

Next steps for form factor calculations and their uncertainties

Christoph Bobeth

TU Munich – IAS

Outline

- I) Form factor (=FF) symmetries & bases
 - II) FF parameterisations
 - III) Overview LCSR determinations
 - IV) Overview Lattice determinations
- !!! Non-expert view of the subject, BUT the experts are in the audience

Introduction

Phenomenological interest in exclusive $B \rightarrow L + (\bar{\ell}\ell, \gamma)$ decays (L = light P or V mesons)

@ parton level $b \rightarrow q + (\bar{\ell}\ell, \gamma)$ ($q = d, s$) FCNC transitions:

$$q = s$$

- ▶ $B \rightarrow K\bar{\ell}\ell$
- ▶ $B \rightarrow K^*\bar{\ell}\ell, B_s \rightarrow \phi\bar{\ell}\ell$
- ▶ $B \rightarrow K^*\gamma$

$$q = d$$

- ▶ $B \rightarrow \pi\bar{\ell}\ell, B_s \rightarrow K\bar{\ell}\ell$
- ▶ $B \rightarrow (\rho, \omega)\bar{\ell}\ell,$
- ▶ $B \rightarrow (\rho, \omega)\gamma$

(in principle also Λ_b baryon decays)

⇒ require various $B \rightarrow$ Pseudoscalar and $B \rightarrow$ Vector FF's
over whole range of dilepton invariant mass

$$q^2 = (p_\ell + p_{\bar{\ell}})^2 = (p - k)^2$$

$$(2m_\ell)^2 \leq q^2 \leq (M_B - M_L)^2$$

$$B(p) \rightarrow L(k) + \bar{\ell}(p_{\bar{\ell}}) + \ell(p_\ell)$$

M_L = mass of light final state meson L

FF definitions: $B \rightarrow P$

3 FF's: f_+ = vector, f_0 = scalar, f_T = tensor

$$\langle P(k) | \bar{q} \gamma_\mu b | B(p) \rangle = (2p - q)_\mu f_+ + \frac{m_B^2 - m_P^2}{q^2} q_\mu [f_0 - f_+] ,$$

$$\langle P(k) | \bar{q} i\sigma_{\mu\nu} q^\nu b | B(p) \rangle = \left[(m_B^2 - m_P^2) q_\mu - q^2 (2p - q)_\mu \right] \frac{f_T}{m_B + m_P}$$

Kinematic limits @ $q^2 = 0$

$$f_0 = f_+$$

Throughout q^2 -dependence of FF's not explicitly shown !!!

FF definitions: $B \rightarrow V$

7 FF's: $V = \text{vector}$, $A_{1,2} = \text{axial-vector}$, $A_0 = \text{scalar}$, $T_{1,2,3} = \text{tensor}$

$$\langle V(k, \epsilon) | \bar{q} \gamma_\mu b | B(p) \rangle = \frac{2V}{m_B + m_V} \epsilon_{\mu\rho\sigma\tau} \epsilon^{*\rho} p^\sigma k^\tau ,$$

$$\begin{aligned} \langle V(k, \epsilon) | \bar{q} \gamma_\mu \gamma_5 b | B(p) \rangle &= i \epsilon^{*\rho} \left[2m_V A_0 \frac{q_\mu q_\rho}{q^2} + (m_B + m_V) A_1 \left(g_{\mu\rho} - \frac{q_\mu q_\rho}{q^2} \right) \right. \\ &\quad \left. - A_2 \frac{q_\rho}{m_B + m_V} \left((p+k)_\mu - \frac{m_B^2 - m_V^2}{q^2} (p-k)_\mu \right) \right] , \end{aligned}$$

$$\langle V(k, \epsilon) | \bar{q} i\sigma_{\mu\nu} q^\nu b | B(p) \rangle = -2T_1 \epsilon_{\mu\rho\sigma\tau} \epsilon^{*\rho} p^\sigma k^\tau ,$$

$$\begin{aligned} \langle V(k, \epsilon) | \bar{q} i\sigma_{\mu\nu} \gamma_5 q^\nu b | B(p) \rangle &= i T_2 \left(\epsilon_\mu^* (m_B^2 - m_V^2) - (\epsilon^* \cdot q)(p+k)_\mu \right) \\ &\quad + i T_3 (\epsilon^* \cdot q) \left(q_\mu - \frac{q^2}{m_B^2 - m_V^2} (p+k)_\mu \right) \end{aligned}$$

Kinematic limits @ $q^2 = 0$

$$A_0 = \frac{m_B + m_V}{2m_V} A_1 - \frac{m_B - m_V}{2m_V} A_2 , \quad T_1 = T_2$$

Form factor symmetries & bases

FF symmetries I: Heavy quark limit $m_b \rightarrow \infty$

[Isgur/Wise PRD 42 (1989) 2388]

