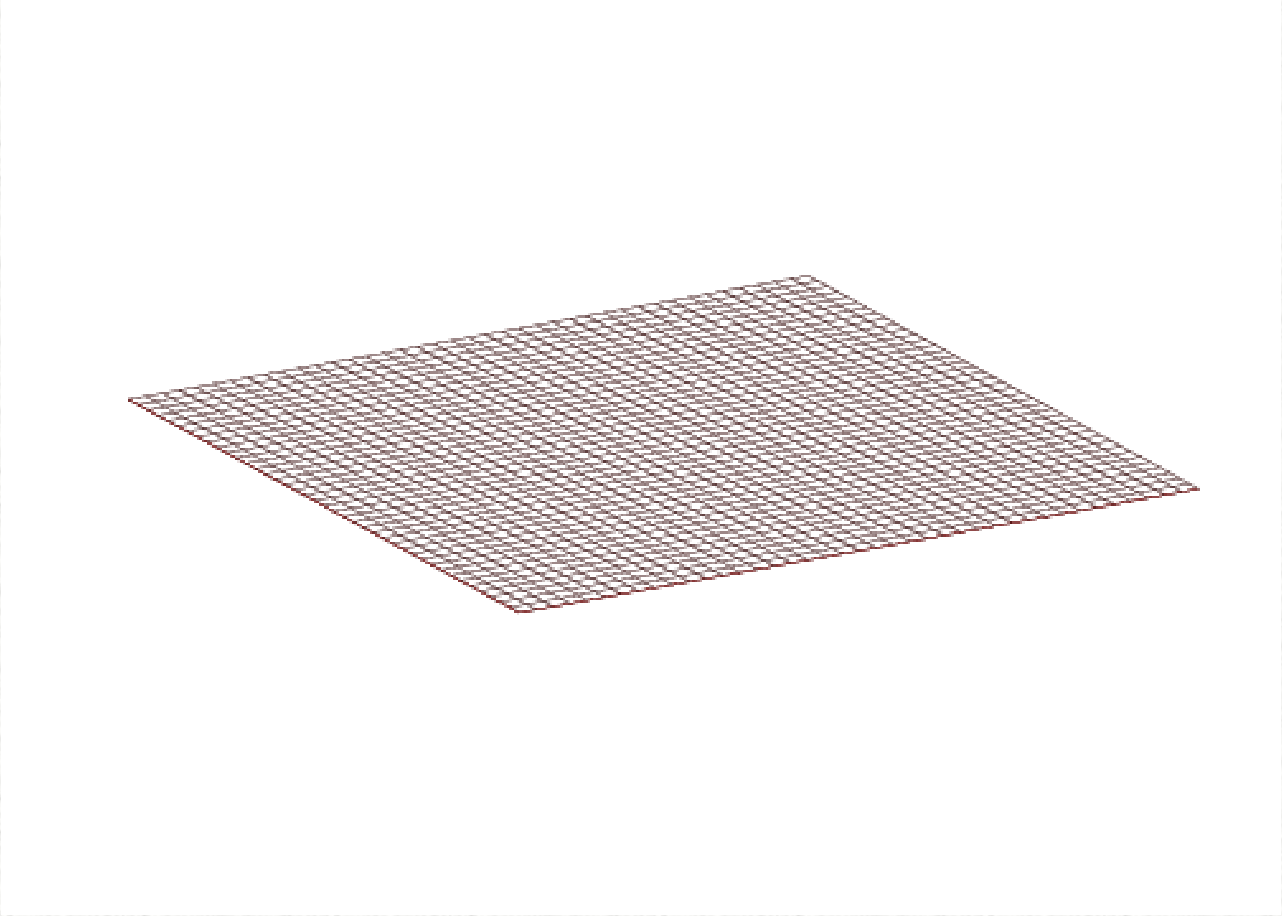
$$\underbrace{\left(\frac{1}{2}\dot{\varphi}^2+V\right)}_{\rho}+3\underbrace{\left(\frac{1}{2}\dot{\varphi}^2-V\right)}_{P}<0$$

$$\Rightarrow V>\dot{\varphi}^2$$



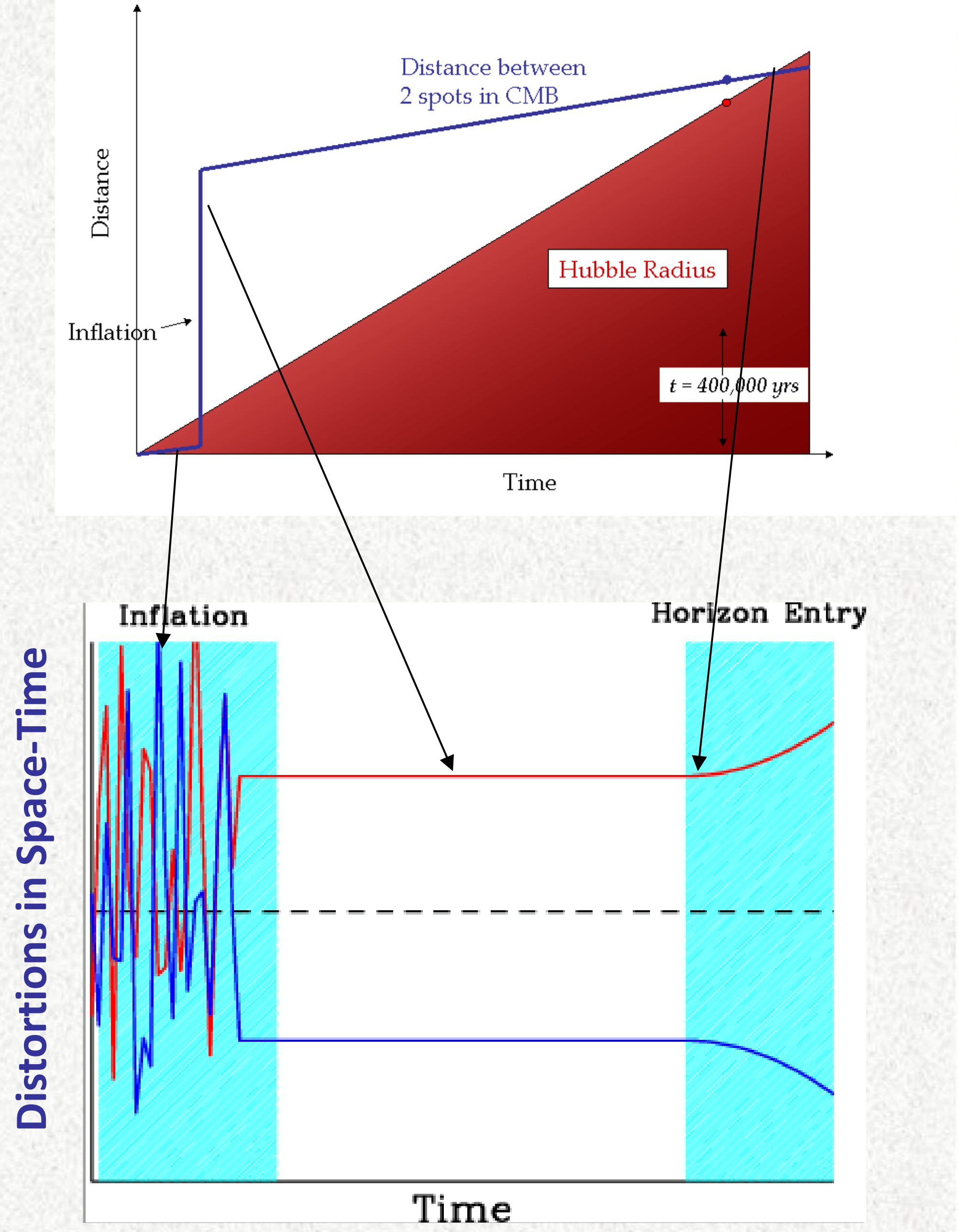
Simplest models are single-field slow-roll models

Perturbations to the FRW metric

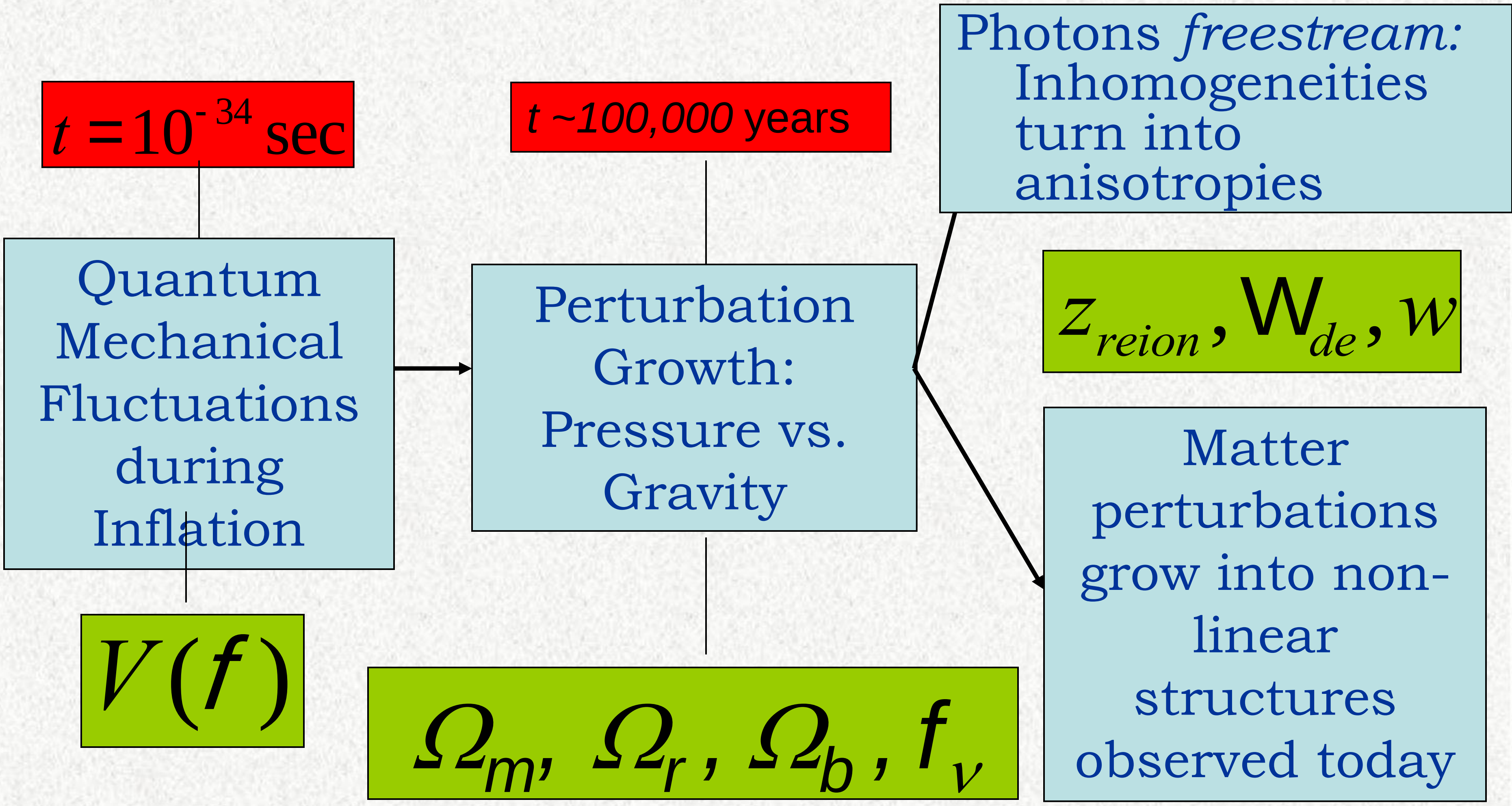


Seeds of Structure

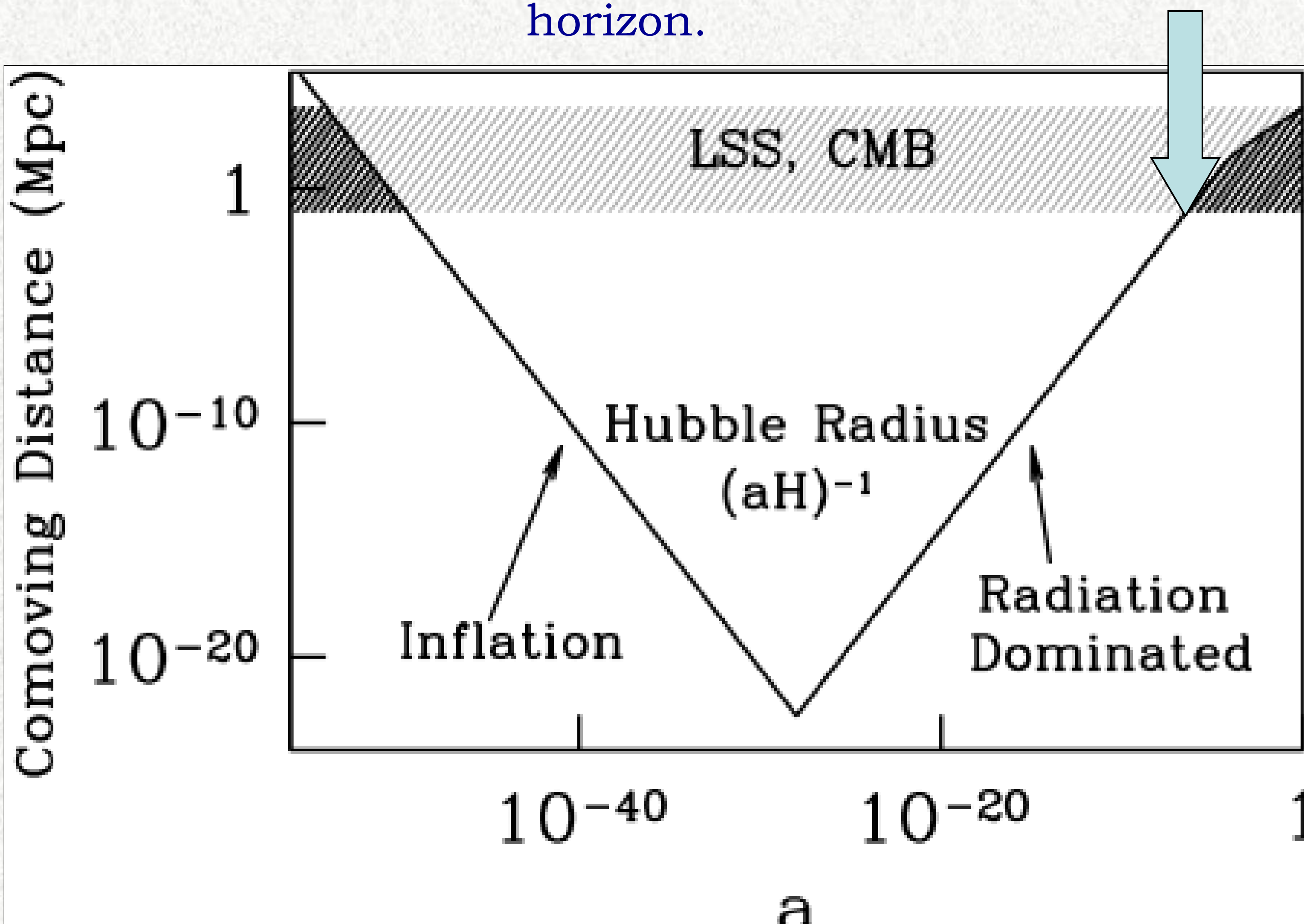
- *Quantum mechanical* fluctuations generated during inflation
- Perturbations *freeze out* when distances get larger than horizon
- *Evolution* when perturbations re-enter horizon



Coherent picture of formation of structure in the universe



Now determine the evolution of perturbations when they re-enter the horizon.



