

Dark Matter

- Phase Space
- The Dark Matter sheet
- Collisionless Boltzmann Equation
- Simulation results

Phase Space

A decorative header image featuring a dark, starry sky with a prominent bright star and nebulae on the right side, and a faint, glowing structure on the left.

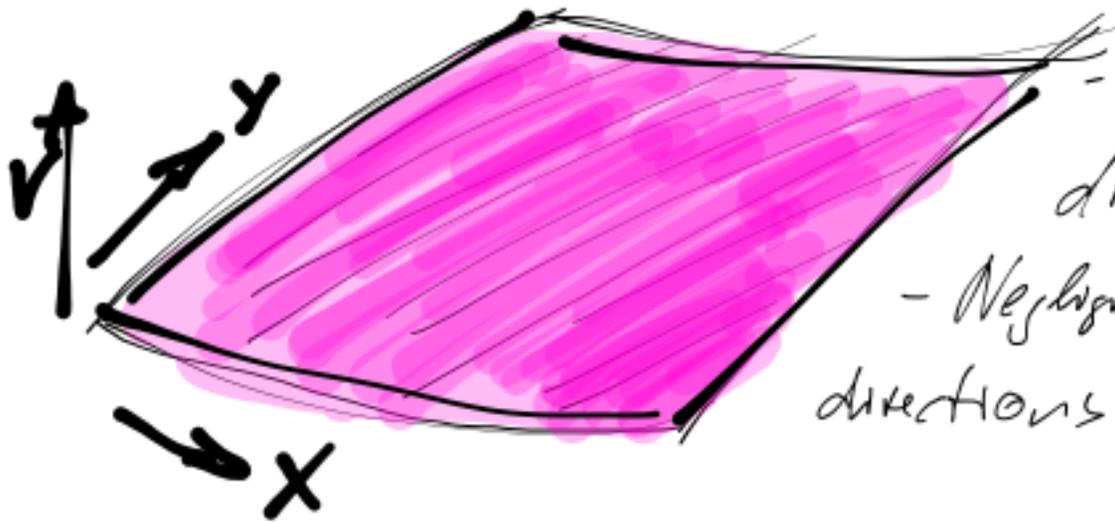
The Dark Matter Sheet?

COOL WIMPS

Dark Matter is commonly hypothesized to originate within seconds after the BIG BANG. If it were moving relativistically today, galaxies and other structures would not exist. We speak of **COLD DARK MATTER**.

Working HYPOTHESIS:

- Weakly interacting massive particle (say ≈ 100 GeV).
- Very cold. Even keV particles would only have $\sim \frac{m}{s}$ speeds today.



- Almost perfectly uniformly distributed initially.
- Negligible extent along velocity directions in phase space.

The Dark Matter Sheet?

Fluid

OF DARK MATTER PARTICLES IN THE MILKY WAY :

$$N_{DM} \approx 10^{67} \left(\frac{100 \text{ GeV}}{m_{DM}} \right) \gg \# \text{ OF STARS IN THE UNIVERSE}$$

\gg # OF PARTICLES THAT FIT ON A COMPUTER
USING ALL THE COMPUTERS IN THE WORLD : $\approx 10^{17}$ particles

SOLVE VLASOV-POISSON SYSTEM INSTEAD.

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f + \vec{a} \cdot \nabla_v f = 0$$

f : distribution function in PHASE SPACE

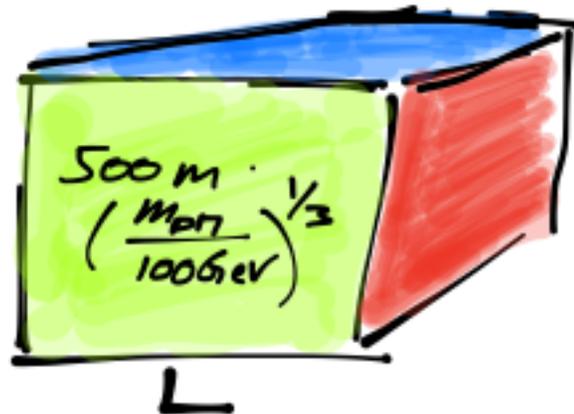
ϕ : potential

$$\vec{a} = -\nabla \phi$$

$$\nabla^2 \phi = 4\pi G \rho$$

FOR PHASE SPACE ELEMENT TO CONTAIN 10^6 PARTICLES @ MEAN DENSITY IT HAS TO BE LARGER THAN

$$L \sim 500 \text{ m} \left(\frac{m_{DM}}{100 \text{ GeV}} \right)^{1/3}$$



GRAVITY:

POISSON EQUATION : $\nabla^2 \phi = 4\pi G \rho$

TOTAL MASS DENSITY

CONTINUUM DESCRIPTION

$$\frac{\vec{F}}{m} = -\nabla \phi$$

VLASOV EQUATION

$$\frac{\partial f}{\partial t} + v \nabla_x f - \nabla \phi \nabla_v f = 0$$

FOR $N \rightarrow \infty$

IDENTICAL

N POINT MASSES : \vec{a}_j

$$= - \sum_{i \neq j} \frac{G m_i}{|\vec{x}_j - \vec{x}_i|^2 + \epsilon^2} \frac{(\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|}$$

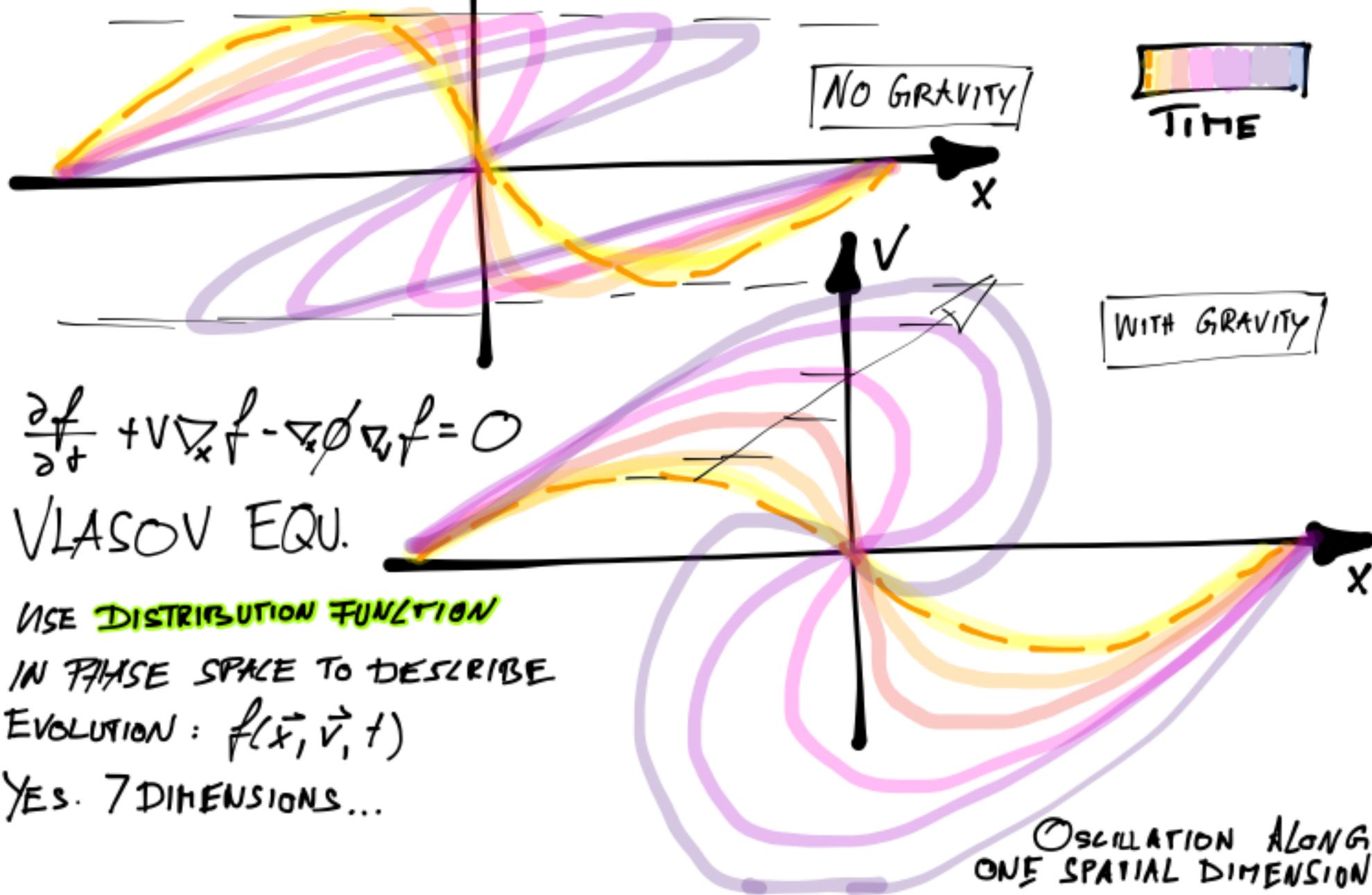
PARTICLE PICTURE

Particle 'advection': $\frac{\partial \vec{x}_j}{\partial t} = \vec{v}$; $\frac{\partial \vec{v}_j}{\partial t} = \vec{a}_j$

DARK MATTER

V m.f.p. \gg SYSTEM

PHASE SPACE



$$\frac{\partial f}{\partial t} + v \nabla_x f - \nabla_x \phi \nabla_v f = 0$$

VLASOV EQU.

USE DISTRIBUTION FUNCTION

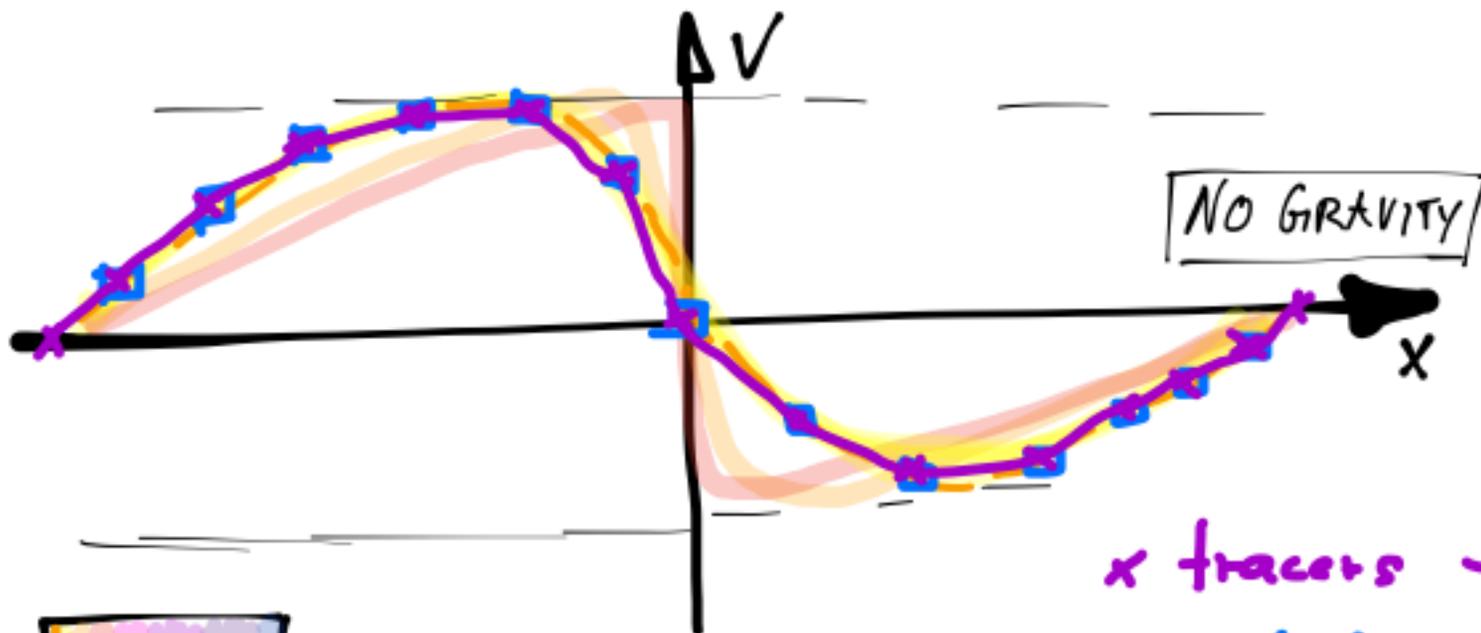
IN PHASE SPACE TO DESCRIBE

EVOLUTION: $f(\vec{x}, \vec{v}, t)$

YES. 7 DIMENSIONS...

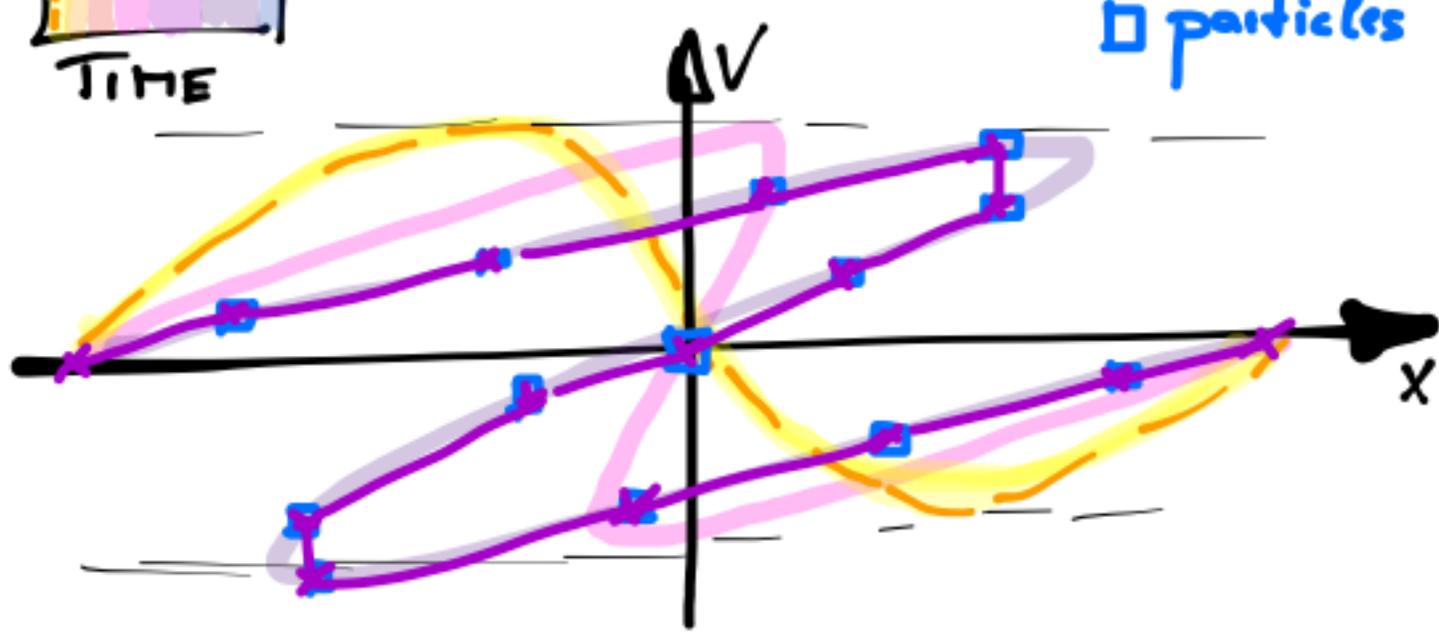
OSILLATION ALONG ONE SPATIAL DIMENSION

DISCRETIZE DARK MATTER DISTRIBUTION: Mass or Volume?

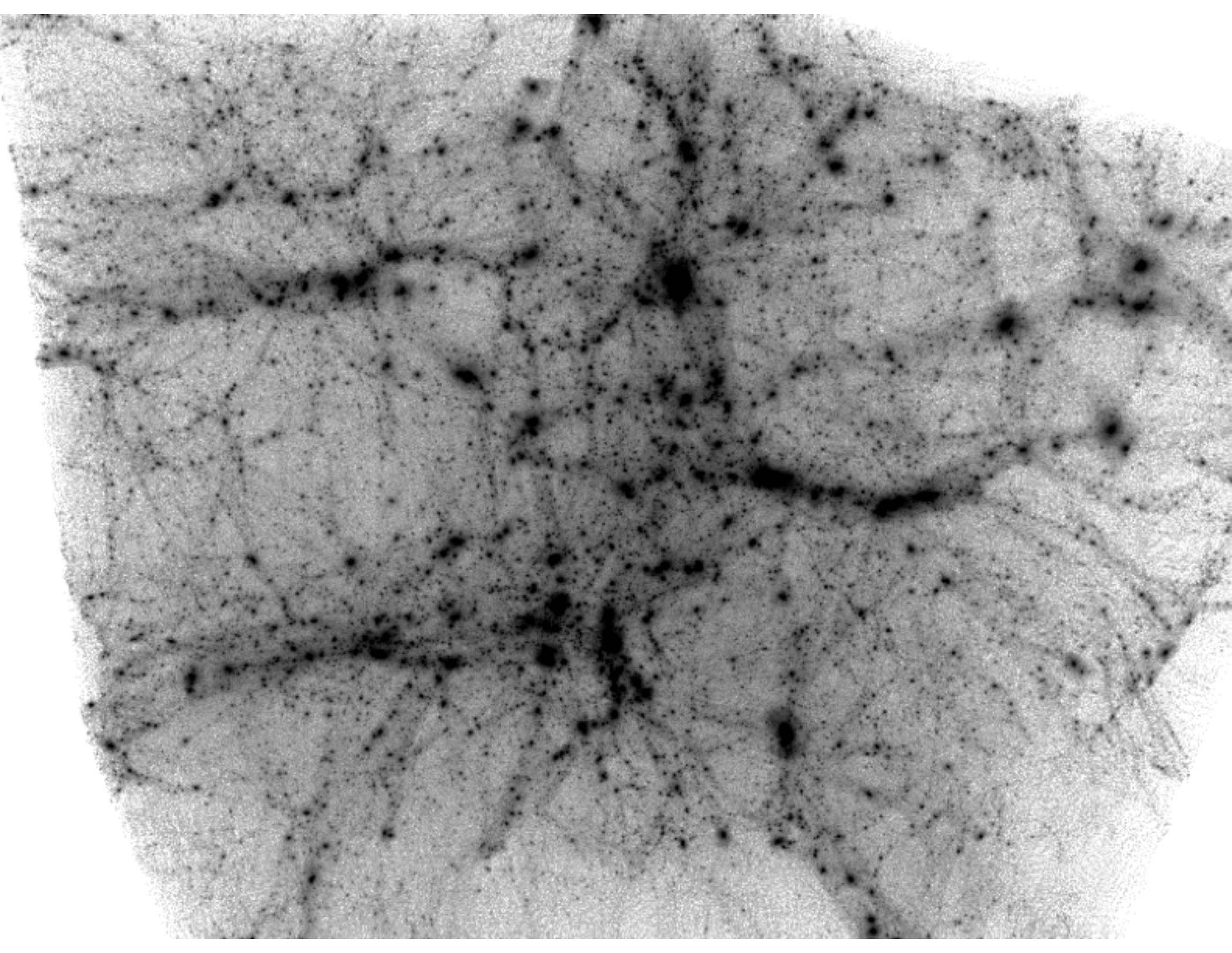


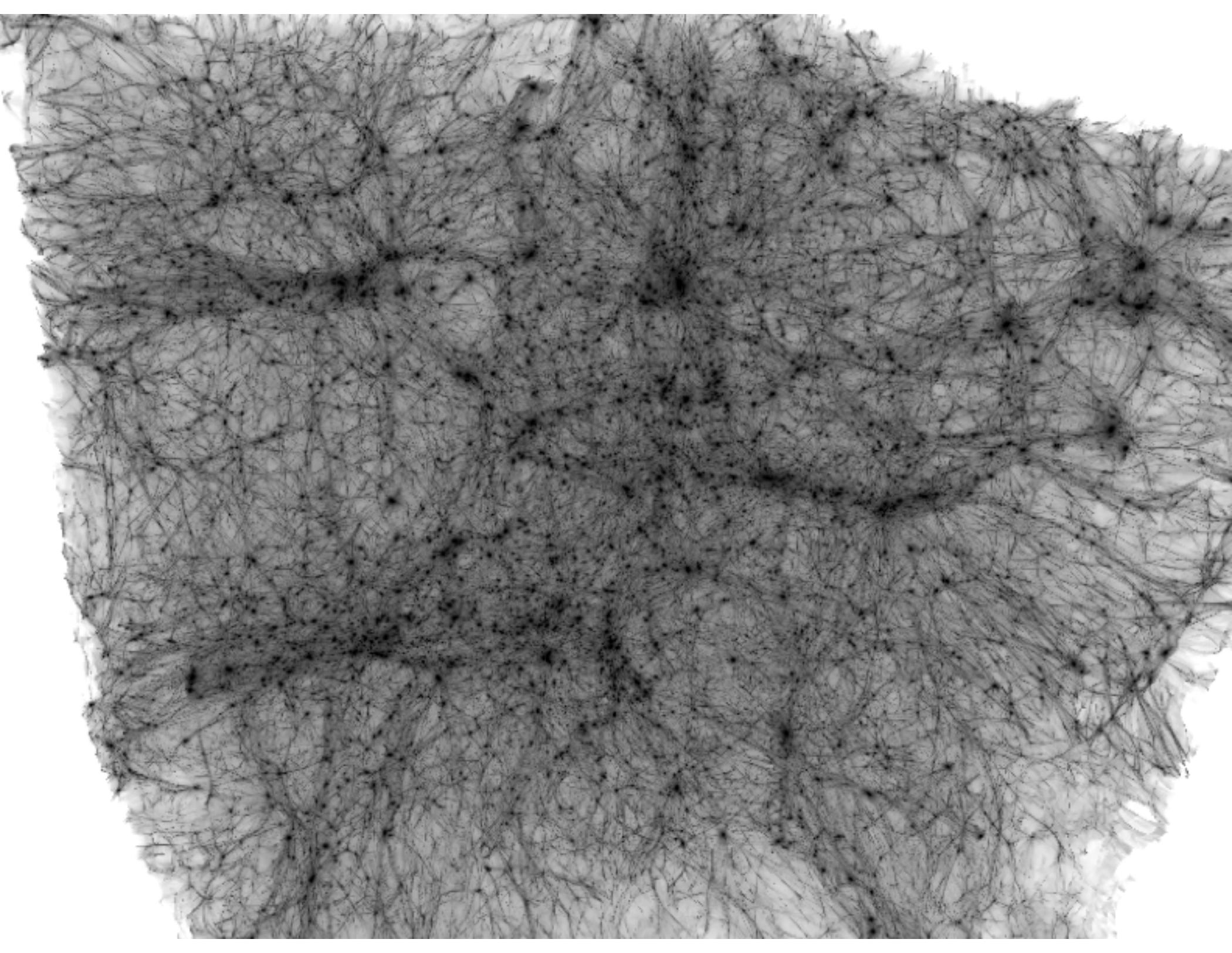
COUNT PARTICLES IN RANDOM VOLUMES GIVES AVERAGE DENSITIES

x tracers — segments
 \square particles



THINK MASS BETWEEN PARTICLES \square
 \Rightarrow DENSITY KNOWN EVERYWHERE!