For all q^2 : relates tensor FF's with vector- & axial-vector FF's

$$B \rightarrow P$$

$$f_T(\mu) \approx \frac{m_B(m_B + m_P)}{q^2} \kappa(\mu) f_+ + \mathcal{O}(\Lambda),$$

$$B \rightarrow V$$

$$T_1(\mu) \approx \kappa(\mu) V + \mathcal{O}(\Lambda),$$

$$\Lambda = \frac{\Lambda_{\text{QCD}}}{m_b}$$

$$T_2(\mu) \approx \kappa(\mu) A_1 + \mathcal{O}(\Lambda),$$

$$T_3(\mu) \approx \frac{m_B^2}{q^2} \kappa(\mu) A_2 + \mathcal{O}(\Lambda)$$

where $\kappa(\mu) = (1 + \alpha_s \dots)$ radiative correction (matching QCD on HQET)

⇒ Improved Isgur-Wise relations:

[Grinstein/Pirjol hep-ph/0404250]

!!! in $b \rightarrow s \bar{\ell} \ell$ decays it seems advantageous to replace in amplitudes tensor FF's

$$\mathcal{M}[B \rightarrow (K^*)_\lambda \bar{\ell} \ell] \sim (C_9^{\text{eff}} \pm C_{10}) V A_\lambda \left(1 + \frac{2m_b}{q^2} \frac{C_7^{\text{eff}}}{(C_9^{\text{eff}} \pm C_{10})} \frac{T_\lambda}{V A_\lambda} \right)$$

such that sub-leading $\mathcal{O}(\Lambda)$ corrections from FF-relations are suppressed by

$$C_7/(C_9 \pm C_{10}) \sim 0.3/(4 \pm 4)$$

works good @ high q^2

(@ low q^2 different FF-relations → next slide)

FF symmetries II: Heavy quark limit + Large recoil

Large recoil = low q^2

$$\Rightarrow \text{energetic light meson } E_L = \frac{m_B^2 + m_L^2 - q^2}{2m_B} \sim \frac{m_B}{2} \text{ for } q^2 \rightarrow 0$$

Soft universal FF's: ξ_i

[Charles et al hep-ph/9812358]

$B \rightarrow P$

$$\xi_P \approx f_+ \approx \frac{m_B}{2E_P} f_0 \approx \frac{m_B}{m_B + m_P} f_T$$

3 QCD FF's \rightarrow 1 soft FF

$B \rightarrow V$

$$\xi_{\perp} \approx \frac{m_B}{m_B + m_V} V \approx \frac{m_B + m_V}{2E_V} A_1 \approx T_1 \approx \frac{m_B}{2E_V} T_2$$

7 QCD FF's \rightarrow 2 soft FF's

$$\xi_{\parallel} \approx \frac{m_B + m_V}{2E_V} A_1 - \frac{m_B - m_V}{m_V} A_2 \approx \frac{m_V}{E_V} A_0 \approx \frac{m_B}{2E_V} T_2 - T_3$$

$\Rightarrow \mathcal{O}(\alpha_s)$ corrections known

[Beneke/Feldmann hep-ph/0008255]

!!! Some freedom of their definition $\mathcal{O}(\Lambda)$ (=renormalisation conventions)

$$\xi_P = f_+, \quad \xi_{\perp} = (m_B + m_V)/m_B V, \quad \xi_{\parallel} = m_V/E_V A_0$$

[Beneke/Feldmann hep-ph/0008255]

$$\xi_{\parallel} = (m_B + m_V)/(2E_V) A_1 - (m_B - m_V)(m_V) A_2$$

[Beneke/Feldmann/Seidel hep-ph/0412400]

$$\xi_{\perp} = T_1, \quad \xi_{\parallel} = m_V/E_V A_0$$

[Jäger/Martin-Camalich arXiv:1212.2263]

Helicity FF's

[Boyd/Savage hep-ph/9702300
 Bharucha/Feldmann/Wick arXiv:1004.3249
 Jäger/Martin-Camalich arXiv:1212.2263]

$B \rightarrow K^* \bar{\ell} \ell$ helicity amplitudes for $(K^*)_\lambda$ ($\lambda = \pm, 0$) are $\propto V_\lambda, T_\lambda$

$$V_\pm = \frac{1}{2} \left[\left(1 + \frac{m_V}{m_B} \right) A_1 \mp \frac{\sqrt{\lambda}}{m_B(m_B + m_V)} V \right]$$

$$V_0 = \frac{1}{2m_V\sqrt{\lambda}(m_B + m_V)} \left[(m_B + m_V)^2(m_B^2 - m_V^2 - q^2)A_1 - \lambda A_2 \right]$$

$$T_\pm = \frac{m_B^2 - m_V^2}{2m_B^2} T_2 \mp \frac{\sqrt{\lambda}}{2m_B^2} T_1$$

$$T_0 = \frac{m_B}{2m_V\sqrt{\lambda}} \left[(m_B^2 + 3m_V^2 - q^2)T_2 - \frac{\lambda}{m_B^2 - m_V^2} T_3 \right]$$

$$S = A_0$$

!!! @ large recoil

$$T_+ = \mathcal{O}(q^2/m_B^2) \times \mathcal{O}(\Lambda)$$

$$V_+ = \mathcal{O}(\Lambda)$$

@ LO in $\mathcal{O}(\Lambda)$ no corrections of $\mathcal{O}(\alpha_s)$

[Burdmann/Hiller hep-ph/0011266, Beneke/Yang hep-ph/0508250]