CMB Acoustic Oscillations

Pressure of radiation acts against clumping

If a region gets overdense, pressure acts to reduce the density: **restoring force**

CMB Acoustic Oscillations

Before recombination, electrons and photons are tightly coupled: equations reduce to

Temperature perturbation

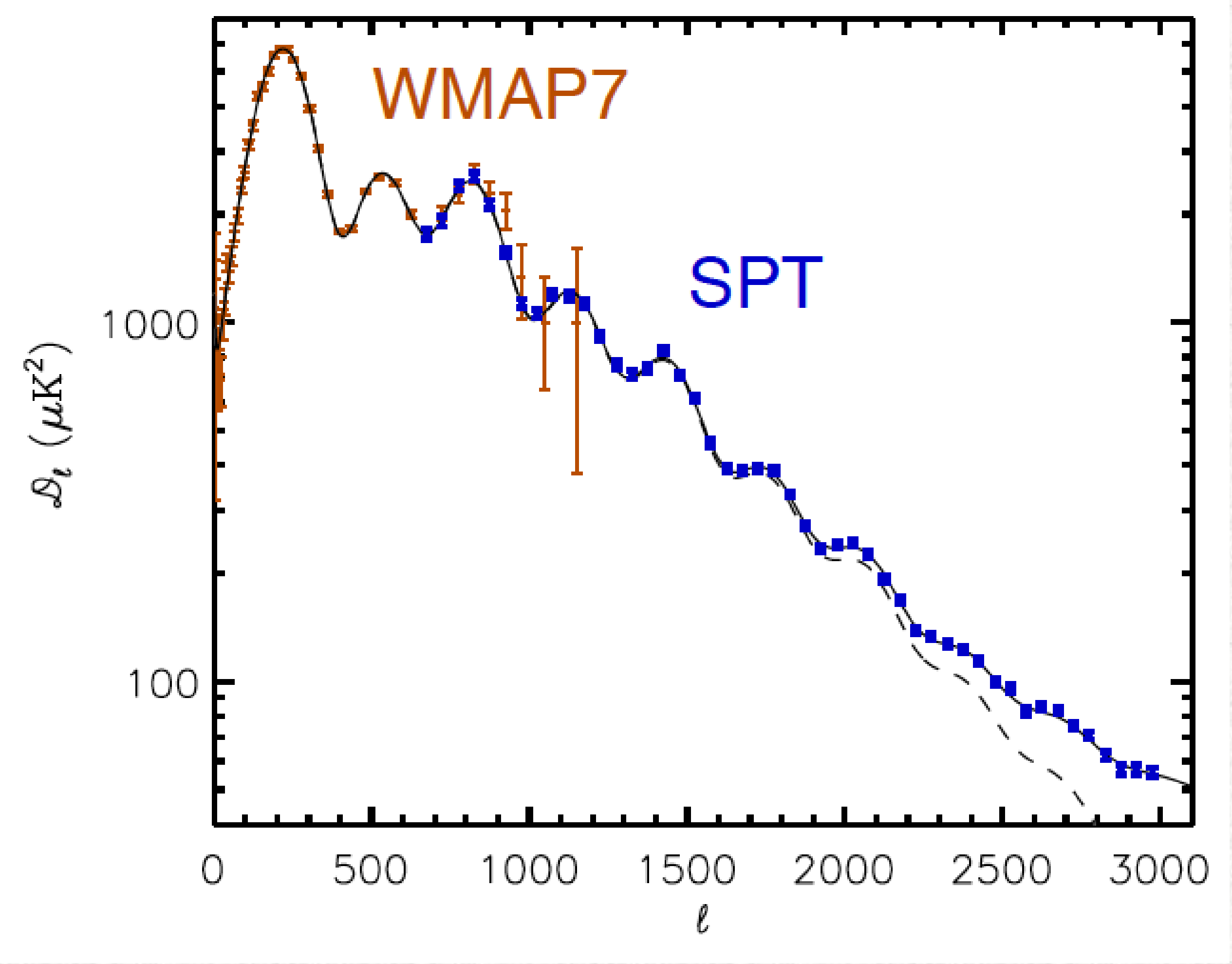
$$\frac{\partial^2 T}{\partial t^2} - c_s^2 \nabla^2 T = F[\Phi]$$

very similar to …

Displacement of a string

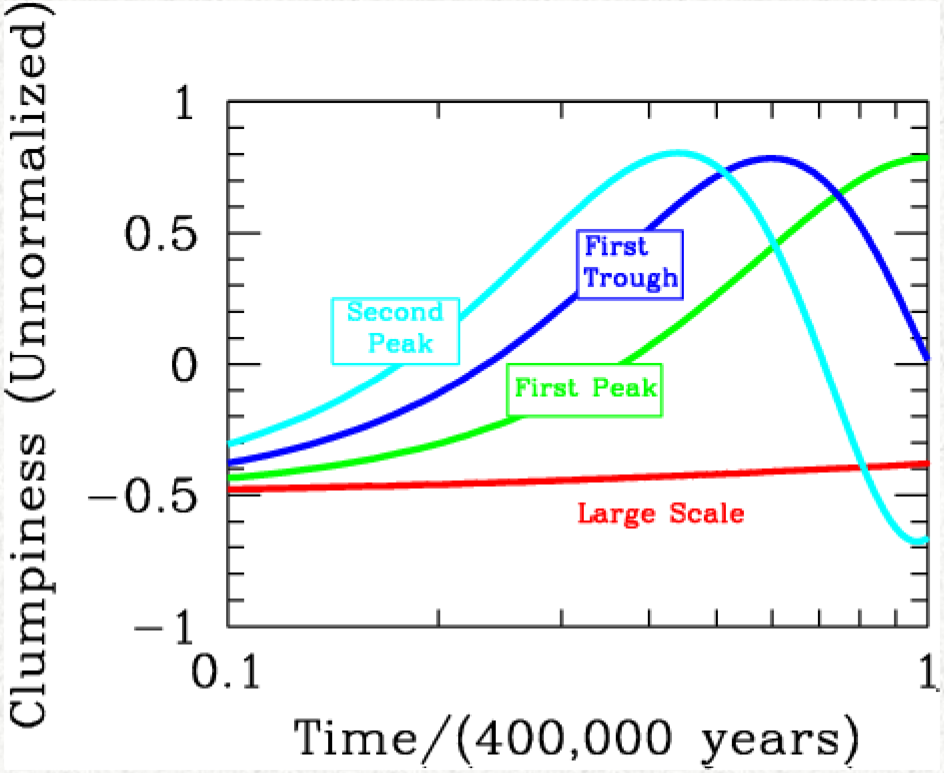
$$\frac{\partial^2 y}{\partial t^2} - c_s^2 \frac{\partial^2 y}{\partial x^2} = F$$

CMB Acoustic Oscillations



Why peaks and troughs?

- Vibrating String: Characteristic frequencies because ends are tied down
- Temperature in the Universe: Small scale modes begin oascillating earlier than large scale modes



CMB Acoustic Oscillations

In Fourier space, this becomes

$$\ddot{x} + w^2 x = F$$

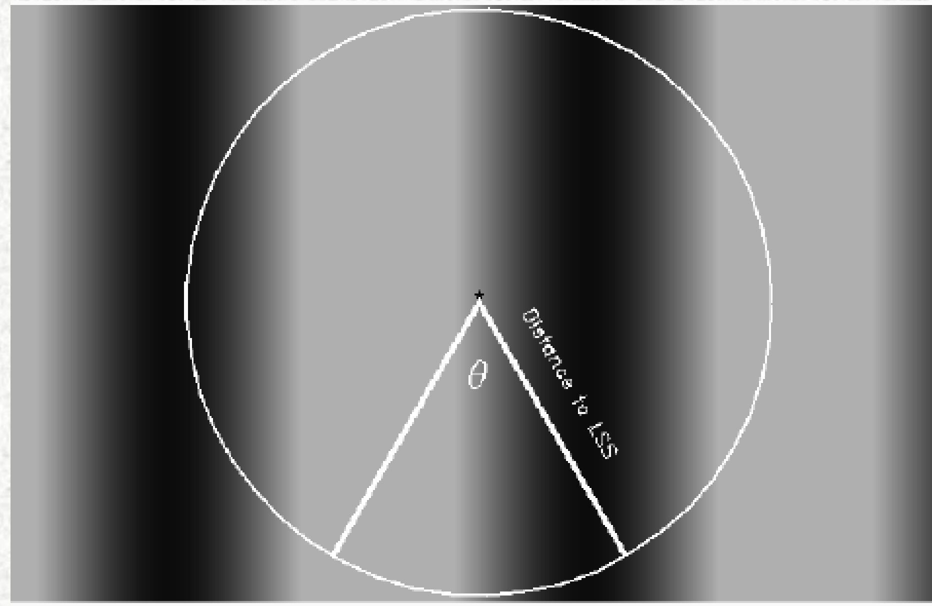
Forced Harmonic Oscillator

with

$$\omega = kc_s = \frac{k}{\sqrt{3(1 + 3\rho_b / 4\rho_\gamma)}}$$

CMB Acoustic Oscillations

$$\ddot{x} + \omega^2 x = F$$



Peaks at:

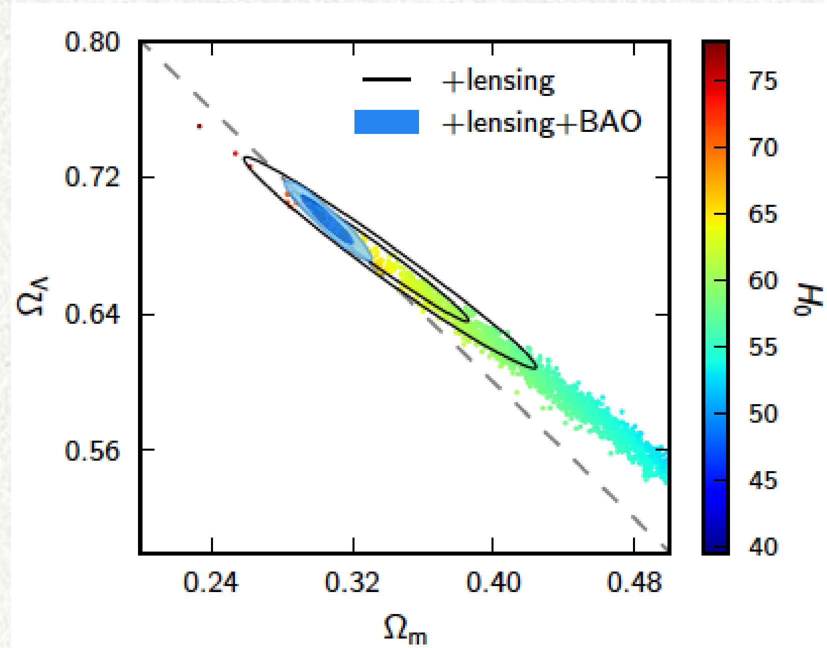
$$\int_0^{t_*} dt\, \omega_n \equiv k_n r_s = n\pi$$

Peaks show up at angular scale $\theta \sim \dfrac{r_s}{D_*}$

CMB Acoustic Oscillations

$$\ddot{x} + \omega^2 x = F$$

The Universe is Flat!



