Spin-parity measurements in CMS/ATLAS
—constraints on Higgs tensor structures

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Higgs Couplings, 03 October 2014

Outline

- Phenomenology of spin-0 HVV interactions
- Hypothesis test vs coupling measurements
- Tool and simulation
- Study of spin-zero HVV couplings
- Conclusions

Phenomenology of anomalous couplings: spin 0

- Interaction between a spin 0 Higgs and two gauge bosons V_1, V_2 (Z, W, γ, g), expansion up to q^2
  - q^4 and higher orders not considered assuming small anomalous couplings

$$A(\mathrm{HV_1V_2}) \sim \left[a_1^{\mathrm{V_1V_2}} + \frac{\kappa_1^{\mathrm{V_1V_2}} q_{\mathrm{V1}}^2 + \kappa_2^{\mathrm{V_1V_2}} q_{\mathrm{V2}}^2}{\left(\Lambda_1^{\mathrm{V_1V_2}}\right)^2} \right] m_{\mathrm{V}}^2 \epsilon_{\mathrm{V1}}^* \epsilon_{\mathrm{V2}}^* + a_2^{\mathrm{V_1V_2}} f_{\mu\nu}^{*(\mathrm{V_1})} f^{*(\mathrm{V_2}),\mu\nu} + a_3^{\mathrm{V_1V_2}} f_{\mu\nu}^{*(\mathrm{V_1})} \tilde{f}^{*(\mathrm{V_2}),\mu\nu}$$

Λ_1 term
leading momentum expansion

a_2 term
CP even state

a_3 term
CP odd state

- SM value for couplings

	a_1	q^2/Λ_1^2	a_2	a_3
HZZ(WW)	2	$10^{-3} - 10^{-2}$	$10^{-3} - 10^{-2}$	$< 10^{-10}$
HZγ	-	$10^{-3} - 10^{-2}$	~ 0.0035	$< 10^{-10}$
H$\gamma\gamma$	-	-	$\sim$ -0.004	$< 10^{-10}$

- Mesuarements

1) hypothesis test, pure 0^- (a_3), 0_h^+ (a_2) states
  - CMS: 99.9% (0^-), 95% CL (0_h^+)
  - ATLAS: 97.8% CL (0^-)

Phys.Lett. B726 (2013) 120–144

2) measure 11 anomalous couplings (10 in CMS-PAS-HIG-14-014 + 1 new)

Phys.Rev.Lett. 110 (2013) 081803
Phys.Rev. D89 (2014) 092007

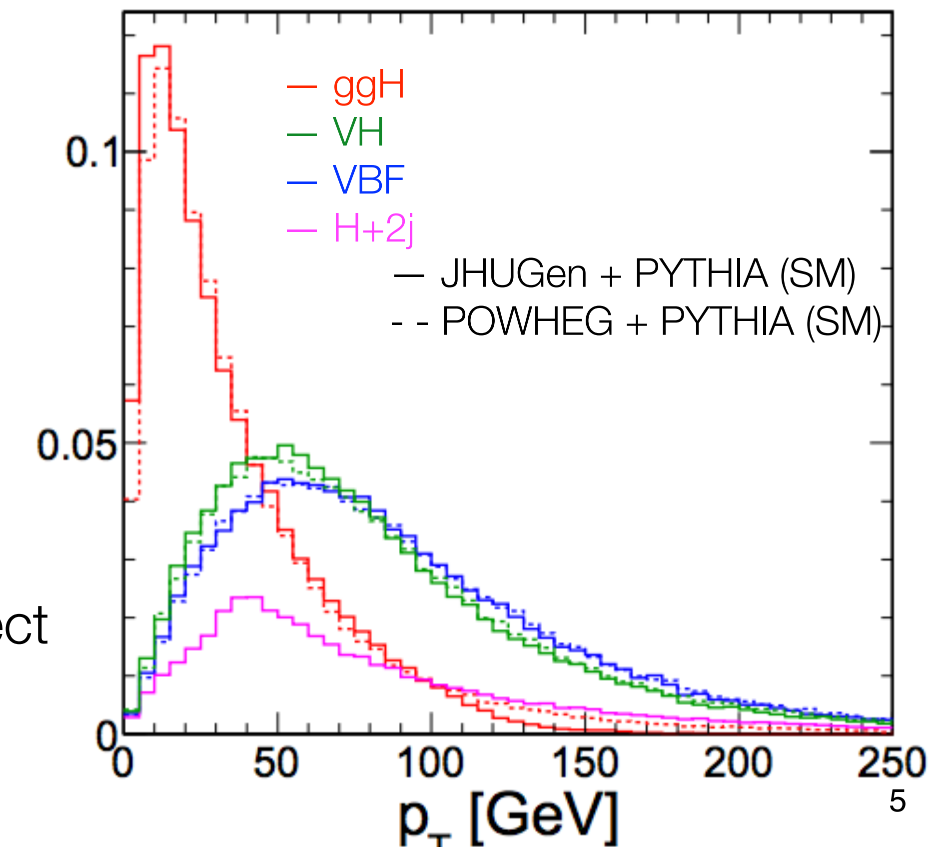
Hypothesis test vs coupling measurements (HZZ)

	Hypothesis test	**Coupling measurement**
Measured parameters	Probability of pure exotic states	Coupling ratio of exotic/SM
Discriminants	ATLAS (1D): SM vs pure J^P (BDT or MELA) CMS (2D): 1. SM vs pure J^P(MELA) 2. signal vs bkg (MELA)	CMS method 1 (3D): 3. SM-BSM interference (MELA) CMS method 2 (8D): 5 angular+3 mass distribution (as cross check method)
Techniques	Templates of discriminants	CMS method 1: templates of discriminants CMS method 2: analytical or 8D templates (gen-level) + transfer function (simulation derived), as a cross-check method
Signal likelihood	Construction: pure states Usage: test statistic	Construction: pure states plus interference Usage: maximum likelihood fit

Tool and Simulation

- CMS and ATLAS simulation
    - ggH spin 0 samples: POWHEG (NLO production)+ JHUGen (H decay) + PYTHIA (parton shower)
    - ggH spin1, 2 & VBF, VH spin 0, 1, 2 samples: JHUGen (LO production + H decay)+ PYTHIA (parton shower)
- JHUGen Monte Carlo Simulation
    - open access: http://www.pha.jhu.edu/spin/
    - complete chain
        - ab-> X -> Z(γ^*)Z(γ^*), WW->($f_1 f'_1$)($f_2 f'_2$), ab-> X -> bb, $\tau\tau$, $\gamma\gamma$
        - production ggH,VBF, VH, Higgs J = 0, 1 , 2
    - interference of couplings in a model independent approach
    - PYTHIA parton shower good approximation for NLO QCD effect

arXiv:1309.4819



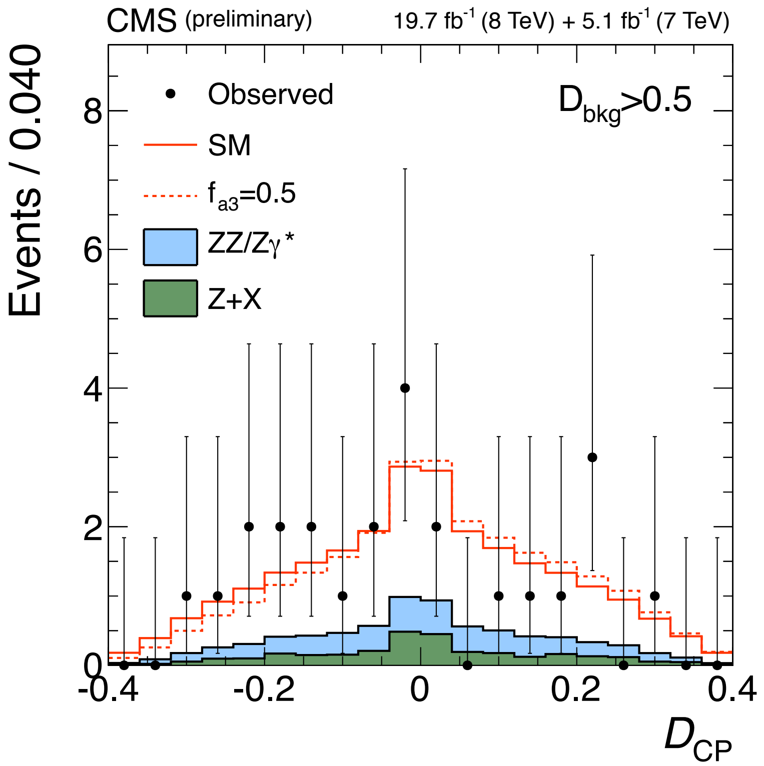
Tool and simulation

- Matrix element used to
  - Construct discriminants, example (interference discriminant):

Mixed state $|ME|^2$ — Pure state 1 $|ME|^2$ — Pure state 2 $|ME|^2$

$$\mathcal{D}_{\rm int} = \frac{\left(\mathcal{P}^{\rm kin}_{\rm SM+J^P}(m_1,m_2,\vec{\Omega}|m_{4\ell}) - \mathcal{P}^{\rm kin}_{J^P}(m_1,m_2,\vec{\Omega}|m_{4\ell}) - \mathcal{P}^{\rm kin}_{\rm SM}(m_1,m_2,\vec{\Omega}|m_{4\ell})\right)}{\mathcal{P}^{\rm kin}_{\rm SM}+\mathcal{P}^{\rm kin}_{J^P}}$$

  - Reweight samples to specific couplings, benefit of increasing statistics
- Angular distribution -> amplitudes (Matrix Element)
  - JHUGenMELA: from JHUGen, for signal, used for CMS results
    - MCFM: for backgrounds, used for CMS results
  - AnalyticalMELA: developed in CMS, cross-check of many above
  - MEKD: MadGraph based, cross-check of many above



Measurements of HVV anomalous couplings

Interaction	Anomalous coupling
HZZ	Λ_1 a_2 a_3
HWW	Λ_1^{WW} a_2^{WW} a_3^{WW}
HZγ	$\Lambda_1^{\mathrm{Z}\gamma}$ $a_2^{\mathrm{Z}\gamma}$ $a_3^{\mathrm{Z}\gamma}$
H$\gamma\gamma$	$a_2^{\gamma\gamma}$ $a_3^{\gamma\gamma}$

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H$\gamma\gamma$	$a_2^{\gamma\gamma}$ $a_3^{\gamma\gamma}$

What to measure

The coupling ratio a_i/a_1-> effective fractional xsec

$$f_{a2} = \frac{|a_2|^2\sigma_2}{|a_1|^2\sigma_1 + |a_2|^2\sigma_2 + |a_3|^2\sigma_3 + \tilde{\sigma}_{\Lambda_1} / \left(\Lambda_1\right)^4}$$

— unique and finite ($0 < f_{ai} < 1$)

$$\frac{|a_i|}{|a_1|} = \sqrt{\frac{f_{ai}}{f_{a1}}} \times \sqrt{\frac{\sigma_1}{\sigma_i}}$$