FF parameterisations

z -Expansion (also Series Expansion)

!!! is one of many parameterisations

Maps $q^2 \rightarrow z(q^2, t_0)$ with $t_{\pm} = (m_B \pm m_L)^2$

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

t_0 is free parameter \Rightarrow determines values that z assumes over kin. range of q^2 on unit disc (usually $-0.15 \leq z \leq 0.15$)

Many variants of z -expansion, FF's are generally parametrised as

$$f(q^2) = \frac{1}{B(q^2)\phi_f(q^2)} \sum_k a_k z^k(q^2, t_0)$$

$B(q^2)$ = Blaschke factor, chosen such that $|z(q^2)| = 1$ in $q^2 \geq t_+$ (pair-production region)

$\phi_f(q^2)$ can be chosen such, that dispersive bound on coefficients reads as

$$\sum_{k=0}^{\infty} (a_k)^2 < 1$$

[Bharucha/Feldmann/Wick arXiv:1004.3249]

Use z -Expansion to combine LCSR @ low q^2 with Lattice @ high q^2 ?

LCSR determinations

LCSR's

2 Types of LCSR's for $B \rightarrow K^*$ FF's

(see yesterday's talk by Alexander Khodjamirian)

interpolated B meson

[Ball/Zwicky hep-ph/0412079]

- ▶ tw-2 + tw-3 @ NLO, tw-4 @ LO, including 2- and 3-particle DA's of K^*
- ▶ errors below 15%

interpolated light meson

[Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]

- ▶ FF's are a by-product in this calculation
- ▶ no radiative corrections included
- ▶ large errors

Possible improvements ?

- ▶ update numerical input
- ▶ progress on K^* LCDA (Gegenbauer, higher-particle DA's)
- ▶ finite K^* -width effects
- ▶ higher order radiative corrections?
- ▶ μ -dependence
- ▶ higher order radiative corrections
- ▶ how well B -DA is known, can be known?
- ▶ many-particle DA's

!!! Provide full correlation among parameters of different FF's

Lattice determinations

- ▶ MILC (2 + 1) asqtad, b quarks in NRQCD, HISQ light and strange quarks
- ▶ 3 coarse and 2 fine ($L^3 \times N_t = 28^3 \times 96$) lattices
- ▶ chiral/continuum extrapolation
- ▶ kinematic extrapolation: z expansion (6 data points in q^2 per FF – highly correlated)
- ▶ determine f_+ , f_0 and f_T in simultaneous fit ($\chi^2/\text{dof} = 8.58/11$)

$$f_0 = a_0^0 + a_1^0 z + a_2^0 z^2 + a_3^0 z^3,$$

$$f_i = \frac{1}{1 - \frac{q^2}{m_B + \Delta_i^*}} \left(a_0^i + a_1^i z + a_2^i z^2 + \frac{z^3}{3} \right) \quad \text{for } i = +, T$$

- ▶ provide full correlation matrix of $a_{0,1,2,3}^0$, $a_{0,1,2}^+$ and $a_{0,1,2}^T$
- ▶ extensive study of systematic uncertainties
- ▶ no el-mgn. nor isospin breaking effects
- ▶ no charm sea quark

⇒ **FF-uncertainties** are below (8 – 10)% for f_T and (6 – 8)% for $f_{+,0}$ for $16 \text{ GeV}^2 \lesssim q^2$

$B \rightarrow K^*$ ($B_s \rightarrow \phi$, $B_s \rightarrow K^*$) FF's

[Horgan/Liu/Meinel/Wingate arXiv:1310.3722+ 1310.3887]

see yesterday's talk by Stefan Meinel

⇒ provide predictions for FF's: V , $A_{0,1,12}$, $T_{1,2,23}$

$$f_i(q^2) = \frac{1}{1 - \frac{q^2}{(m_B + \Delta_i)^2}} \left(a_0^i + a_1^i z(q^2) \right)$$

!!! so far only correlations of z -exp. coefficients for each FF separately, but no
“inter-FF”-correlations

Possible improvements ?

- ▶ provide also correlation between coefficients $a_{0,1}^i$ of different FF's, i.e., $i \neq j$
- ▶ domain wall light quarks?
- ▶ physical meson masses to avoid linear extrapolations
- ▶ improve perturbative lattice matching (renormalisation scheme conversions)
- ▶ calculate directly $B \rightarrow K\pi$ FF's ?
- ▶ finite width effects of K^* ?