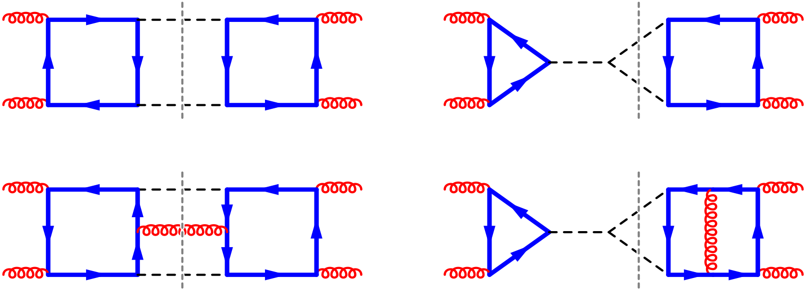
NLO and NNLO corrections to Higgs Boson Pair Production

Higgs Couplings 2014, October 1-3, Torino

Matthias Steinhauser | TTP Karlsruhe



Outline

- $gg \to HH$ at NLO with finite m_t
- $gg \to HH$ at NNLO for $m_t \to \infty$

H discovery and open questions

- July 4, 2012: discovery of a Higgs boson

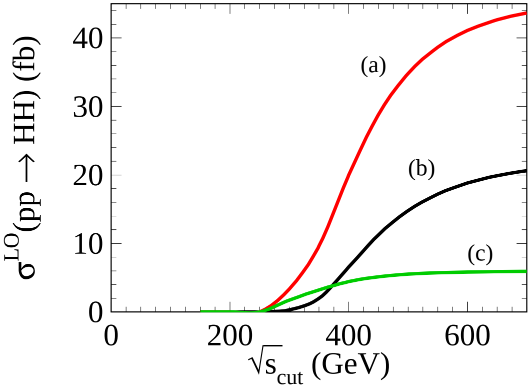
- SM Higgs boson?
- Couplings to fermions?
- Couplings to bosons?
- Self-coupling?
- $V_{\rm Higgs} = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{1}{4} \lambda H^4$ $\qquad \lambda^{\rm SM} = \frac{m_H^2}{2v^2} \approx 0.13$

$gg \to HH$

$qq' \to HHqq'$, $q\bar{q}' \to ZHH/WHH$, $q\bar{q}, gg \to t\bar{t}HH$

➪ more than 10 times smaller

$gg \to HH$ at LO



box

triangle

box + triangle

Promising channels/strategies

Note: single-H: “pb” $\longrightarrow$ double-H: “fb”

HH production possible with luminosity-upgraded LHC

- $gg \to HH \to b\bar{b}\gamma\gamma$: $\approx$ 50 signal events for $\int \mathcal{L} = 3000$ fb^{-1}

  [Baglio,Djouadi,Gröber,Mühlleitner,Quevillon,Spira’13]

  similar results: [Baur,Plehn,Rainwater’04]

  $gg \to HH \to b\bar{b}\tau\tau$: “promising”; $gg \to HH \to b\bar{b}WW$??
- $\frac{\sigma(gg\to HH)}{\sigma(gg\to H)}$ ➪ $\lambda > 0$ at 95% C.L. with 600 fb^{-1} [Goertz,Papaefstathiou,Yang,Zurita’13]
- $gg \to HH \to b\bar{b}\gamma\gamma$ ➪ λ_{HHH} with 40% accuracy for $\int \mathcal{L} = 3000$ fb^{-1}

  [Barger,Everett,Jackson,Shaughnessy’14]
- $gg \to HH \to b\bar{b}\tau\tau$ ➪ λ_{HHH} with 60% accuracy for $\int \mathcal{L} = 3000$ fb^{-1}

  [Barr,Dolan,Englert,Spannowsky’14]
- $gg \to HH + 2j$ [Dolan,Englert,Greiner,Spannowsky’14]
- $gg \to HHt\bar{t}$: important in case $\lambda > \lambda_{\rm SM}$ [Englert,Krauss,Spannowsky,Thompson’14]

[Dolan,Englert,Spannowsky’12; Dawson,Furlan,Lewis’13,. . .]