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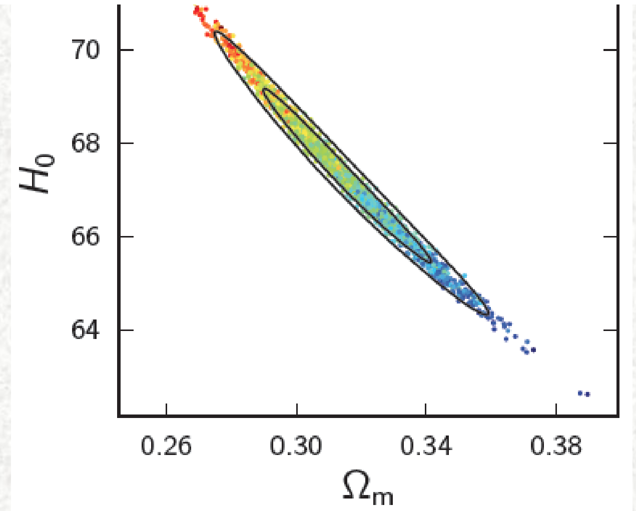
CMB Acoustic Oscillations

$$\ddot{x} + \omega^2 x = F$$

In flat models, peak locations determine one combination of parameters



$$\Omega_m h^3 = 0.0959 \pm 0.0006 \quad (68\%; \textit{Planck})$$

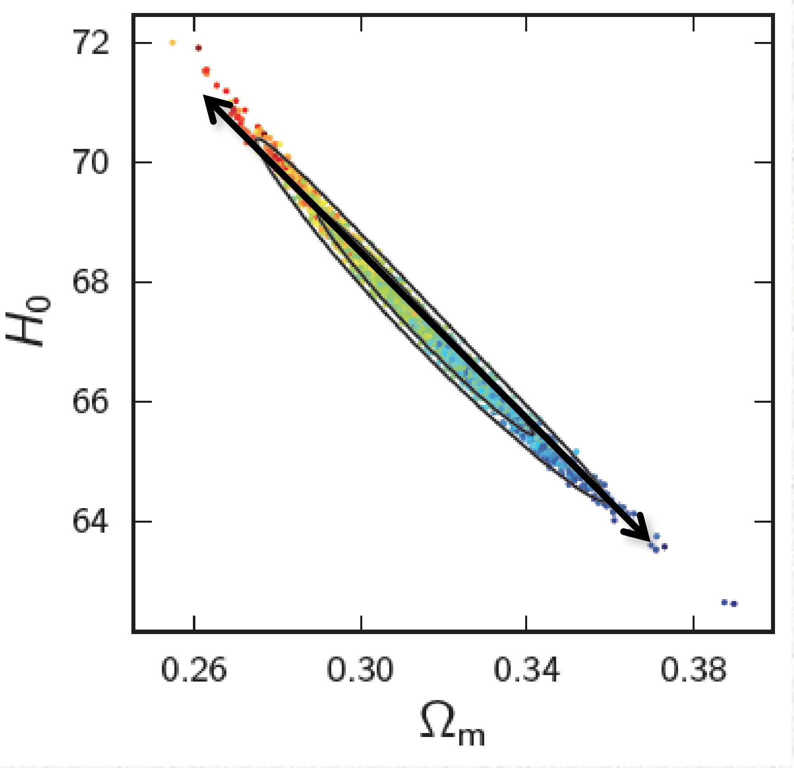
Peaks at:

$$\int_0^{t_*} dt\, \omega_n \equiv k_n r_s = n\pi$$

Peaks show up at angular scale $\theta \sim \dfrac{r_s}{D_*}$

Constraints in Degeneracy Direction come from peak heights

Much more difficult to pin down – especially when combining experiments – as sensitive calibration is required



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with

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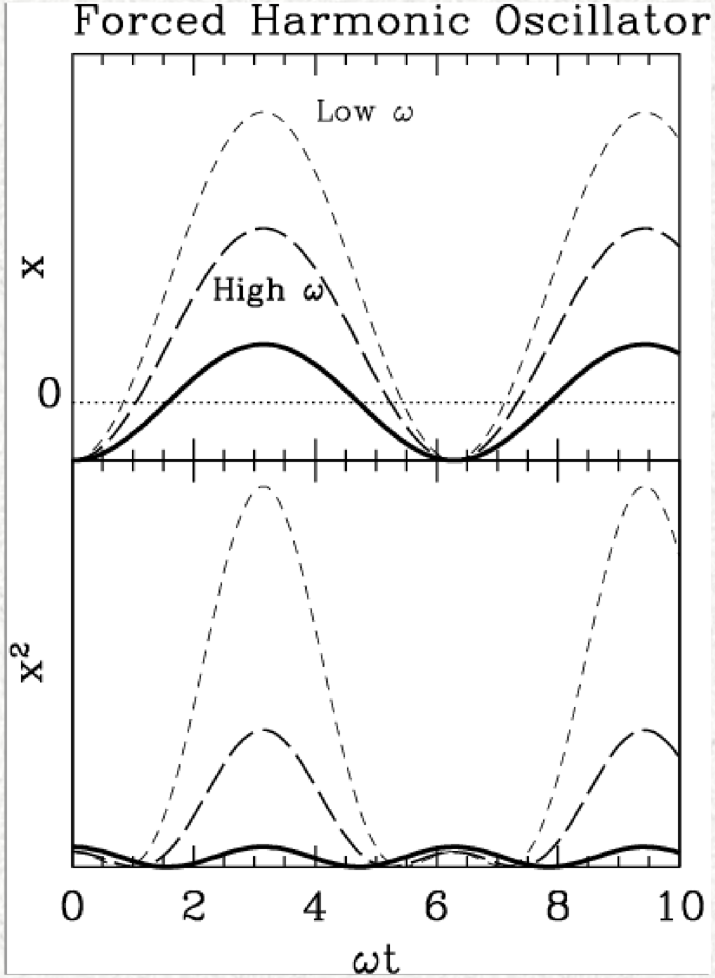
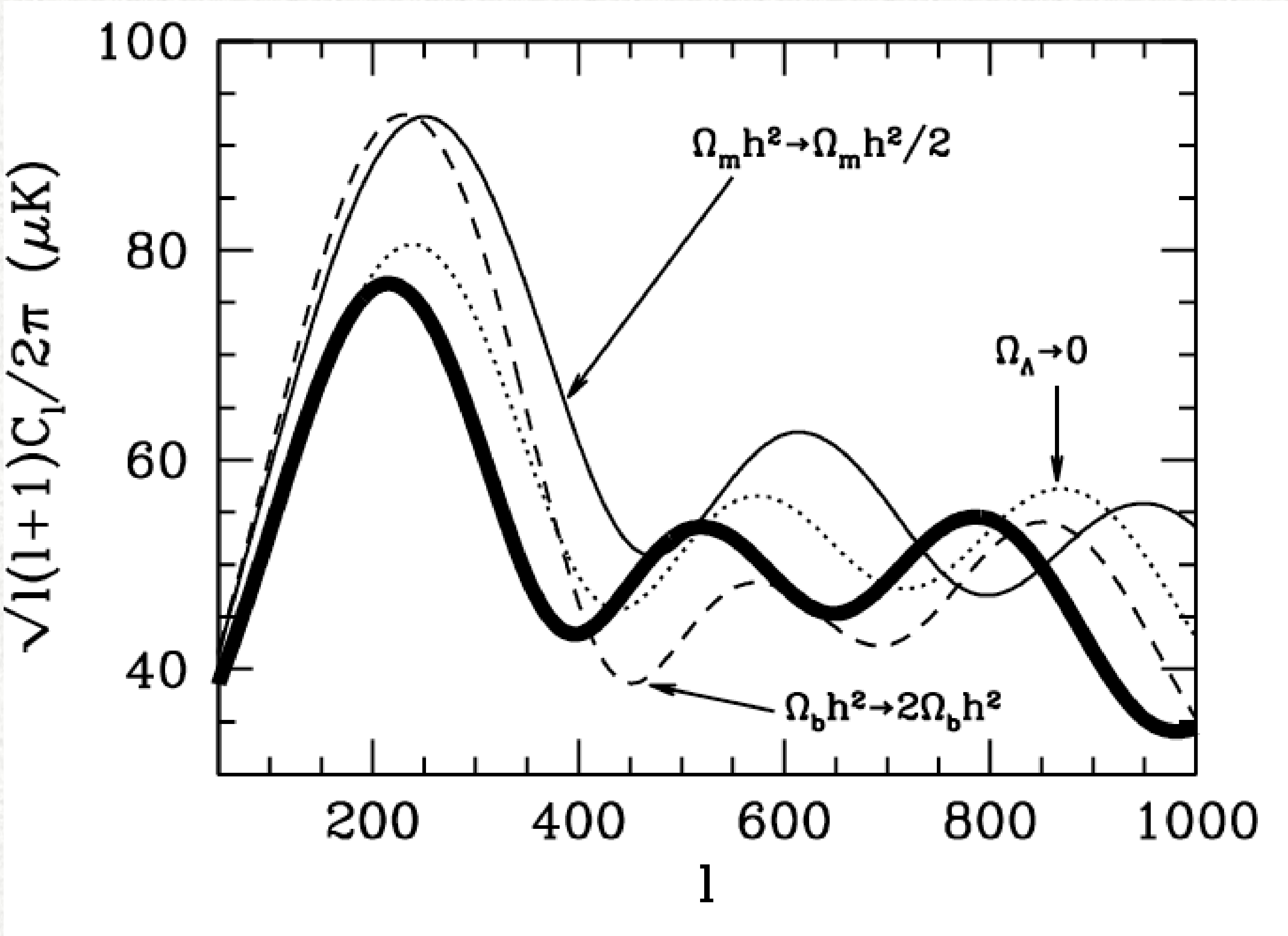


Immediately see: lower ω (e.g. with more baryons) -> greater odd/even peak disparity.

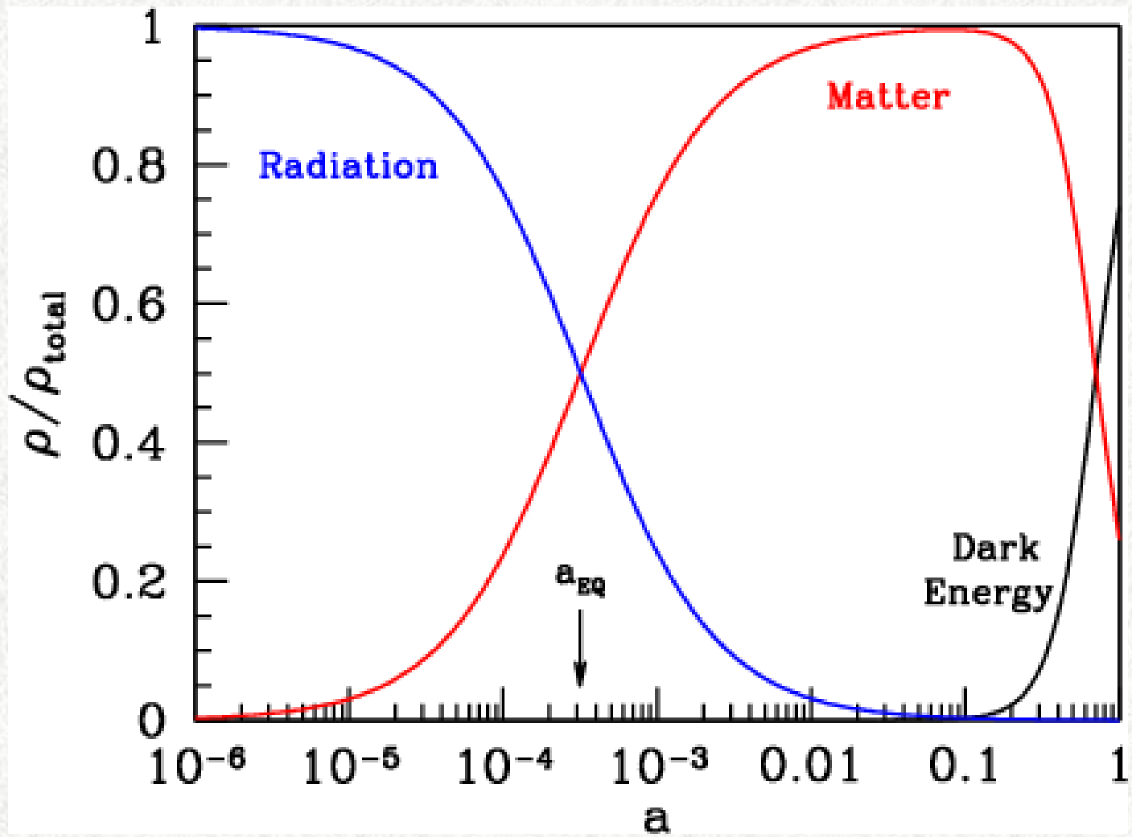
CMB Acoustic Oscillations



Reducing the matter density brings the epoch of equality closer to recombination

$$\frac{\partial^2 T}{\partial t^2} - c_s^2 \nabla^2 T = F[\mathrm{F}]$$

- Forcing term larger when potentials decay
- During the radiation era, potentials decay, leading to a *larger* anisotropy at first peak