Measurements of HVV anomalous couplings

Interaction	Anomalous coupling	Coupling phase	Effective fraction	Translation constant
HZZ	Λ_1	$\phi_{\Lambda 1}$	$f_{\Lambda 1}$	$\sigma_1/\tilde{\sigma}_{\Lambda 1} = 1.45 \times 10^{-8}\ \mathrm{GeV}^{-4}$
	a_2	ϕ_{a2}	f_{a2}	$\sigma_1/\sigma_2 = 2.68$
	a_3	ϕ_{a3}	f_{a3}	$\sigma_1/\sigma_3 = 6.36$
HWW	Λ_1^{WW}	$\phi_{\Lambda 1}^{\mathrm{WW}}$	$f_{\Lambda 1}^{\mathrm{WW}}$	$\sigma_1^{\mathrm{WW}}/\tilde{\sigma}_{\Lambda 1}^{\mathrm{WW}} = 1.87 \times 10^{-8}\ \mathrm{GeV}^{-4}$
	a_2^{WW}	ϕ_{a2}^{WW}	f_{a2}^{WW}	$\sigma_1^{\mathrm{WW}}/\sigma_2^{\mathrm{WW}} = 1.25$
	a_3^{WW}	ϕ_{a3}^{WW}	f_{a3}^{WW}	$\sigma_1^{\mathrm{WW}}/\sigma_3^{\mathrm{WW}} = 3.01$
HZγ	$\Lambda_1^{\mathrm{Z}\gamma}$	$\phi_{\Lambda 1}^{\mathrm{Z}\gamma}$	$f_{\Lambda 1}^{\mathrm{Z}\gamma}$	$\sigma_1/\tilde{\sigma}_{\Lambda 1}^{\mathrm{Z}\gamma} = 5.76 \times 10^{-9}\ \mathrm{GeV}^{-4}$
	$a_2^{\mathrm{Z}\gamma}$	$\phi_{a2}^{\mathrm{Z}\gamma}$	$f_{a2}^{\mathrm{Z}\gamma}$	$\sigma_1/\sigma_2^{Z\gamma} = 22.4 \times 10^{-4}$
	$a_3^{\mathrm{Z}\gamma}$	$\phi_{a3}^{\mathrm{Z}\gamma}$	$f_{a3}^{\mathrm{Z}\gamma}$	$\sigma_1/\sigma_3^{Z\gamma} = 27.2 \times 10^{-4}$
H$\gamma\gamma$	$a_2^{\gamma\gamma}$	$\phi_{a2}^{\gamma\gamma}$	$f_{a2}^{\gamma\gamma}$	$\sigma_1/\sigma_2^{\gamma\gamma} = 28.2 \times 10^{-4}$
	$a_3^{\gamma\gamma}$	$\phi_{a3}^{\gamma\gamma}$	$f_{a3}^{\gamma\gamma}$	$\sigma_1/\sigma_3^{\gamma\gamma} = 28.8 \times 10^{-4}$

What to measure

The coupling ratio a_i/a_1-> effective fractional xsec

$$f_{a2} = \frac{|a_2|^2\sigma_2}{|a_1|^2\sigma_1 + |a_2|^2\sigma_2 + |a_3|^2\sigma_3 + \tilde{\sigma}_{\Lambda_1}/\left(\Lambda_1\right)^4}$$

— unique and finite ($0 < f_{ai} < 1$)

$$\frac{|a_i|}{|a_1|} = \sqrt{\frac{f_{ai}}{f_{a1}}} \times \sqrt{\frac{\sigma_1}{\sigma_i}}$$

Measurements of HVV anomalous couplings

Interaction	Anomalous coupling	Coupling phase	Effective fraction		Translation constant
HZZ	Λ_1	$\phi_{\Lambda 1}$	$f_{\Lambda 1}$	A,B,C,D	$\sigma_1/\tilde{\sigma}_{\Lambda 1} = 1.45 \times 10^{-8}\ \mathrm{GeV}^{-4}$
	a_2	ϕ_{a2}	f_{a2}	A,B,C,D	$\sigma_1/\sigma_2 = 2.68$
	a_3	ϕ_{a3}	f_{a3}	A,B,C,D	$\sigma_1/\sigma_3 = 6.36$
HWW	Λ_1^{WW}	$\phi_{\Lambda 1}^{\mathrm{WW}}$	$f_{\Lambda 1}^{\mathrm{WW}}$	A	$\sigma_1^{\mathrm{WW}}/\tilde{\sigma}_{\Lambda 1}^{\mathrm{WW}} = 1.87 \times 10^{-8}\ \mathrm{GeV}^{-4}$
	a_2^{WW}	ϕ_{a2}^{WW}	f_{a2}^{WW}	A	$\sigma_1^{\mathrm{WW}}/\sigma_2^{\mathrm{WW}} = 1.25$
	a_3^{WW}	ϕ_{a3}^{WW}	f_{a3}^{WW}	A	$\sigma_1^{\mathrm{WW}}/\sigma_3^{\mathrm{WW}} = 3.01$
HZγ	$\Lambda_1^{\mathrm{Z}\gamma}$	$\phi_{\Lambda 1}^{\mathrm{Z}\gamma}$	$f_{\Lambda 1}^{\mathrm{Z}\gamma}$	A	$\sigma_1/\tilde{\sigma}_{\Lambda 1}^{\mathrm{Z}\gamma} = 5.76 \times 10^{-9}\ \mathrm{GeV}^{-4}$
	$a_2^{\mathrm{Z}\gamma}$	$\phi_{a2}^{\mathrm{Z}\gamma}$	$f_{a2}^{\mathrm{Z}\gamma}$	A	$\sigma_1/\sigma_2^{Z\gamma} = 22.4 \times 10^{-4}$
	$a_3^{\mathrm{Z}\gamma}$	$\phi_{a3}^{\mathrm{Z}\gamma}$	$f_{a3}^{\mathrm{Z}\gamma}$	A	$\sigma_1/\sigma_3^{Z\gamma} = 27.2 \times 10^{-4}$
H$\gamma\gamma$	$a_2^{\gamma\gamma}$	$\phi_{a2}^{\gamma\gamma}$	$f_{a2}^{\gamma\gamma}$	A	$\sigma_1/\sigma_2^{\gamma\gamma} = 28.2 \times 10^{-4}$
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What to measure

The coupling ratio a_i/a_1 -> effective fractional xsec

$$f_{a2} = \frac{|a_2|^2\sigma_2}{|a_1|^2\sigma_1 + |a_2|^2\sigma_2 + |a_3|^2\sigma_3 + \tilde{\sigma}_{\Lambda_1}/\left(\Lambda_1\right)^4}$$

— unique and finite ($0 < f_{ai} < 1$)

$$\frac{|a_i|}{|a_1|} = \sqrt{\frac{f_{ai}}{f_{a1}}} \times \sqrt{\frac{\sigma_1}{\sigma_i}}$$

Measurement Scenarios

— 1 non-zero anomalous coupling

A. real, $\phi_{ai}=0,\pi$

B. complex, ϕ_{ai} unconstrained

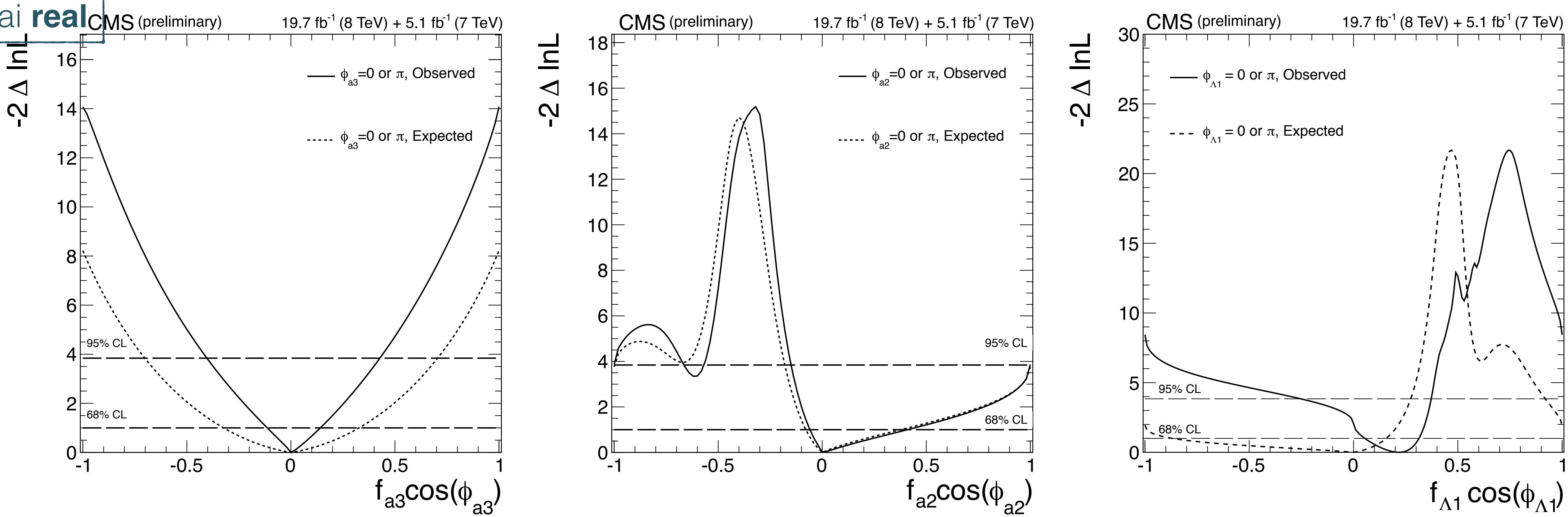
— 2 non-zero anomalous couplings

C. real, $\phi_{ai,aj}=0,\pi$

D. complex, $\phi_{ai,aj}$ unconstrained

CMS results: HZZ anomalous coupling

ai **real**



destructive interference | constructive interference

pure 0$^-$ — SM — pure 0$^-$

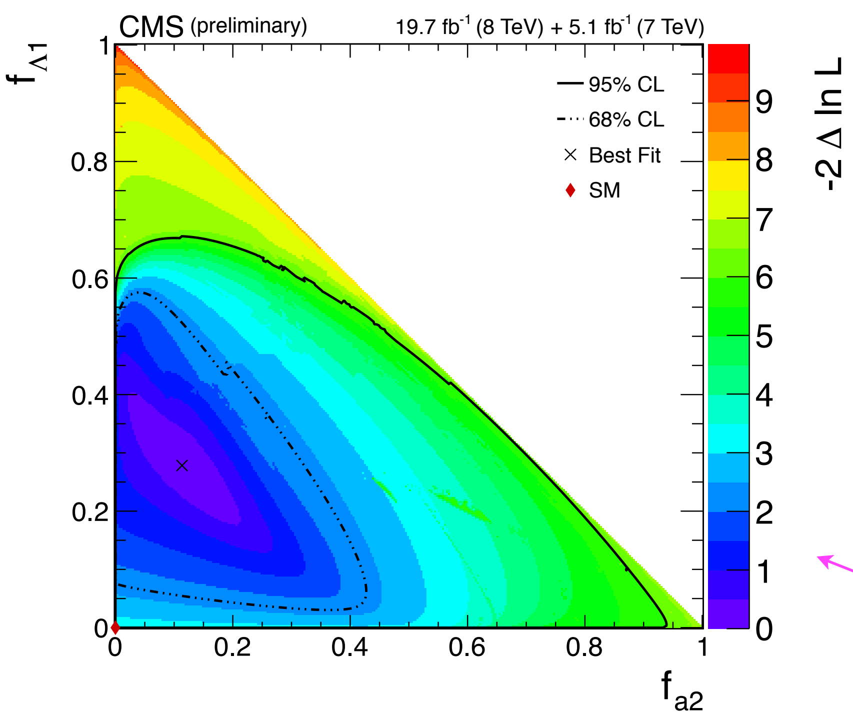
cross-check plots in backup

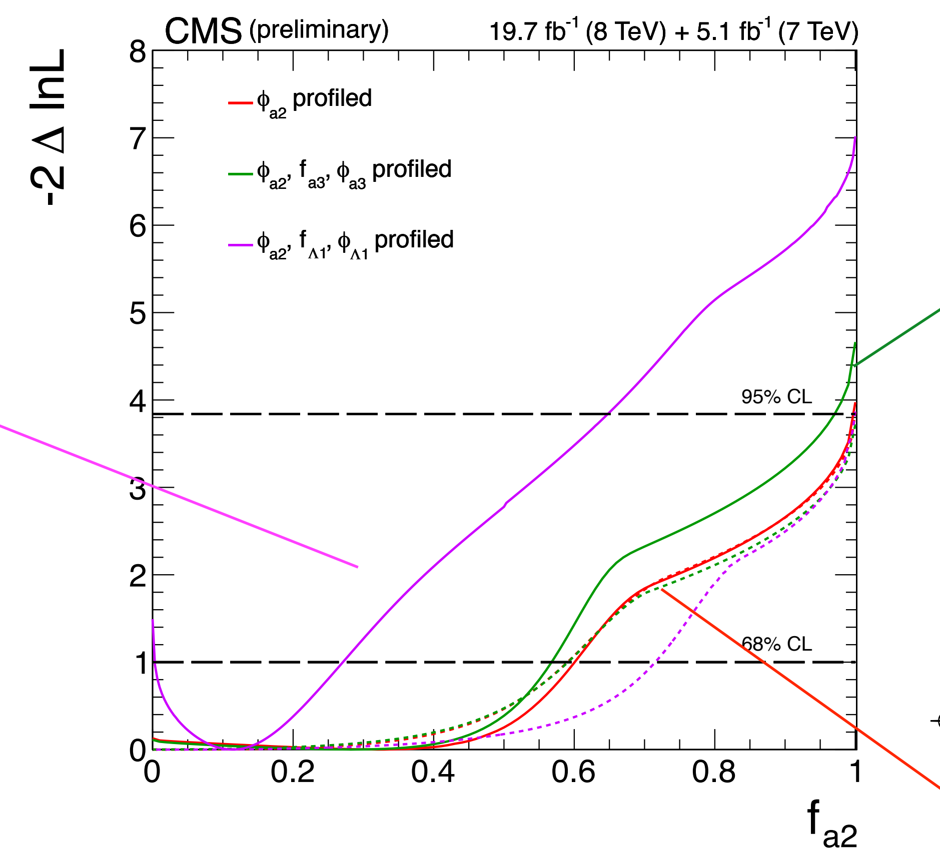
- Consistent with SM
- Precision ~20%
- Pure 0- excluded at 99.98% CL
- Pure 0_h^+ excluded at 95% CL