$gg \to HH$: known results

- LO [Glover, van der Bij'88; Plehn,Spira,Zerwas'96]
- NLO for $m_t \to \infty$ [Dawson,Dittmaier,Spira'98] $\approx +100\%$
- NLO incl. $1/m_t$ terms [Grigo,Hoff,Melnikov,Steinhauser'13] $\approx +10\%$
- NNLO $m_t \to \infty$ [de Florian,Mazzitelli'13; Grigo,Melnikov,Steinhauser'14] $\approx +20\%$
- NNLL resummation [Shao,Li,Li,Wang'13] $\approx$ NLO $+20\%$

- matching to parton showers [Li,Yan,Zhao'14; Maierhöfer,Papaefstathiou'14; Frederix,Hirschi,Maltoni,Mattelaer,Torrielli,Vryonidou,Zaro'14; Maltoni,Vryonidou,Zaro'14]

- VBF NNLO [Liu-Sheng,Ren-You,Wen-Gan,Lei,Wei-Hua,Xiao-Zhou'14; . . .]

$gg \to HH$: known results

Karlsruhe Institute of Technology

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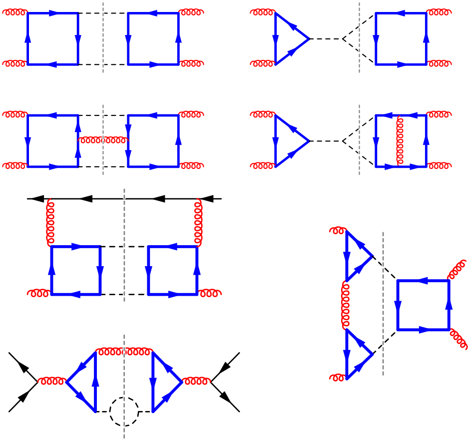
$gg \to HH$ at NLO with finite m_t

Technique

- calculation in full theory
- forward scattering amplitude + optical theorem

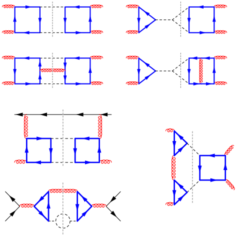


Technique

- calculation in full theory
- forward scattering amplitude + optical theorem
- Asymptotic expansion for $m_t^2 \gg m_H^2, s$

$\Rightarrow$

$$\sigma(gg \to HH) = \sum_n c(x) \left(\frac{m_H^2}{m_t^2}\right)^n, \quad x = \frac{4m_H^2}{s}$$



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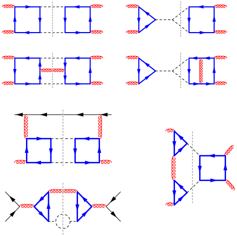

- many Feynman integrals $\longrightarrow$ few "master integrals"

Technique

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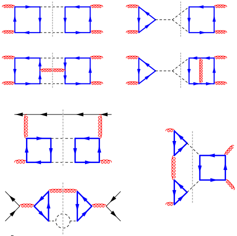
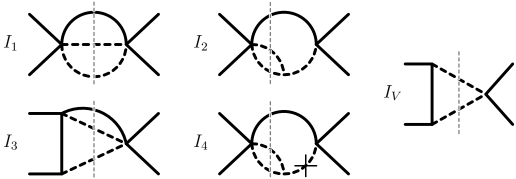
  ⇨

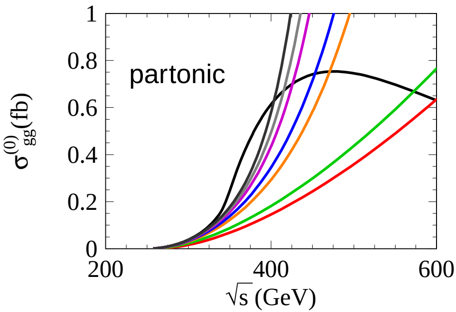
$$\sigma(gg \to HH) = \sum_n c(x) \left(\frac{m_H^2}{m_t^2} \right)^n, \quad x = \frac{4m_H^2}{s}$$



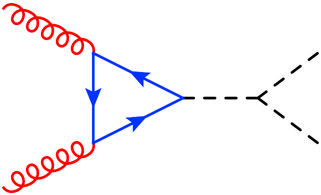
- many Feynman integrals $\longrightarrow$ few "master integrals"
- Master integrals (+ 2-loop tadpoles, ...)