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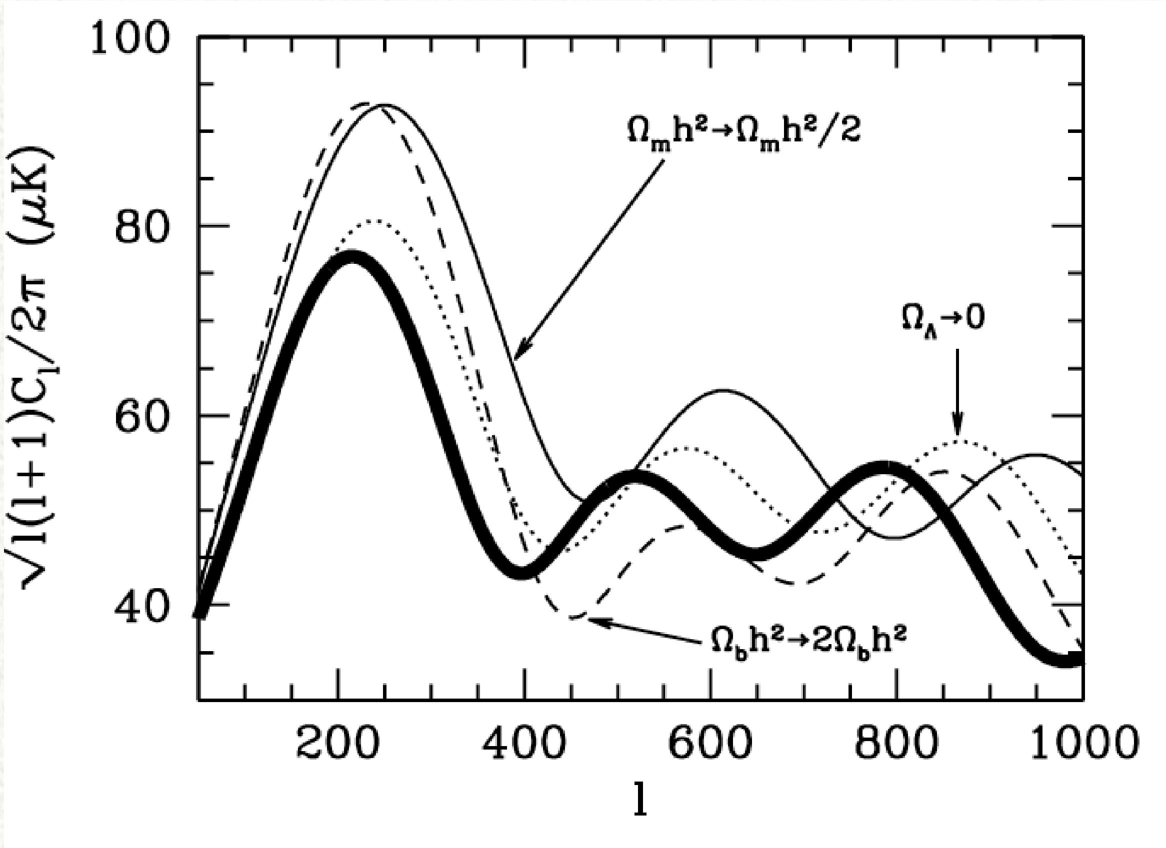
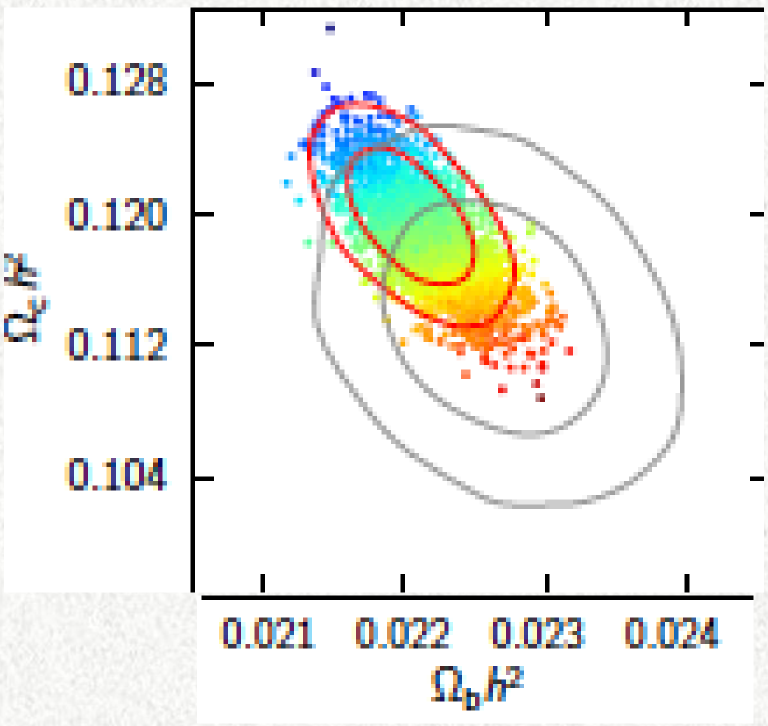


CMB Constraints on Baryonic and Dark Matter Densities

Some movement from WMAP (gray contours) to Planck (red) but … bottom line is: strong evidence for non-baryonic dark matter



Growth of Structure: Gravitational Instability

Define overdensity:

$$\delta(\vec{x},t) \equiv \frac{\rho(\vec{x},t) - \overline{\rho}(t)}{\overline{\rho}(t)}$$

Fundamental equation governing overdensity in a matter-dominated universe when scales are within horizon:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\overline{\rho}_m \delta = 0$$

Growth of Structure: Gravitational Instability

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m \delta = 0$$

Example 1: No expansion (H=0,energy density constant)

$$\delta \propto e^{\pm t\sqrt{4\pi G\bar{\rho}_m}}$$

- Two modes: growing and decaying
- Growing mode is exponential (the more matter there is, the stronger is the gravitational force)

Gravitational Instability in an Expanding Universe

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m \delta = 0$$

Example 2: Matter density equal to the critical density in an expanding universe.

The coefficient of the 3rd term is then *$3H^2/2$*, so

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\delta = 0$$

Exercise: Show that, in this universe, *$a=(t/t_0)^{2/3}$* so *$H=2/(3t)$*

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

Gravitational Instability in an Expanding Universe

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

Insert solution of the form: $\delta \sim t^p$

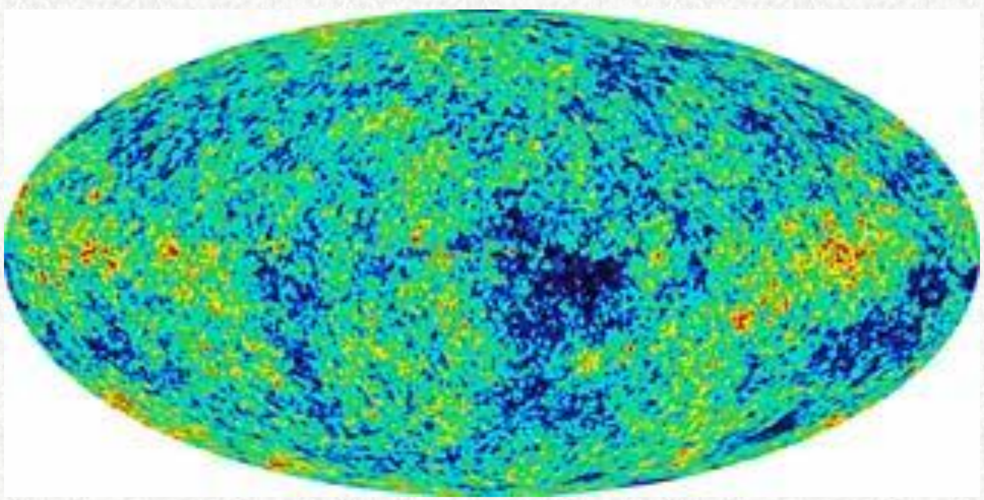
Growing mode: $\delta \sim a$. Dilution due to expansion counters attraction due to overdensity. Result: power law growth instead of exponential growth

$$p = \frac{\quad}{6} \pm \frac{\quad}{2}\sqrt{\frac{\quad}{9} + \frac{8}{3}} = \begin{cases} 2/3 \\ -1 \end{cases}$$

Argument for Dark Matter

$$\frac{d_0}{d(t_*)} = 1000$$

Perturbations have grown by (at most) a factor of 1000 since recombination.



If this map accurately reflected the level of inhomogeneity at recombination, over-densities today should be less than 10%. I.e. we should not exist

MOdified Newtonian Dynamics (MOND) (Milgrom 1983):

$$a_g F(a_g / a_0) = a_N = \frac{MG}{r^2}$$

Acceleration due to gravity (v^2/r for circular orbit)

New,fundamental scale

For a point mass

$$F(x) = \begin{cases} 1, & x \gg 1 \\ x, & x \ll 1 \end{cases}$$

$$\boxed{\frac{(v^2/r)^2}{a_0} = \frac{MG}{r^2}}$$

This leads to a simple prediction

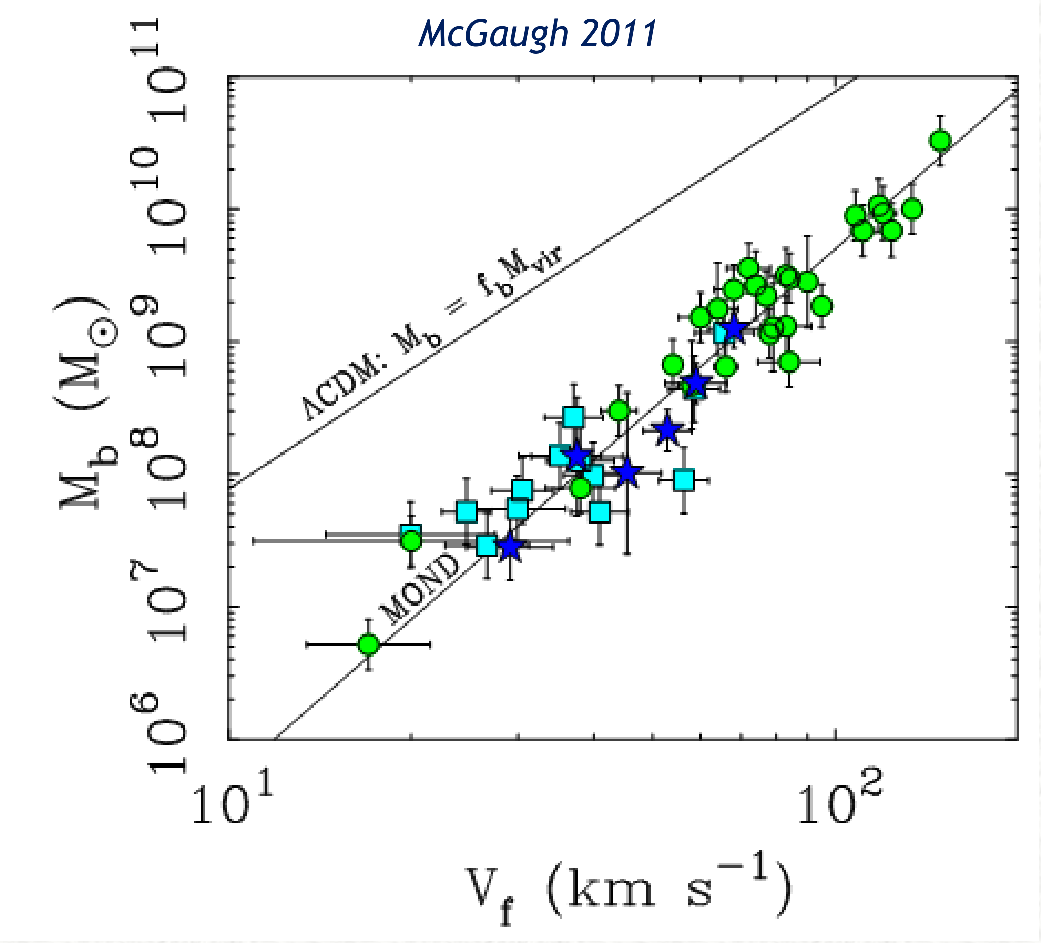
$$\frac{v^2}{r} = \sqrt{\frac{a_0 M G}{r^2}} \Rightarrow v^4 = a_0 M G$$

So MOND predicts

$$M = \frac{v^4}{a_0 G}$$

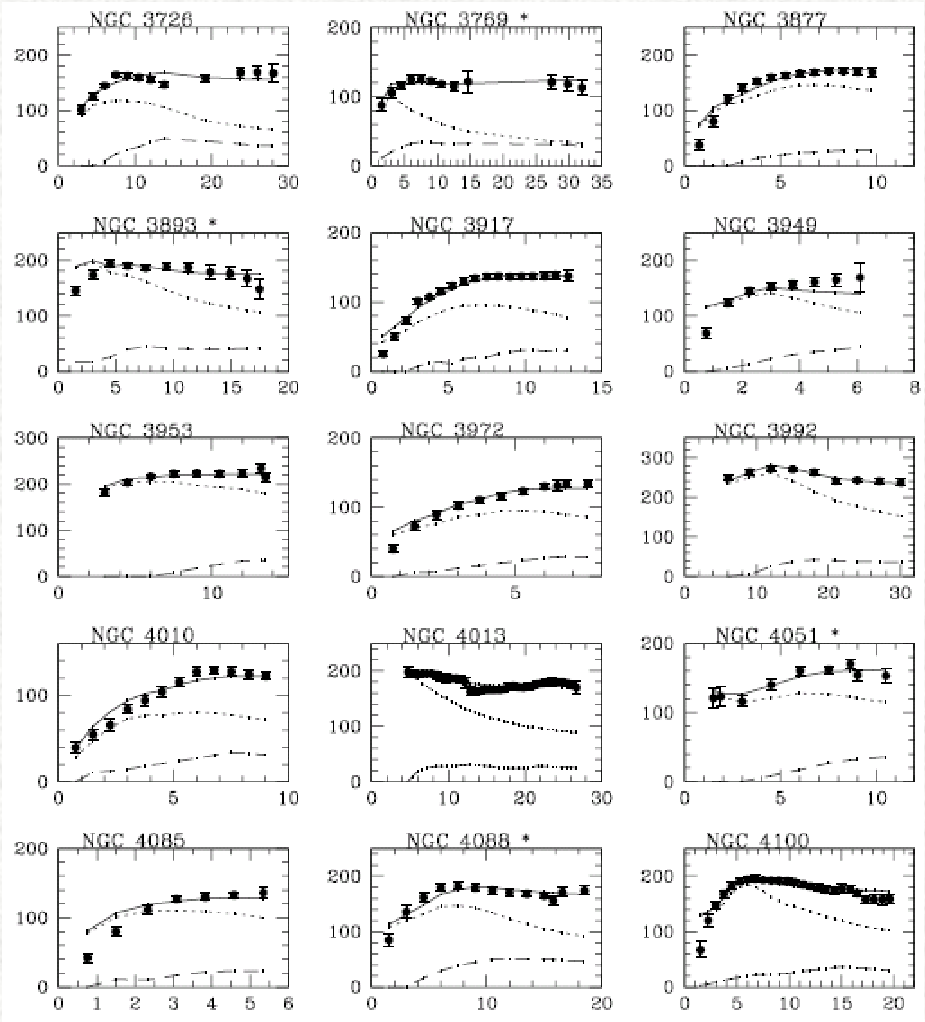
When the acceleration scale is fixed from rotation curves, this is a zero-parameter prediction!

… which has been verified* (Tully-Fisher Law)



* but see Gnedin (1108.2271) , Foreman & Scott (1108.5734)

MOND does a good job doing what it was constructed to do



McGaugh

Fit Rotation Curves of many galaxies w/ only one free parameter (instead of 3 used in CDM).