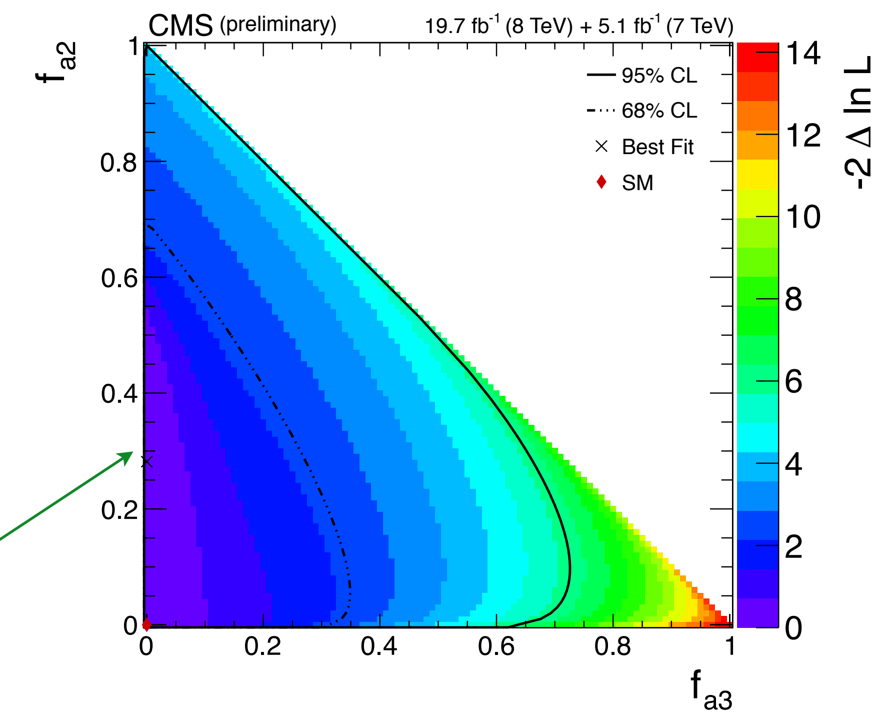
	Parameter	Observed	Expected	$f_{ai}^{VV}=1$
HZZ	$f_{\Lambda1}\cos(\phi_{\Lambda1})$	$0.22^{+0.10}_{-0.16}$ $[-0.25, 0.37]$	$0.00^{+0.16}_{-0.87}$ $[-1.00, 0.27]$ $\cup[0.92, 1.00]$	1.1% (16%)
	$f_{a2}\cos(\phi_{a2})$	$0.00^{+0.41}_{-0.06}$ $[-0.66, -0.57]$ $\cup[-0.15, 1.00]$	$0.00^{+0.38}_{-0.08}$ $[-0.18, 1.00]$	5.2% (5.0%)
	$f_{a3}\cos(\phi_{a3})$	$0.00^{+0.14}_{-0.11}$ $[-0.40, 0.43]$	$0.00^{+0.33}_{-0.33}$ $[-0.70, 0.70]$	0.02% (0.41%)

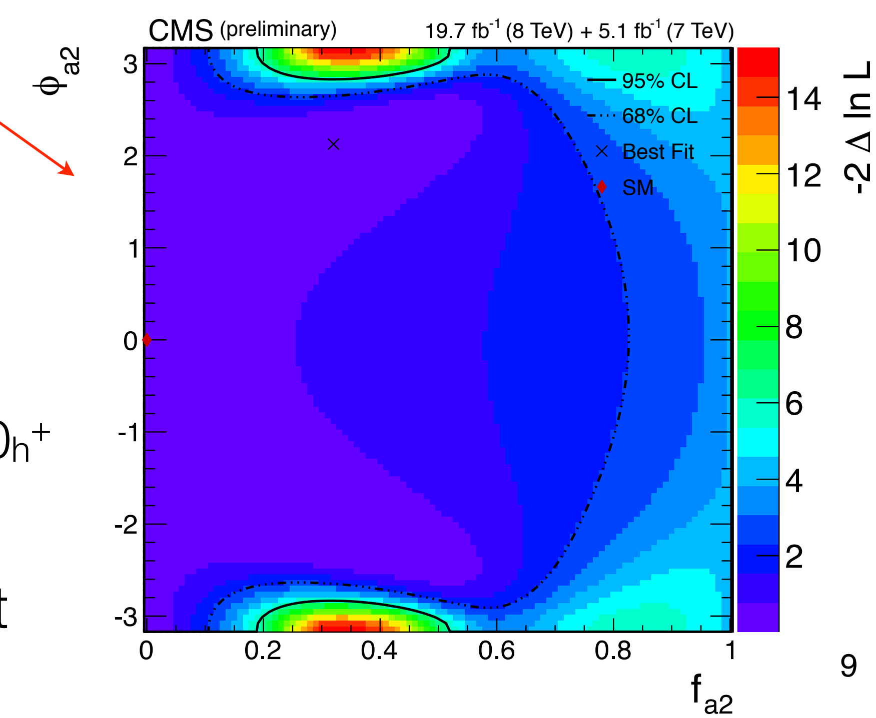
CMS results: HZZ anomalous coupling

ai **complex**









SM

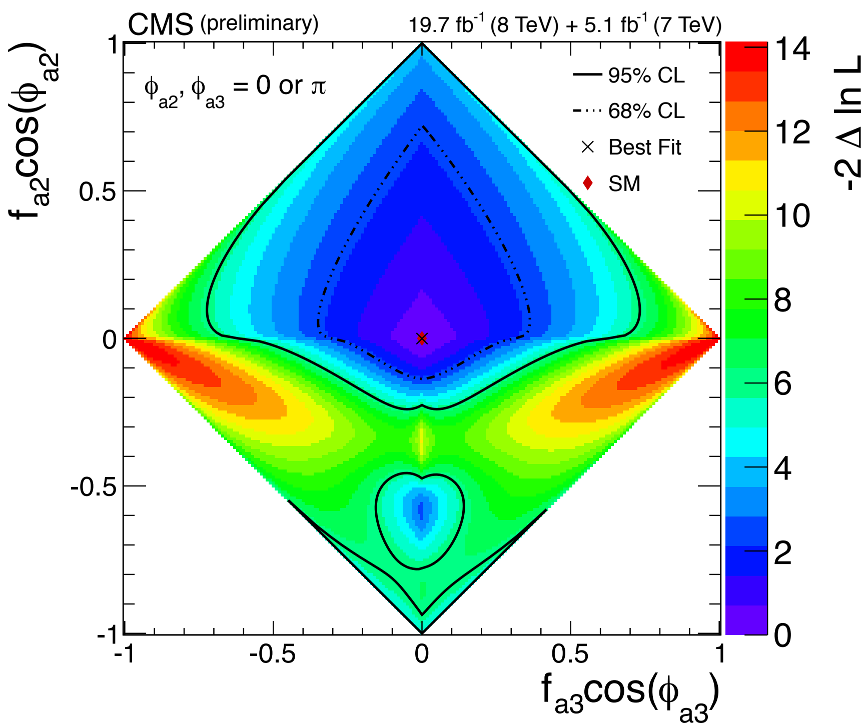
pure 0_h^+

More plots for all the other parameters in backup

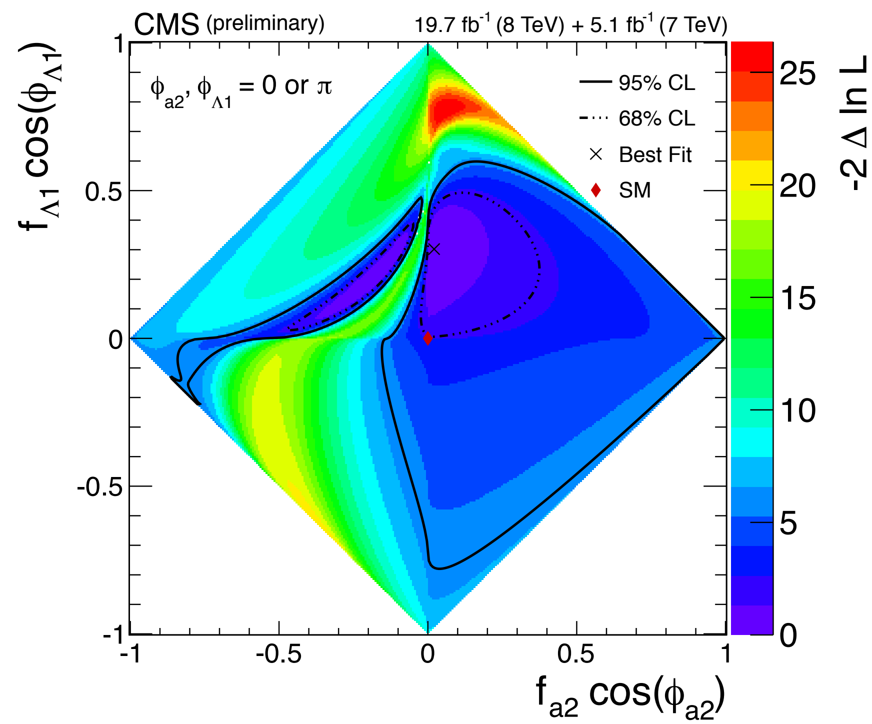
- Consistent with SM
- Less precision when phase not constrained