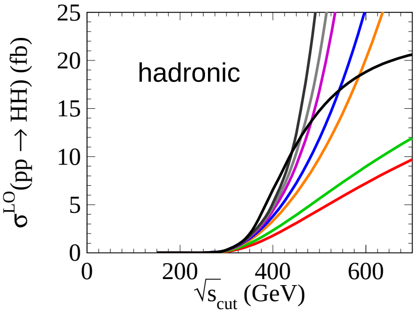


LO result

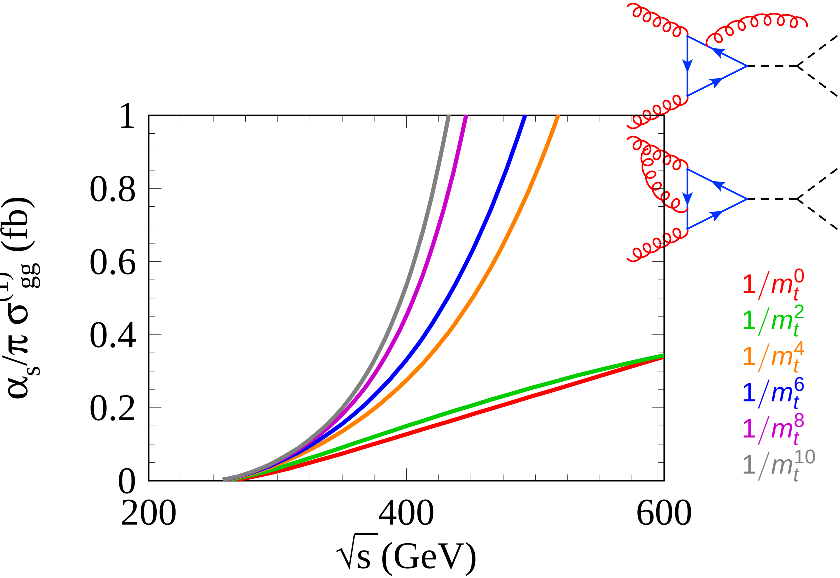


$1/m_t^0$
$1/m_t^2$
$1/m_t^4$
$1/m_t^6$
$1/m_t^8$
$1/m_t^{10}$
$1/m_t^{12}$
exact





NLO contribution: $gg \to HH$, partonic



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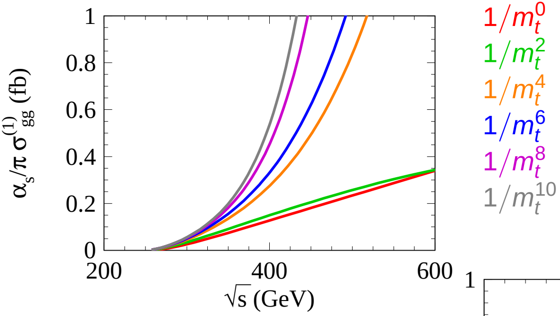
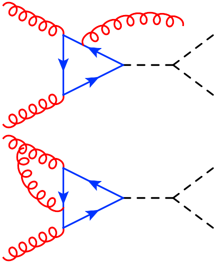
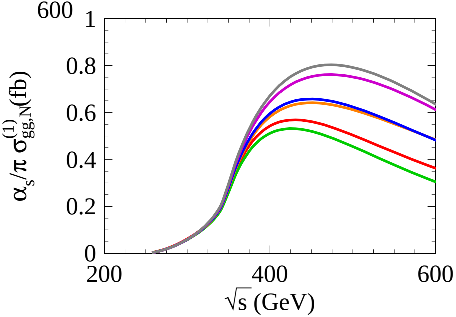
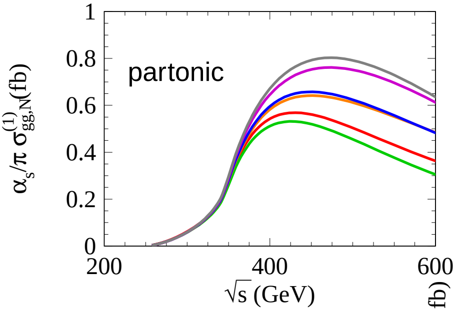


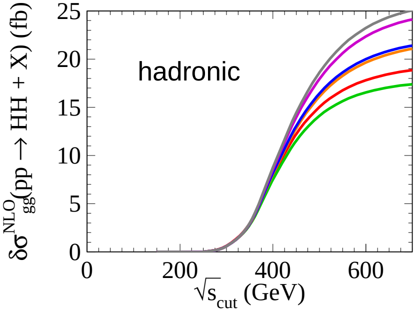


$$\sigma^{(1)} = \sigma^{(0)}_{\rm exact}\,\frac{\sigma^{(1)}_{\rm exp}}{\sigma^{(0)}_{\rm exp}}$$