Making MOND Serious

- MOND is not a covariant theory that can be used to make predictions for cosmology
- Challenge the implicit assumption of General Relativity that the metric in the Einstein-Hilbert action is the same as the metric that couples to matter

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}R[\tilde{g}] \qquad S_m = \int d^4x\sqrt{-g}L_m$$

- Allow $\tilde{g} = e^{-2\varphi/m_P} g$

- Scalar-Tensor model is defined by dynamics *S[Φ]*

Making MOND Serious

$$L_\varphi = \frac{a_0{}^2}{8\pi G} F(\varphi_{,\mu}\varphi^{,\mu} / a_0{}^2)$$

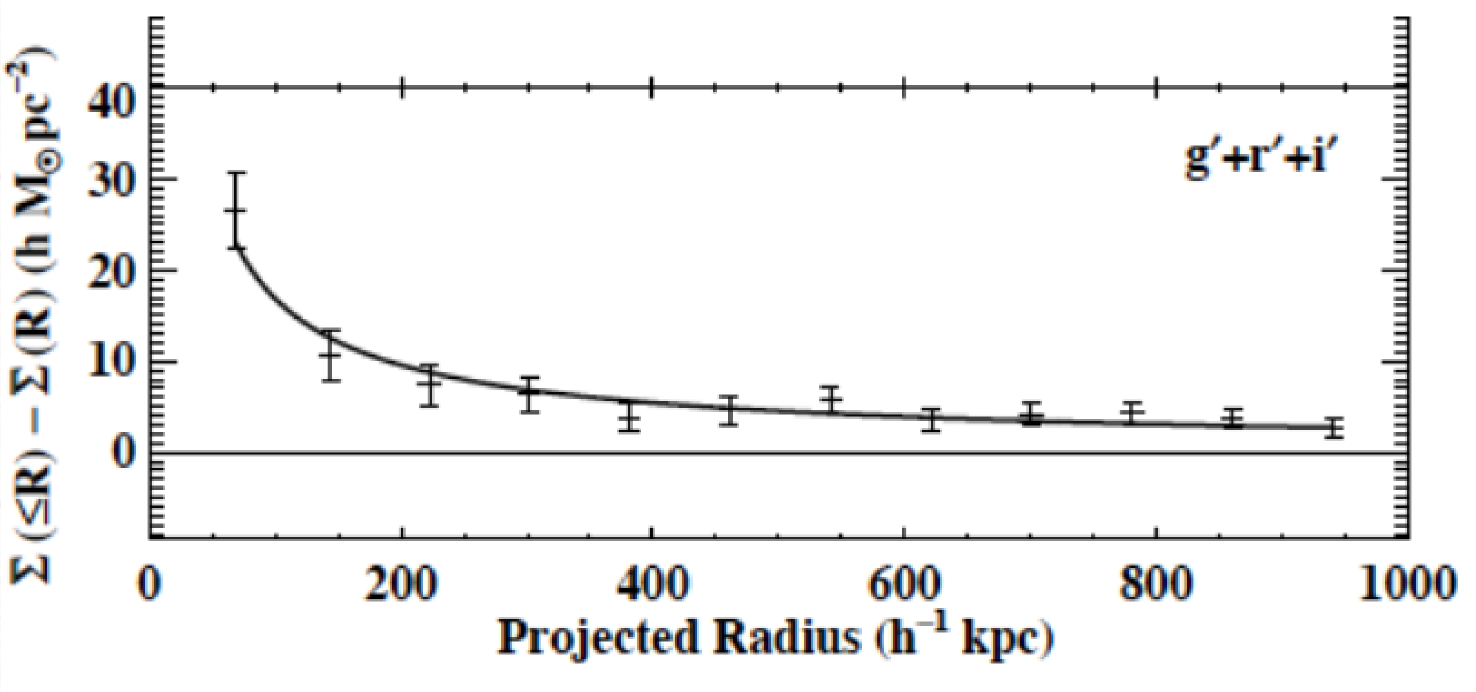
Bekenstein & Milgrom 1984

There is a new fundamental mass scale in the Lagrangian

$$\frac{a_0}{c} \approx \frac{v_{gal}{}^2}{c r_{gal}} \approx \frac{(200\, km/\sec)^2}{(3\times10^5\, km/\sec)(0.005\, Mpc)} = 27 \frac{km}{\sec Mpc} \approx H_0 \approx 10^{-33} eV$$

Same scale used for quintessence or any other approach to acceleration!

Making MOND Serious



- Scalar-Tensor models give same light deflection prediction as GR. ***Exercise:*** Prove this.
- Far from the center of galaxies, signal should fall of as $1/R^2$. ***Exercise:*** Prove this.

Making MOND Serious

- Scalar-Tensor Models fail because of lensing constraints
- Add a vector field (to get more lensing w/o dark matter) to get TeVeS (Bekenstein 2004)
- Relation between 2 metrics now more complex

$$g_{\mu\nu} \equiv e^{-2\phi}\left(\tilde{g}_{\mu\nu} + A_\mu A_\nu\right) - e^{2\phi} A_\mu A_\nu$$

Breaks theorem about light deflection

- We can now do cosmology: is there enough clustering w/o dark matter?

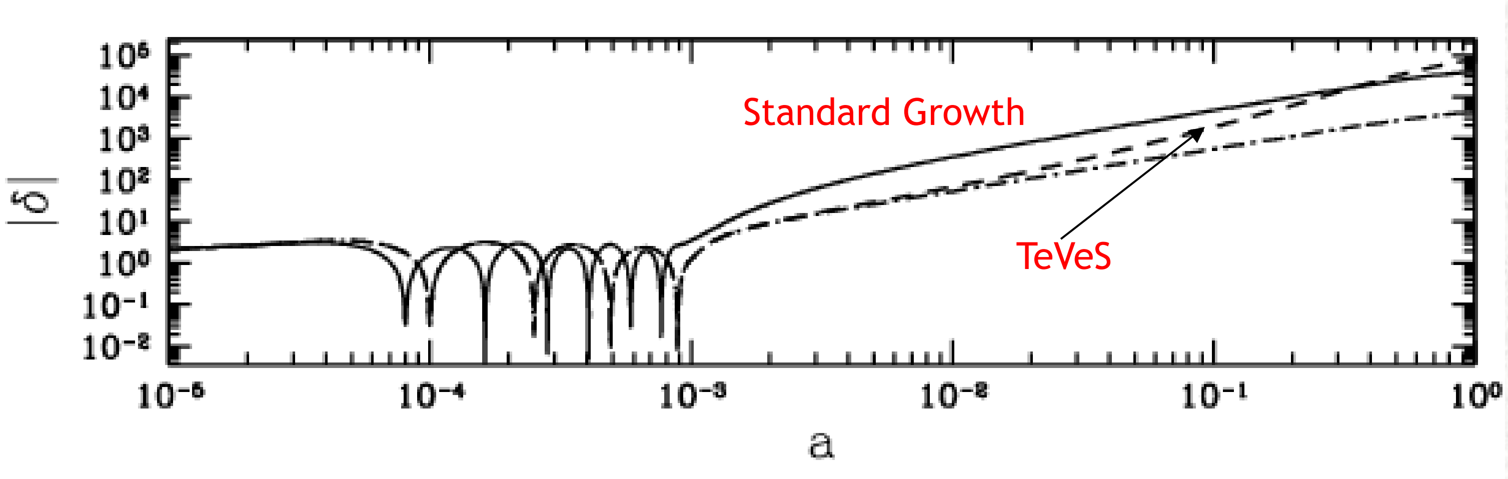
Inhomogeneities in TeVeS

Perturb all fields: (metric, matter, radiation) + (scalar field, vector field)

E.g., the perturbed metric is

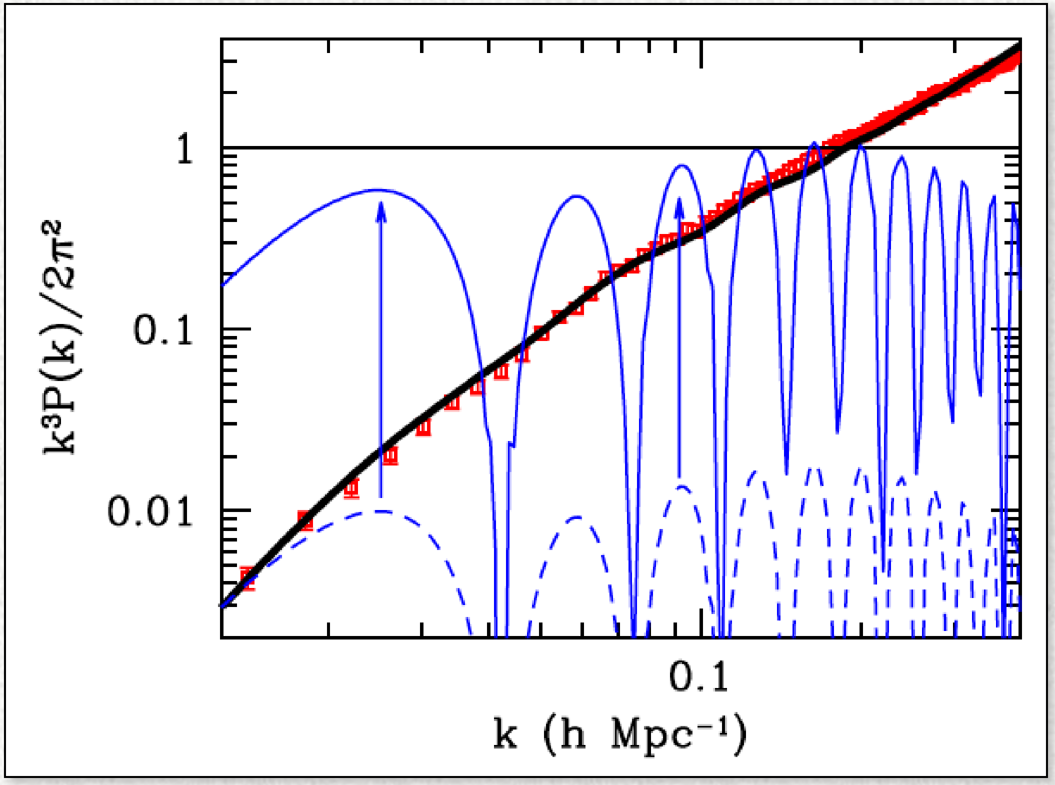
$$g_{\mu\nu} = diag[-a^2(1-2\Psi), a^2(1+2\Phi), a^2(1+2\Phi), a^2(1+2\Phi)]$$

where a depends on time only and the two potentials depend on space and time.



Scorecard for Modified Gravity

- Appears to do at least as well as DM on small scales
- Two coincidences/successes: (i) requires same scale as MG models that drive acceleration (ii) fix to get lensing also enhances growth of structure
- Problems on large scales persist



Neutrinos affect large scale structure

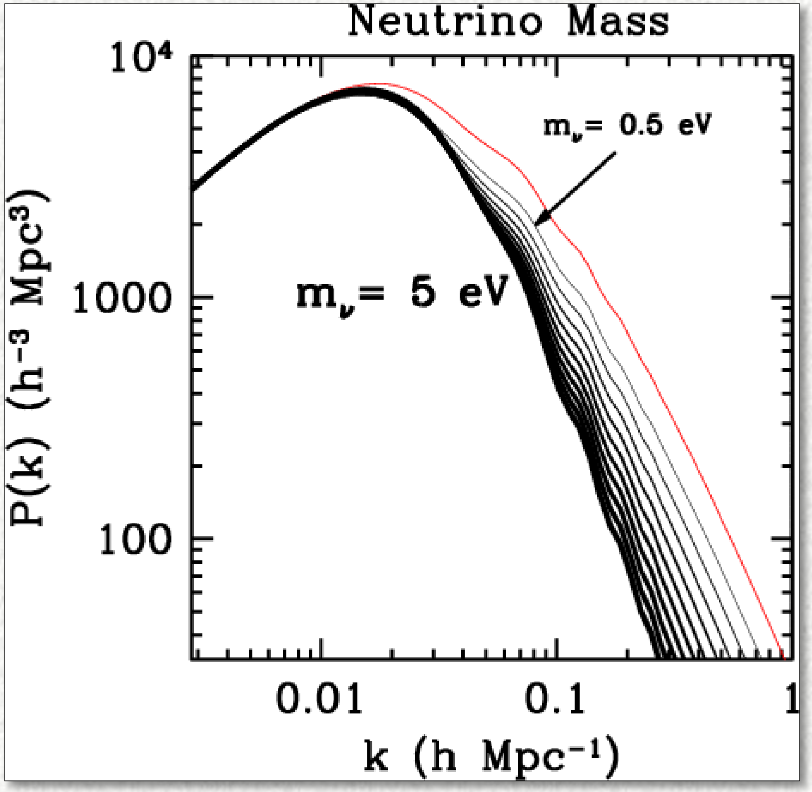
Recall $$\Omega_\nu = 0.02 \frac{m_\nu}{1\,\mathrm{eV}}$$

This fraction of the total density does **not** participate in collapse on scales smaller than the freestreaming scale

$$k_{\mathrm{fs}}^{-1} \simeq \frac{vt}{a} \simeq \frac{(T/m)H^{-1}}{a}$$

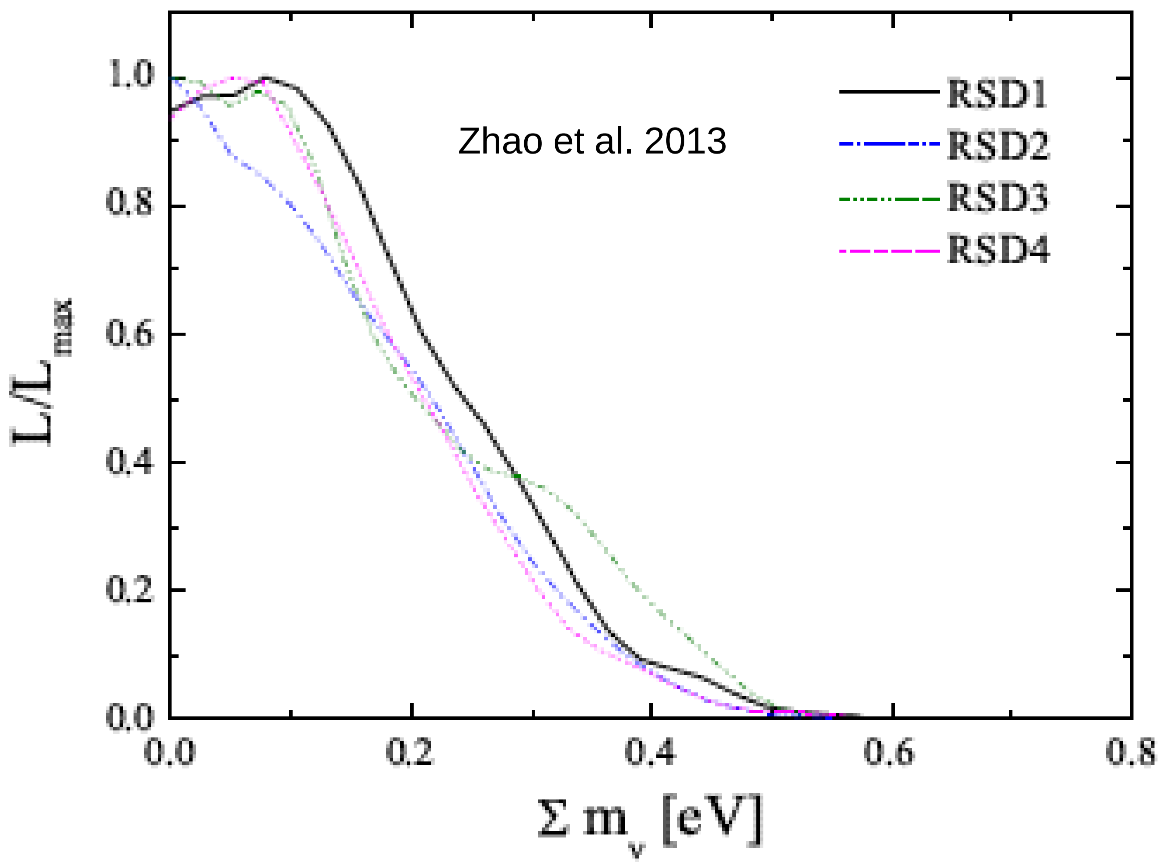
At the relevant time, this scale is 0.02 Mpc^{-1} for a 1 eV ν; power on scales smaller than this is suppressed.