CMS results: HZZ anomalous couplings

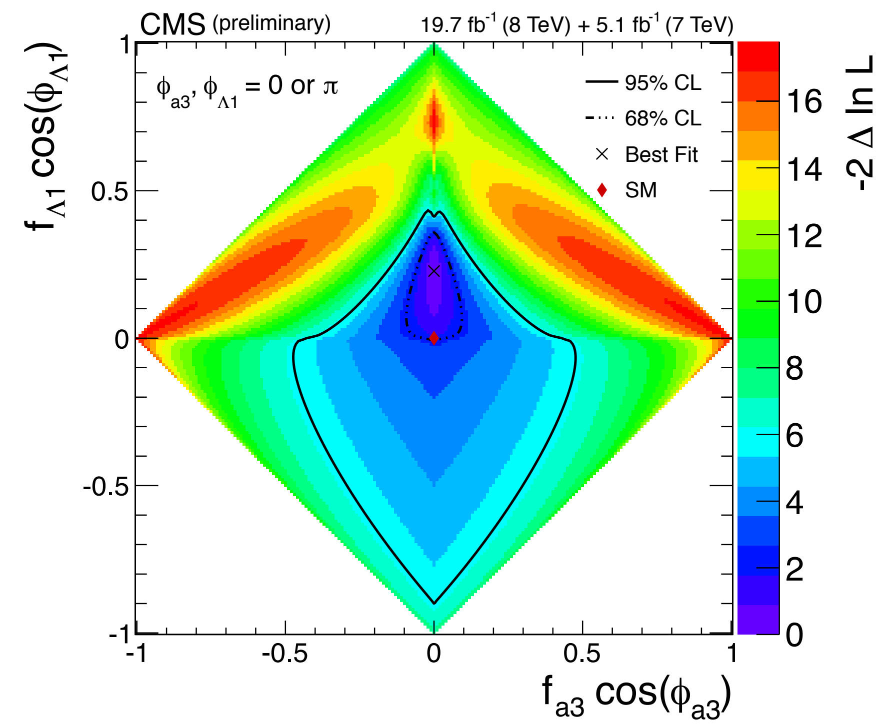
ai, aj **real**



f_{a2} vs f_{a3}



$f_{\Lambda1}$ vs f_{a2}

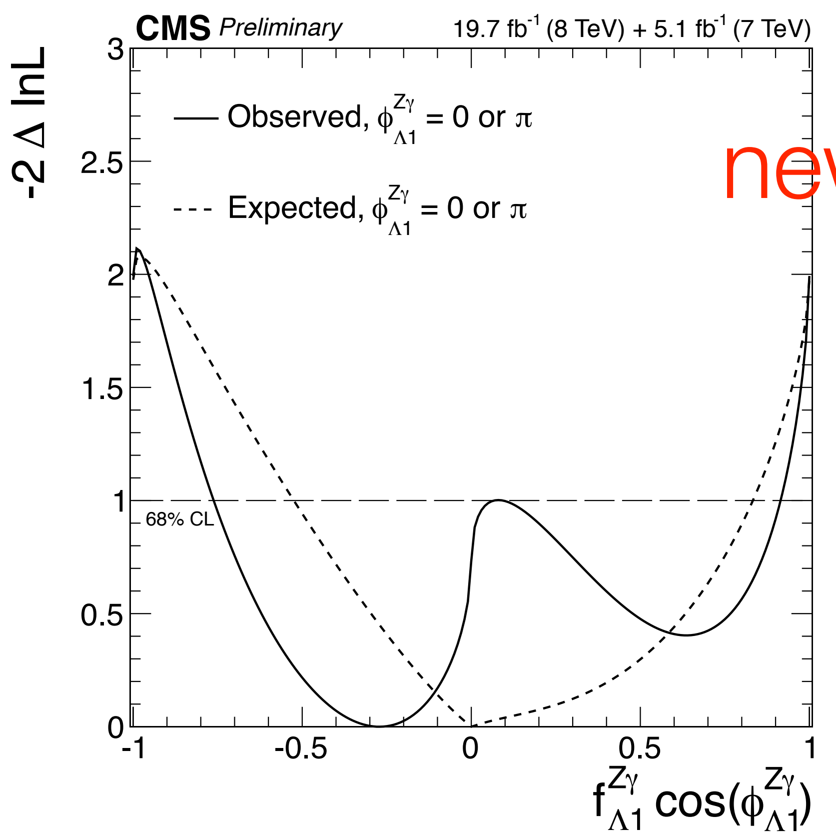


$f_{\Lambda1}$ vs f_{a3}

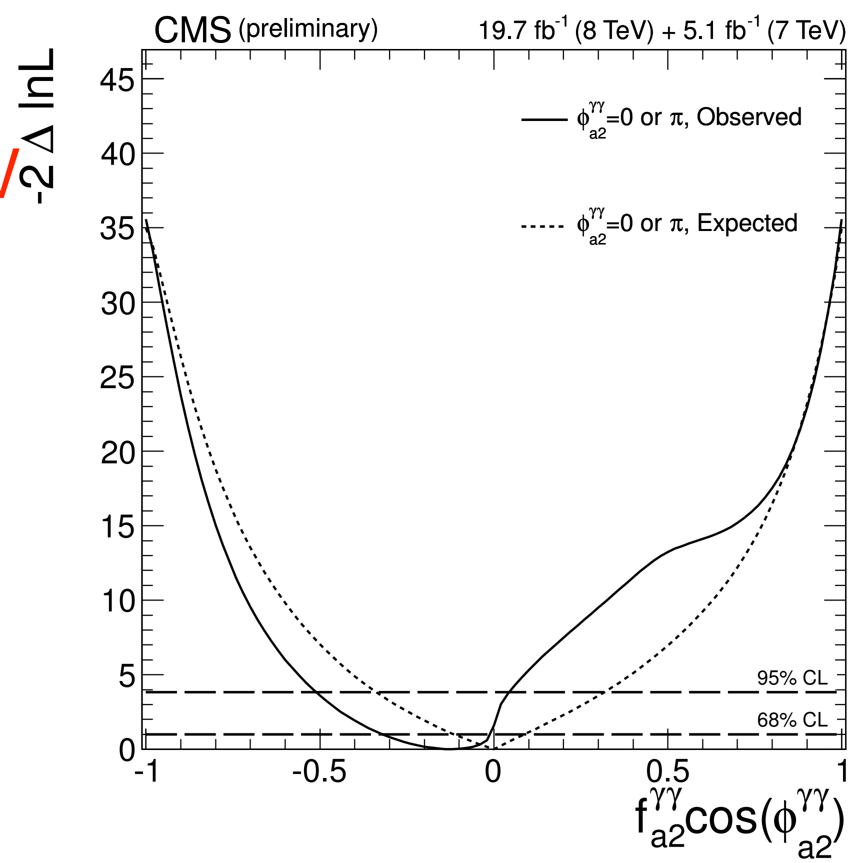
- Consistent with SM
- Interference effect reflected in different regions

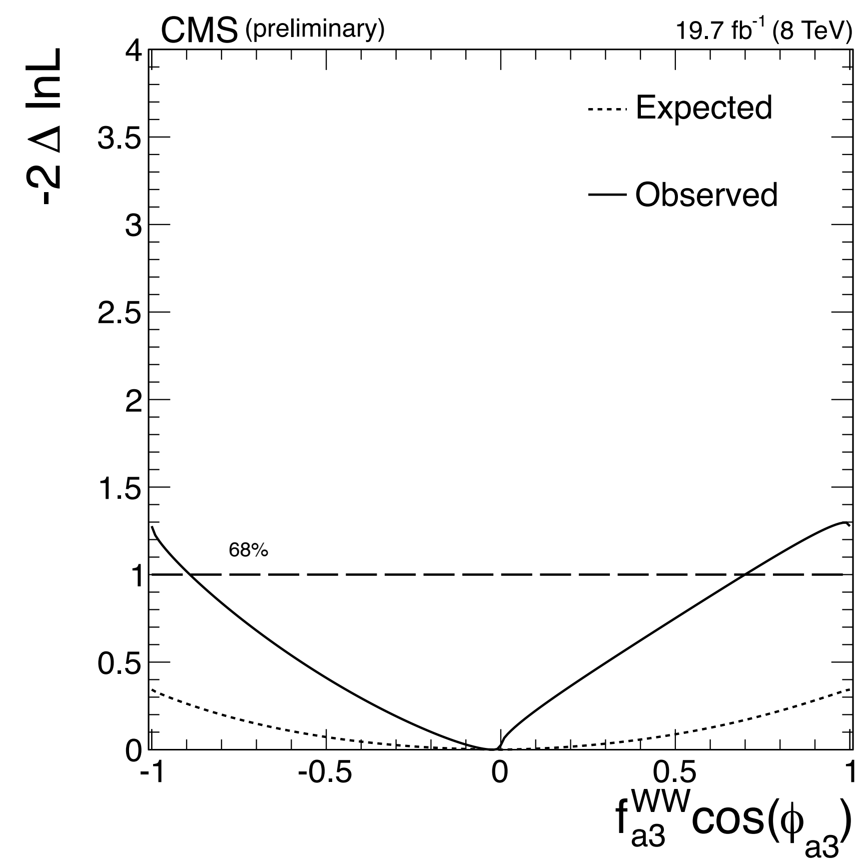
cross-check plots in backup

CMS results: HWW, HZγ, Hγγ anomalous couplings



new





	Parameter	Observed	Expected	$f_{ai}^{VV}=1$ vs 0
HWW	$f_{\Lambda1}^{WW}\cos(\phi_{\Lambda1}^{WW})$	$0.64^{+0.36}_{-0.11}$ $[-1.00,0.44]\cup[0.49,1.00]$	$0.00^{+0.36}_{-1.00}$ $[-1.00,0.43]\cup[0.48,1.00]$	100% (81%)
	$f_{a2}^{WW}\cos(\phi_{a2}^{WW})$	$0.44^{+0.56}_{-0.53}$ $[-1.00,-0.58]\cup[-0.25,1.00]$	$0.00^{+1.00}_{-0.13}$ $[-1.00,-0.56]\cup[-0.24,1.00]$	100% (50%)
	$f_{a3}^{WW}\cos(\phi_{a3}^{WW})$	$-0.03^{+0.73}_{-0.87}$ $[-1.00,1.00]$	$0.00^{+1.00}_{-1.00}$ $[-1.00,1.00]$	26% (56%)
HZγ	$f_{\Lambda1}^{Z\gamma}\cos(\phi_{\Lambda1}^{Z\gamma})$	$-0.27^{+0.34}_{-0.49}$ $[-1.00,1.00]$	$0.00^{+0.83}_{-0.53}$ $[-1.00,1.00]$	26% (16%)
	$f_{a2}^{Z\gamma}\cos(\phi_{a2}^{Z\gamma})$	$0.00^{+0.14}_{-0.20}$ $[-0.49,0.46]$	$0.00^{+0.51}_{-0.51}$ $[-0.78,0.79]$	<0.01% (0.01%)
	$f_{a3}^{Z\gamma}\cos(\phi_{a3}^{Z\gamma})$	$0.02^{+0.21}_{-0.13}$ $[-0.40,0.51]$	$0.00^{+0.51}_{-0.51}$ $[-0.75,0.75]$	<0.01% (<0.01%)
Hγγ	$f_{a2}^{\gamma\gamma}\cos(\phi_{a2}^{\gamma\gamma})$	$-0.12^{+0.11}_{-0.20}$ $[-0.51,0.04]$	$0.00^{+0.09}_{-0.11}$ $[-0.34,0.32]$	<0.01% (<0.01%)
	$f_{a3}^{\gamma\gamma}\cos(\phi_{a3}^{\gamma\gamma})$	$0.02^{+0.13}_{-0.06}$ $[-0.32,0.35]$	$0.00^{+0.11}_{-0.15}$ $[-0.40,0.37]$	<0.01% (<0.01%)

- a_i real, ϕ_{ai}=0,π
- Consistent with SM
- H->4l from pure HZγ*, Hγ*γ* excluded at > 99.99% CL

More plots for all the other parameters in backup

CMS results: HZZ+HWW combination

- HZZ and HWW anomalous couplings

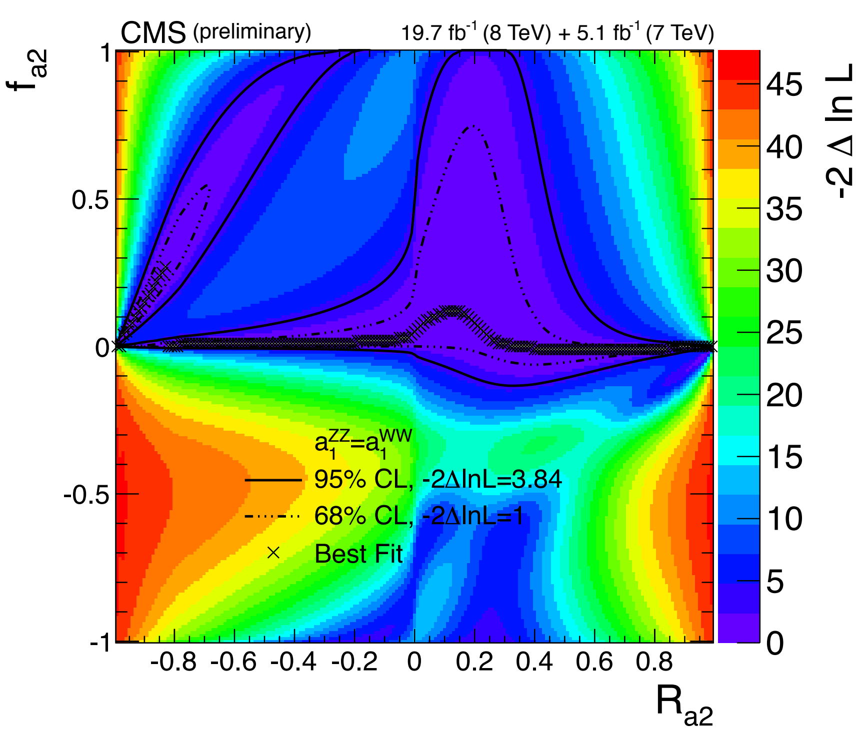
$$r_{ai} = \frac{a_i^{\mathrm{WW}}/a_1^{\mathrm{WW}}}{a_i/a_1} \quad R_{ai} = \frac{r_{ai}|r_{ai}|}{1+r_{ai}^2}$$