NLO contribution: gluon channel

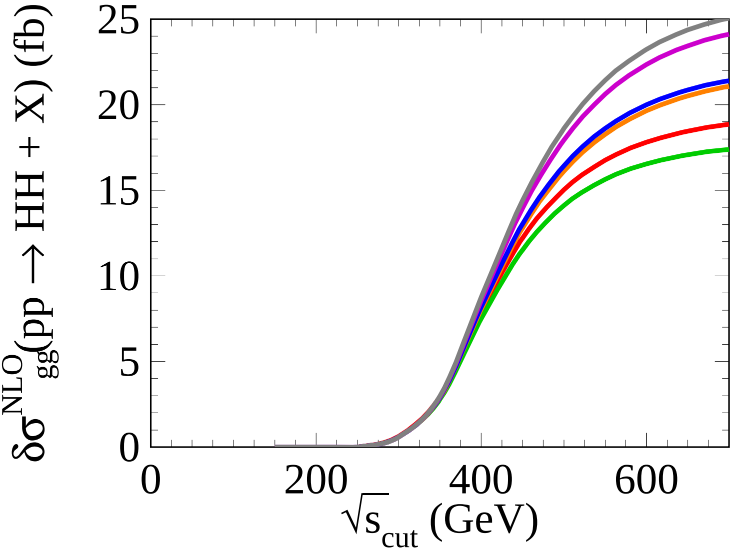


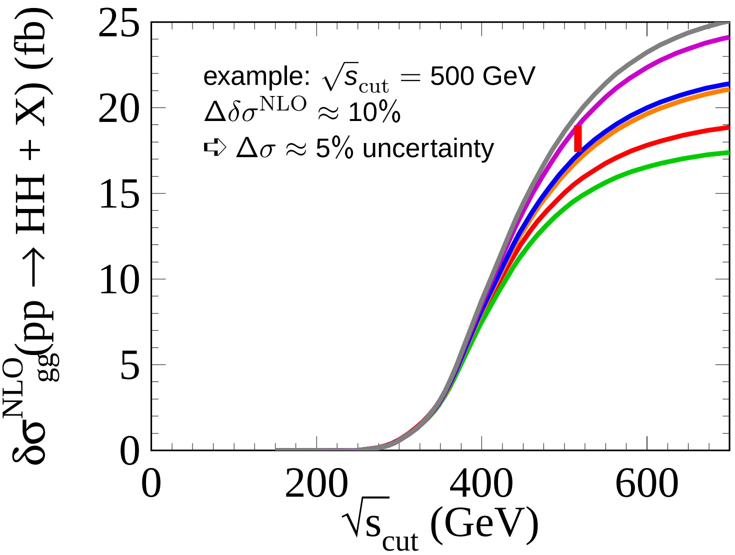


$\sqrt{s_{\rm cut}}$: upper limit on partonic center-of-mass energy

$\approx m(HH)$ invariant mass

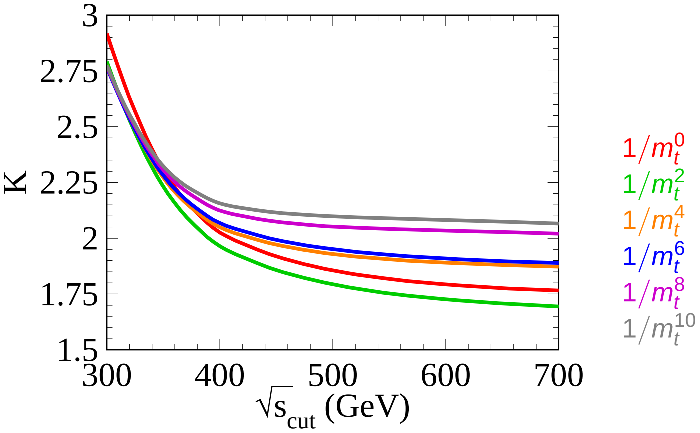
NLO contribution: gluon channel




example: $\sqrt{s}_{\rm cut} = 500$ GeV
$\Delta\delta\sigma^{\rm NLO} \approx 10\%$
➪ $\Delta\sigma \approx 5\%$ uncertainty
$\delta\sigma^{\rm NLO}_{gg}(pp \to HH + X)$ (fb)
0
5
10
15
20
25
0
200
400
600
$\sqrt{s}_{\rm cut}$ (GeV)