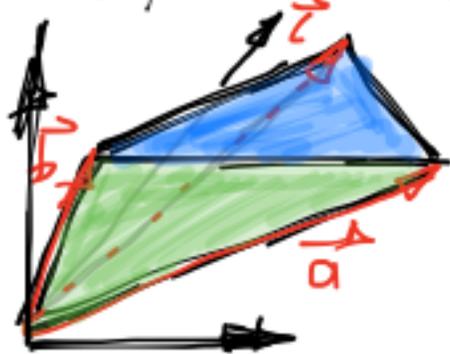




3 dimensional manifold in 6D Phase Space

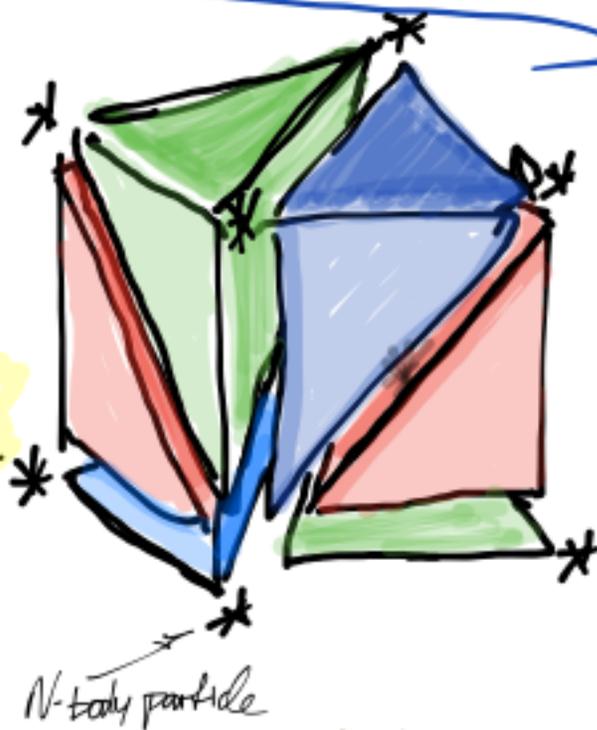
- Natural tessellation takes unit cube & splits it into six equal size tetrahedra.

- mass per tetrahedron = $1/6$ of M_P particle mass.



$$V = \frac{|\vec{a} \cdot (\vec{b} \times \vec{c})|}{6}$$

$$\Rightarrow \rho = \frac{M_P}{6V} = \frac{M_P}{|\vec{a} \cdot (\vec{b} \times \vec{c})|}$$



- Number of edges of the cube

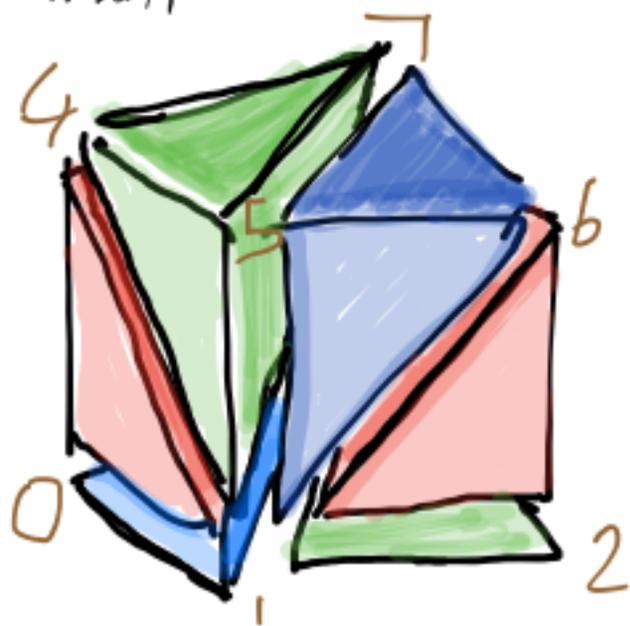
- think of lattice

- Looping over

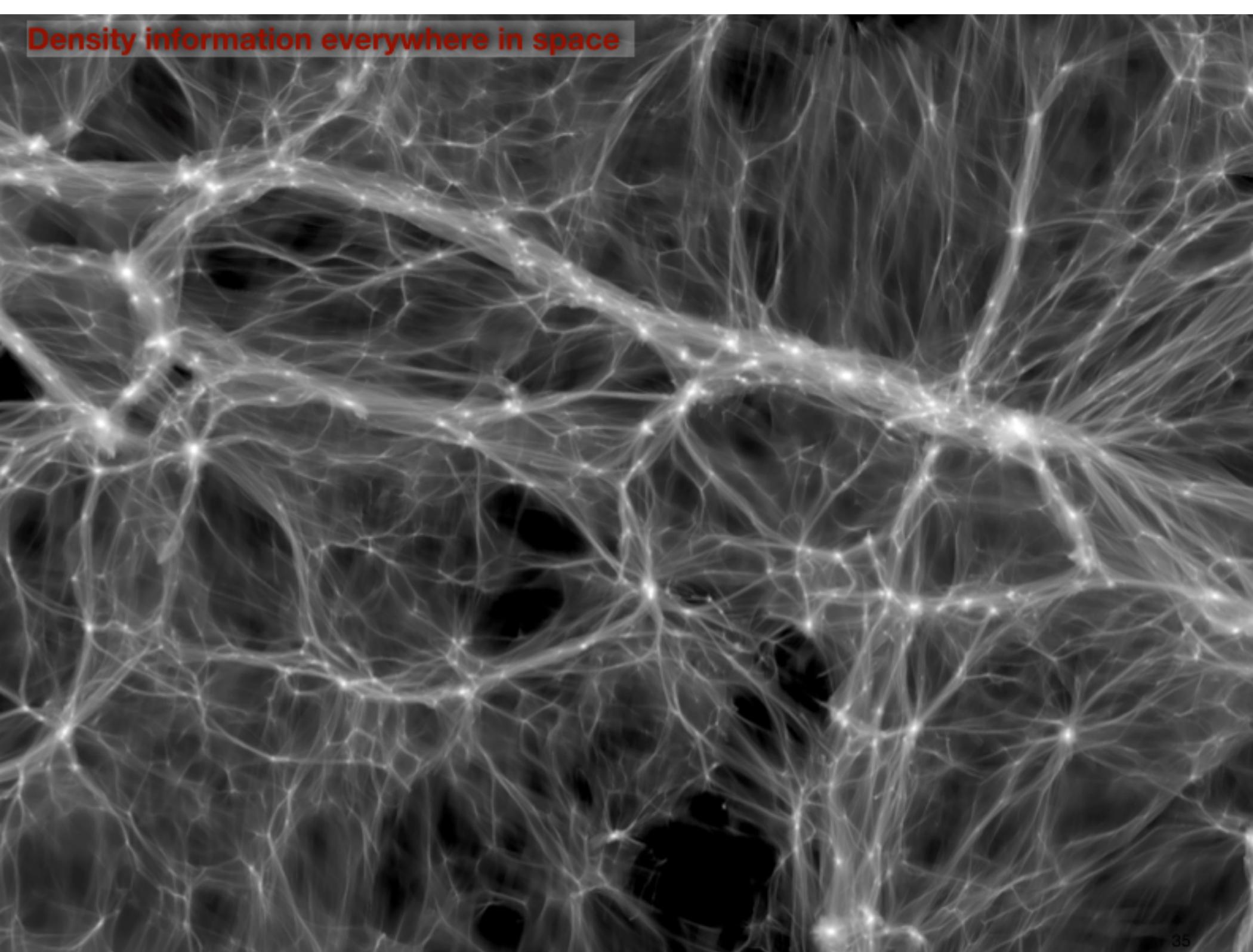


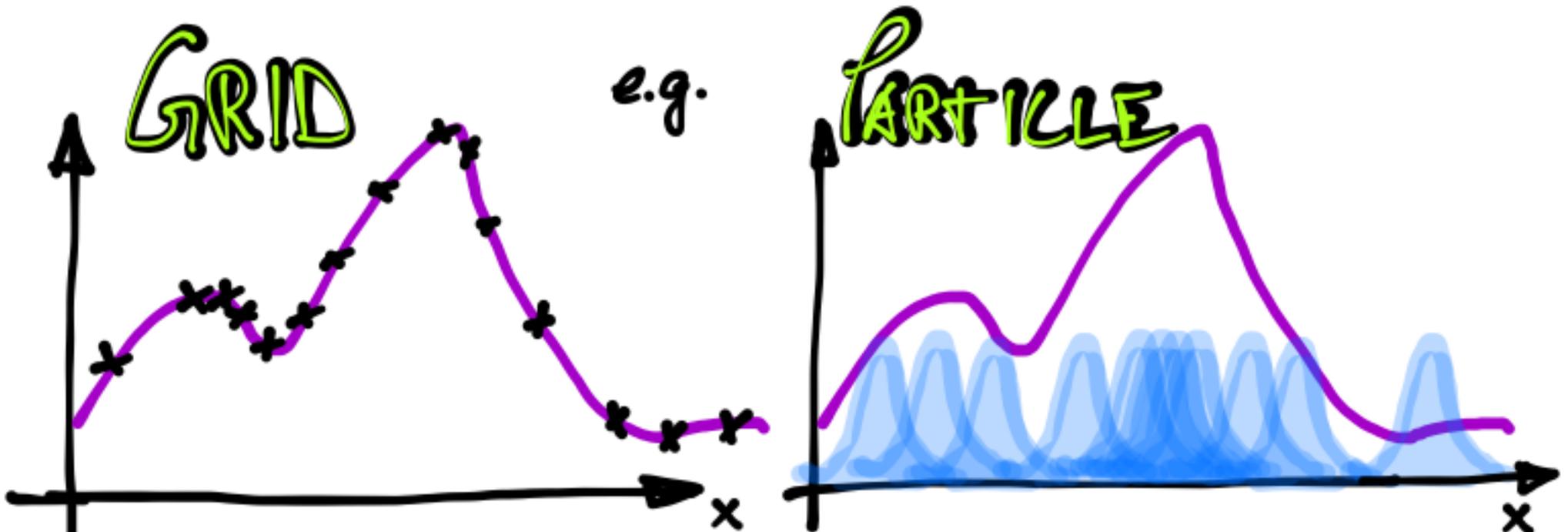
the initial cartesian (LAGRANGIAN)

lattice generates the 6N tetrahedra.



Density information everywhere in space





DISCRETIZATION: REPRESENT CONTINUUM SOLUTION WITH FINITE NUMBER OF ELEMENTS ("COMPRESSION")

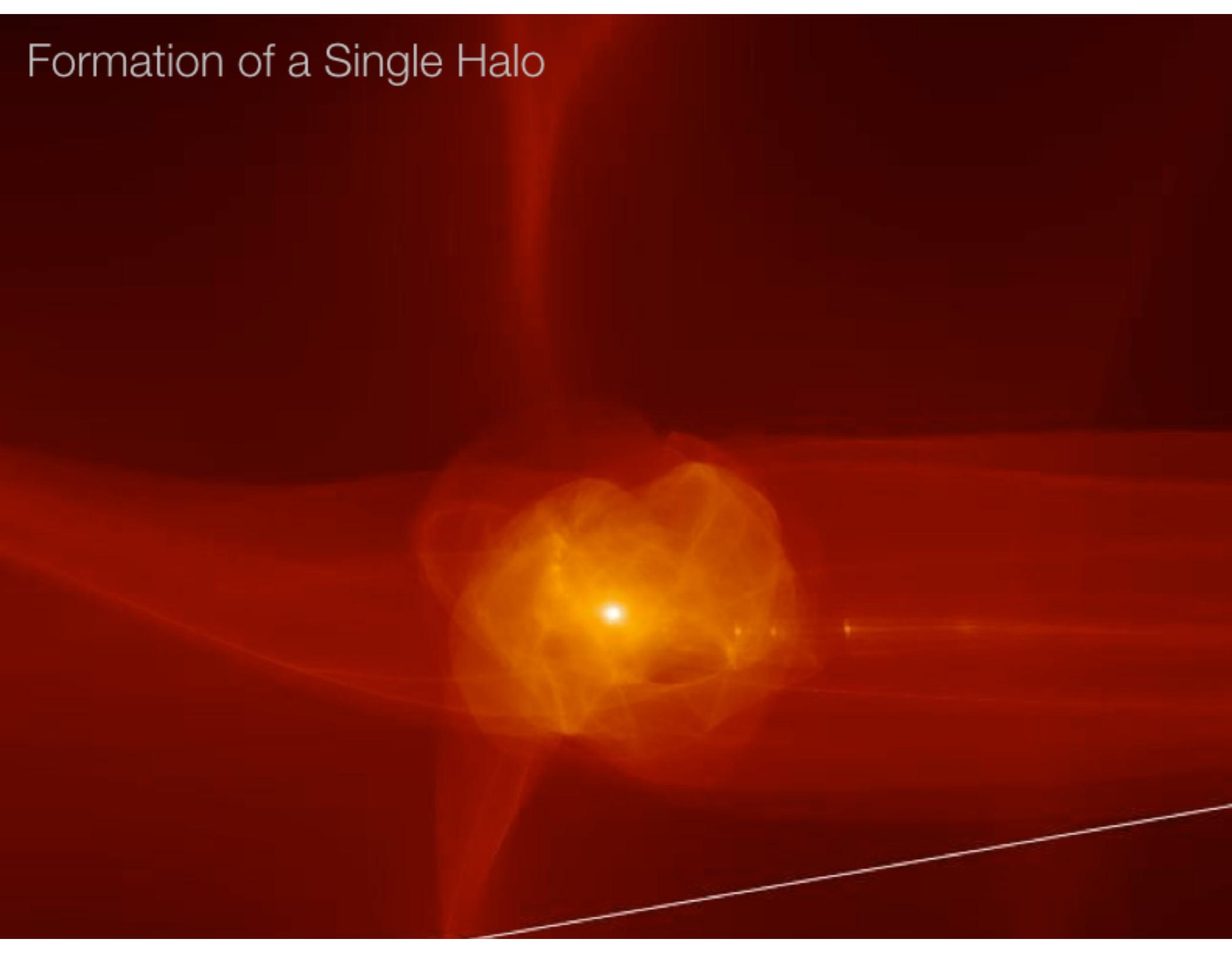
Uniform, adaptive, moving, structured, unstructured, tetrahedral, cartesian, cylindrical

MESH

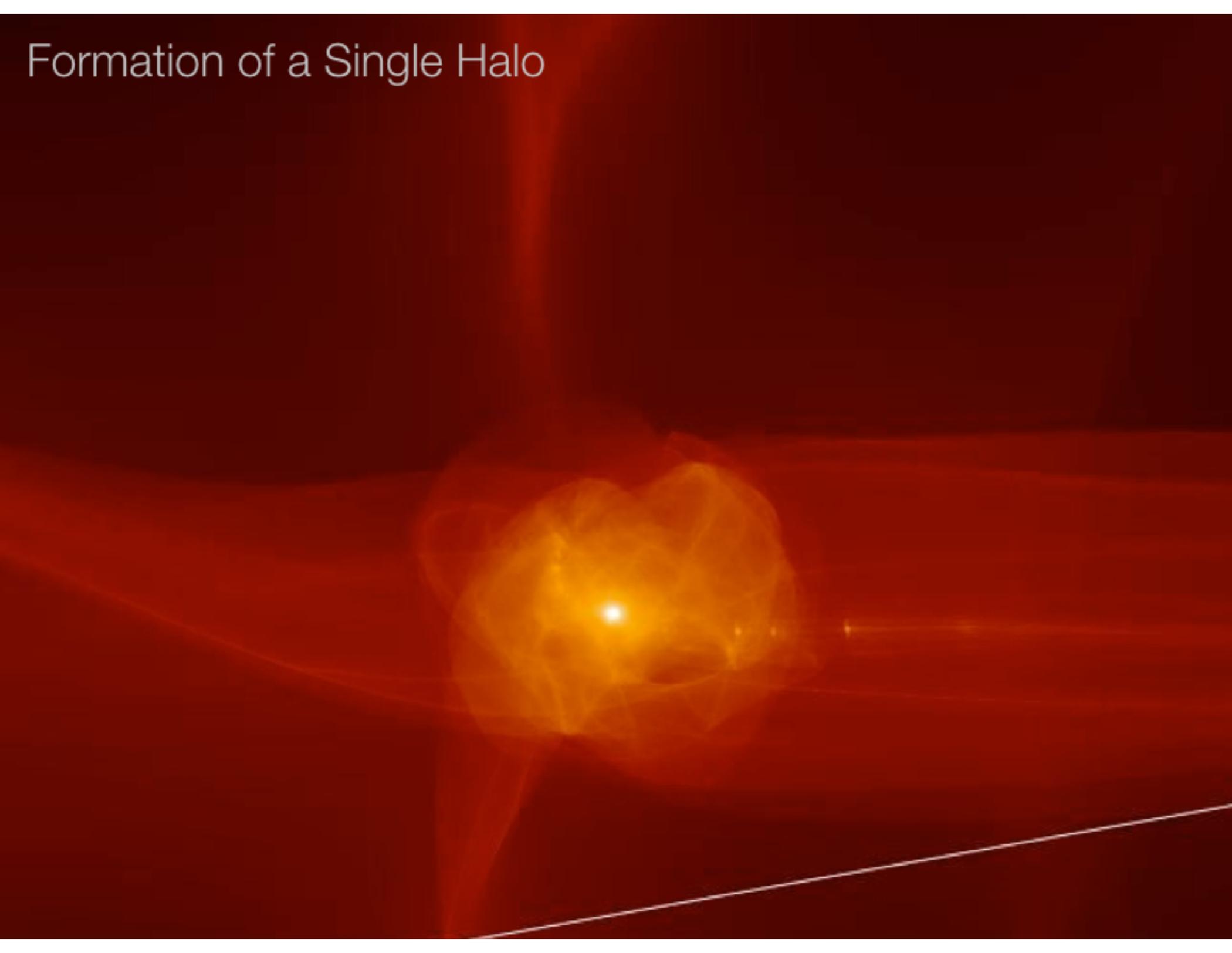
fixed, adaptive, high order, asymmetric, ...

KERNEL

Formation of a Single Halo

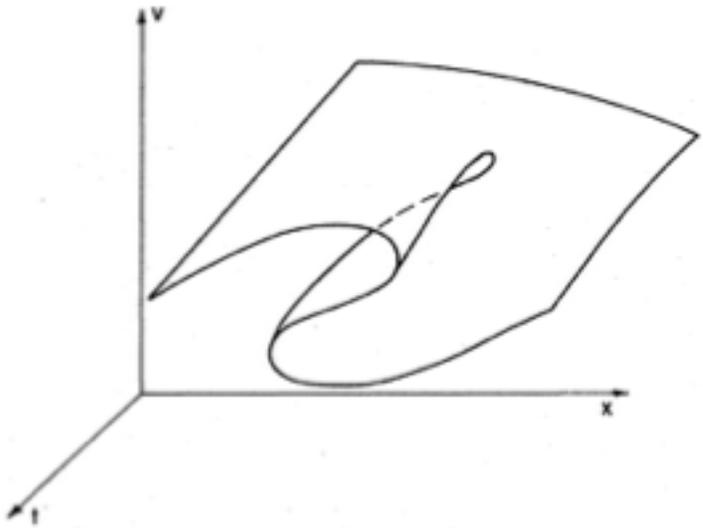


Formation of a Single Halo

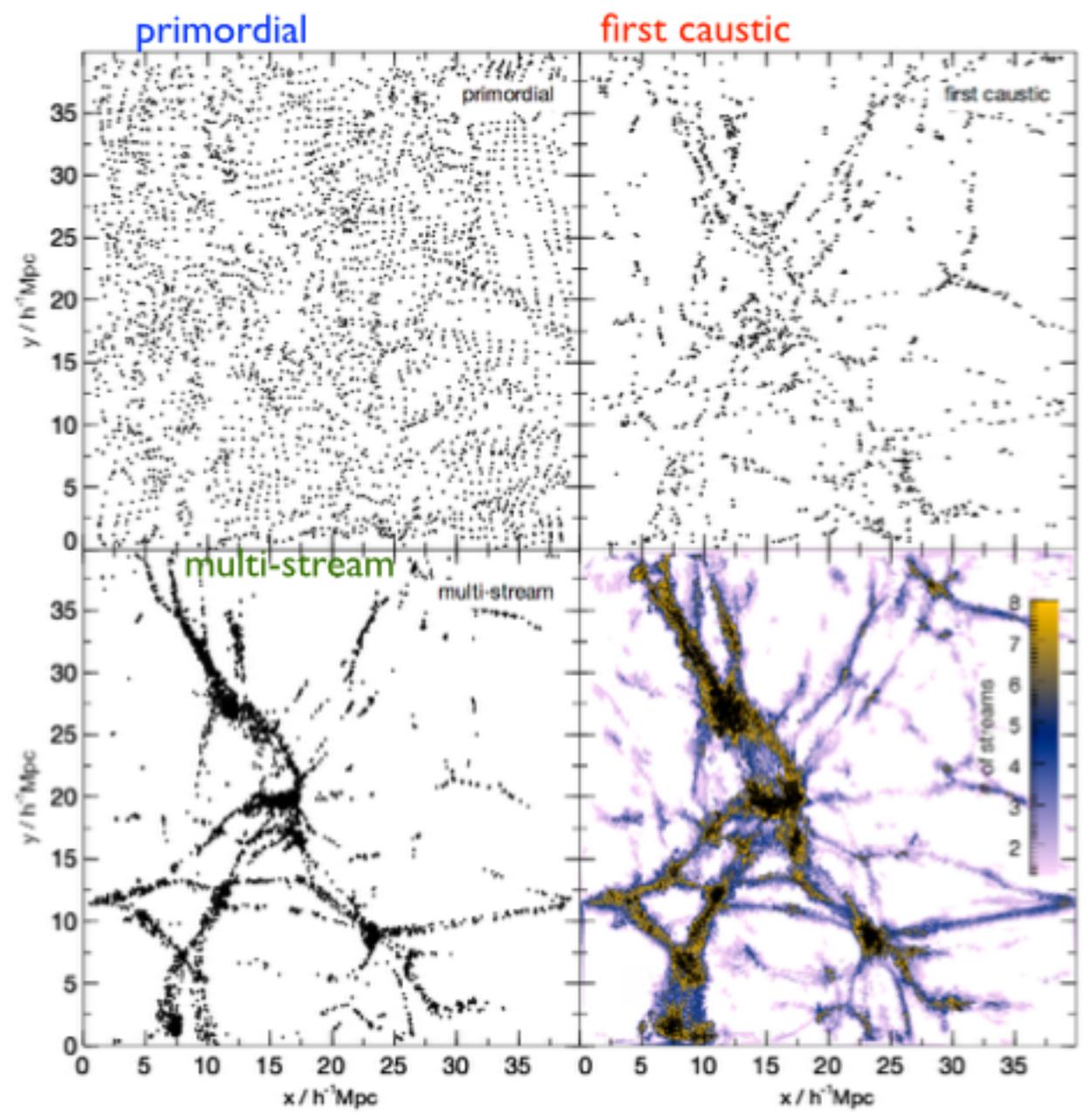


From caustics to multistream...