Neutrinos affect large scale structure

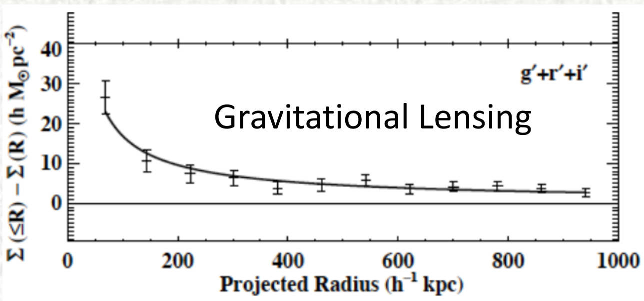


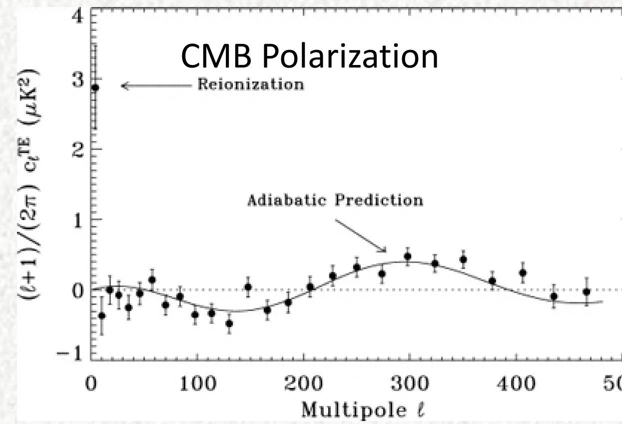
Even for a small neutrino mass, get large impact on structure: power spectrum is excellent probe of neutrino mass

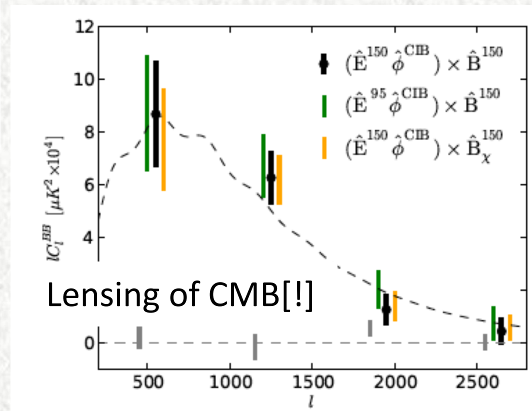
Neutrinos affect large scale structure

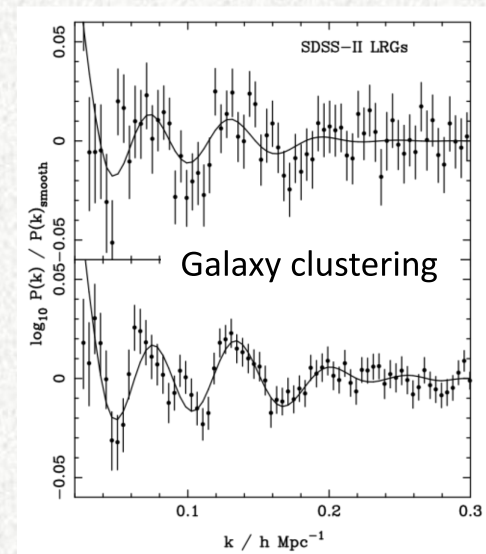


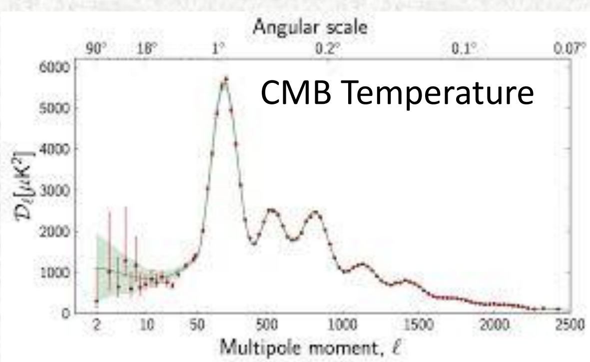
I. Stunning Agreement with a wide variety of observations

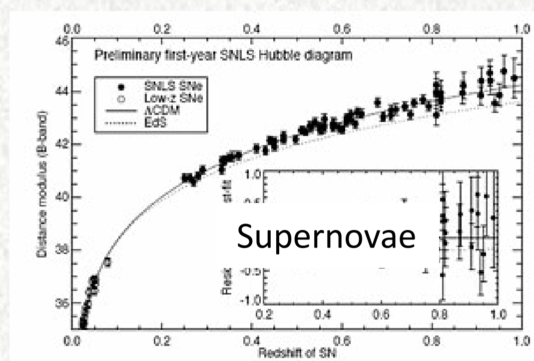












II. Requires New Physics

- Non-Baryonic Dark Matter (SUSY?)
- Inflation (Grand Unified Theory?)
- Dark Energy (Cosmological Constant?)
- Right-Handed Neutrinos (Grand Unified Theory?)

III. *You* need to come up with creative alternatives!

- Modified Gravity to drive acceleration?
- Modified Gravity instead of dark matter?
- Moving beyond WIMPs?
- Complexity in the Neutrino Sector?
- …

Other Alternatives

Sterile neutrinos → Neff
Sterile neutrinos as DM
Asymmetric DM
Modified Gravity for DE

Alternatives: Light Sterile Neutrinos

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

- Majorana Mass: $m_L = m;\ m_D = m_R = 0$
- Dirac Mass: $m_D = m;\ m_L = m_R = 0$
- See-Saw Mechanism: $m_D = m;\ m_R = M;\ m_L = 0$

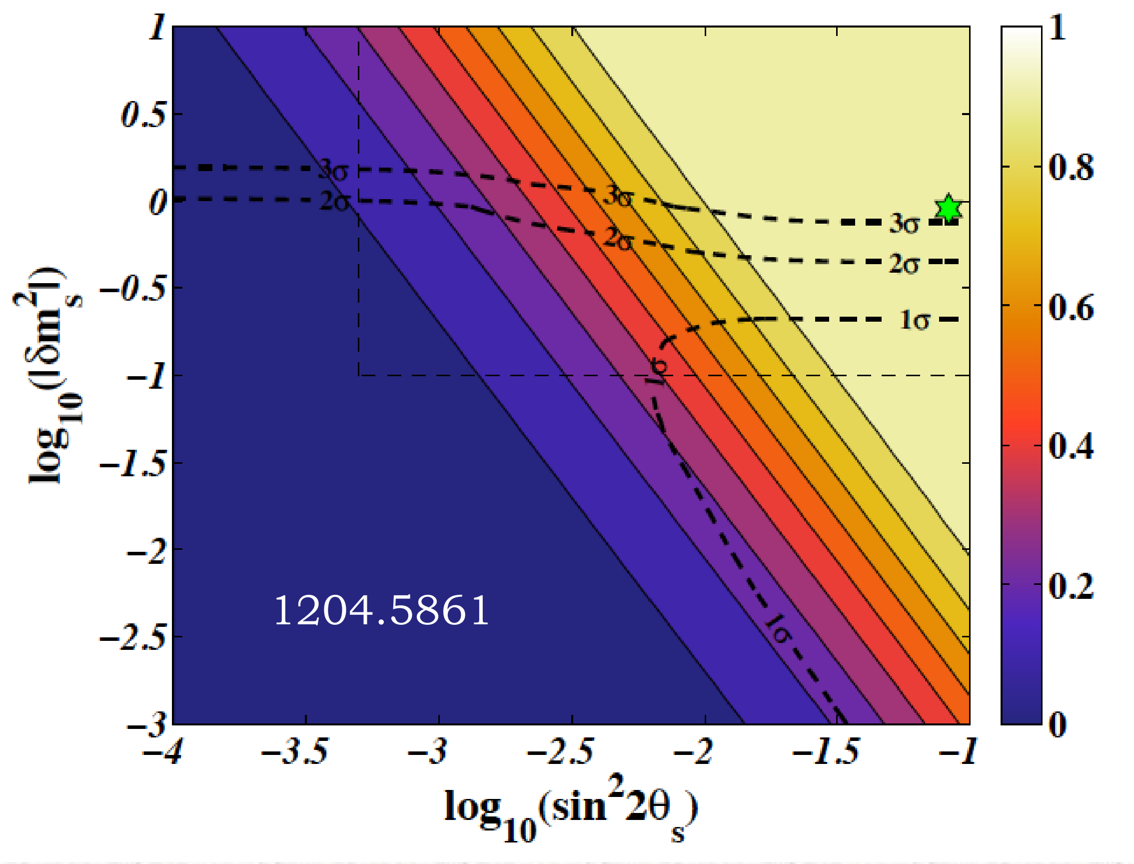
Standard See-Saw mechanism has $M >> m$. Explains why observed neutrino masses (m^2/M) are so small. But M could be small as well; in that case, sterile neutrinos might be observable, both in the Lab and in the cosmos

Alternatives: Light Sterile Neutrinos

Sterile neutrinos can be produced via oscillations

$$Rate = \frac{1}{2}\sin^2(2q_m)\Gamma_{weak}$$

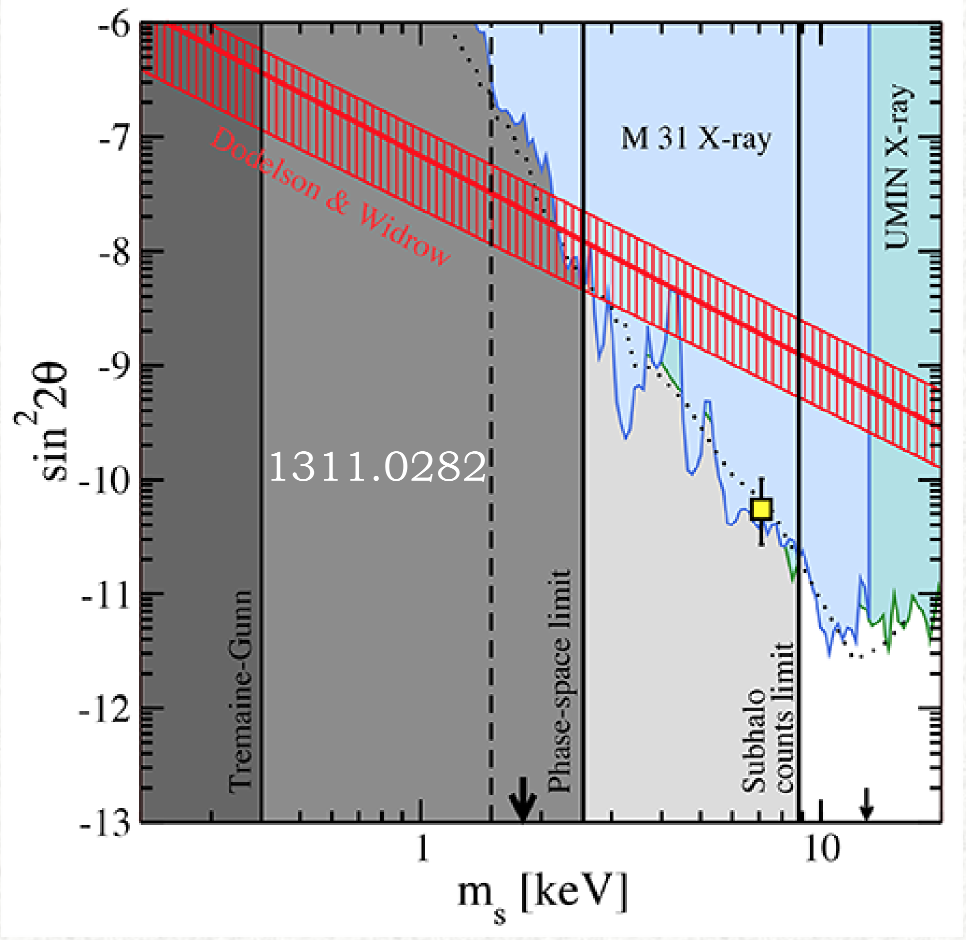
where the mixing angle needs to be computed in matter and the usual $\sin^2(\Delta Et)$ term averages to ½ since the interaction time is very fast