More plots in backup

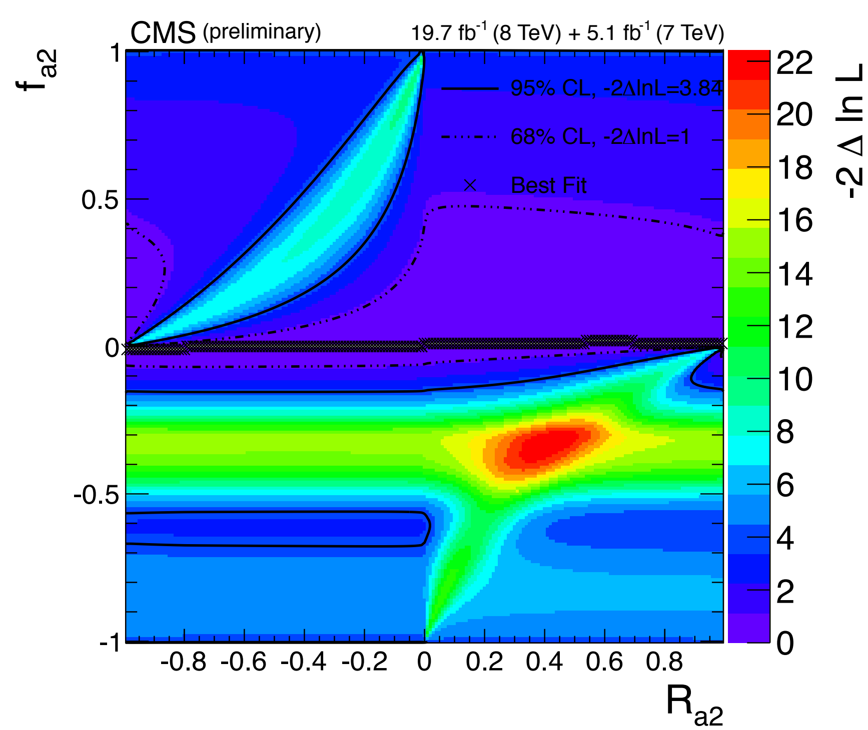
- Scenarios for combinations
  - $a_1^{ZZ} = a_1^{WW}$, custodial symmetry
  - $a_1^{ZZ} \neq a_1^{WW}$

custodial symmetry constrains more due to the correlation

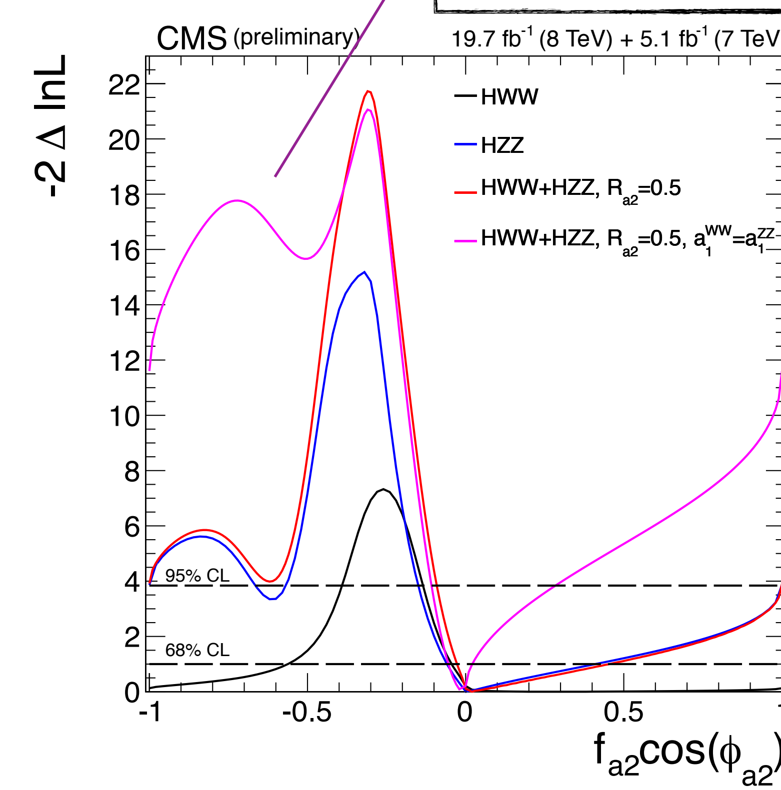
$a_1^{ZZ} = a_1^{WW}$



$a_1^{ZZ} \neq a_1^{WW}$



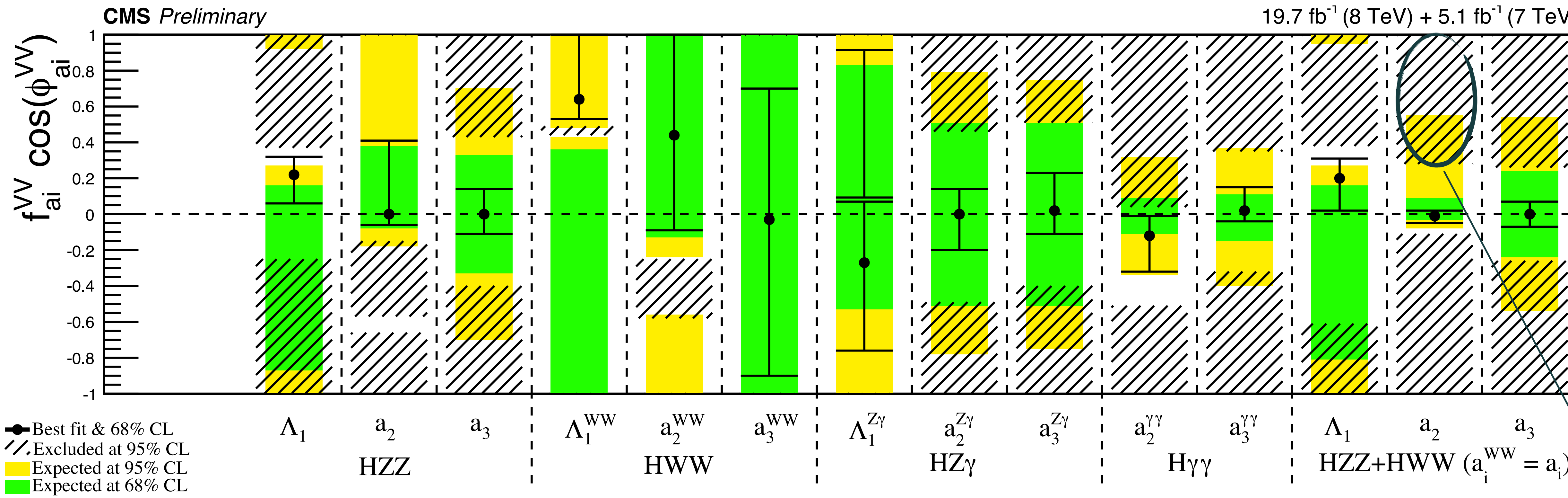
$a_i^{ZZ} = a_i^{WW}$ slice



CMS results summary

HWW+ZZ, $a_i^{ZZ} = a_i^{WW}$

Parameter	Observed	Expected	$f_{ai}^{VV} = 1$ vs 0
$f_{\Lambda 1}\cos(\phi_{\Lambda 1})$	$0.20^{+0.11}_{-0.18}$ $[-0.61, 0.38]$	$0.00^{+0.16}_{-0.81}$ $[-1, 0.27] \cup [0.95, 1]$	0.86% (13%)
$f_{a2}\cos(\phi_{a2})$	$-0.01^{+0.03}_{-0.04}$ $[-0.11, 0.28]$	$0.00^{+0.09}_{-0.03}$ $[-0.08, 0.55]$	0.07% (0.31%)
$f_{a3}\cos(\phi_{a3})$	$0.00^{+0.07}_{-0.07}$ $[-0.26, 0.26]$	$0.00^{+0.24}_{-0.24}$ $[-0.54, 0.54]$	<0.01% (0.09%)



Region not excluded in either HZZ or HWW
Constraint from HZZ HWW correlation

- Consistent with SM
- when assuming $a_i^{ZZ} = a_i^{WW}$
  - pure 0_h^+ excluded at 99.93% CL
  - pure 0^- excluded at 99.99% CL

Conclusion

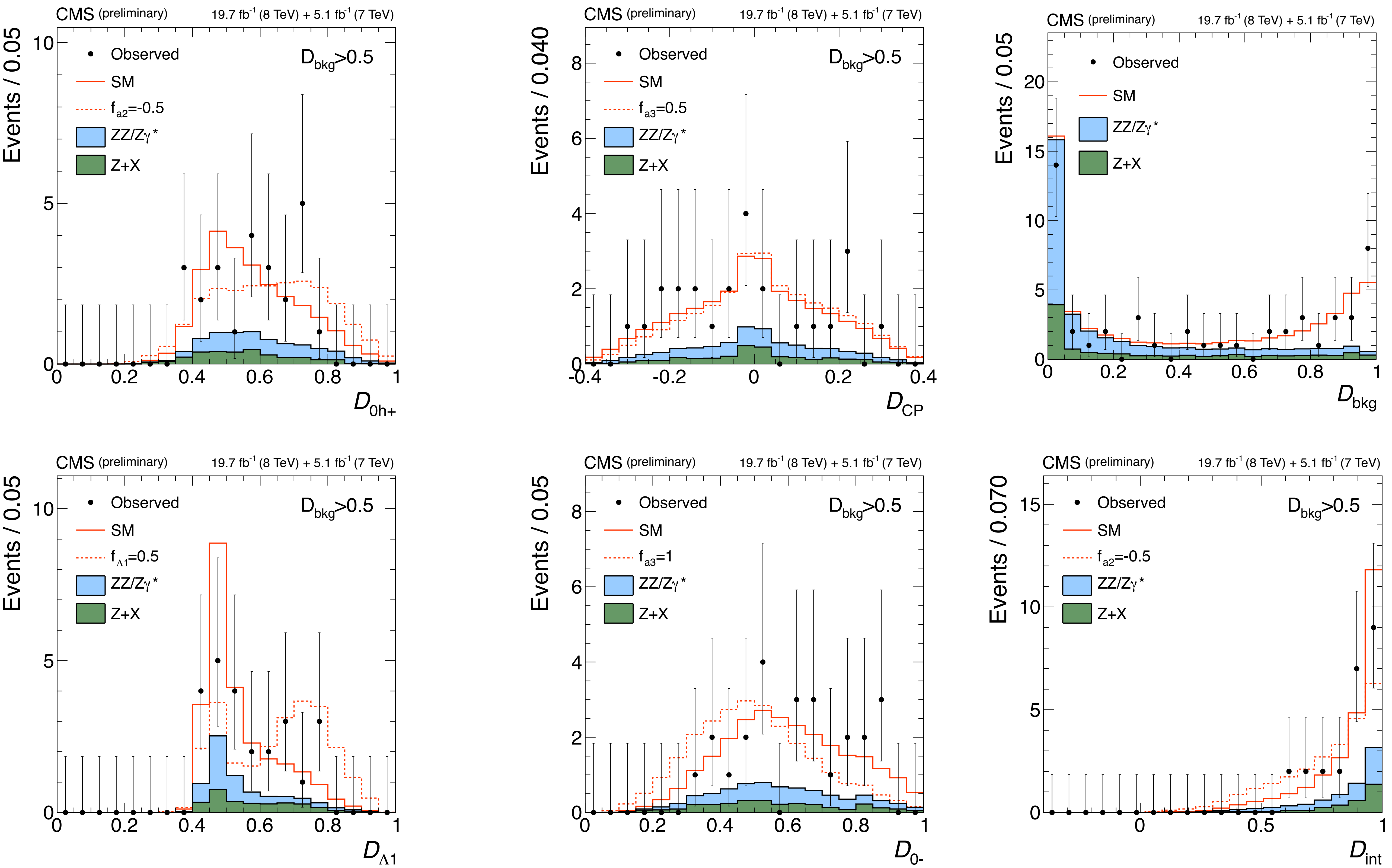
- A comprehensive study of the the Higgs boson spin-parity properties (ATLAS & CMS) and tensor structure (CMS) through
    - H->Z(γ*)Z(γ*)->4l, H->WW->lνlν and H->γγ
- Exotic-spin hypotheses excluded at 99% or higher confidence level
- Constraints on 11 anomalous couplings under spin-0 assumption
    - ZZ, WW, Zγ,γγ, couplings measured
        - f_{ai} with precision ~20% order, pure 0_h^+ and 0^- excluded at 95% and 99.98% CL (ZZ)
- All observations consistent with the SM expectation