Use the local number of foldings



easy tool to help the understand the dynamics of LSS



So, what volume fraction is multi-stream?

or, how much volume is LSS?

approaches power-law

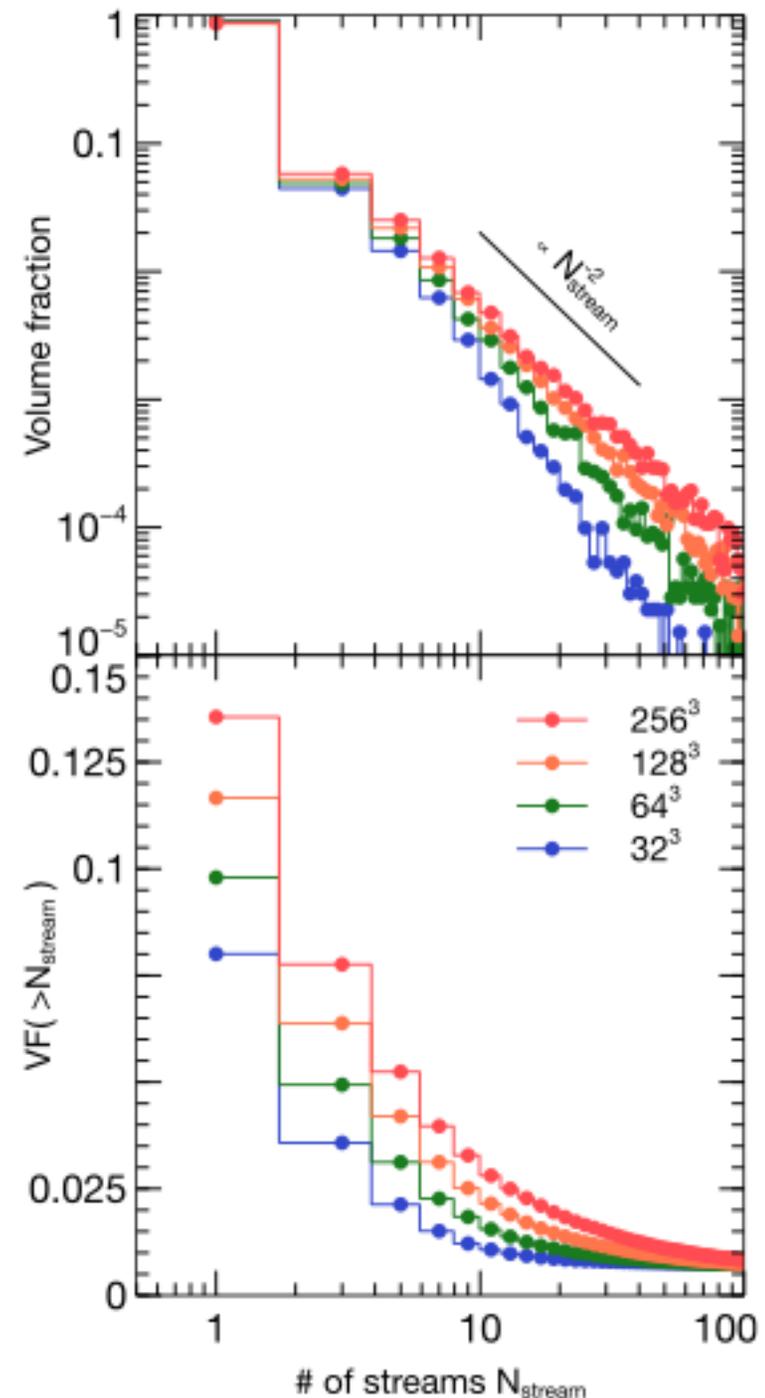
Continues to change with resolution

In particular:

The volume fraction of voids cannot even be determined.

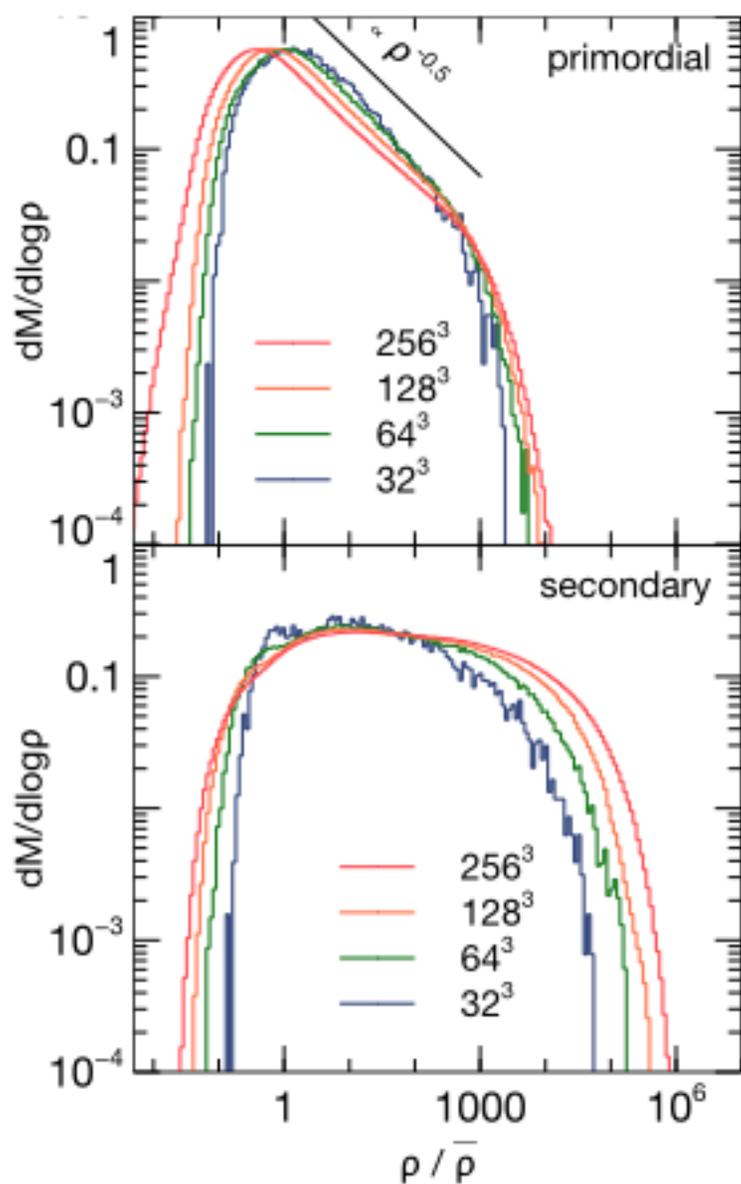
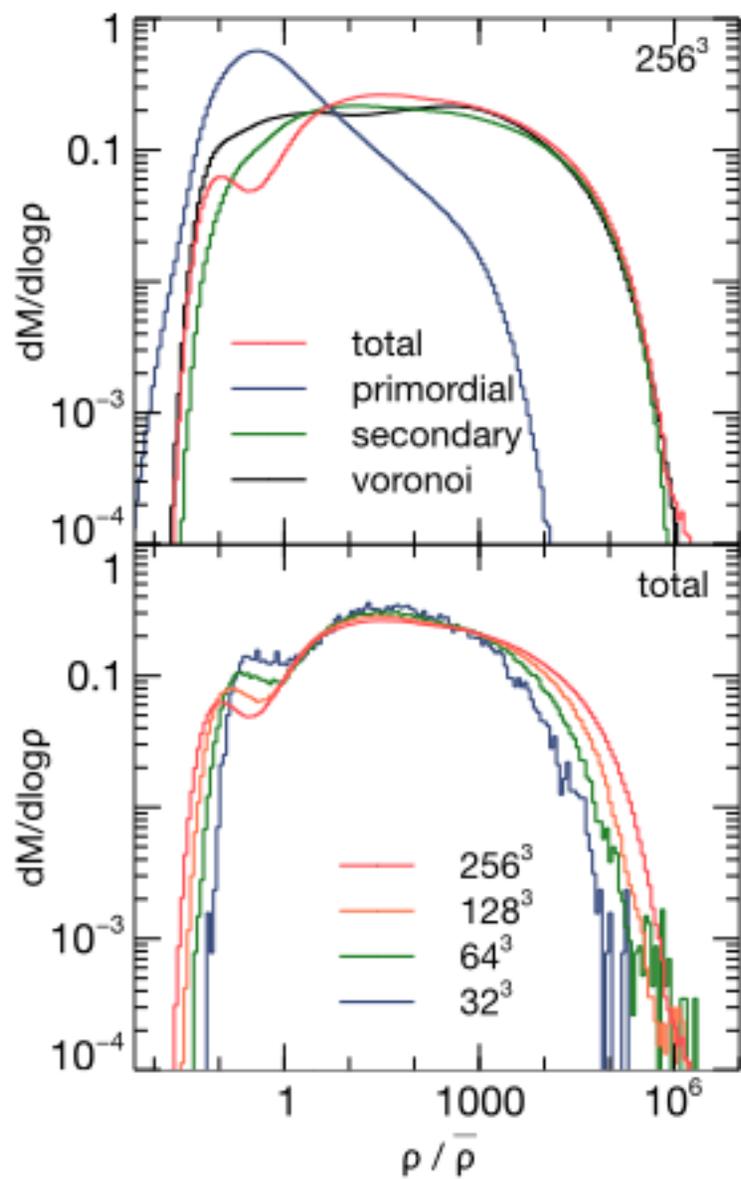
This is CDM : clumps on all scales, maybe down to earth masses.

Voids, Sheets, Filaments can be sensibly defined only for a given spatial scale.

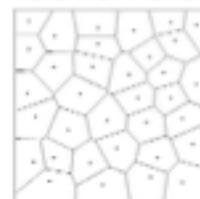


The density distributions

(mass weighted)

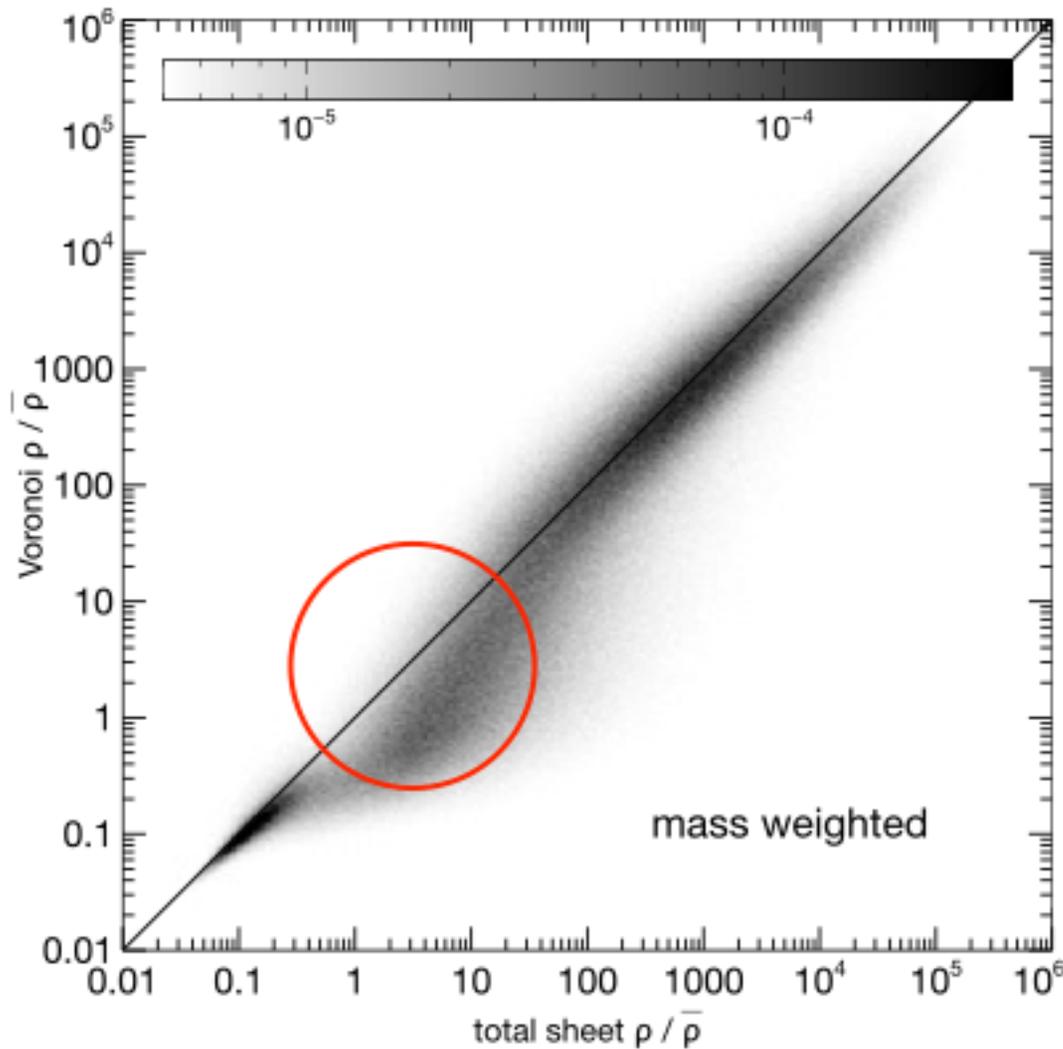


Voronoi:

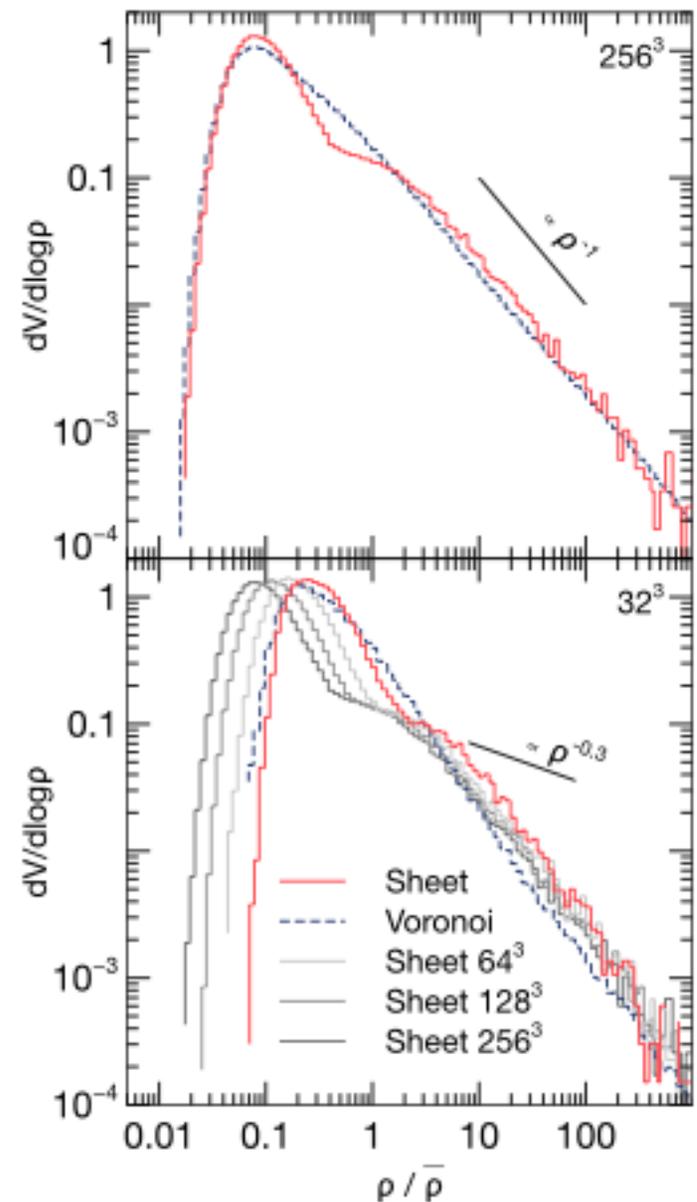


Comparison with Voronoi densities

Much of the difference is at modest overdensities!

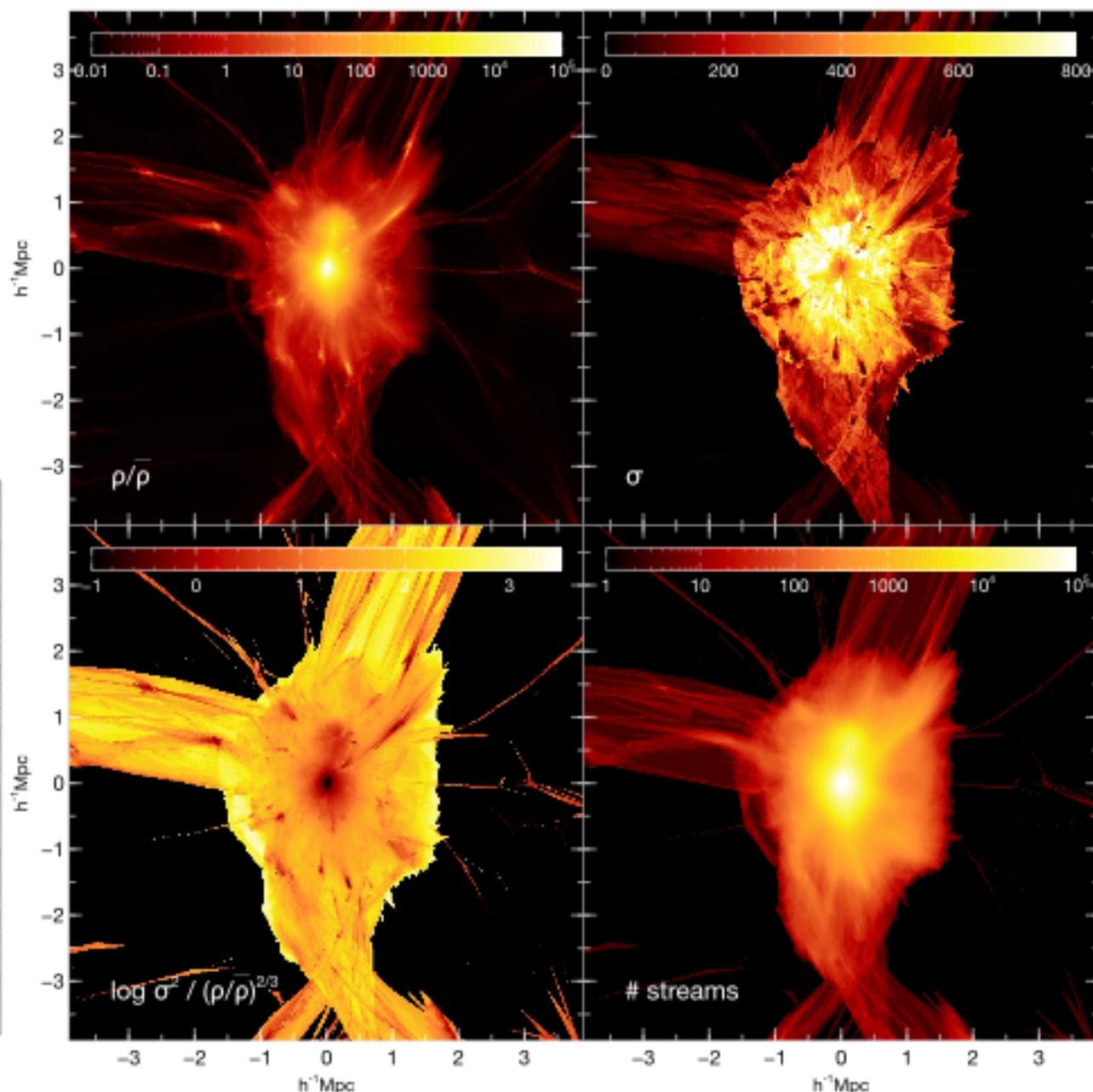
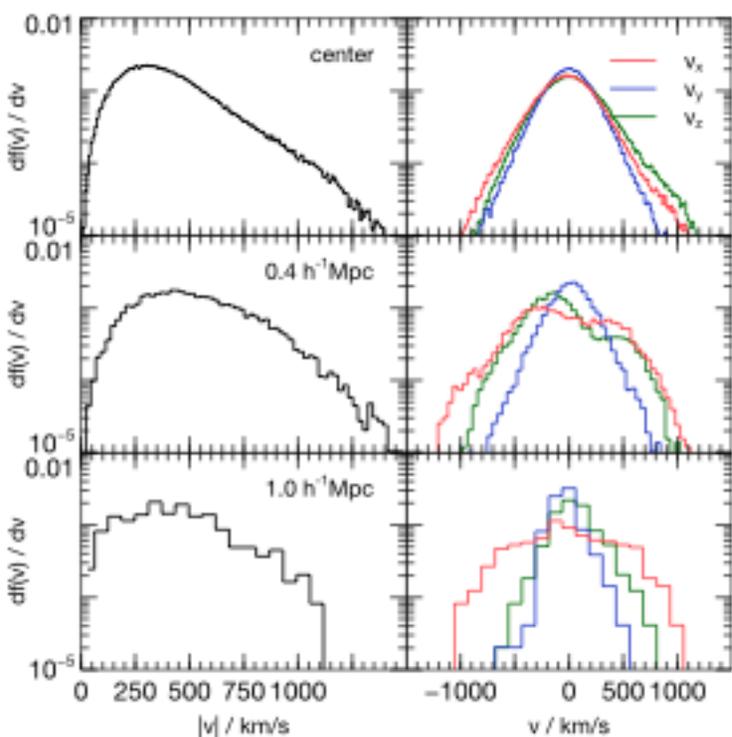