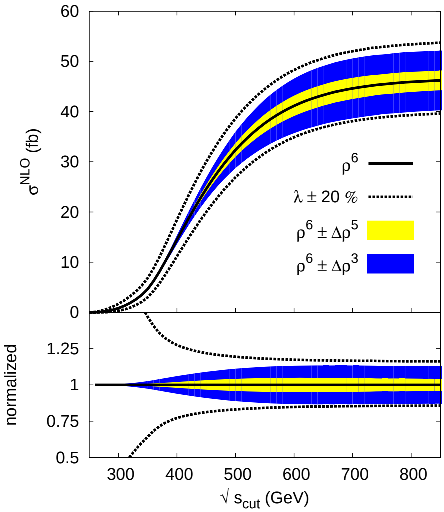
K factor

$K = \frac{\sigma^{\mathrm{NLO}}}{\sigma^{\mathrm{LO}}}$



➪ enhancement close to threshold due to suppression of LO result

Implication on extraction of λ_{HHH}

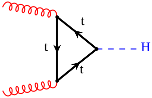


$$\rho = M_H^2 / m_t^2$$

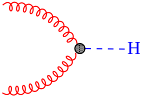
➪ sensitivity to "$\lambda \pm 20\%$"

$gg \to HH$ at NNLO for $m_t \to \infty$

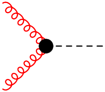
Effective Higgs-gluon coupling

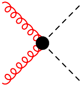


$$\xrightarrow{m_t \gg M_H} \quad C(M_t, \alpha_s) \times$$



$$\mathcal{L}_{\rm eff} = -\frac{H}{4v} C_H G_{\mu\nu}^2 + \frac{1}{2}\left(\frac{H}{4v}\right)^2 C_{HH} G_{\mu\nu}^2$$





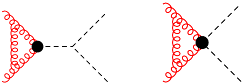
$$\sim G_{\mu\nu}^2$$

➪ same structure

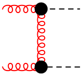
➪ take over results from single-H production

Virtual corrections

NLO



$\longrightarrow$ from single-H [Dawson'91; Spira,Djouadi,Graudenz,Zerwas'91]



$\longrightarrow$ new but only tree-level [Dawson,Dittmaier,Spira'98; de Florian,Mazzitelli'13; Grigo,Melnikov,Steinhauser'14]

Virtual corrections

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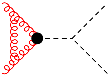
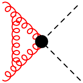

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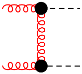

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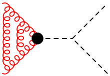
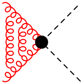
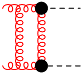



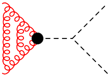
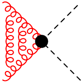
$\longrightarrow$ from single-H [Harlander'00,. . .]

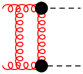


$\longrightarrow$ new but only 1-loop [de Florian,Mazzitelli'13; Grigo,Melnikov,Steinhauser'14]

Virtual corrections

NNLO

  $\longrightarrow$ from single-H [Harlander'00,...]

 $\longrightarrow$ new but only 1-loop [de Florian,Mazzitelli'13; Grigo,Melnikov,Steinhauser'14]

2 independent calculations:

[de Florian,Mazzitelli'13] effective theory; C_{HH} not computed $[\mathcal{L}_{\rm eff} \sim H^2\, C_{HH}\, G^2_{\mu\nu}]$

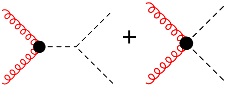
[Grigo,Melnikov,Steinhauser'14] full theory in the limit $m_t \to \infty$; result for C_{HH}

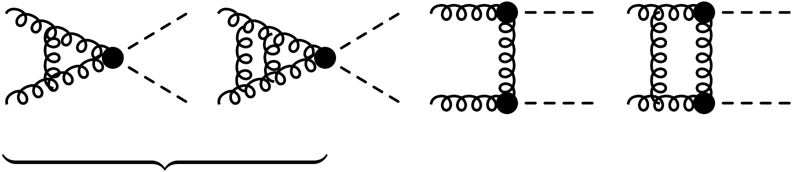
Virtual corrections: results

$$\frac{\mathrm{d}\sigma^{(2)}_{v,\mathrm{fin}}}{\mathrm{d}t} = \Sigma_{\mathrm{LO}}\left[C_{\mathrm{LO}}^2\mathcal{F}^{(2)} + C_{\mathrm{LO}}\mathcal{R}^{(2)} + \mathcal{V}^{(2)}\right] + \mathcal{O}\left(\epsilon\right)$$

$$\Sigma_{\mathrm{LO}} = \frac{\alpha_s^2[(tu-m_H^4)/s]^{-\epsilon}}{2^{11}3^2v^4\pi^3(1-\epsilon)^2\Gamma(1-\epsilon)(4\pi)^{-\epsilon}}$$