Backup

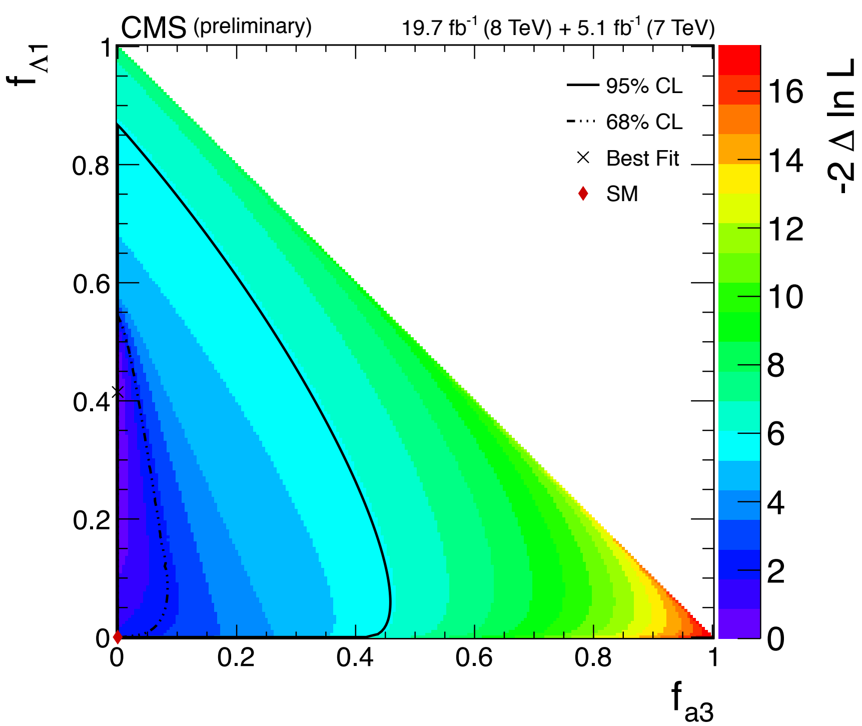
Spin 0 Lagrangian

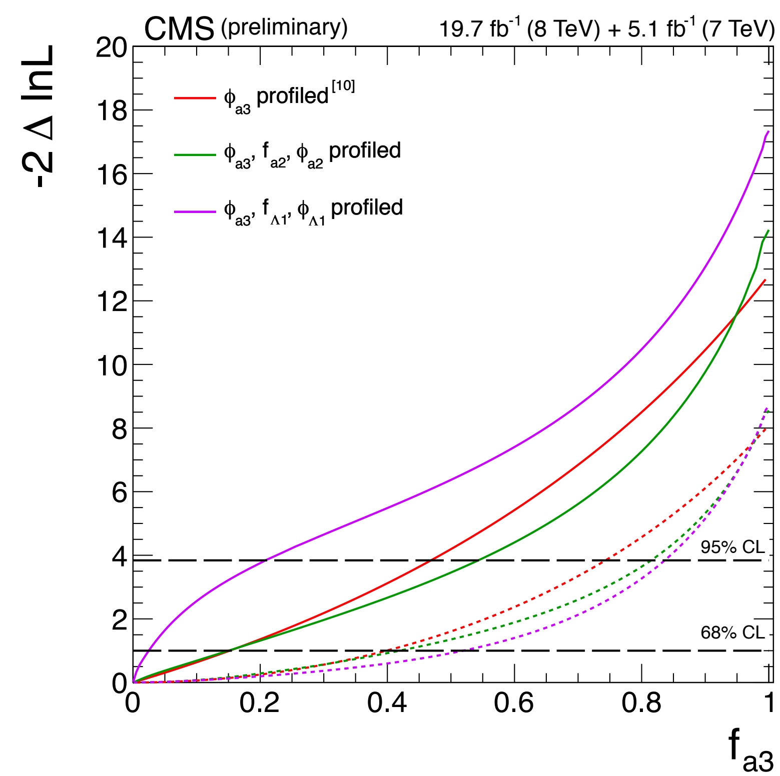
$$
\begin{aligned}
L(\mathrm{HV_1V_2}) \;\sim\;& a_1 \frac{m_Z^2}{2}\mathrm{HZ}^\mu \mathrm{Z}_\mu + \frac{1}{(\Lambda_1)^2} m_Z^2 \mathrm{HZ}_\mu \Box \mathrm{Z}^\mu - \frac{1}{2} a_2 \mathrm{HZ}^{\mu\nu}\mathrm{Z}_{\mu\nu} - \frac{1}{2} a_3 \mathrm{HZ}^{\mu\nu}\tilde{\mathrm{Z}}_{\mu\nu} \\
&+ a_1^{\mathrm{WW}} \frac{m_W^2}{2}\mathrm{HW}^\mu \mathrm{W}_\mu + \frac{1}{(\Lambda_1^{\mathrm{WW}})^2} m_W^2 \mathrm{HW}_\mu \Box \mathrm{W}^\mu - \frac{1}{2} a_2^{\mathrm{WW}} \mathrm{HW}^{\mu\nu}\mathrm{W}_{\mu\nu} - \frac{1}{2} a_3^{\mathrm{WW}} \mathrm{HW}^{\mu\nu}\tilde{\mathrm{W}}_{\mu\nu} \\
&+ \frac{1}{\left(\Lambda_1^{Z\gamma}\right)^2} m_Z^2 \mathrm{HZ}_\mu \partial_\nu \mathrm{F}^{\mu\nu} - a_2^{Z\gamma}\mathrm{HF}^{\mu\nu}\mathrm{Z}_{\mu\nu} - a_3^{Z\gamma}\mathrm{HF}^{\mu\nu}\tilde{\mathrm{Z}}_{\mu\nu} - \frac{1}{2} a_2^{\gamma\gamma}\mathrm{HF}^{\mu\nu}\mathrm{F}_{\mu\nu} - \frac{1}{2} a_3^{\gamma\gamma}\mathrm{HF}^{\mu\nu}\tilde{F}_{\mu\nu},
\end{aligned}
$$