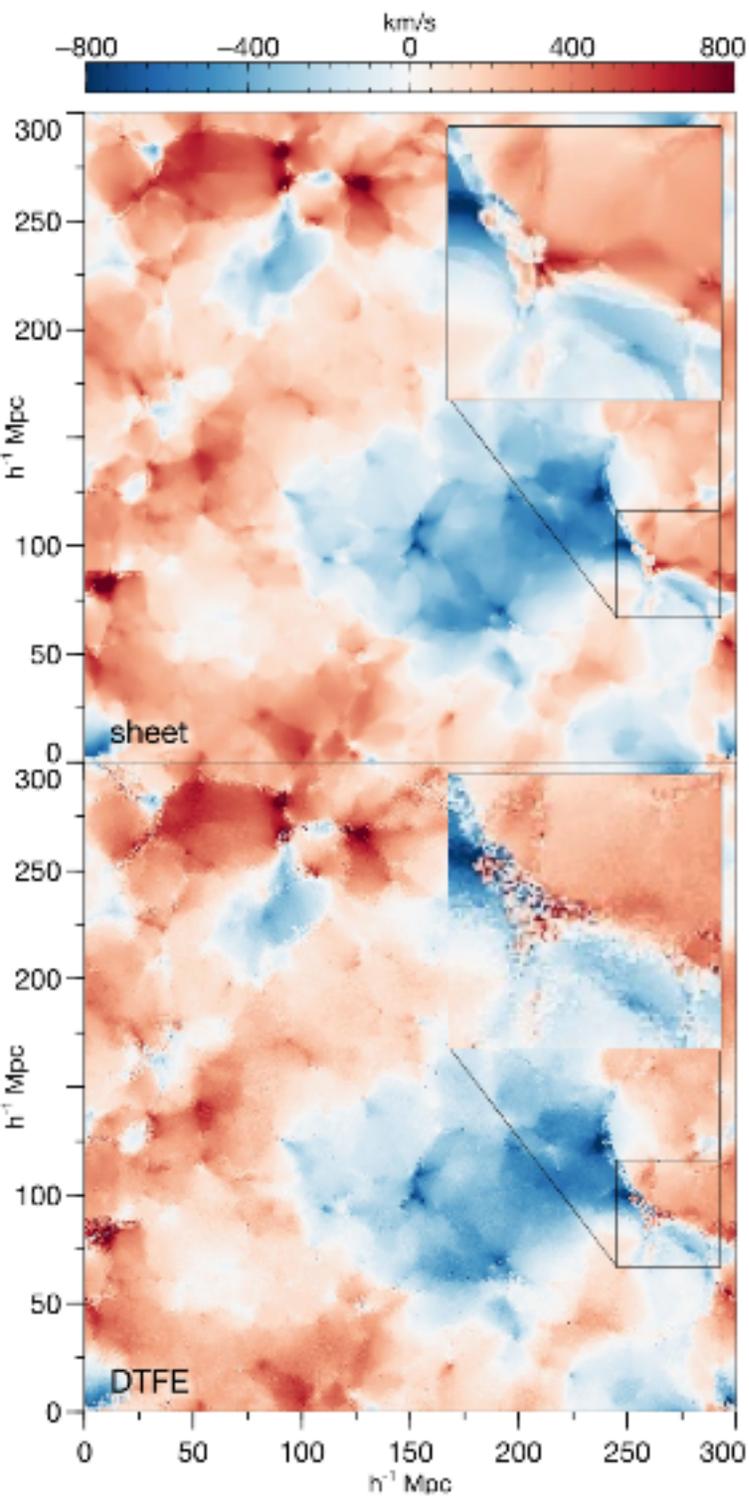
they occupy a lot of volume.



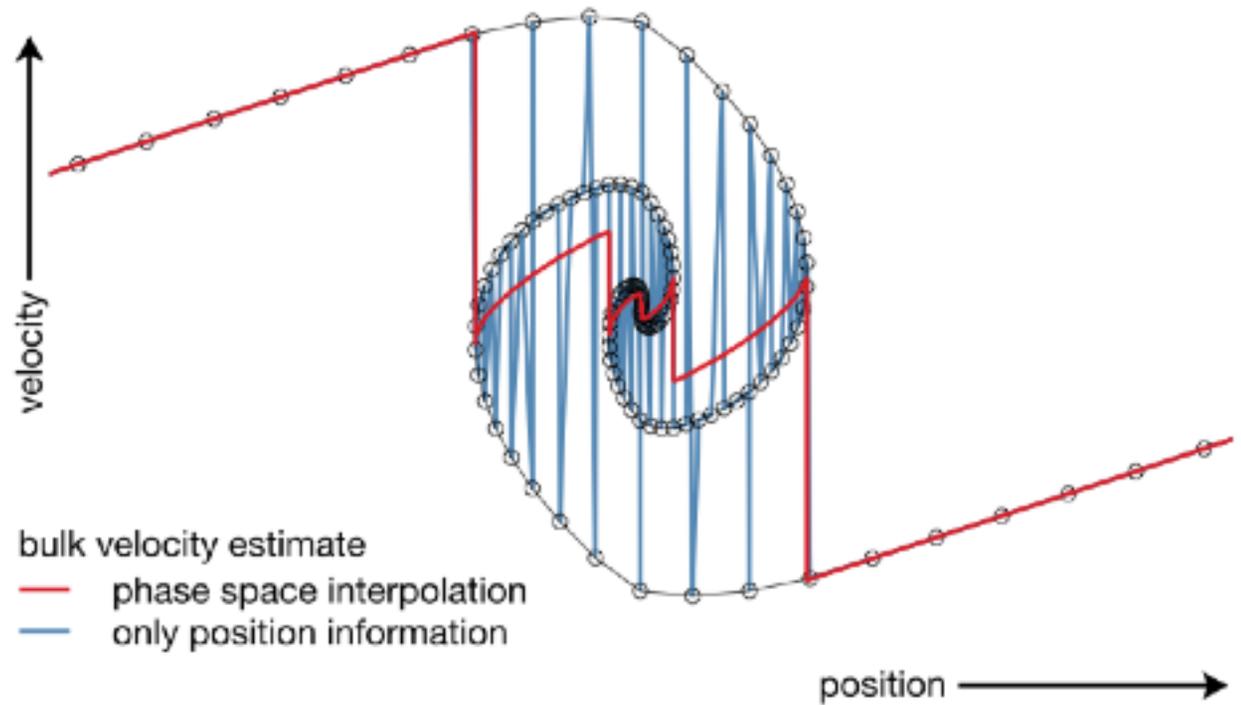
All microphysical phase space information available

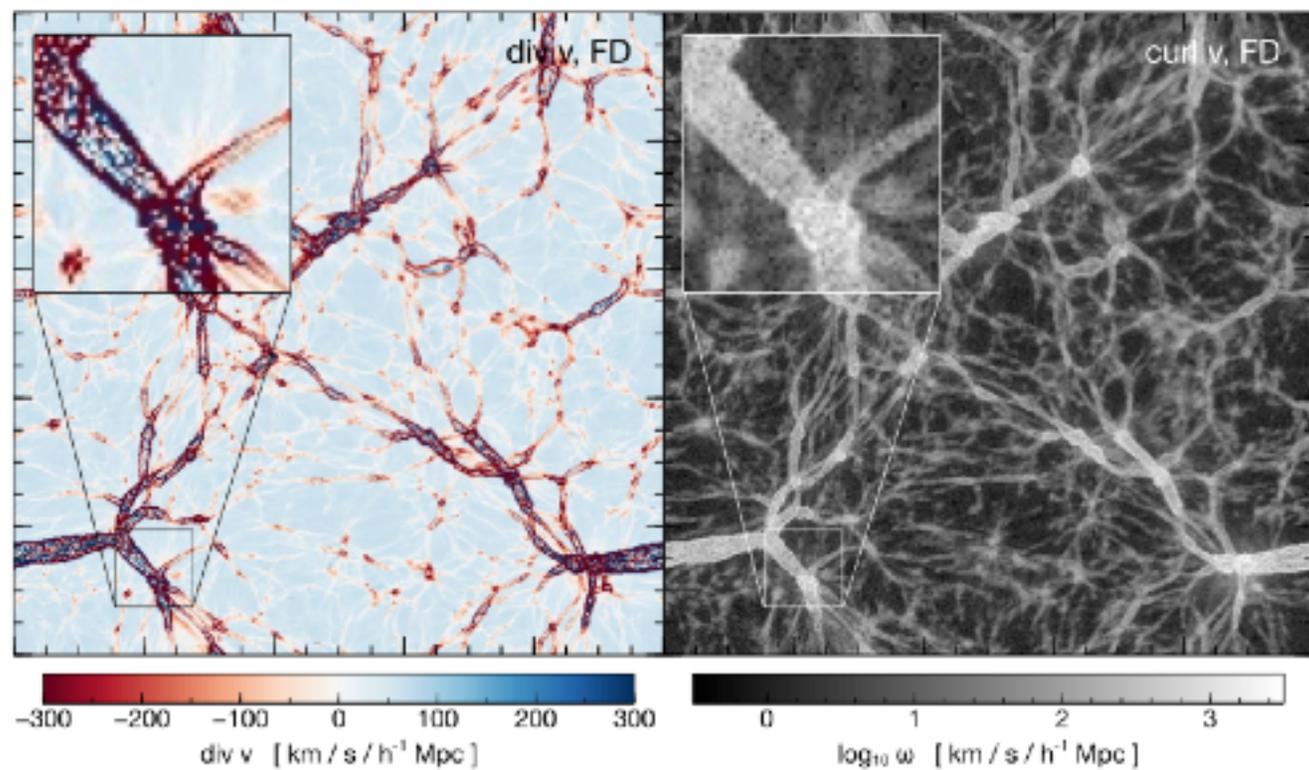
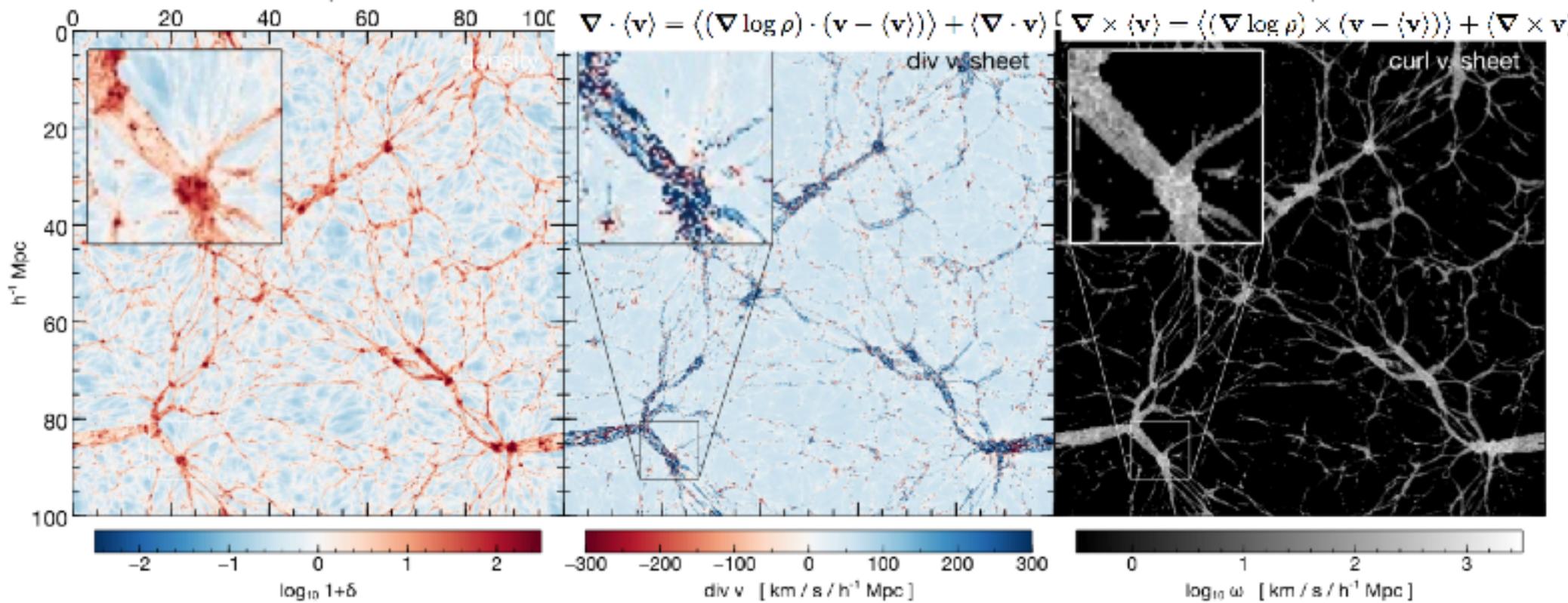
can probe
fine-grained
phase space
structure.

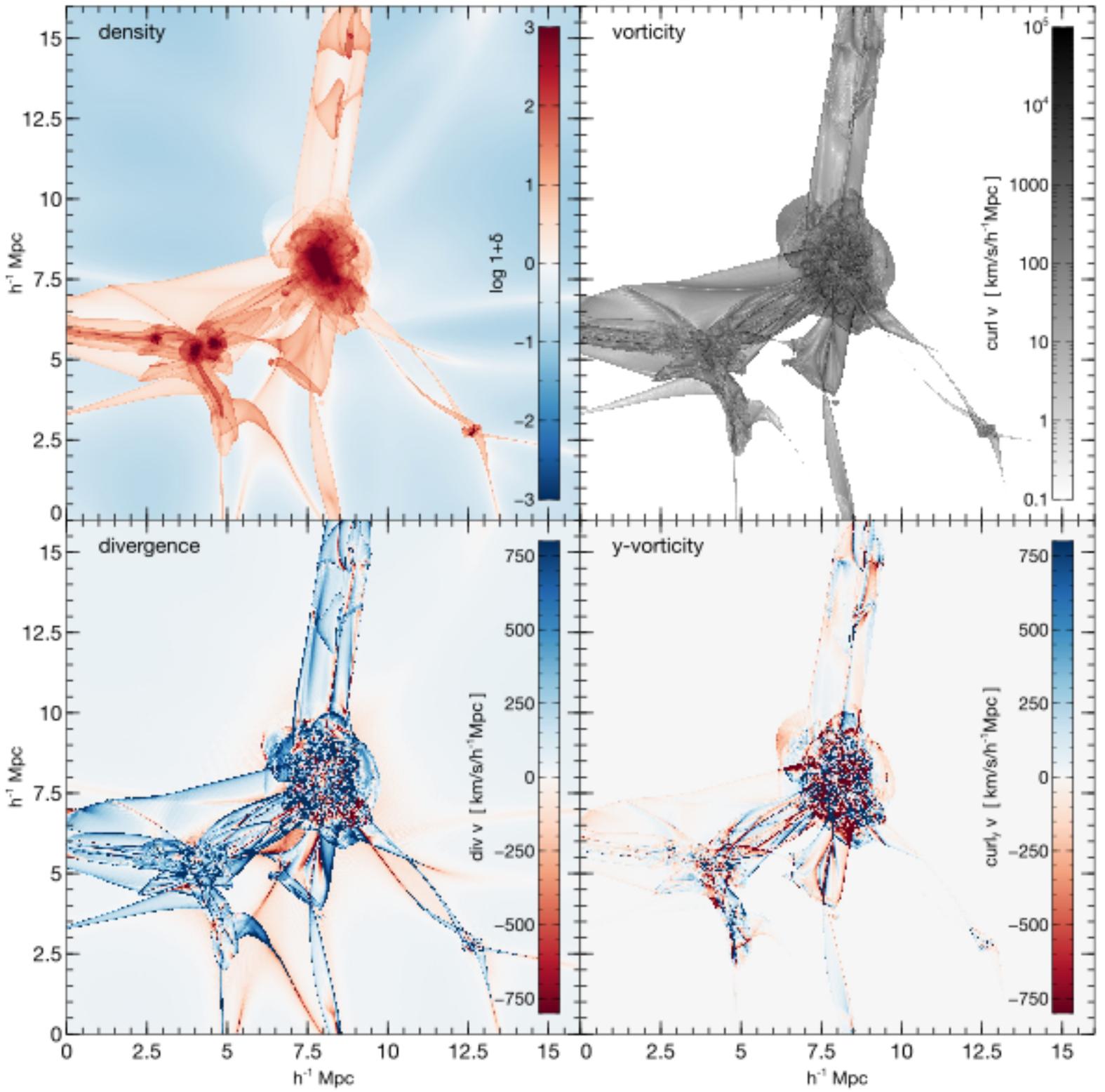




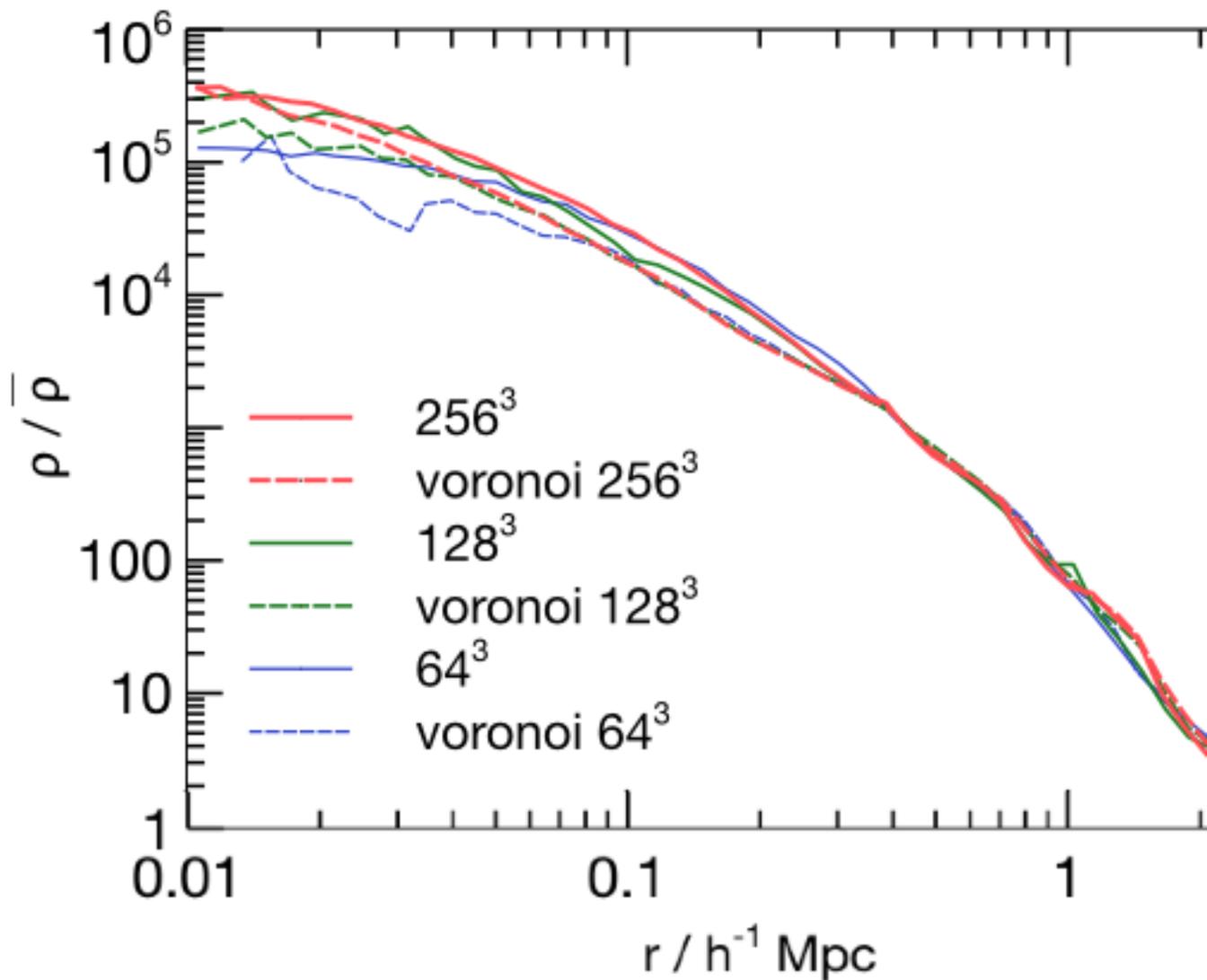
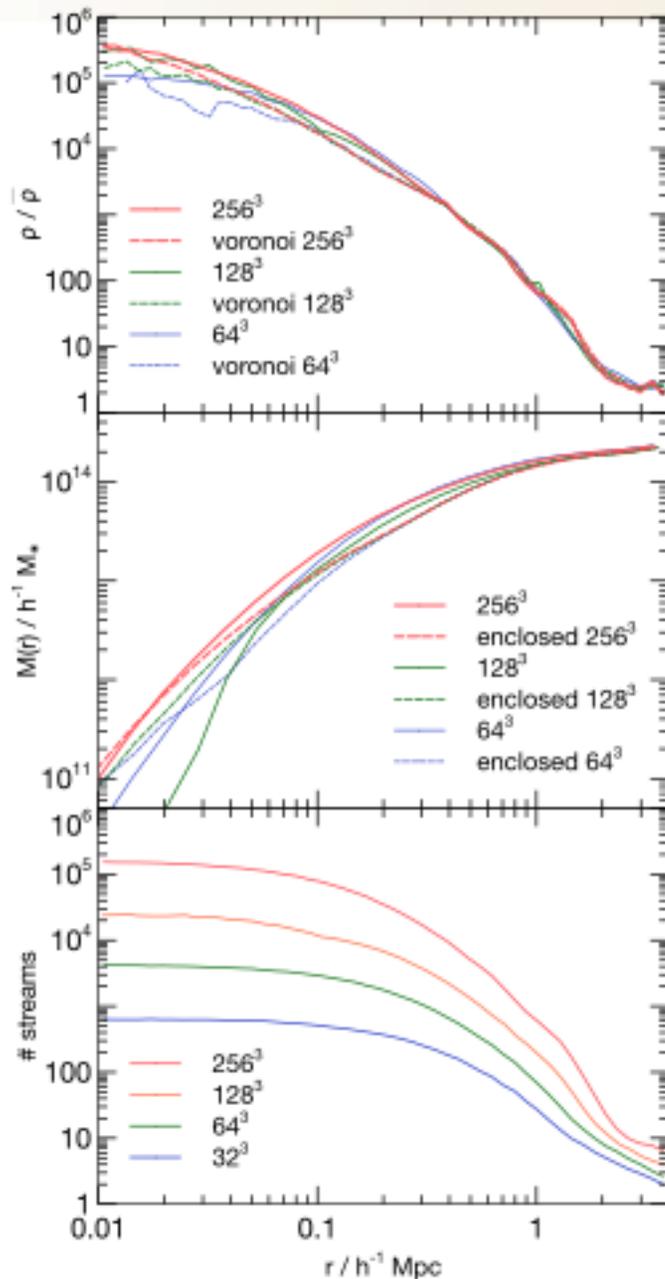
Cosmic Velocity Fields