$$C_{\mathrm{LO}} = \frac{6\lambda v^2}{s-m_H^2} - 1 \sim$$





$\sim C_{\mathrm{LO}}$

Virtual corrections: results

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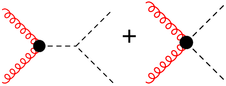
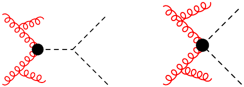


$$\mathcal{R}^{(2)} = -7C_A^2 + 11C_AC_F - 8n_lC_FT_F + \frac{1}{3}C_A\left[\frac{476}{9} + \frac{11}{3}(4L_s + L_t + L_u) + \frac{4m_H^2}{s}\right]$$
$$-8C_F - \frac{4}{9}T_Fn_l\left[\frac{10}{3} + 4L_s + L_t + L_u\right] - \frac{C_A}{3}\left(1 + \frac{2m_H^4}{s^2}\right)\left[2\mathrm{Li}_2(1 - \frac{m_H^4}{tu})\right.$$
$$+4\mathrm{Li}_2(\frac{m_H^2}{t}) + 4\mathrm{Li}_2(\frac{m_H^2}{u}) + 4\ln\left(1 - \frac{m_H^2}{t}\right)\ln\left(-\frac{m_H^2}{t}\right)$$
$$\left. +4\ln\left(1 - \frac{m_H^2}{u}\right)\ln\left(-\frac{m_H^2}{u}\right) - 8\zeta_2 - \ln^2\left(\frac{t}{u}\right)\right]$$

Real corrections

NNLO real corrections

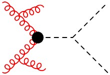
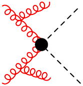


$\longrightarrow$ from single-H

[Harlander,Kilgore'02; Anastasiou,Melnikov'02; Ravindran,Smith,v.Neerven'03]

Real corrections

NNLO real corrections

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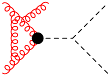
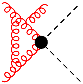
[Harlander,Kilgore'02; Anastasiou,Melnikov'02; Ravindran,Smith,v.Neerven'03]

NNLO real-virtual

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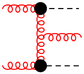
[Harlander,Kilgore'02; Anastasiou,Melnikov'02; Ravindran,Smith,v.Neerven'03]



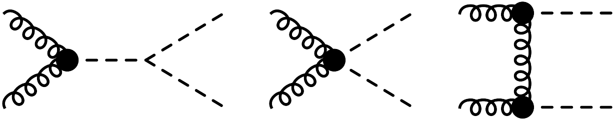
$\longrightarrow$ new but only tree level [de Florian,Mazzitelli'13]

C_{HH}

$$\mathcal{L}_{\text{eff}} = -\frac{H}{4v} C_H G_{\mu\nu}^2 + \frac{1}{2}\left(\frac{H}{4v}\right)^2 C_{HH} G_{\mu\nu}^2$$

NEW 2014: C_{HH} to NNLO

- compute amplitude $\mathcal{A}_{gg\to HH}$ in full and effective theory
  choose convenient kinematical configuration
- $q_1 \sim q_2 \sim q_3 \sim q_4 \sim m_H \ll m_t$
  ➪ only tree-level diagrams in effective theory



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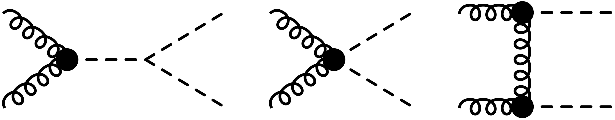
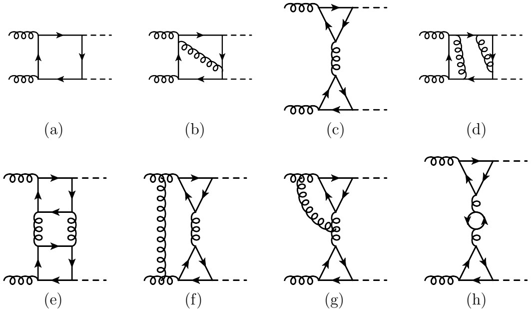
$$C_H Z_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{\text{tree,1PR},\lambda\neq 0} + C_{HH} Z_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{\text{tree,1PI}} + C_H^2 Z_{\mathcal{O}_1}^2 \mathcal{A}^{\text{eff}}_{\text{tree,1PR},\lambda=0}$$
$$= \frac{1}{\zeta_3^0}\left(\mathcal{A}^{\text{hard}}_{\text{1PI}} + \mathcal{A}^{\text{hard}}_{\text{1PR},\lambda=0} + \mathcal{A}^{\text{hard}}_{\text{1PR},\lambda\neq 0}\right)$$