Alternatives: Light Sterile Neutrinos

Sterile neutrinos can be the Dark Matter

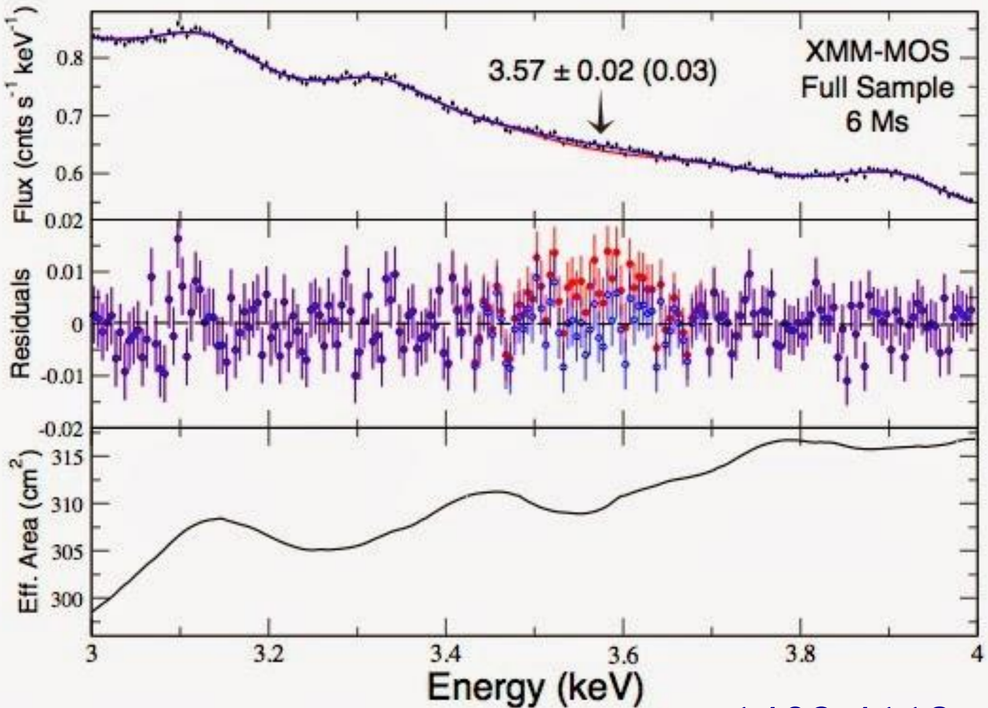
To be cold, need a mass *M* greater than ~ keV. Thermal production does not quite work, but there are other possibilities



Alternatives: Light Sterile Neutrinos

Sterile neutrinos can be the Dark Matter

To be cold, need a mass M greater than ~ keV. Thermal production does not quite work, but there are other possibilities … and we might have seen a signature of this over the past year



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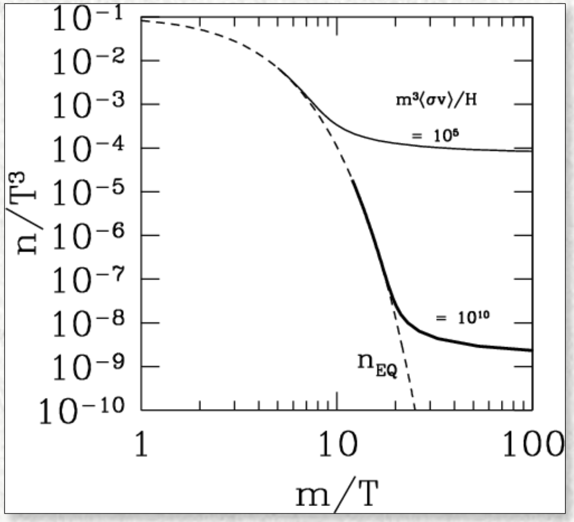
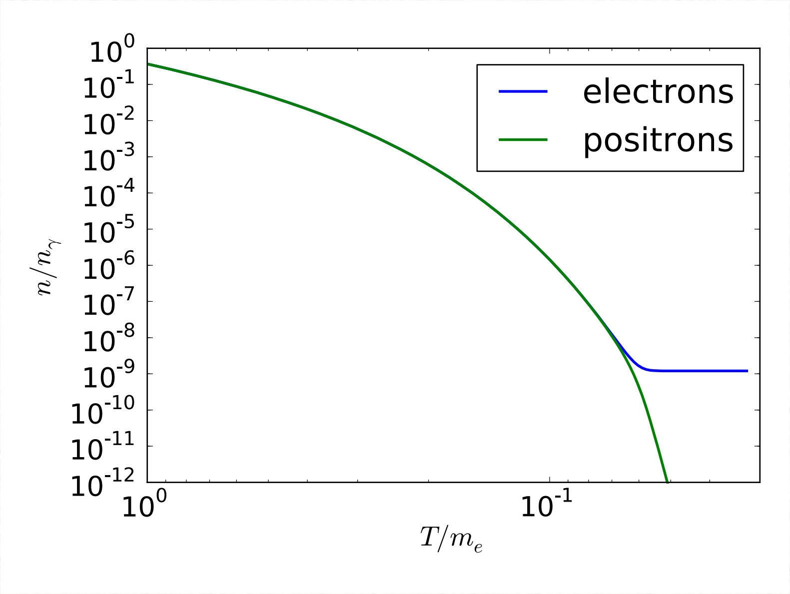
Alternative: Asymmetric Dark Matter

Instead of annihilation freeze-out



Abundance could be fixed by asymmetry



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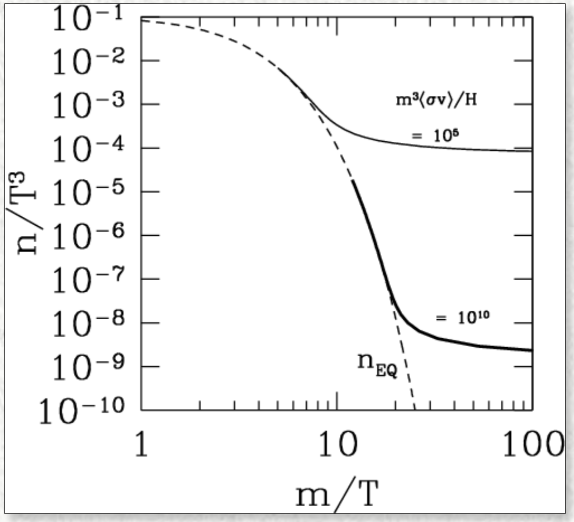

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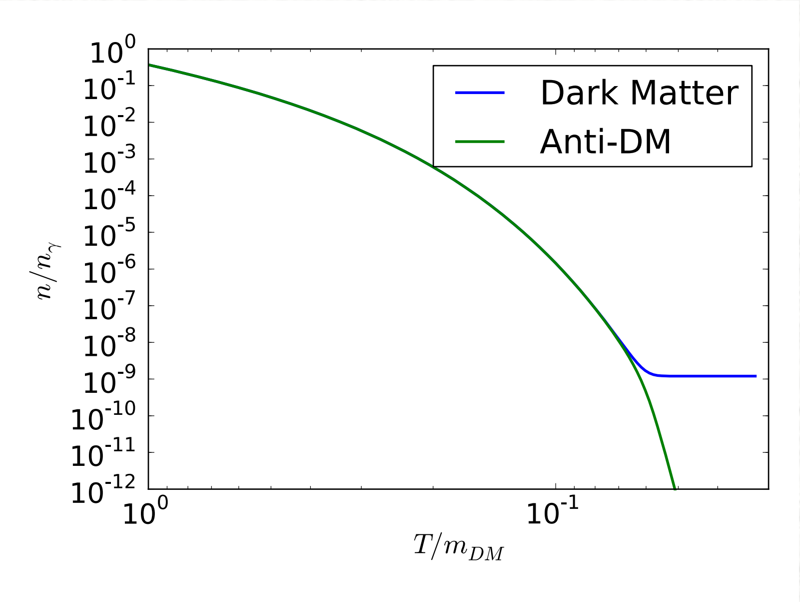


Natural value is the same as baryon asymmetry → m_{DM}~5-10 eV

Perturbations to the FRW metric

$$g_{00} = -1 - 2\Psi(\vec{x},t) \; ; \; g_{ij} = \delta_{ij} a^2(t)\left(1 + 2\Phi(\vec{x},t)\right)$$

The scalar potentials typically satisfy Φ = - Ψ (but beware sign conventions) so we will use them interchangably.

During inflation, □□fluctuates quantum mechanically around a smooth background

The mean value of □ is zero, but its variance is

$$\langle \Psi^2(\vec{x}) \rangle = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} e^{i\vec{k}'\cdot\vec{x}} \langle \tilde{\Psi}(\vec{k}) \tilde{\Psi}(\vec{k}') \rangle$$

$$= \int \frac{dk}{k} \frac{k^3 P_\Psi(k)}{2\pi^2}$$

$$\left\langle \tilde{\Psi}(\vec{k}) \tilde{\Psi}(\vec{k}') \right\rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') P_\Psi(k)$$

Get contributions from all scales equally if

$P_\Psi \propto k^{-4+n}$ with *n=1* (scale-invariant spectrum)

Inflation predicts …

Two regions with the scalar field taking the same value at slightly different times have relative potential

$$\Psi \sim \frac{\delta a}{a} \sim \frac{\dot{a}}{a}\delta t = H\delta t$$

But

$$\delta t = \frac{\delta\phi}{\dot{\phi}} \sim \frac{\delta\phi}{V'/H}$$

The last equality following from the equations of motion

Leading to …

$$Y \sim \frac{df\, H^2}{V'}$$

RMS fluctuations in the scalar field are roughly equal to the Hubble rate, and the Friedmann equation tells us that $H^2 \sim GV$

$$Y_{RMS} \sim \frac{(GV)^{3/2}}{V'}$$

Each Fourier mode is associated with φ [the value of φ when k exits the horizon].

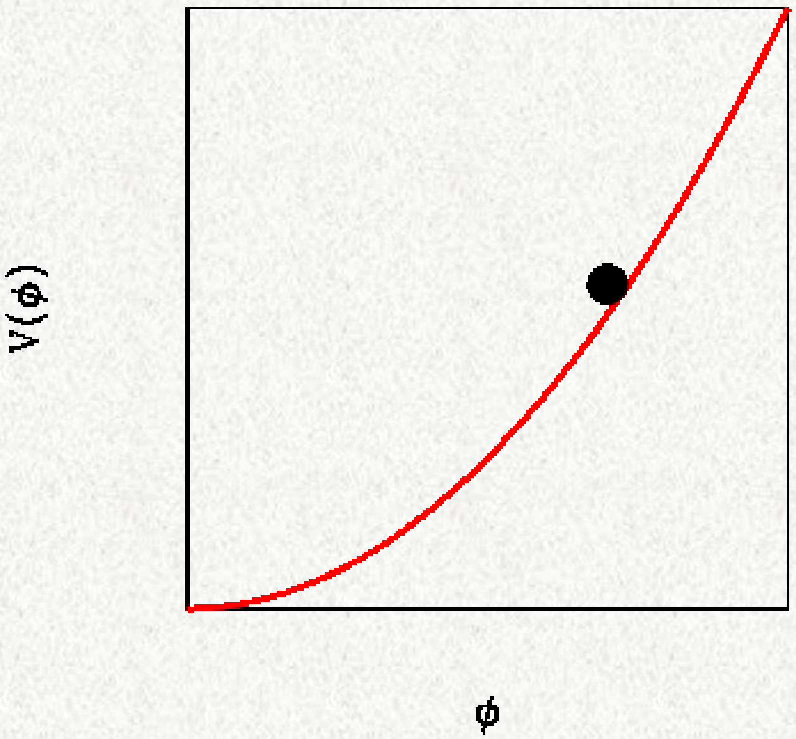
Inflation: Scalar Field

Equation of motion for a scalar field in an expanding Universe

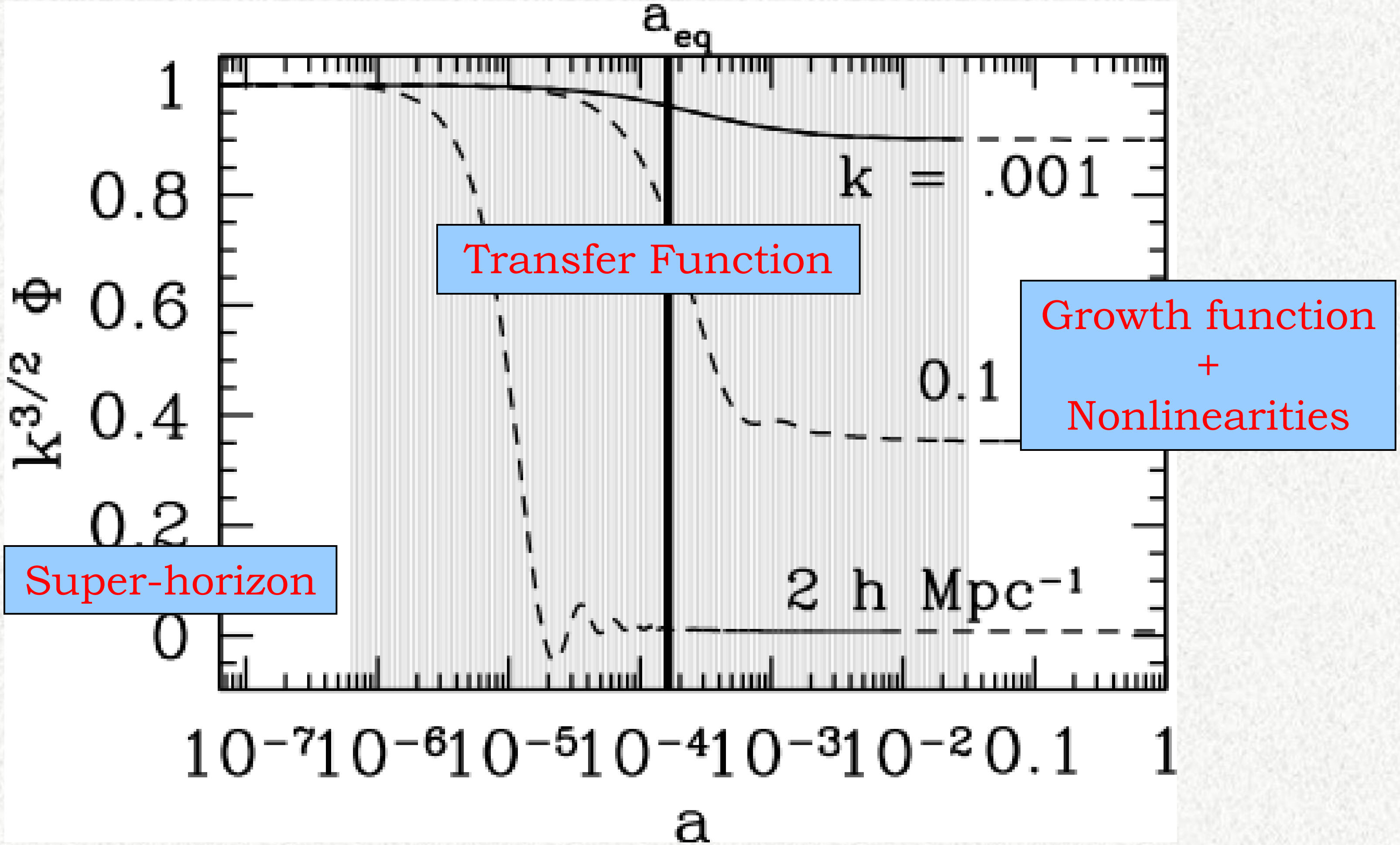
$$\ddot{\varphi} + 3H\dot{\varphi} + V' = 0$$

Slow roll approximation sets
dφ/dt=-V'/3H

Since *H* is very large,
$V \sim H^2 m_{Pl}^2$ is also large,
typically of order $(10^{16}\text{GeV})^4$,
much larger than the dark energy today



Earlier than a_{EQ}, most of the energy density is relativistic; afterwards non-relativistic



Gravitational Potential

Poisson's Equation: $\nabla^2 \Phi = 4\pi G \bar{\rho} \delta$

In Fourier space, this becomes: $-\frac{k^2}{a^2}\tilde{\Phi} \propto \frac{\tilde{\delta}}{a^3}$

So the gravitational potential remains constant! Delicate balance between attraction due to gravitational instability and dilution due to expansion.