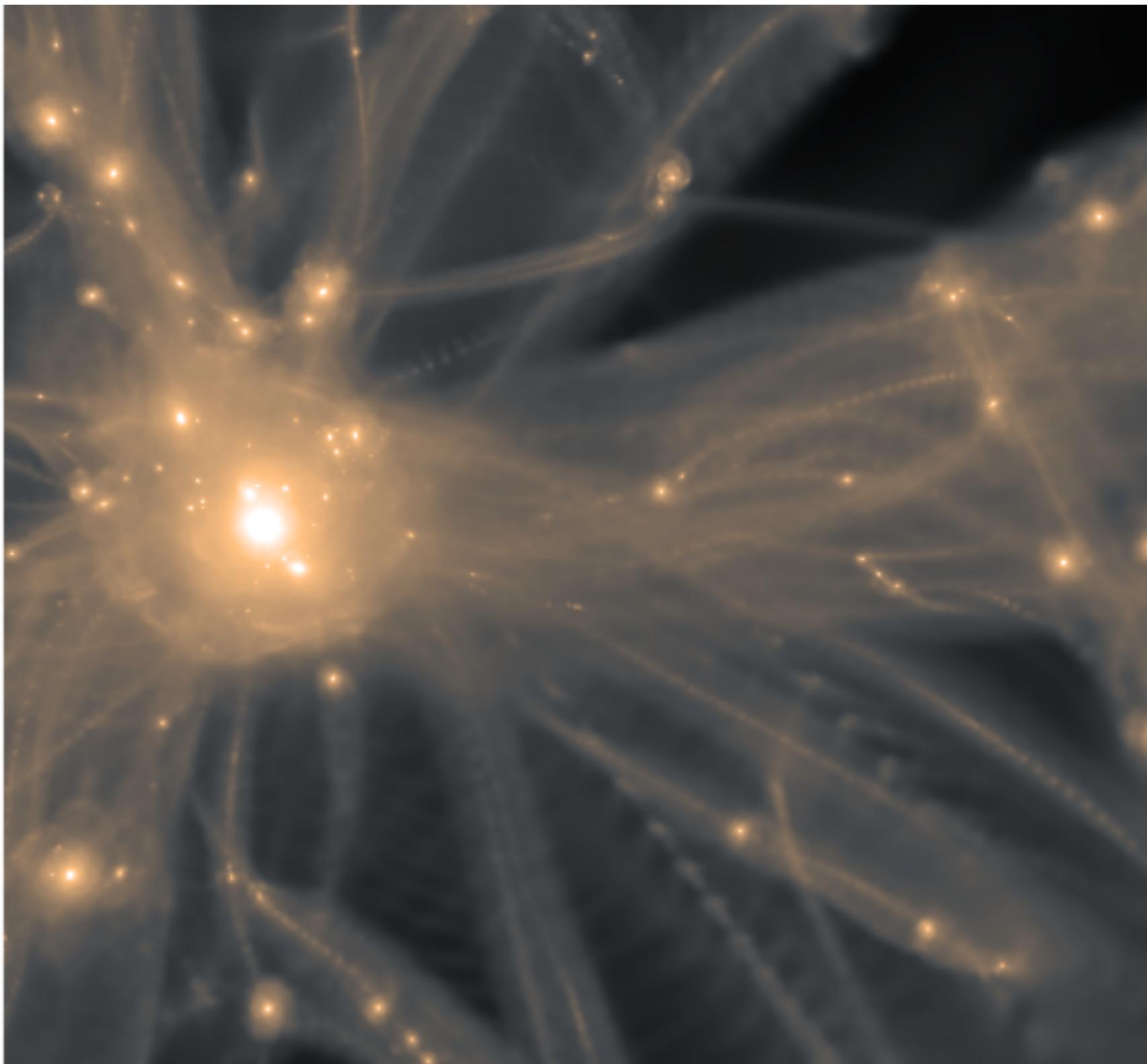


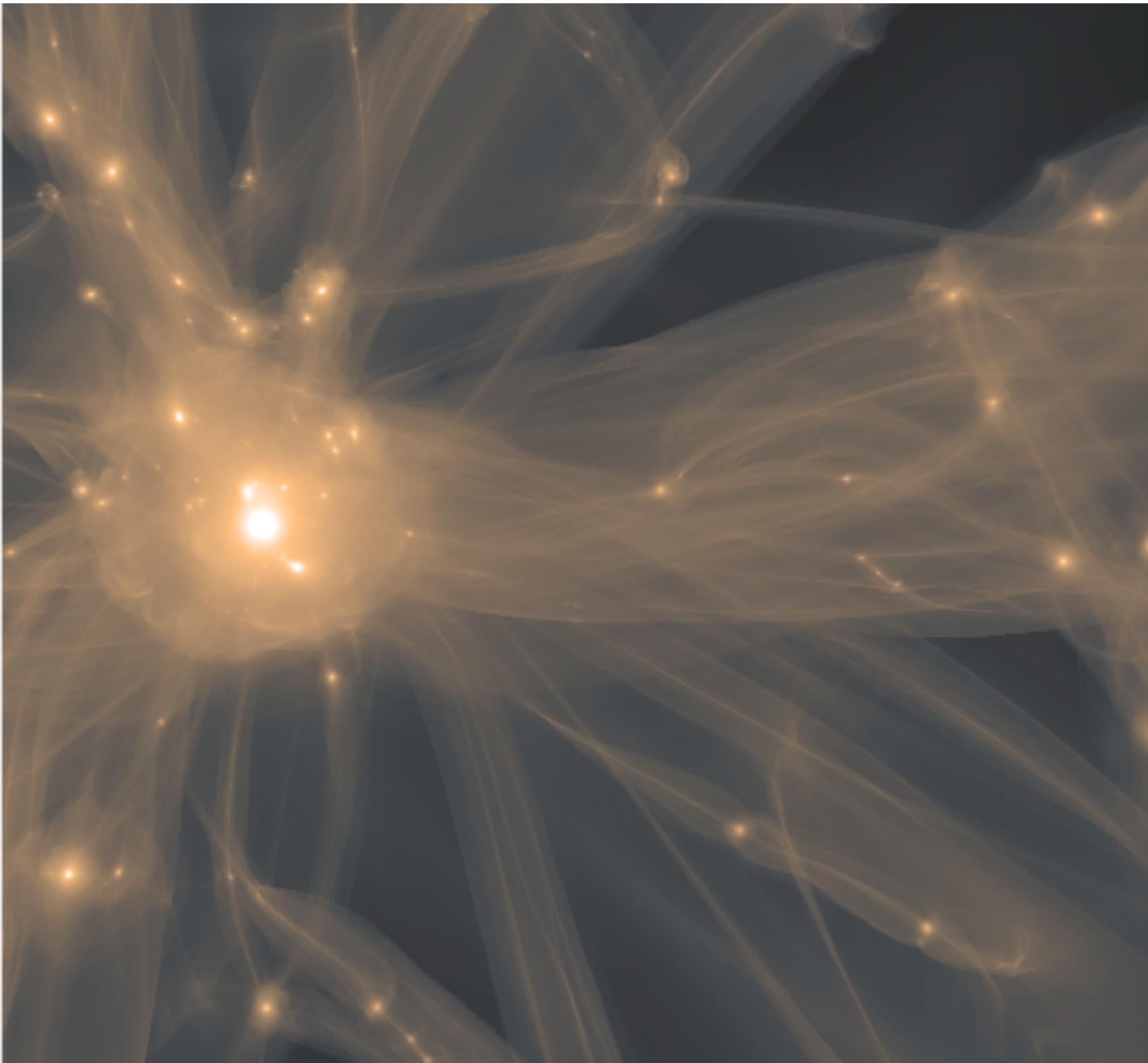




Radial profiles reveal slight density bias







TET-PM

- NEW WAY TO DO N-BODY SIMULATIONS
- **MASSLESS TRACERS** moving along characteristics.
- These span tetrahedra.
- FIRST IMPLEMENTATION: **MONOPOLE APPROXIMATION**

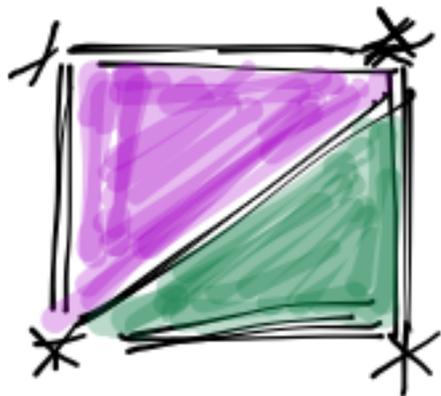
NEW

HAIN, KAEHLER
ABEL 2012 in prep.

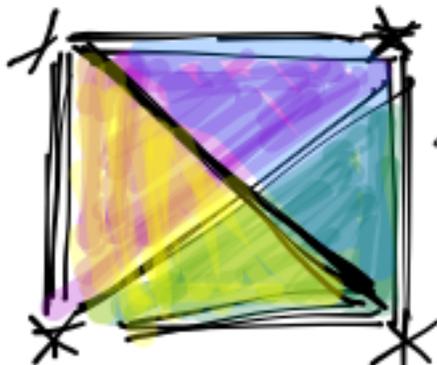
2 TYPES OF
PARTICLES

→ **MASS** OF TET DEPOSITED AS POINT PARTICLE @ **CENTROID LOCATION**

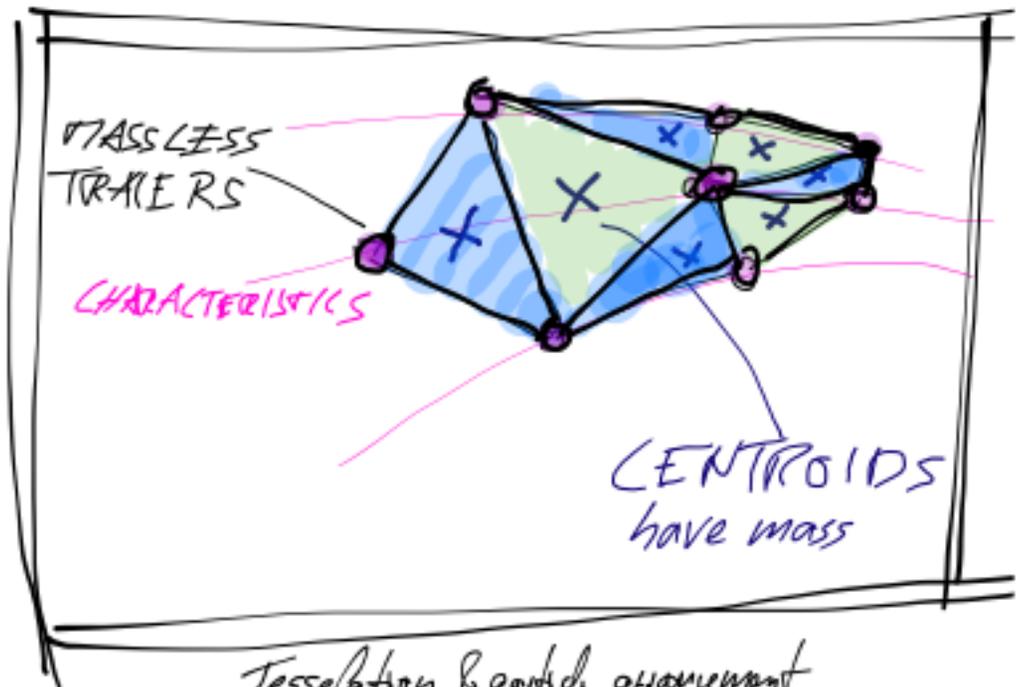
- OTHERWISE IDENTICAL TO A **PARTICLE MESH CODE**



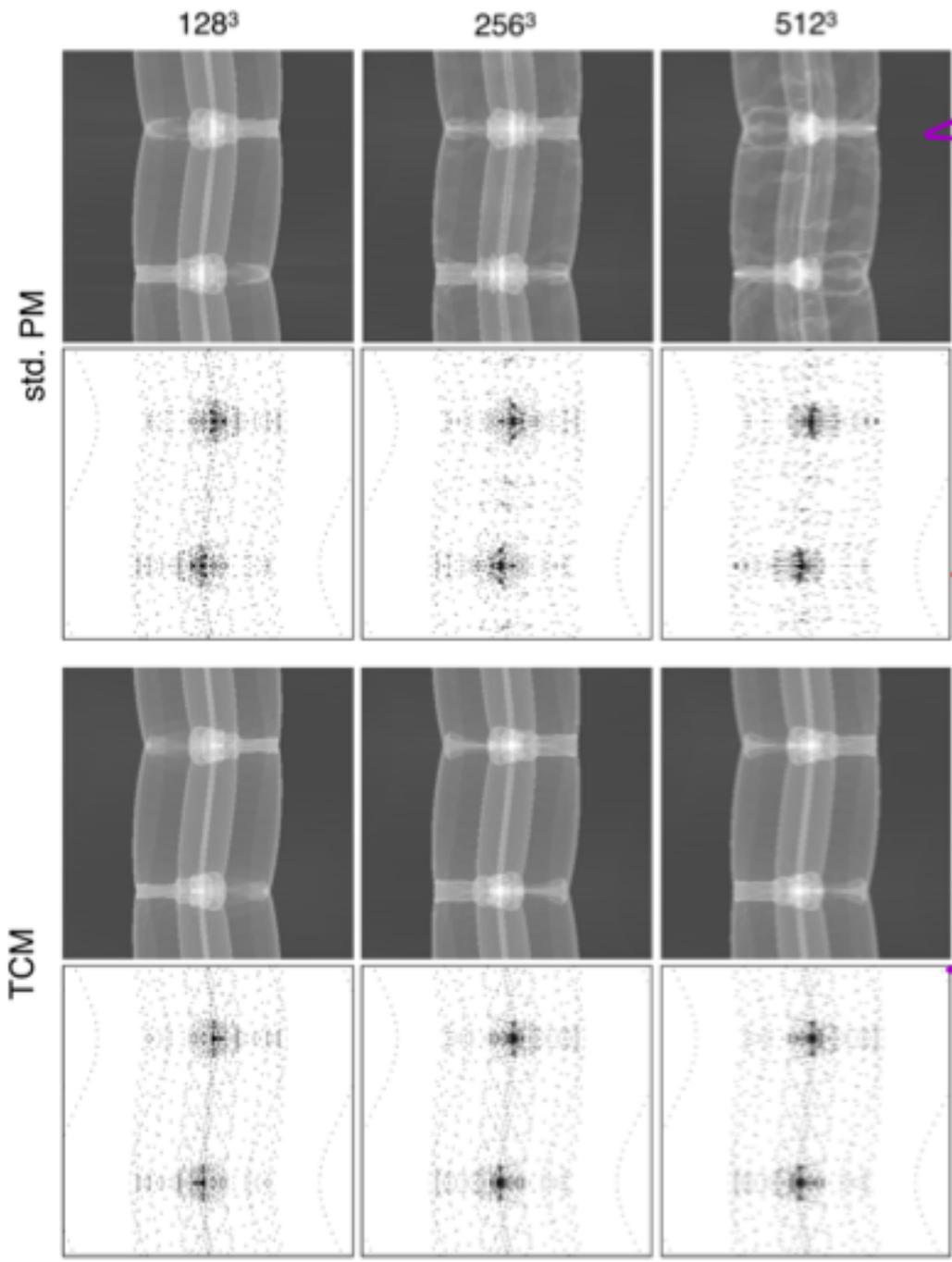
⇐ PROPER
TESSELLATION



⇐ SYMMETRIC
VERSION
2D : 4 Δ 's per \square
3D : 8 Δ 's per \square



Tessellation & particle arrangement
in TET-PM



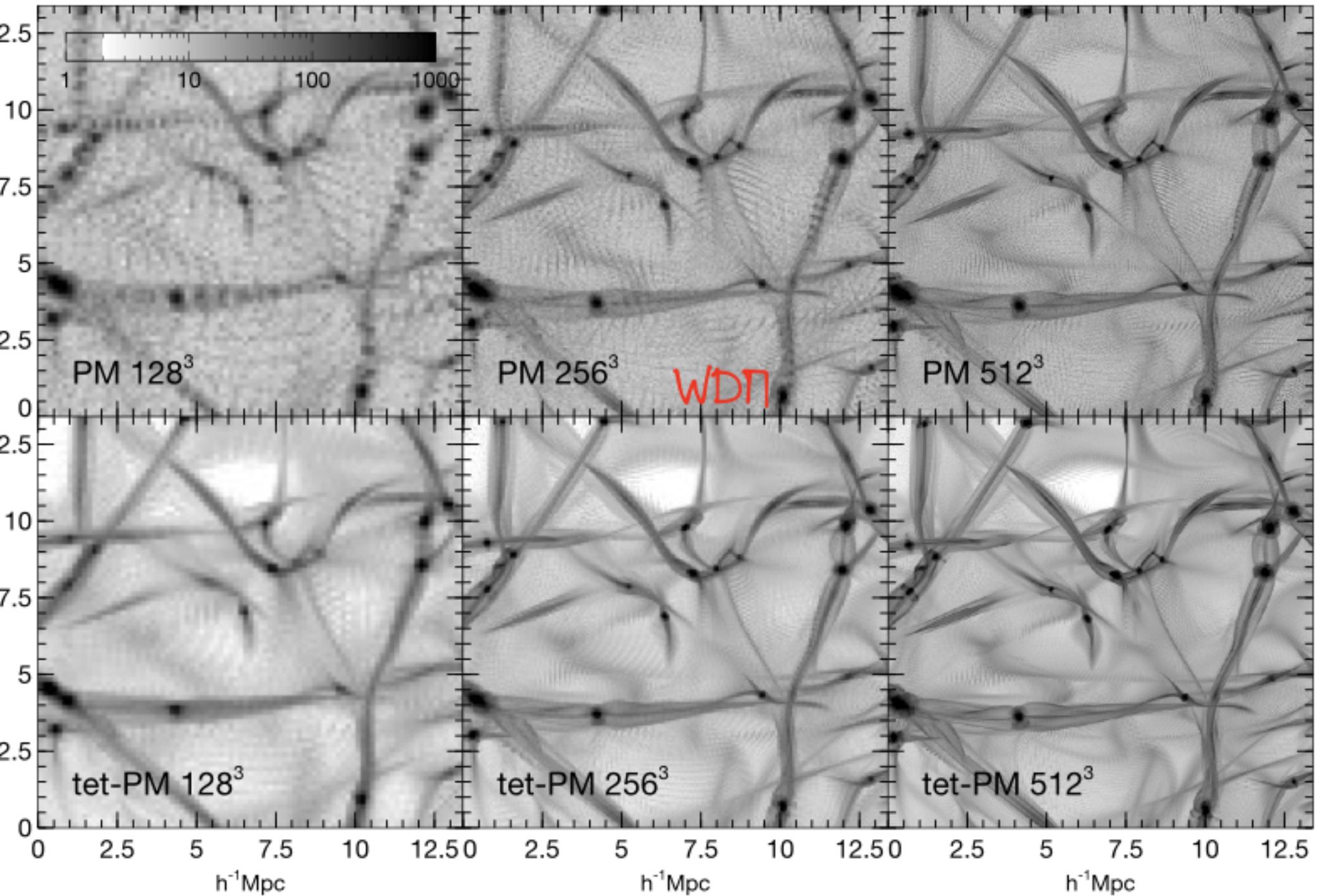
← two body scattering undesirable ...

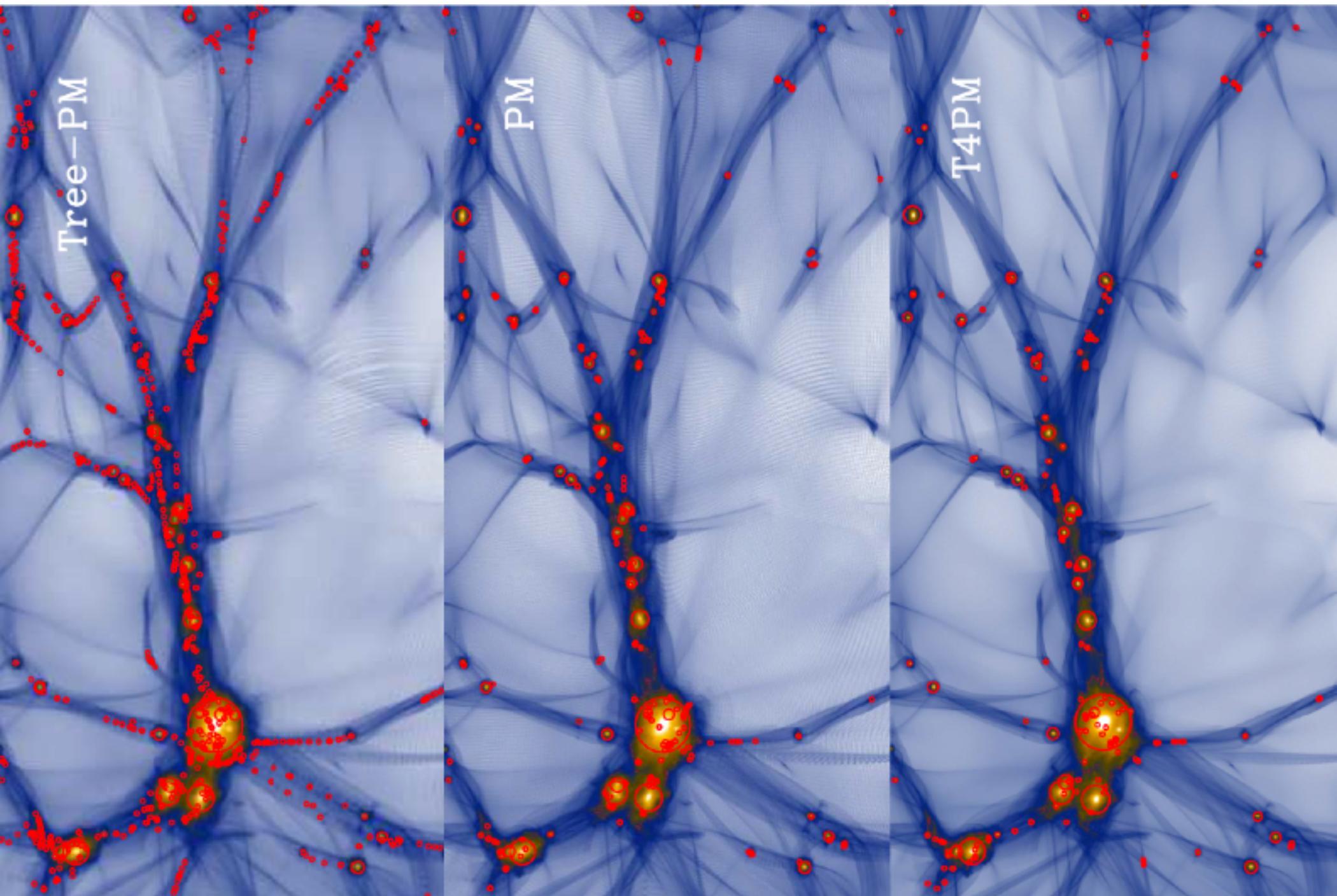
Note how the new visualization technique helps in spotting errors in the N-body integration

Constant mass resolution
Vary force resolution

← excellent convergence behavior of our new method.