C_{HH}

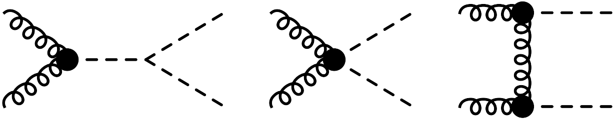


$$C_H Z_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{\text{tree},1\text{PR},\lambda\neq 0} + C_{HH} Z_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{\text{tree},1\text{PI}} + C_H^2 Z_{\mathcal{O}_1}^2 \mathcal{A}^{\text{eff}}_{\text{tree},1\text{PR},\lambda=0}$$
$$= \frac{1}{\zeta_3^0} \left(\mathcal{A}^{\text{hard}}_{1\text{PI}} + \mathcal{A}^{\text{hard}}_{1\text{PR},\lambda=0} + \mathcal{A}^{\text{hard}}_{1\text{PR},\lambda\neq 0} \right)$$



(a) (b) (c) (d)
(e) (f) (g) (h)

C_{HH}



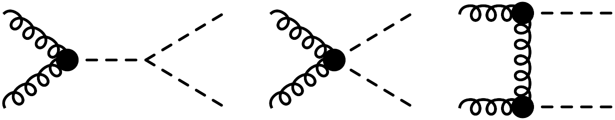
$$C_H Z_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{\text{tree,1PR},\lambda\neq 0} + C_{HH} Z_{\mathcal{O}_1} \mathcal{A}^{\text{eff}}_{\text{tree,1PI}} + C_H^2 Z_{\mathcal{O}_1}^2 \mathcal{A}^{\text{eff}}_{\text{tree,1PR},\lambda=0}$$
$$= \frac{1}{\zeta_3^0} \left(\mathcal{A}^{\text{hard}}_{\text{1PI}} + \mathcal{A}^{\text{hard}}_{\text{1PR},\lambda=0} + \mathcal{A}^{\text{hard}}_{\text{1PR},\lambda\neq 0} \right)$$

- $\sigma^{m_t \to \infty}$ complete [Grigo,Melnikov,Steinhauser'14]

$$C_{HH}^{(1)} = C_H^{(1)}$$
$$C_{HH}^{(2)} = C_H^{(2)} + \frac{35}{24} + \frac{2n_l}{3}$$
$$C_{HH}^{(2)}/C_H^{(2)} \approx 1.8 \qquad \mu = m_t, n_l = 5$$

⇨ σ_{tot} changes by 1% as compared to "$C_{HH}^{(2)} = C_H^{(2)}$"

C_{HH}



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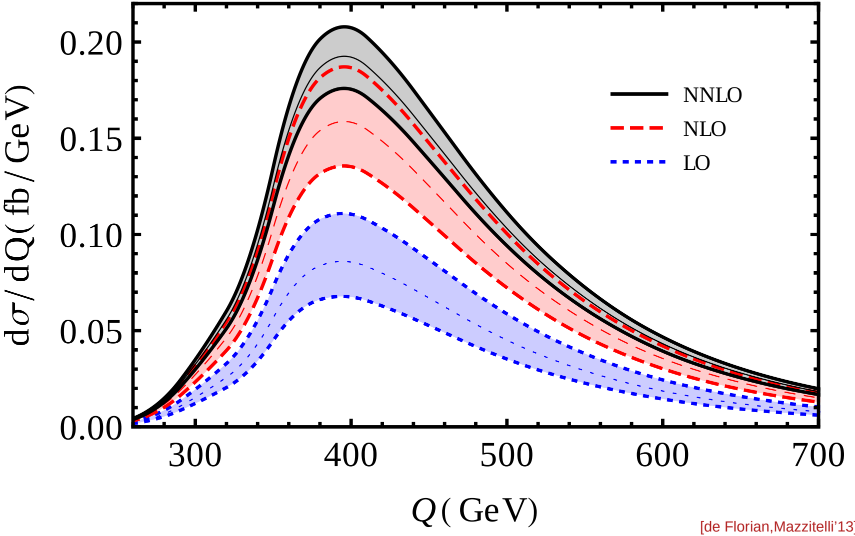
$$\frac{\mathrm{d}\sigma_{C_H\neq C_{HH}} - \mathrm{d}\sigma_{C_H=C_{HH}}}{\mathrm{d}\sigma_{C_H=C_{HH}}} = 0.0117\left(\frac{\alpha_s}{0.11}\right)^2 \frac{(s-m_H^2)}{(s-4m_H^2)}$$