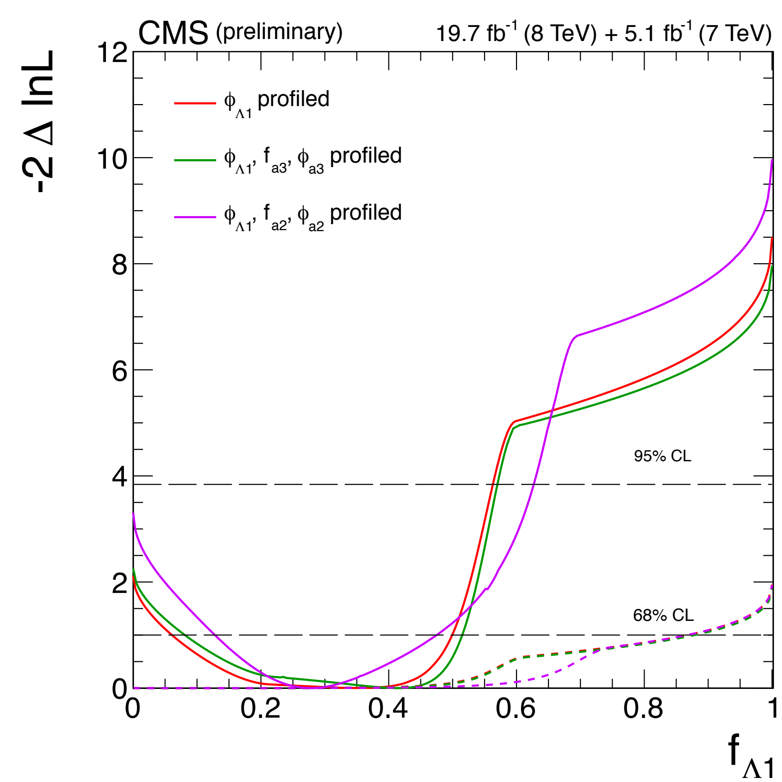
Discriminants



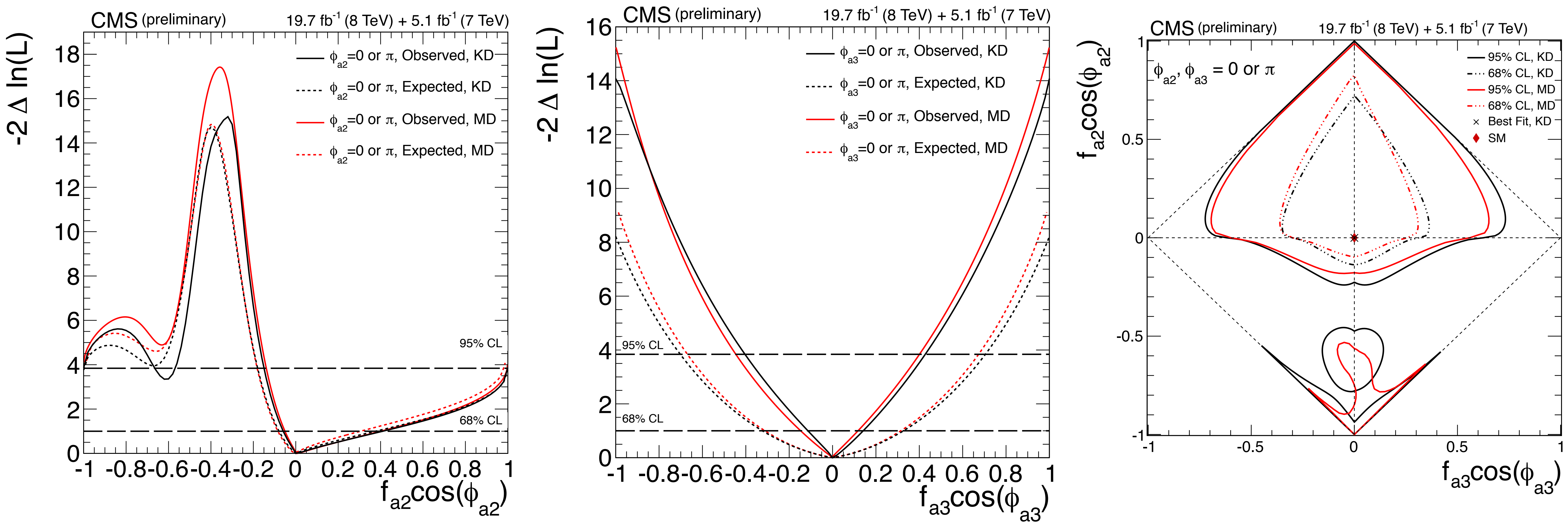
HZZ couplings



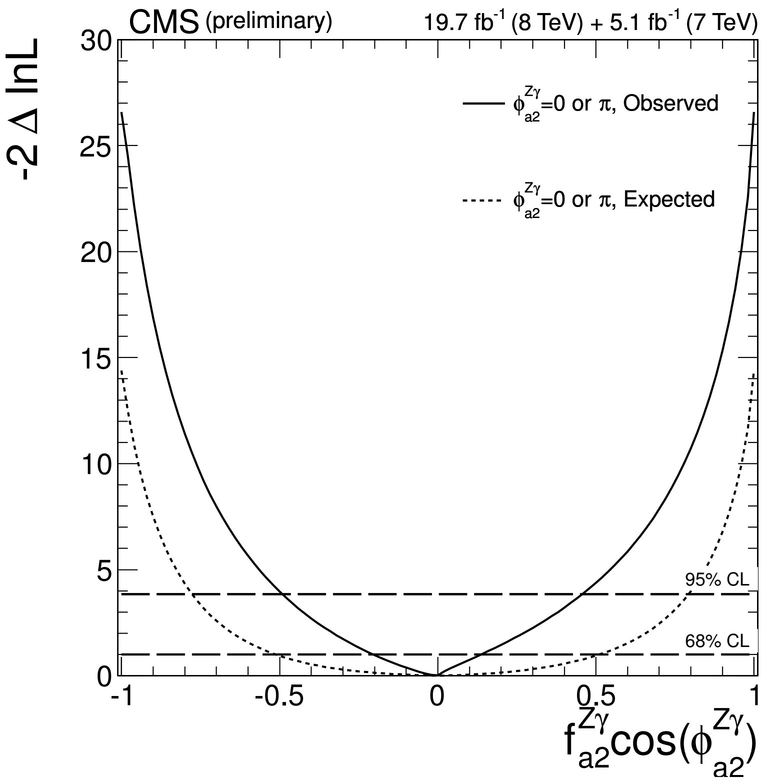


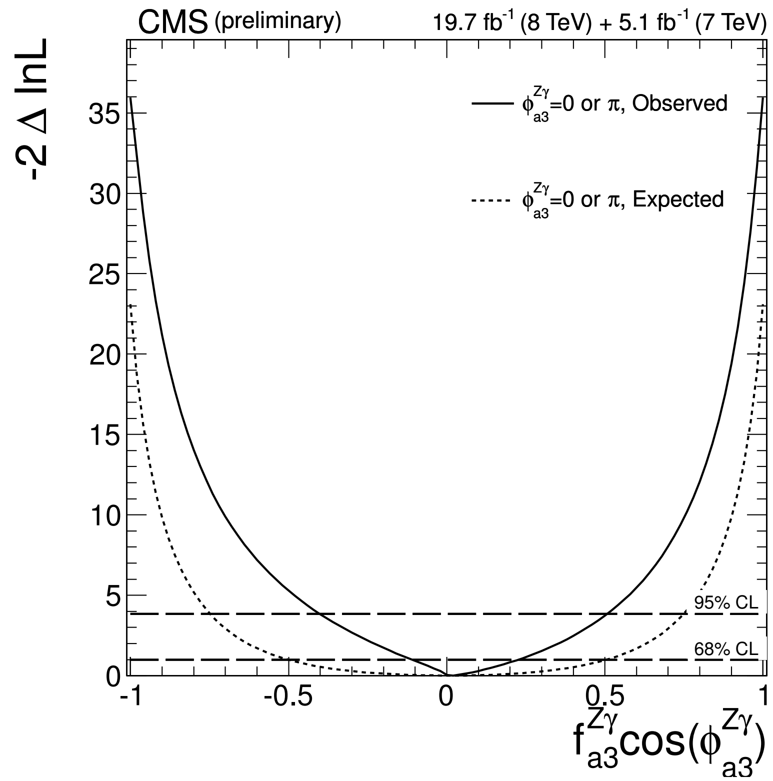


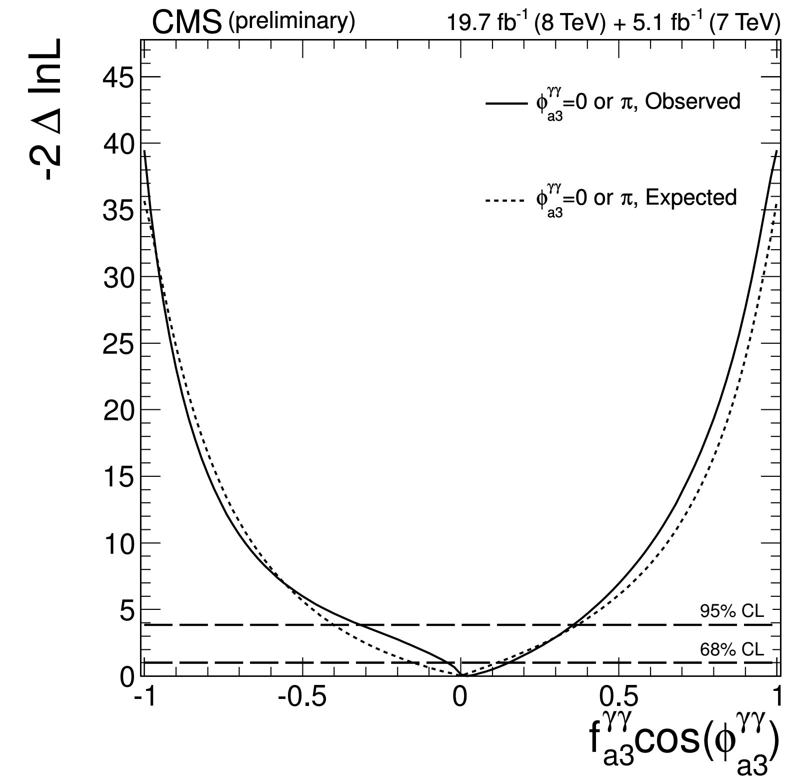
8D validation

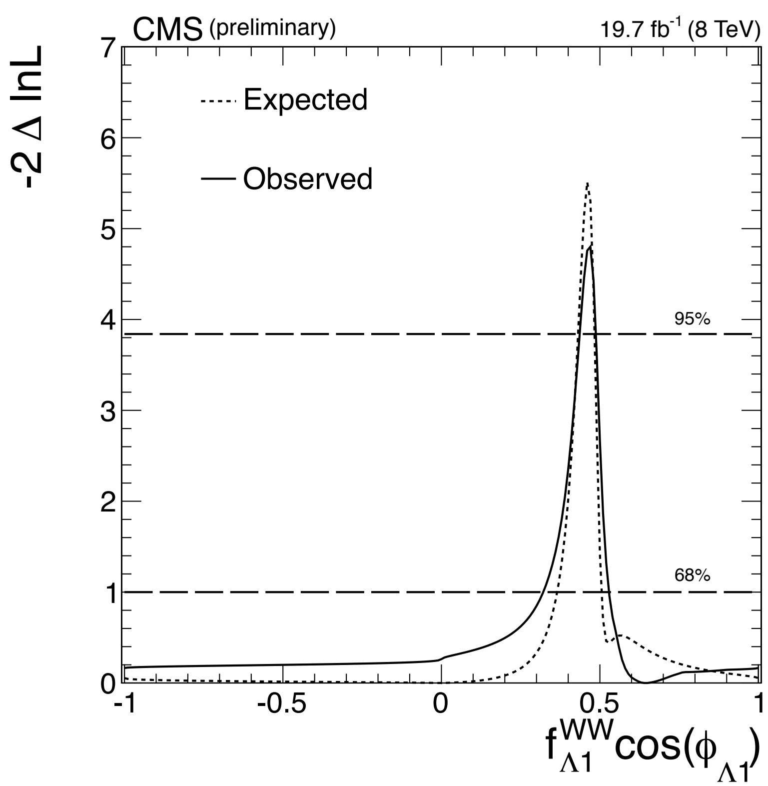


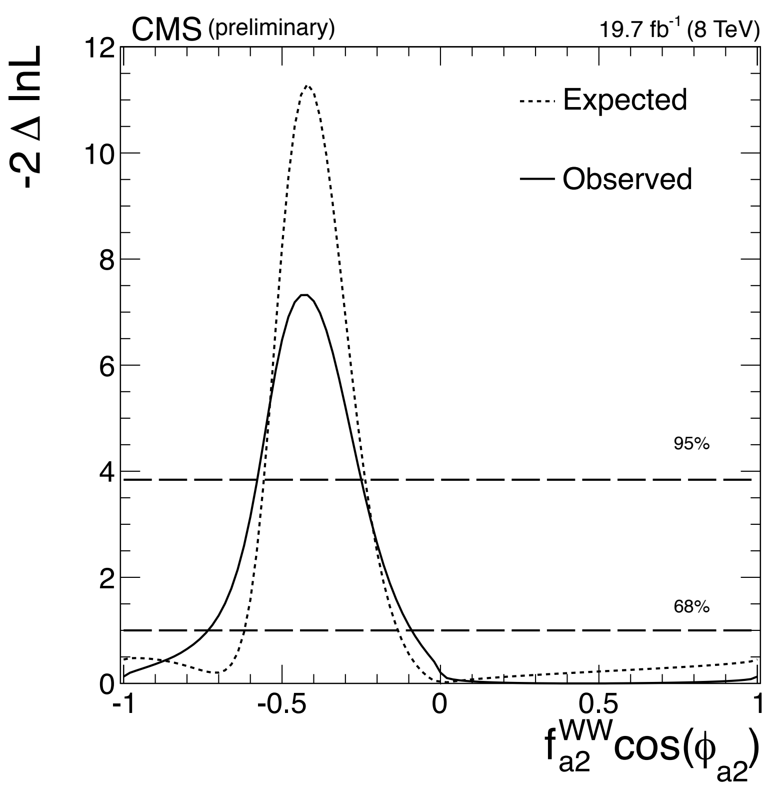
HWW, HZγ, Hγγ couplings



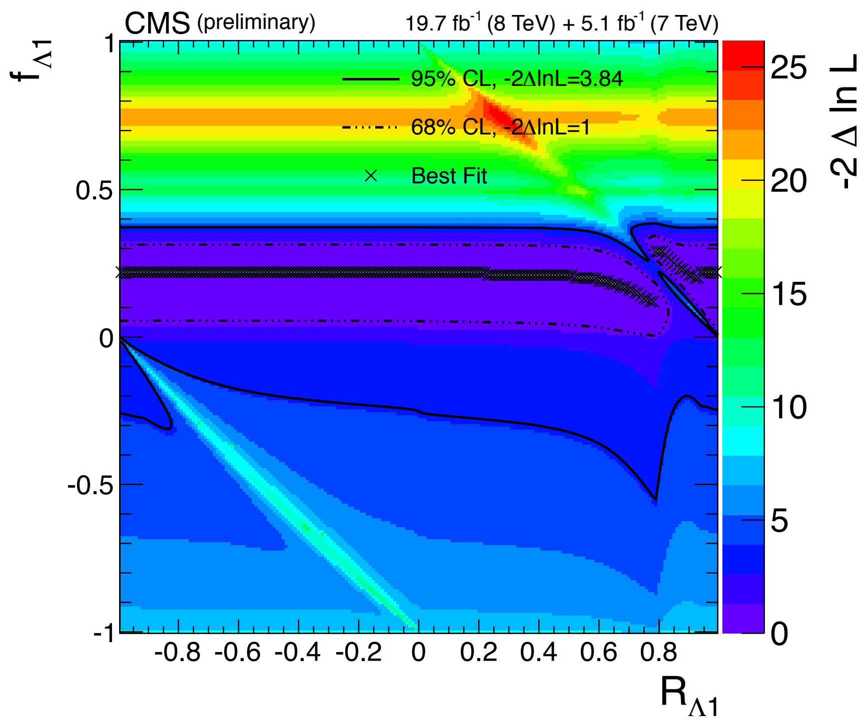


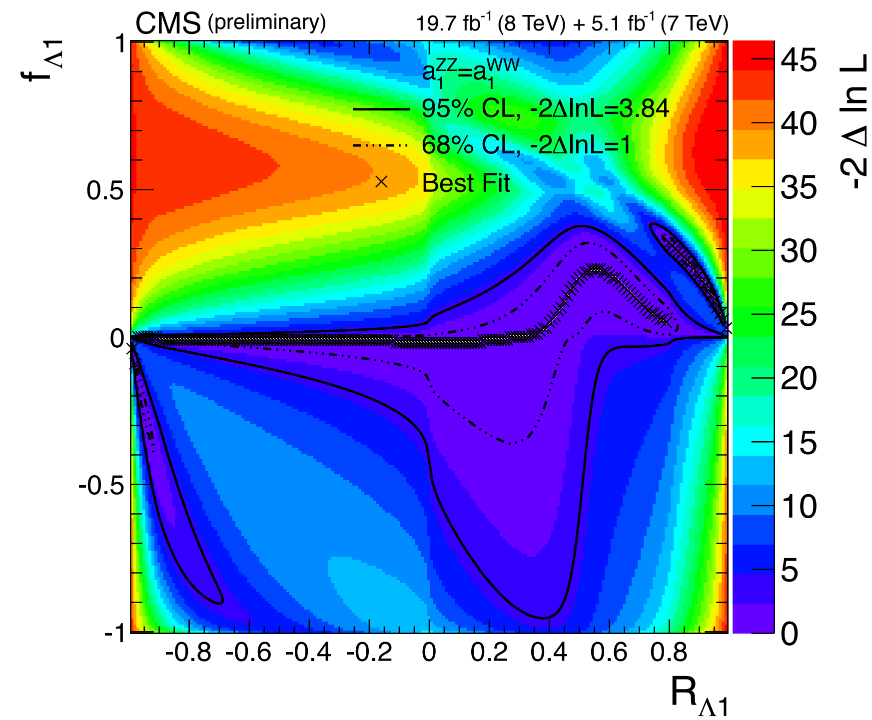


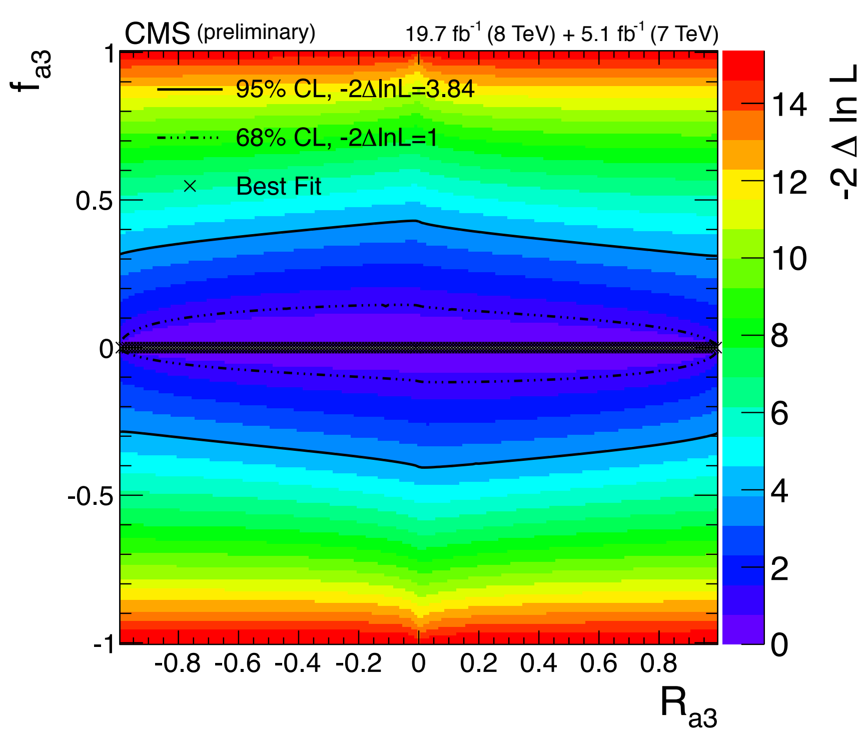


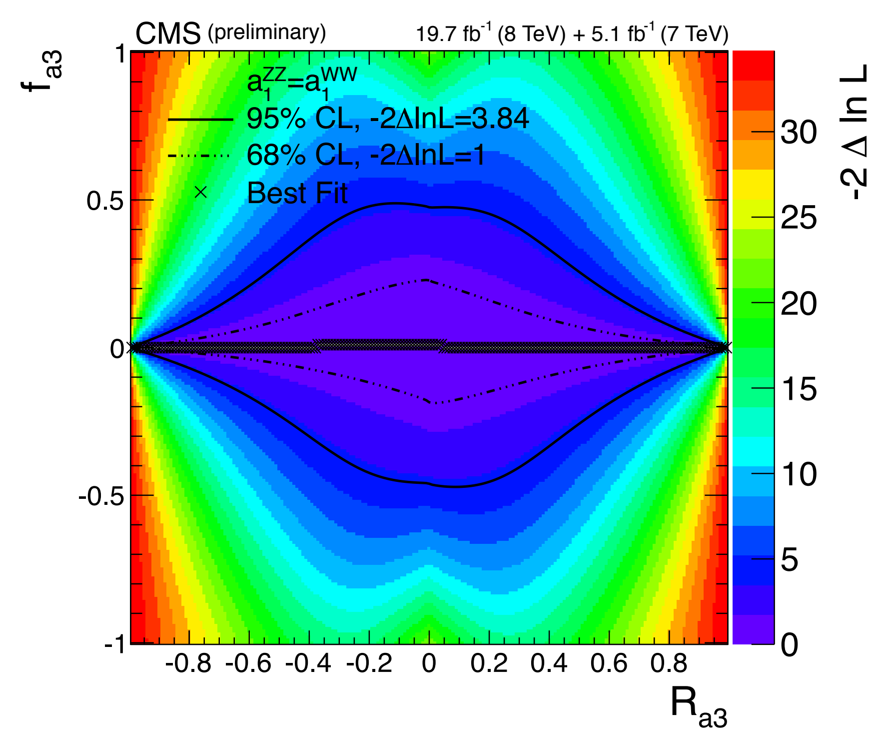


HWW and HZZ combination

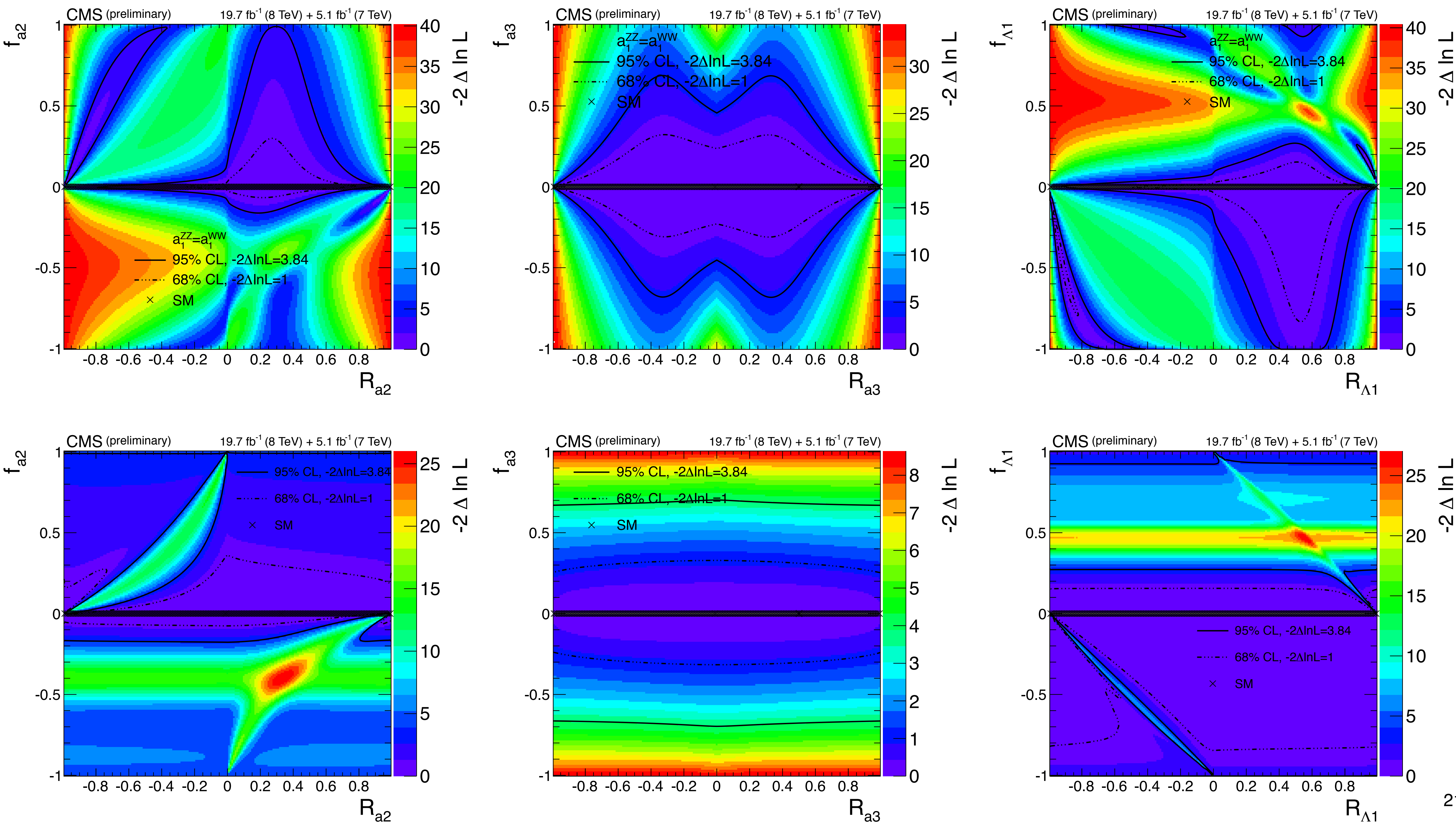








HWW and HZZ combination, expected



Allowed 95% CL intervals (real couplings)		
Kinematic discriminants method		
Parameter	Observed	Expected
$(\Lambda_1 \sqrt{\lvert a_1 \rvert}) \cos(\phi_{\Lambda_1})$	$[-\infty, -119\,\mathrm{GeV}] \cup [104\,\mathrm{GeV}, \infty]$	$[-\infty, 50\,\mathrm{GeV}] \cup [116\,\mathrm{GeV}, \infty]$