Solution to 30 yr old artificial fragmentation problem in warm and hot DM models

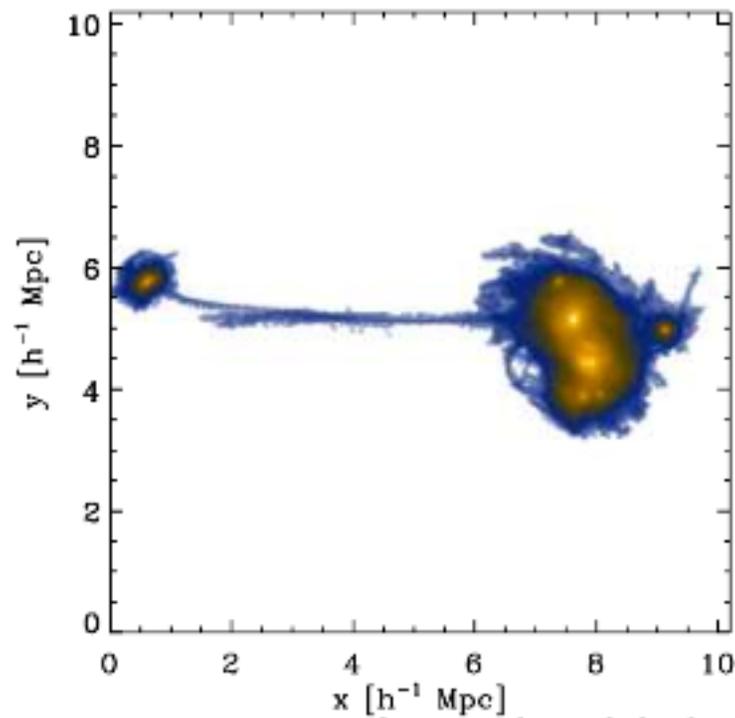
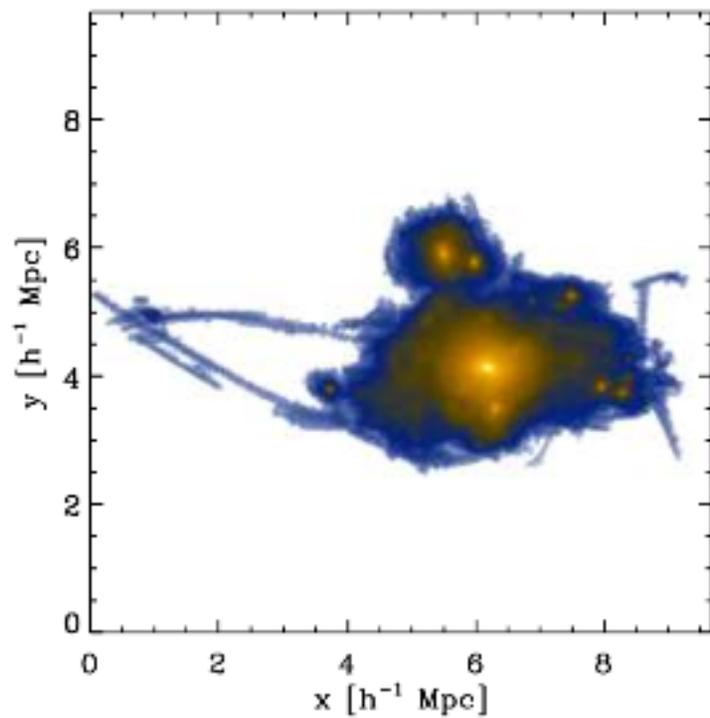
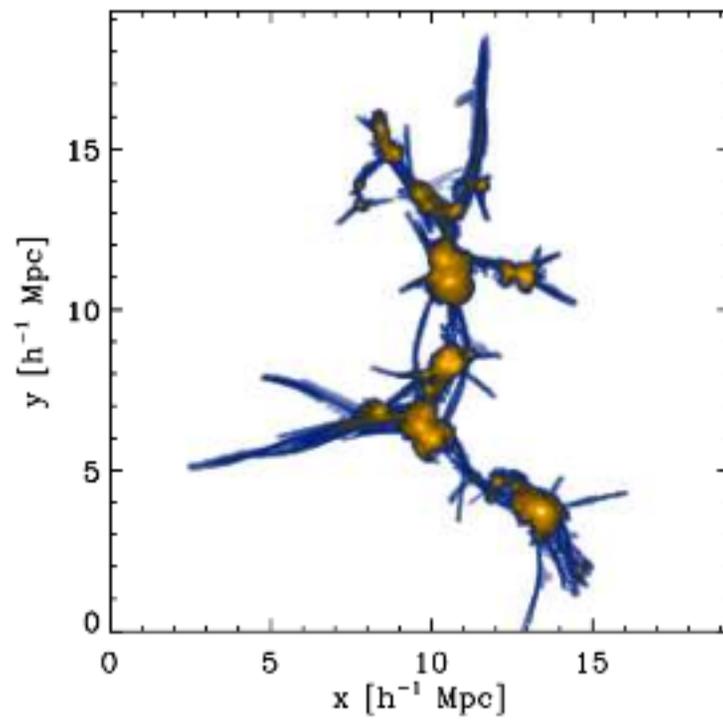
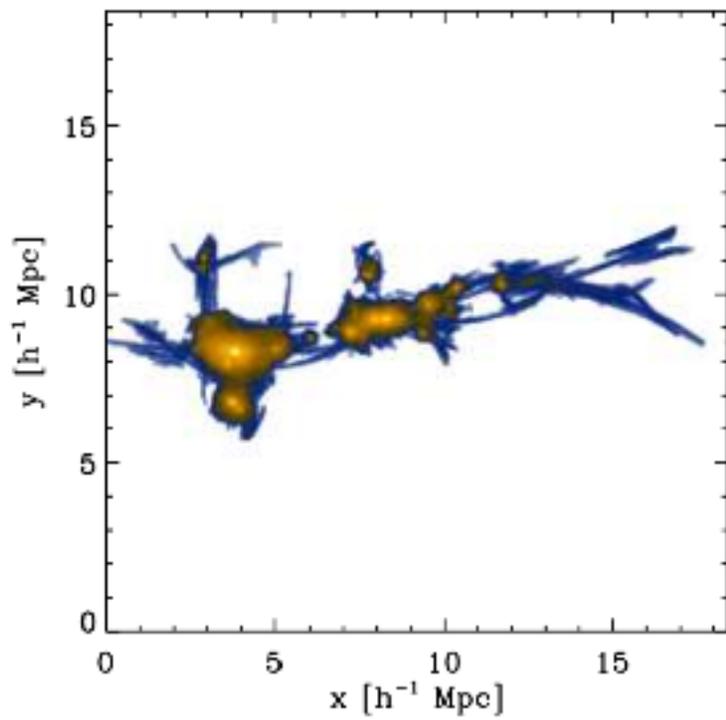




[arXiv:1304.2406](https://arxiv.org/abs/1304.2406)

The Warm DM halo mass function below the cut-off scale

Raul E. Angulo (1), Oliver Hahn (1 and 2), Tom Abel (1) ((1) KIPAC, Stanford University, (2) ETH, Zurich)
(Submitted on 8 Apr 2013)



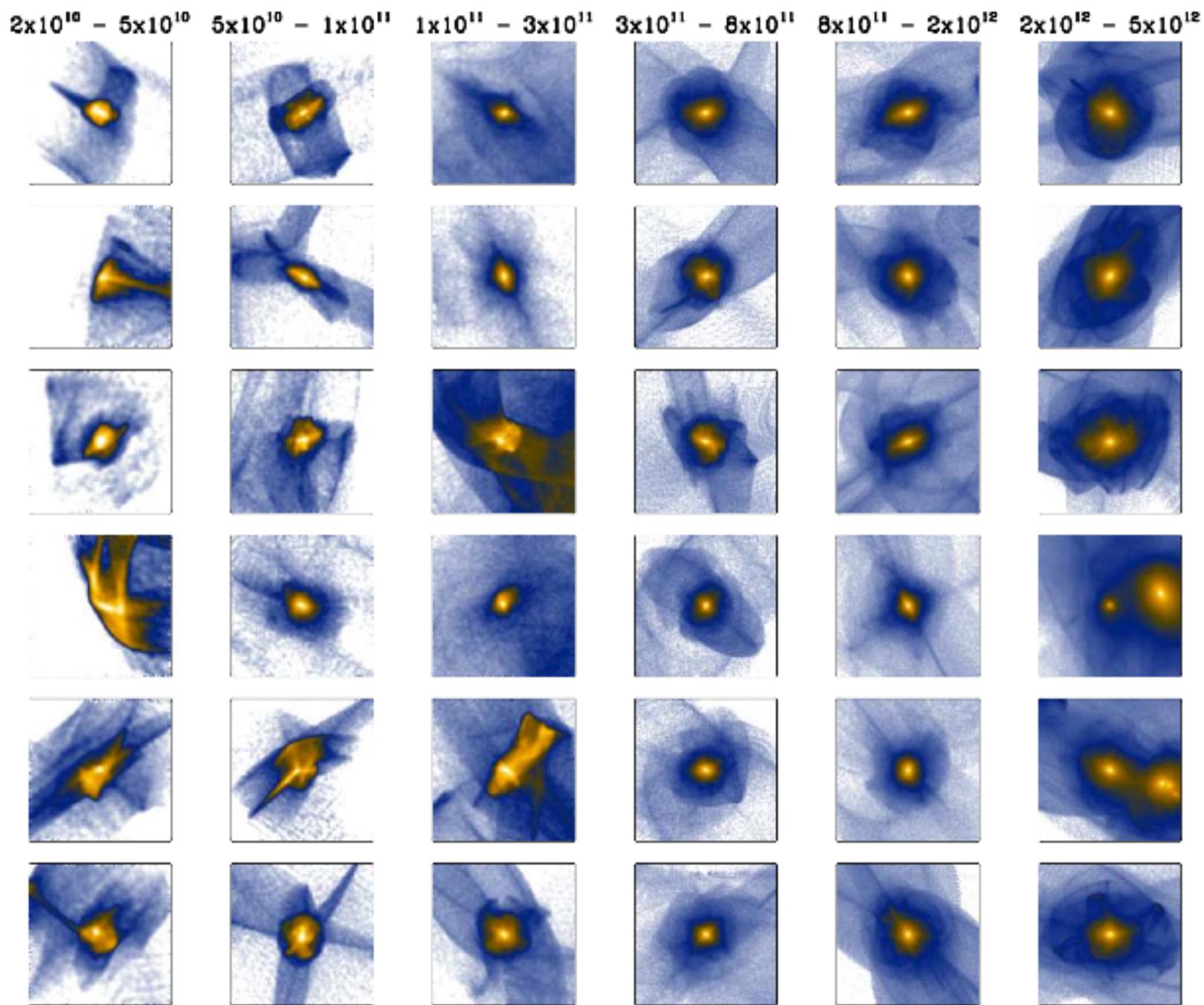
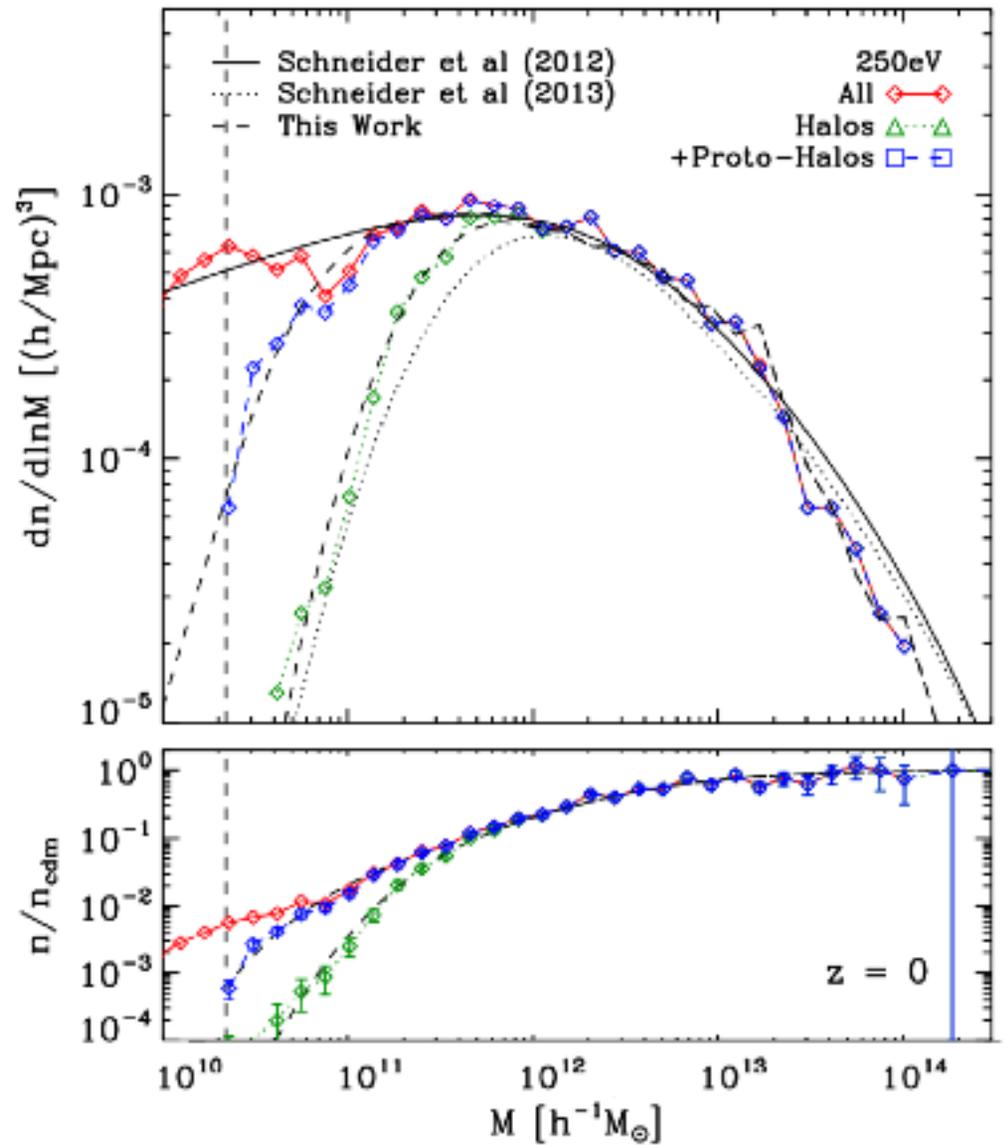
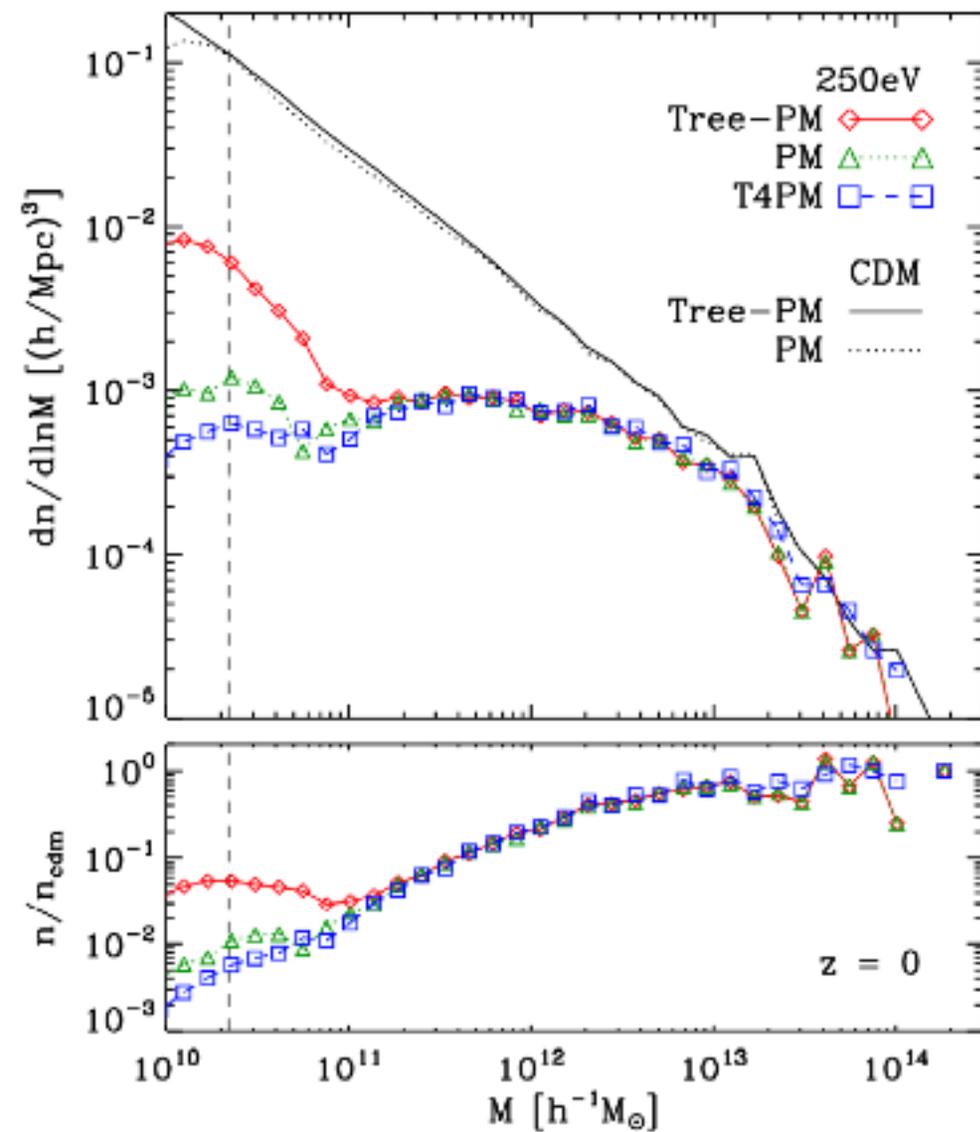


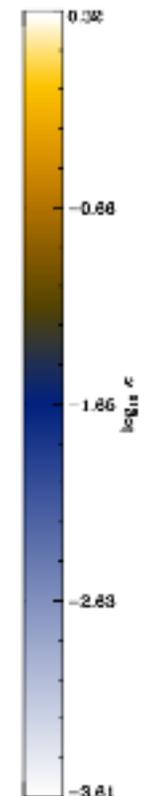
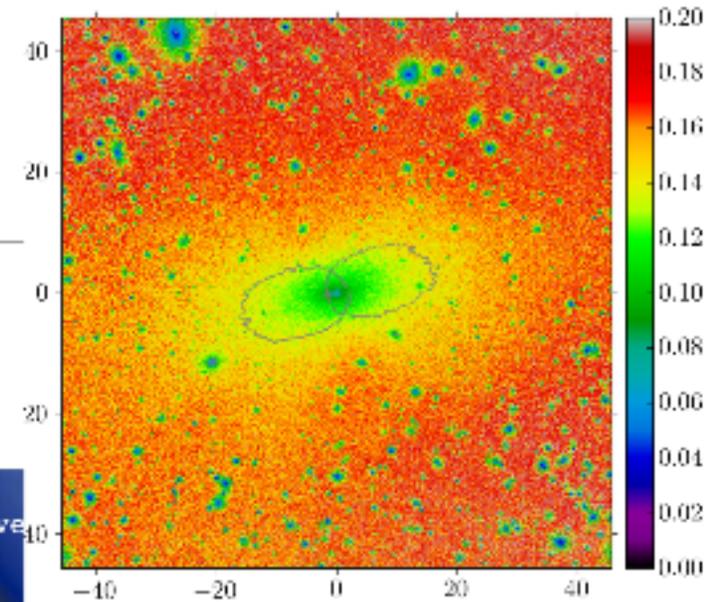
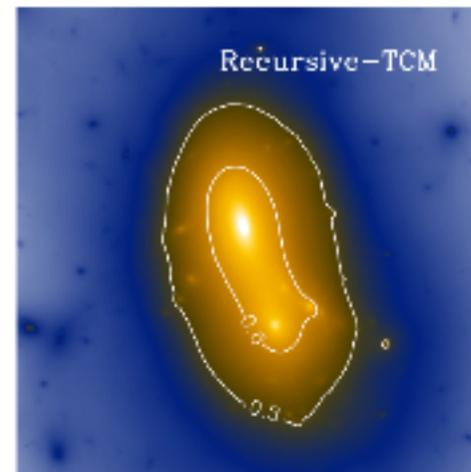
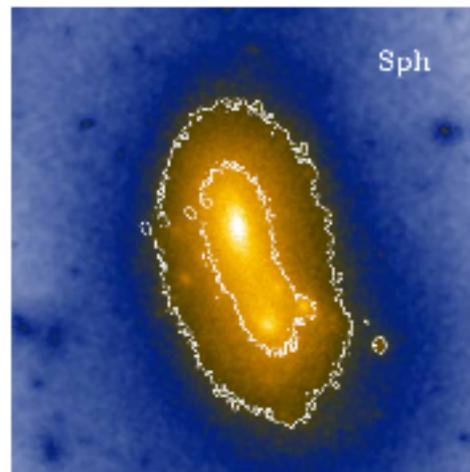
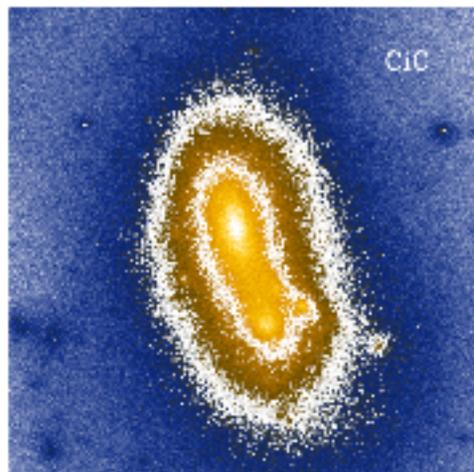
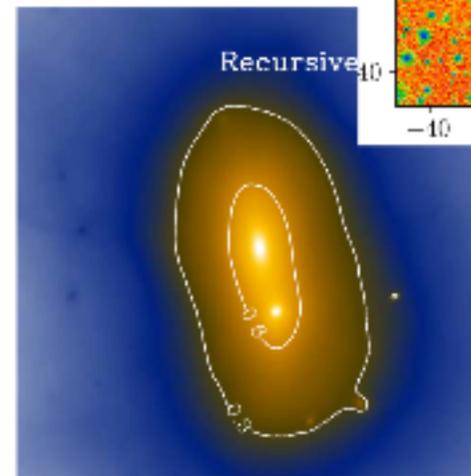
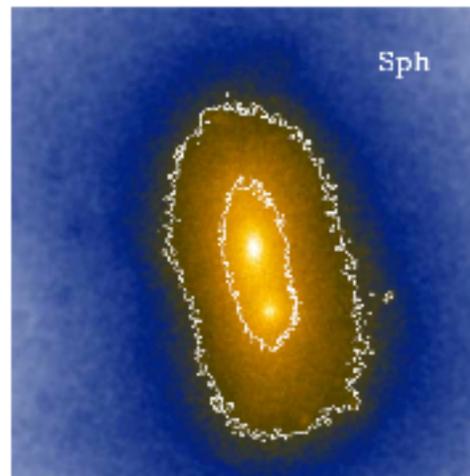
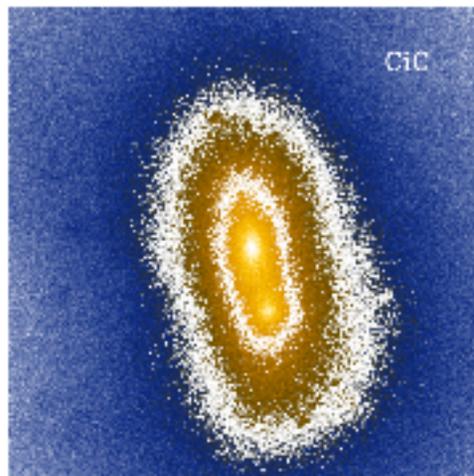
Figure 4. Images of randomly-chosen objects in six disjoint mass bins. These mass bins are equally spaced in $\log M$ over the mass range $2 \times 10^{10} < M_{200}/(h^{-1}M_{\odot}) < 5 \times 10^{12}$. Each image displays the logarithmic projected density field computed using the T4PM method. The extent of each image is equal to $2 \times R_{200}$, i.e. twice the virial radius of the respective halo.