Only if all the energy is in non-relativistic matter.
Dark energy or massive neutrinos lead to potential decay.

Matter Power Spectrum

Poisson says: $$k^2\tilde{\Phi} \propto \tilde{\delta}$$

So the power spectrum of matter (which measures the density *squared)* scales as:

$$P_\delta \propto k^4 P_\Phi \propto k^n$$

Valid on large scales which entered the horizon at late times when the universe was matter dominated.

Sub-horizon modes oscillate and decay in the radiation-dominated era

Newton's equations - with radiation as the source - reduce to

Here using η as time variable

$$\ddot{\Phi} + \frac{4}{\eta}\dot{\Phi} + \frac{k^2}{3}\Phi = 0$$

with analytic solution

$$\Phi(\eta) = 3\Phi(0)\frac{\sin(k\eta/\sqrt{3}) - (k\eta/\sqrt{3})\cos(k\eta/\sqrt{3})}{(k\eta/\sqrt{3})^3}$$

Expect less power on small scales

For scales that enter the horizon well before equality,

$$\Phi(\eta_{\mathrm{EQ}}) \to \Phi(0)\,\frac{\cos(k\eta_{\mathrm{EQ}}/\sqrt{3})}{(k\eta_{\mathrm{EQ}}/3)^2}$$

So, we expect the transfer function to fall off as

$$\lim_{k\to\infty} T(k) \equiv \lim_{k\to\infty} \frac{\Phi_{today}(k)}{\Phi_{initial}(k)} \propto k^{-2}$$

Shape of the Matter Power Spectrum

$$P(k) \propto k^n T^2(k) \propto \begin{cases} k^n & \text{Large scales} \\ k^{n-3} \ln^2(k) & \text{Small scales} \end{cases}$$

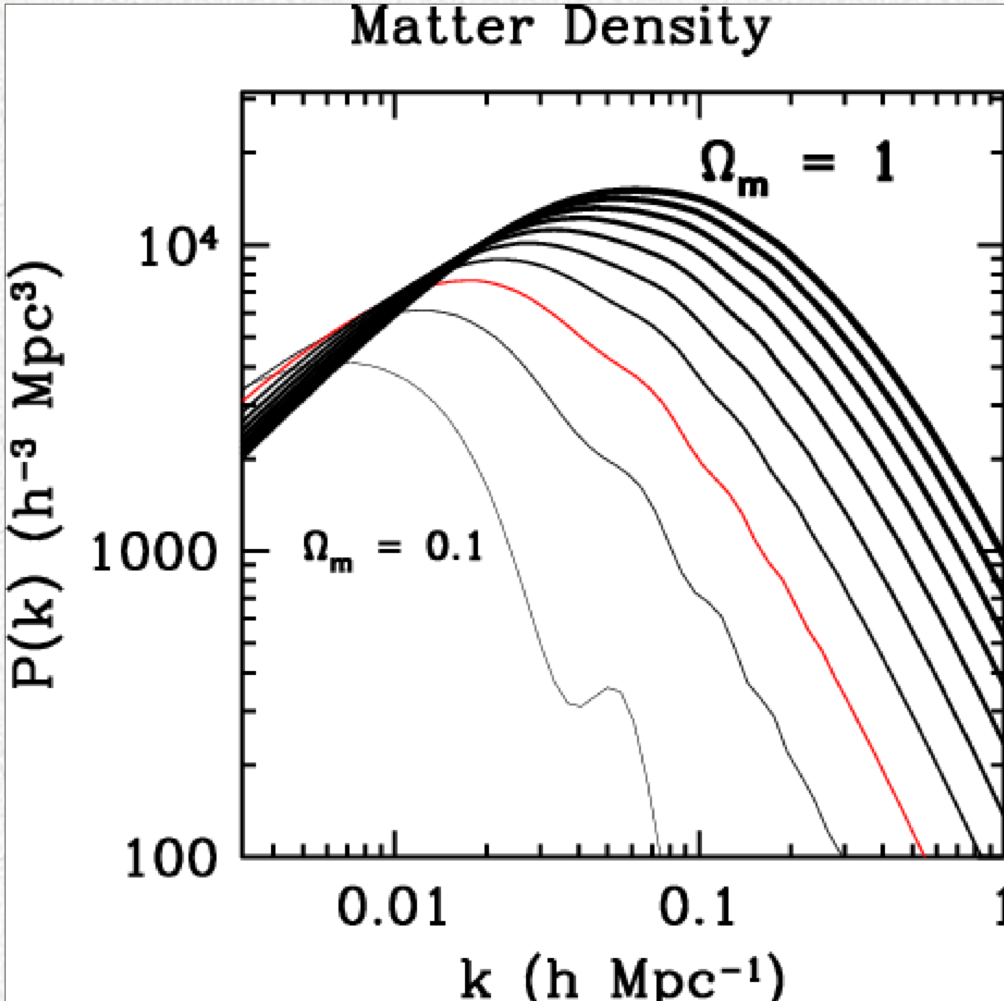
Log since structure grows slightly during radiation era when potential decays

The *turnover* scale is the one that enters the horizon at the epoch of matter-radiation equality:

$$k_{EQ} = 0.073 \Omega_m h^2 \text{Mpc}^{-1}$$

Therefore, measuring the shape of the power spectrum will give a precise estimate of Ω_m

Turnover scale sensitive to the matter density



Inhomogeneities in TeVeS

Skordis 2006
Skordis, Mota, Ferreira, & Boehm 2006
Dodelson & Liguori 2006

Perturb all fields: (metric, matter, radiation)
+ (scalar field, vector field)

E.g., the perturbed metric is

$$g_{\mu\nu} = diag[-a^2(1-2\Psi), a^2(1+2\Phi), a^2(1+2\Phi), a^2(1+2\Phi)]$$

where *a* depends on time only and the **two** potentials depend on space and time.

Inhomogeneities in TeVeS

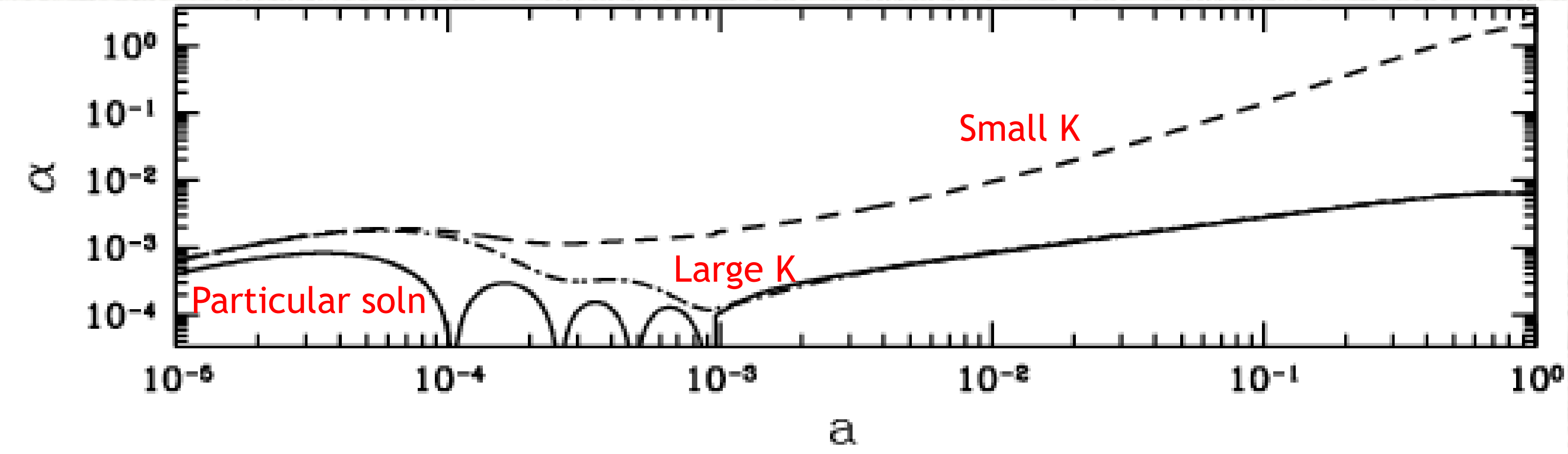
Other fields are perturbed in the standard way; only the vector perturbation is subtle.

$$A_\mu = ae^{-\varphi}\left(1+\Psi+\delta\varphi, \vec{\alpha}\right)$$

Constraint leaves only 3 DOF's. Two of these decouple from scalar perturbations, so we need track only the longitudinal component defined via:

$$\vec{\nabla}\alpha \equiv \vec{\alpha}$$

Inhomogeneities in TeVeS

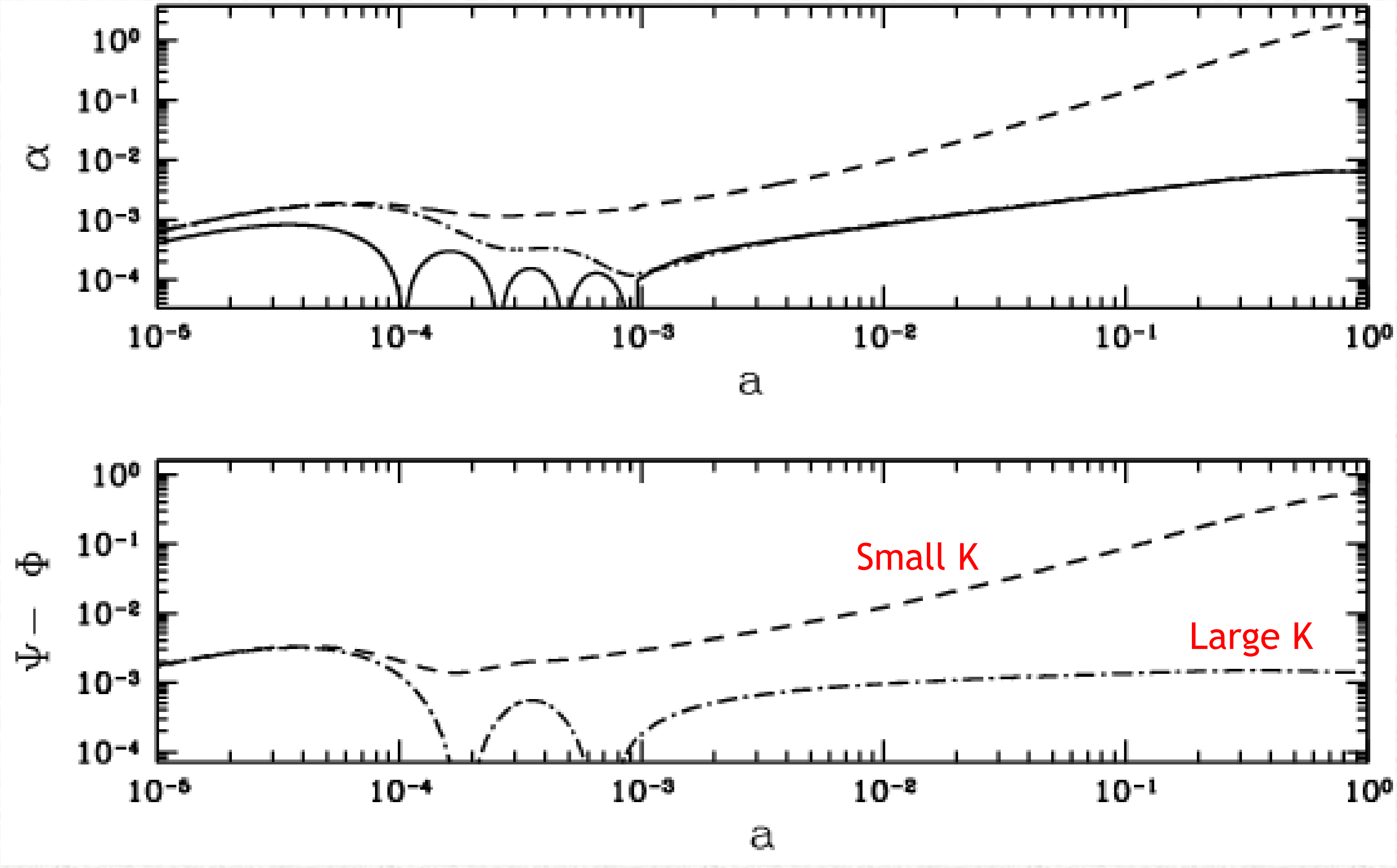


For large K, no growing mode: vector follows particular solution.

For small K, growing mode comes to dominate.

Inhomogeneities in TeVeS

This drives difference in the two gravitational potentials …



Inhomogeneities in TeVeS

… which leads to enhanced growth in matter perturbations!