a_2/a_1	$[-2.28, -1.88] \cup [-0.69, \infty]$	$[-0.77, \infty]$
a_3/a_1	$[-2.05, 2.19]$	$[-3.85, 3.85]$
$a_2^{Z\gamma}/a_1$	$[-0.046, 0.044]$	$[-0.089, 0.092]$
$a_3^{Z\gamma}/a_1$	$[-0.042, 0.053]$	$[-0.090, 0.090]$
$a_2^{\gamma\gamma}/a_1$	$[-0.054, 0.011]$	$[-0.038, 0.036]$
$a_3^{\gamma\gamma}/a_1$	$[-0.037, 0.039]$	$[-0.044, 0.041]$
$(\sigma_2^{Z\gamma}/\sigma_2^{Z\gamma})(\frac{a_1}{2})^2 \cos(\phi_{a2}^{Z\gamma})$	$[-702, 622]$	$[-2590, 2748]$
$(\sigma_3^{Z\gamma}/\sigma_2^{Z\gamma})(\frac{a_1}{2})^2 \cos(\phi_{a2}^{Z\gamma})$	$[-487, 760]$	$[-2192, 2192]$
$(\sigma_2^{\gamma\gamma}/\sigma_2^{\gamma\gamma})(\frac{a_1}{2})^2 \cos(\phi_{a2}^{\gamma\gamma})$	$[-734, 29.4]$	$[-363, 332]$
$(\sigma_3^{\gamma\gamma}/\sigma_2^{\gamma\gamma})(\frac{a_1}{2})^2 \cos(\phi_{a3}^{\gamma\gamma})$	$[-332, 380]$	$[-470, 414]$
Multidimensional distributions method		
Parameter	Observed	Expected
a_2/a_1	$[-0.66, \infty]$	$[-0.77, 9.31]$
a_3/a_1	$[-2.23, 2.06]$	$[-3.59, 3.59]$