Warm Dark Matter Mass functions



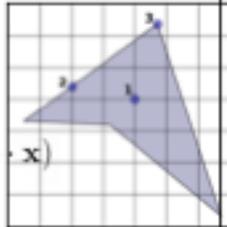
Particle Noise in Gravitational Lensing Predictions

- Noiseless but biased estimator



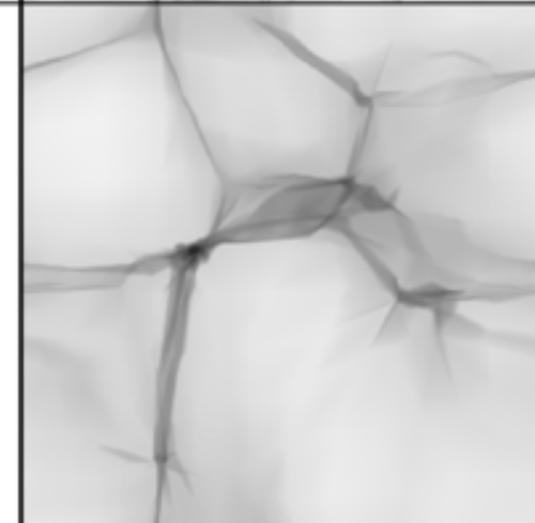
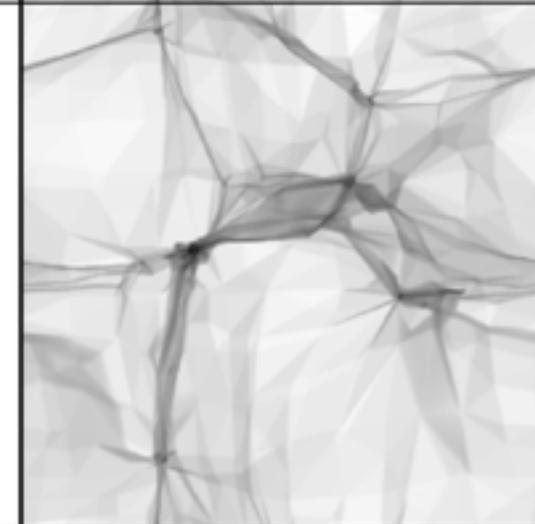
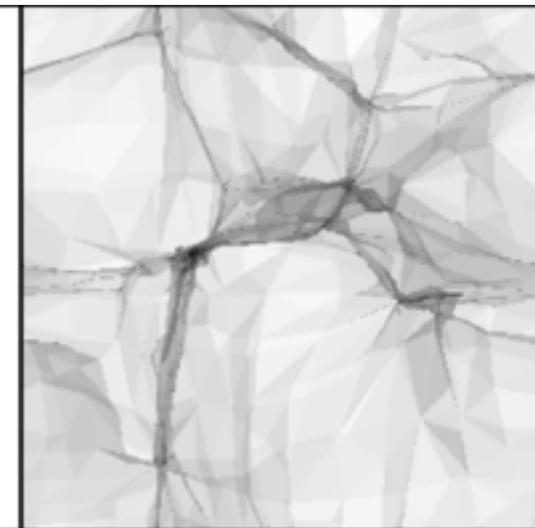
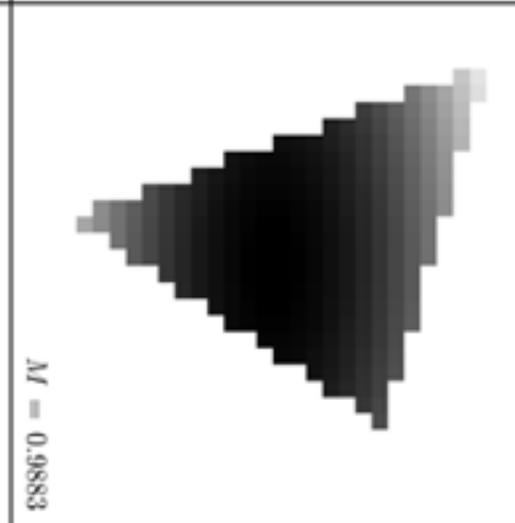
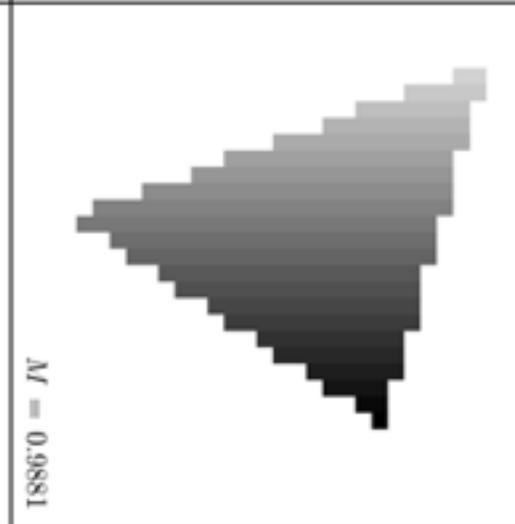
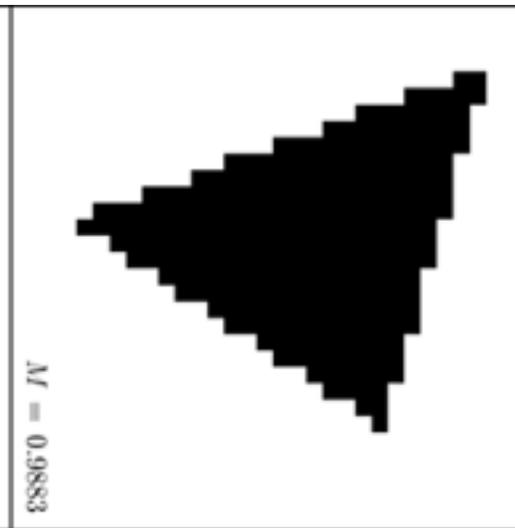
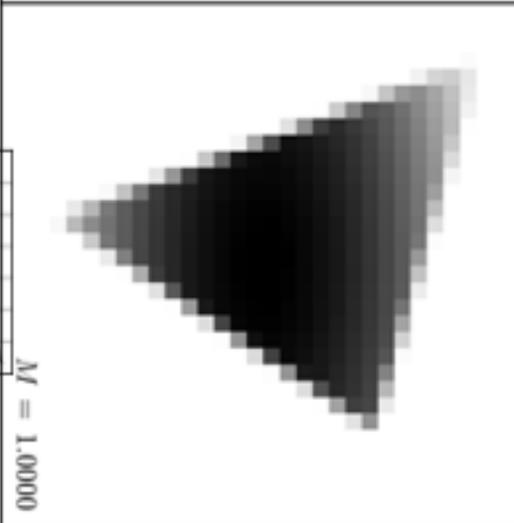
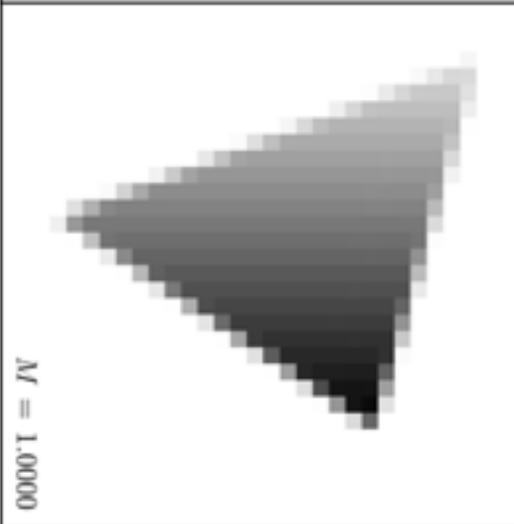
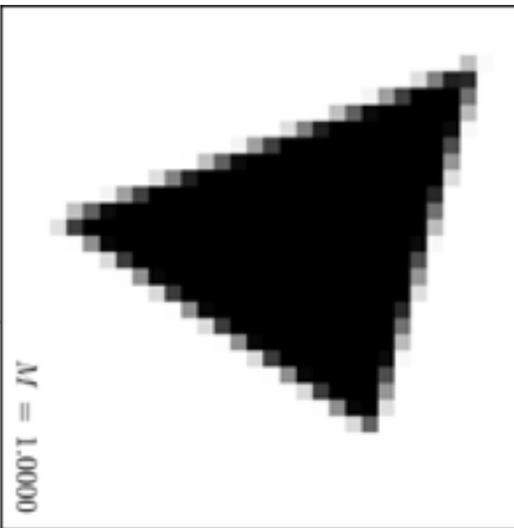
Exact Deposit

- any polyhedra intersections without constructing the overlap
- linear and quadratic function defined over polyhedra
- fundamental building block for many novel algorithms
- 30 times faster than a recursive algorithm
- Computational Geometry - Patent?
- Powell and Abel (2014) submitted to JCompP



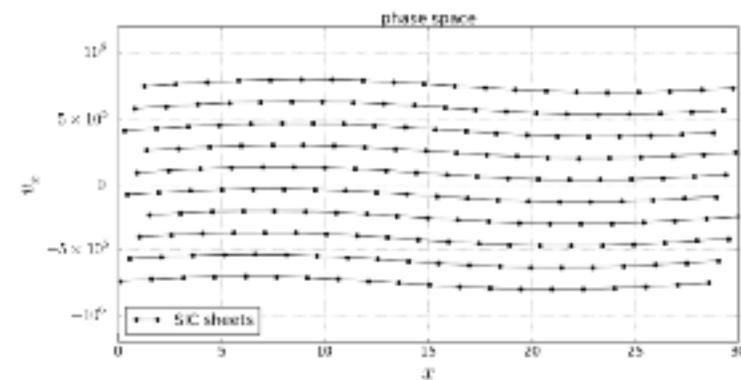
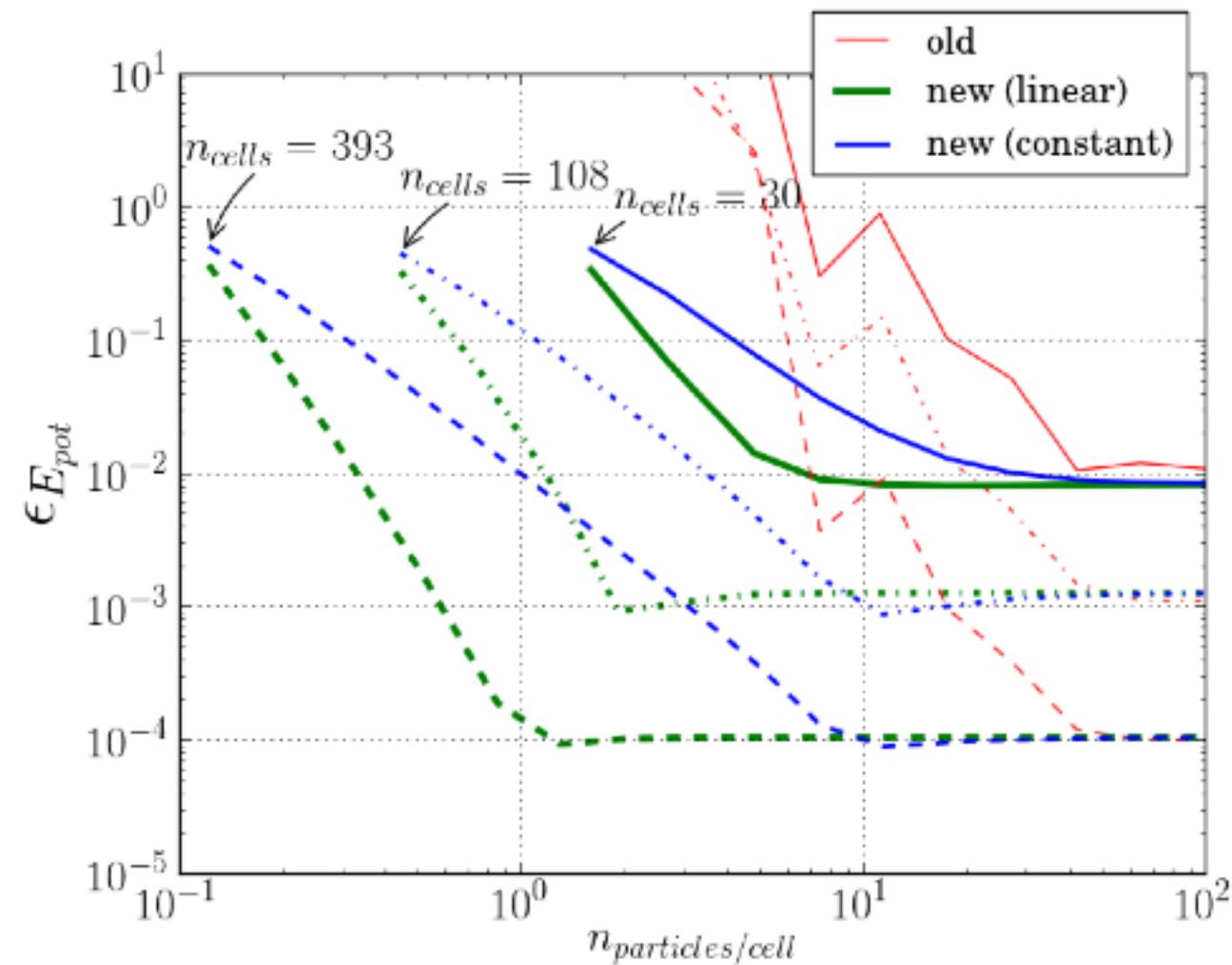
$$A = \frac{1}{2} \sum_{e,p} (\hat{\mathbf{n}}_e \cdot \mathbf{x})(\hat{\mathbf{n}}_p \cdot \mathbf{x})$$

$$V = -\frac{1}{6} \sum_{f,e,p} (\hat{\mathbf{n}}_f \cdot \mathbf{x})(\hat{\mathbf{n}}_e \cdot \mathbf{x})(\hat{\mathbf{n}}_p \cdot \mathbf{x})$$

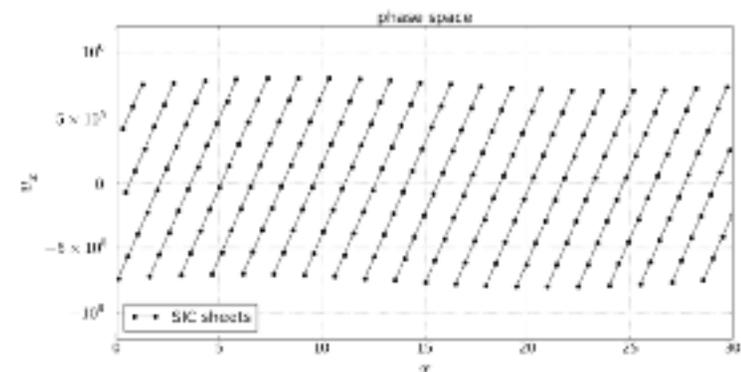


Also applicable to Collisionless Plasmas -> Neutrinos?