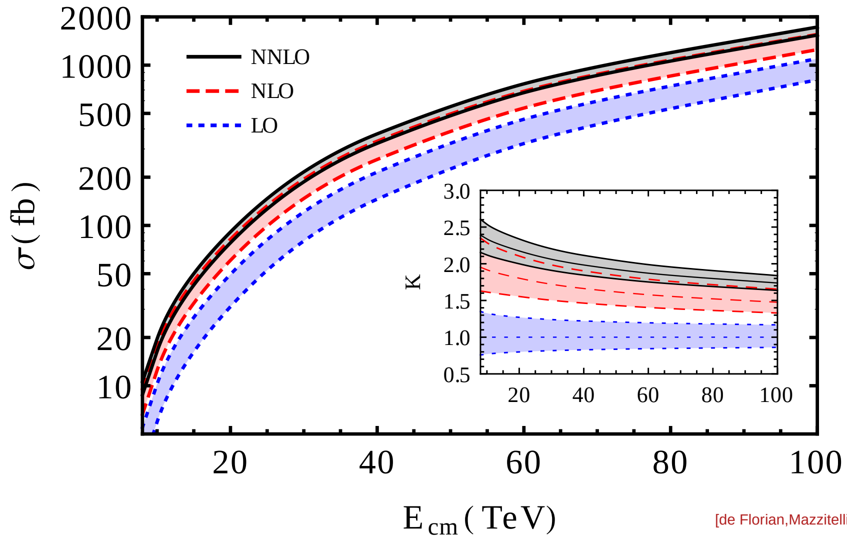
➪ 6.4% for $\sqrt{s} = 270\ \mathrm{GeV}$

$d\sigma/dQ$ Q: HH invariant mass



[de Florian,Mazzitelli'13]

σ



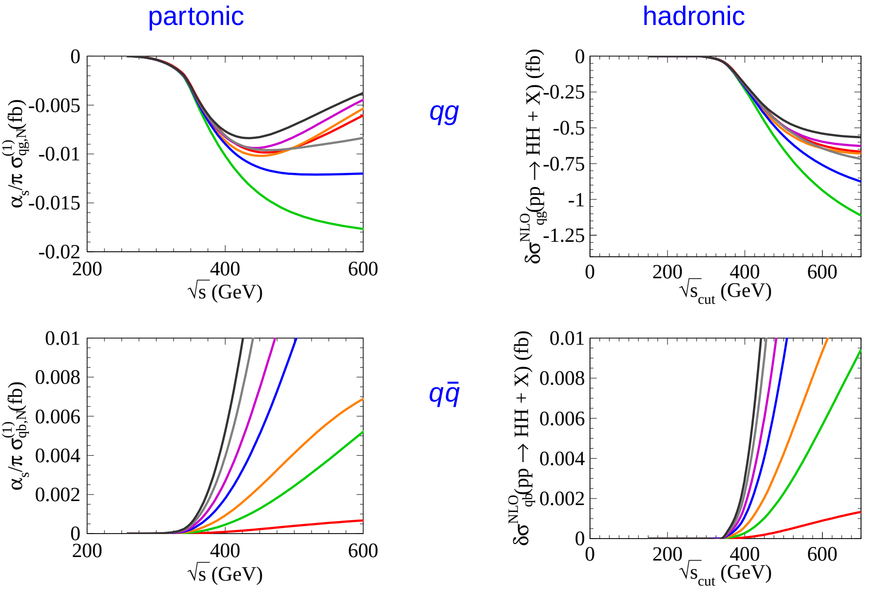
[de Florian,Mazzitelli'13]

Conclusions

- $\sigma(pp \to HH)$ SM cross section; dominated by gluon fusion
- determination of λ_{HHH} (?)
- NLO $1/m_t$ corrections ➪ σ increases by $\approx$ 10%
- $K \nearrow$ for $s \to$ threshold ($1/m_t$ terms important!)
  ($\longleftrightarrow$ extraction of λ_{HHH})
- reliable error estimate at NLO
- NNLO for $m_t \to \infty$
- matching coefficient C_{HH} to 3 loops

BACKUP

NLO result: quark channels



NLO contribution: gluon channel