Example: Landau Damping in 1D



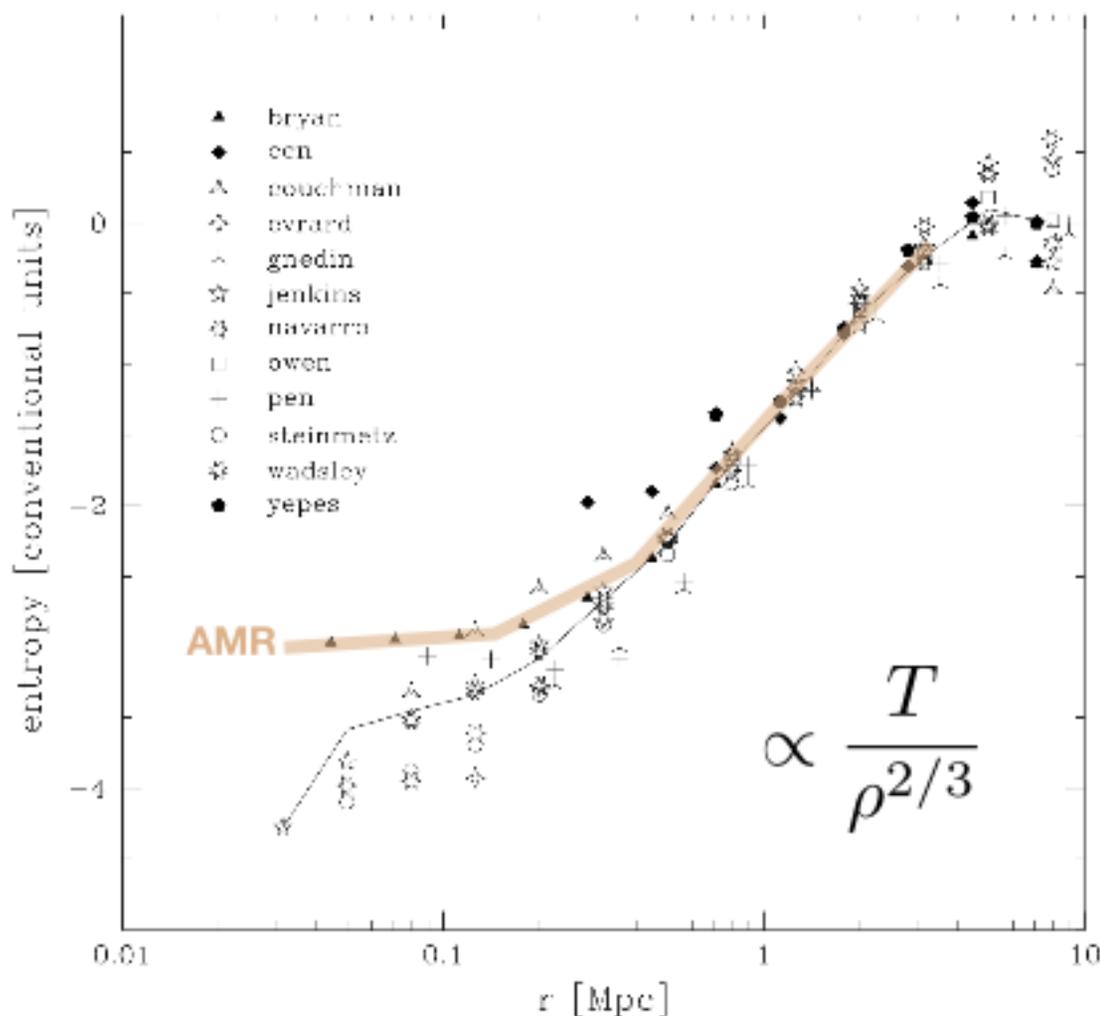
(a) horizontal streams



(b) vertical streams

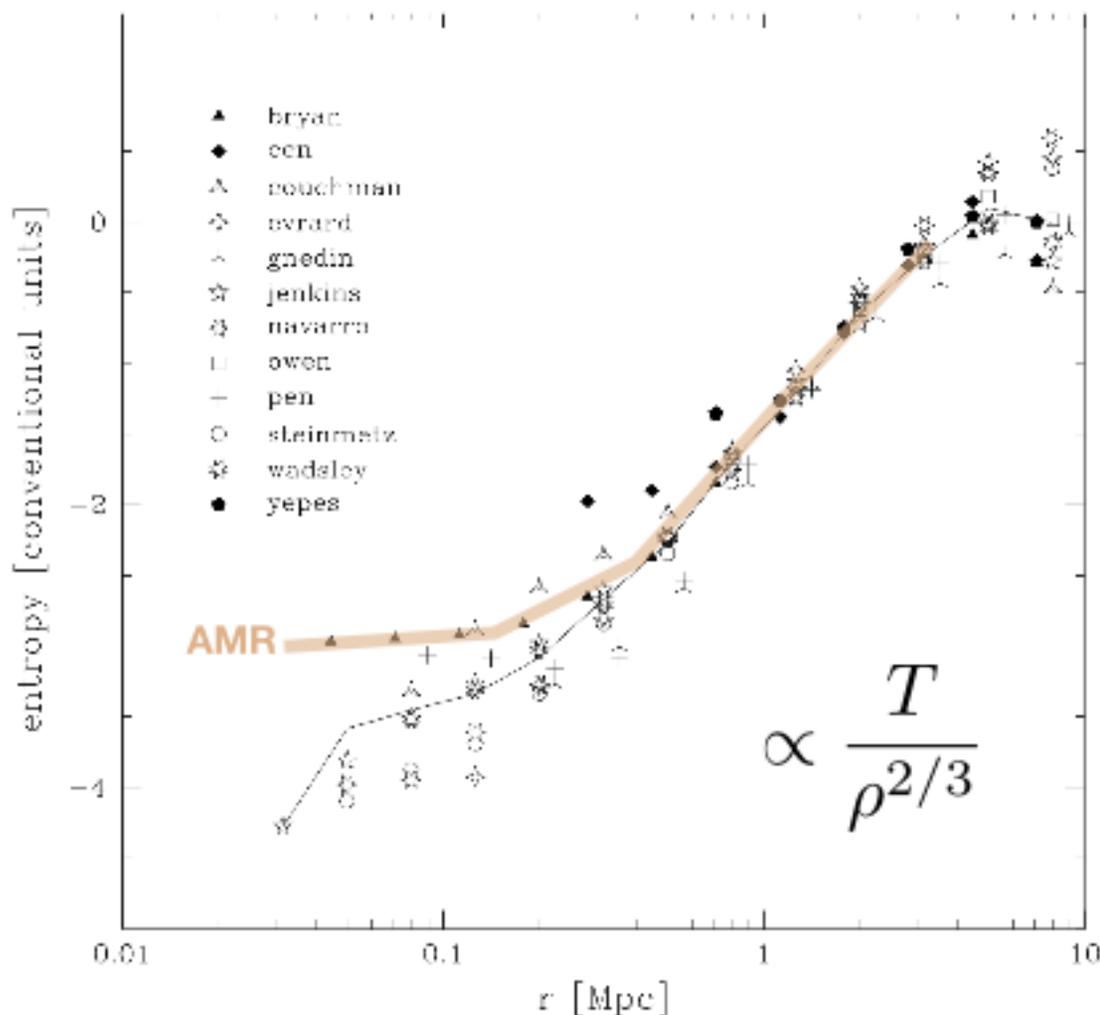
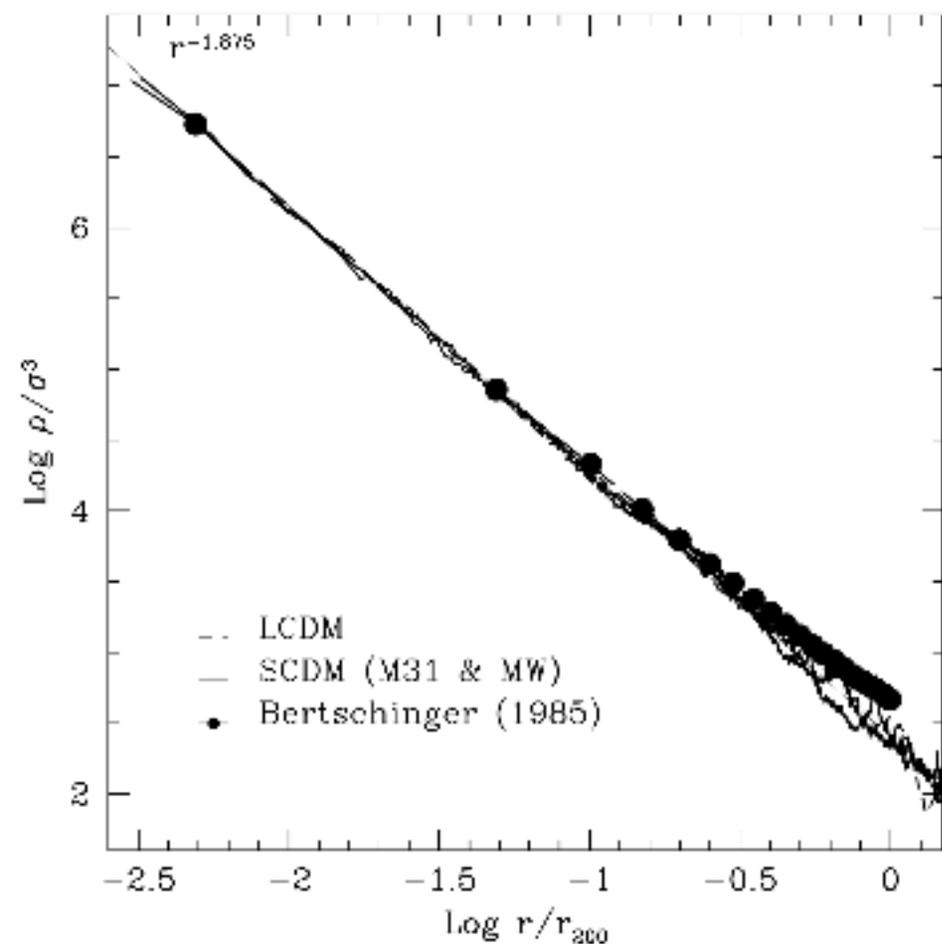
Numerical Hydrodynamics & Mixing

- In comparison of 12 cosmological hydrodynamics codes on simple non-radiative simulations of galaxy clusters, the only grid based result looked significantly divergent in the entropy profile
- Over many years detailed follow up studies concluded that the inability of traditional Smoothed Particle Hydrodynamics techniques was the culprit
- Discretized mass elements which cannot shear and mix with surrounding fluid stay trapped in cluster centers



Frenk 1999, Santa Barbara
Galaxy Cluster Comparison Project

Pseudo-Phase Space Density perfect power law in Dark Matter Simulations

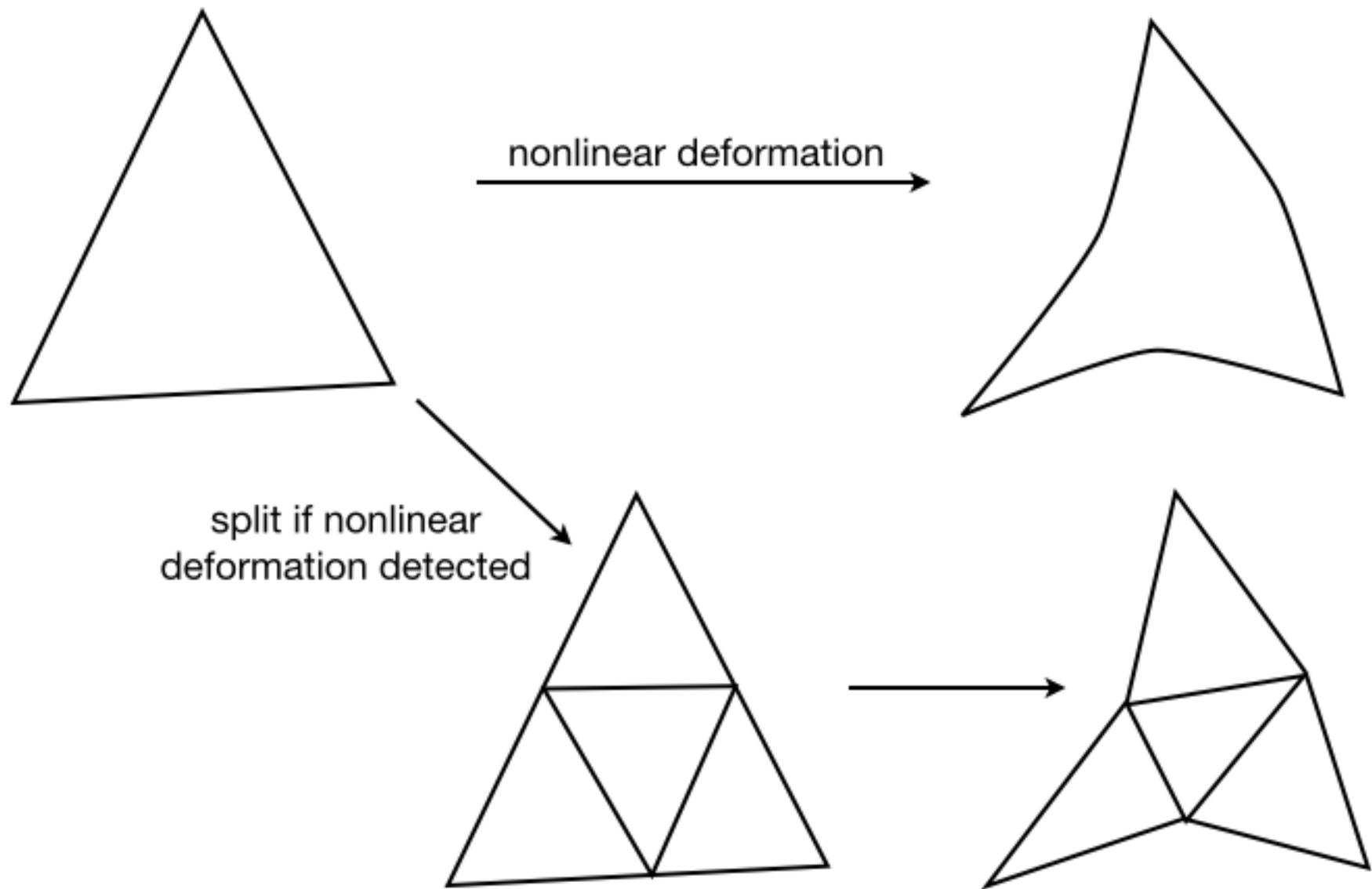


THE PHASE-SPACE DENSITY PROFILES OF COLD DARK MATTER HALOS

JAMES E. TAYLOR AND JULIO F. NAVARRO¹
 Department of Physics and Astronomy, University of Victoria, Victoria, BC V8P 1A1, Canada
 Received 2001 April 2; accepted 2001 August 27

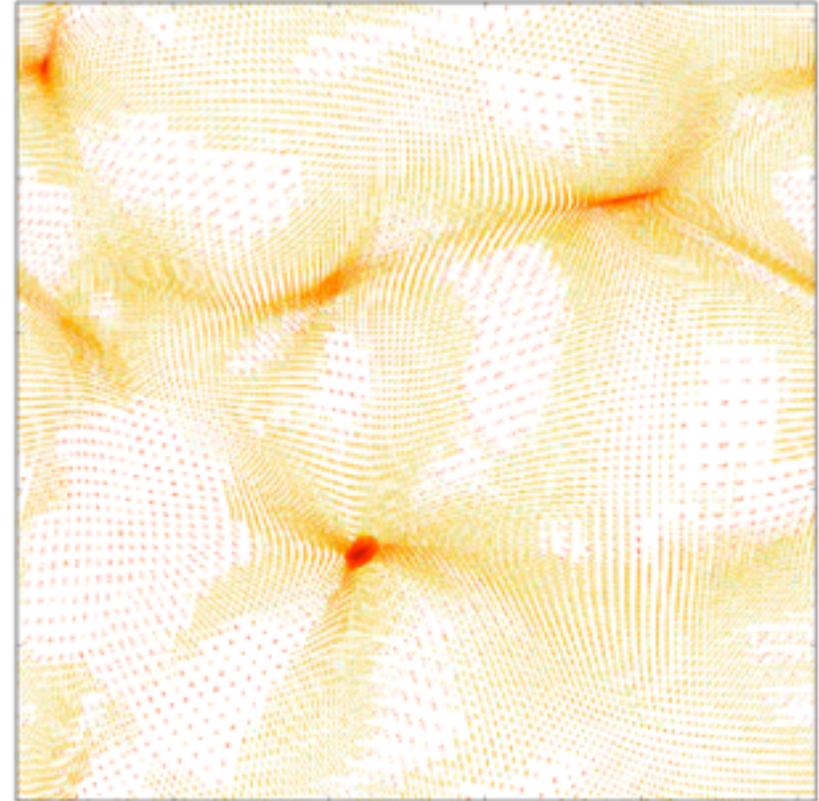
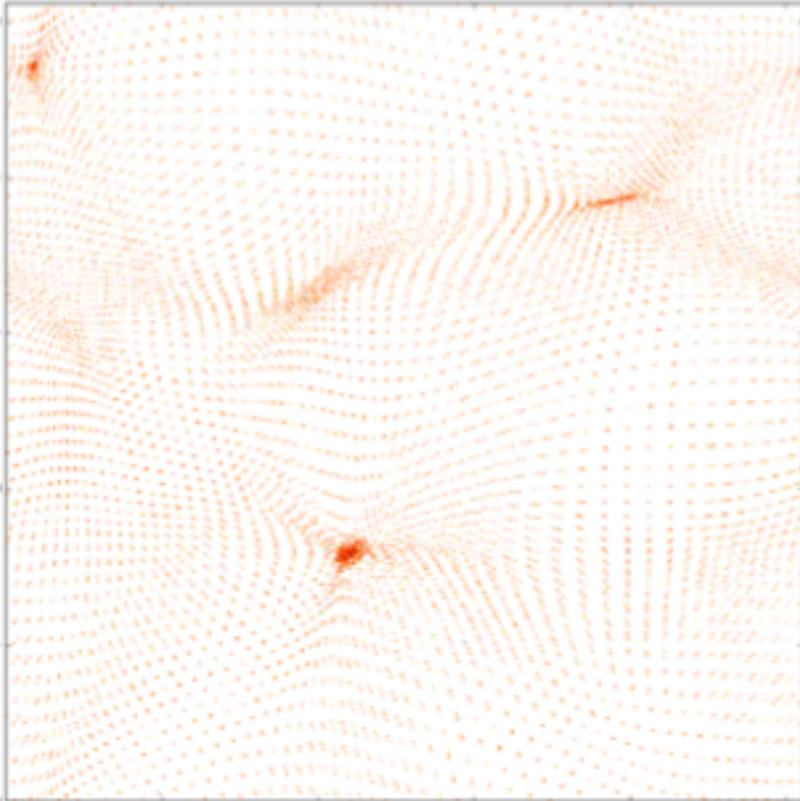
Frenk 1999, Santa Barbara
 Galaxy Cluster Comparison Project

Limits of the sheet: need for refinement



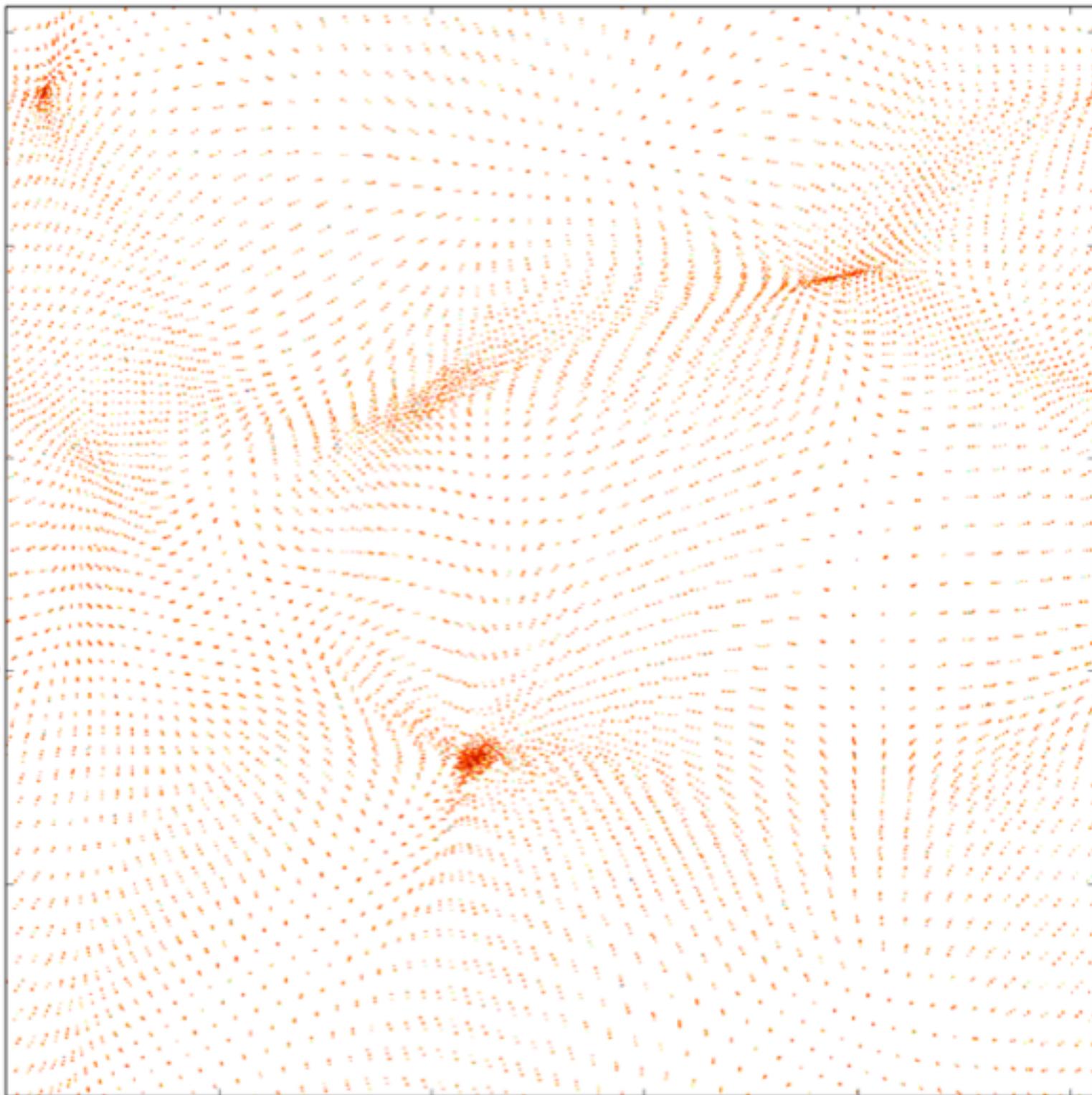
Adaptive Refinement II - work in progress

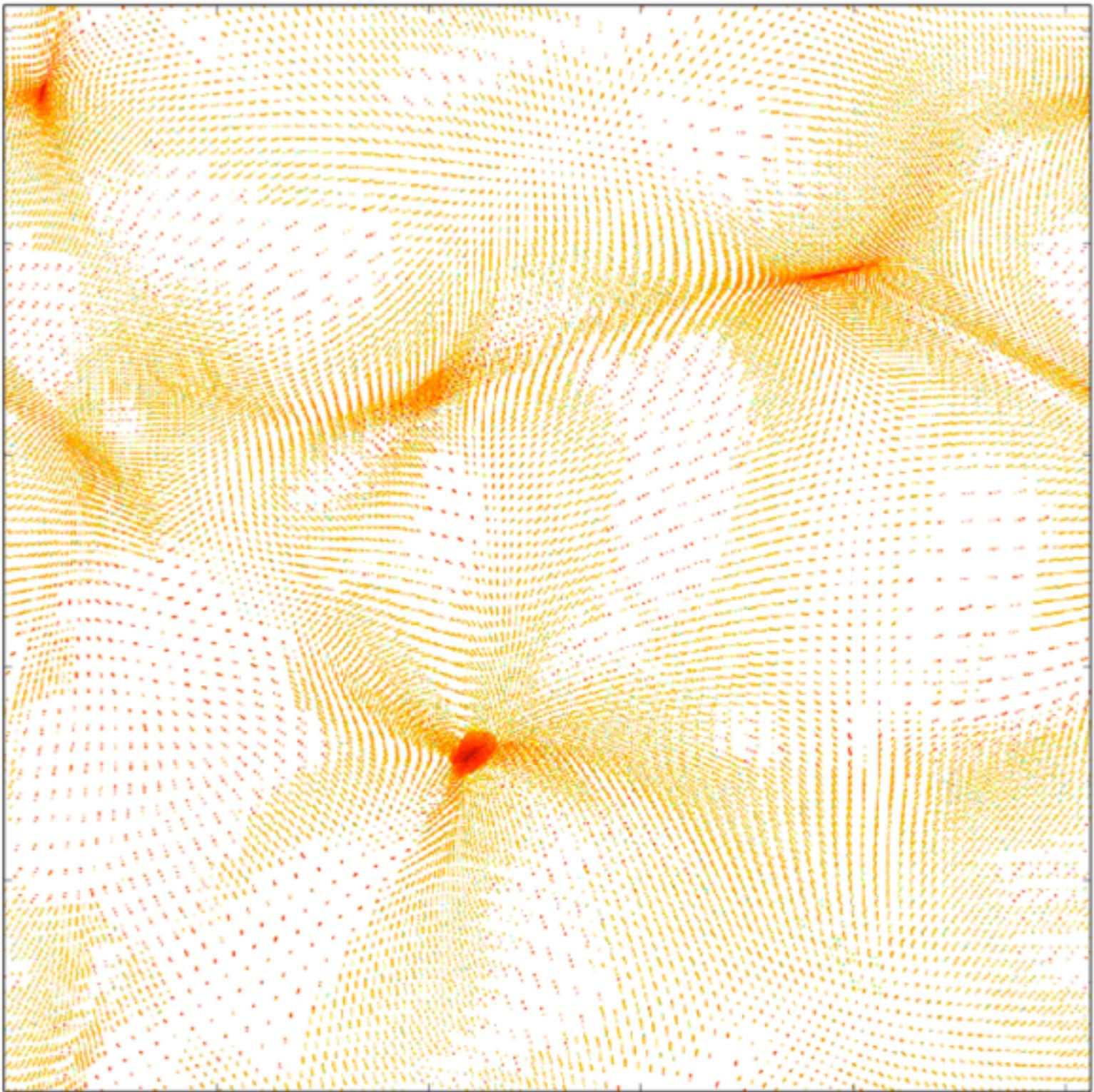
3 levels of
additional refinement



stay tuned...

Hahn, Angulo & Abel ongoing





Lagrangian Tessellation: What's it good for?

- Analyzing N-body sims, including web classification, velocity dispersion, profiles, resolution study (Abel, Hahn, Kaehler 2012)
- DM visualization (Kaehler, Hahn, Abel 2012)
- Better Numerical Methods (Hahn, Abel & Kaehler 2013, Hahn, Angulo & Abel 2014-)
- Finally reliable WDM mass functions below the cutoff scale (Angulo, Hahn, Abel 2013)
- Gravitational Lensing predictions (Angulo, Chen, Hilbert & Abel 2014)
- Cosmic Velocity fields (Hahn, Angulo, Abel 2014)
- Plasma simulations (Vlasov/Poisson and relativistic Vlasov/Maxwell) (Kates-Harbeck, Totorica, Zrake & Abel 2014, in prep.)
- Exact overlap integrals of Polyhedra (Powell & Abel 2014 submitted.)
- your application here ...