

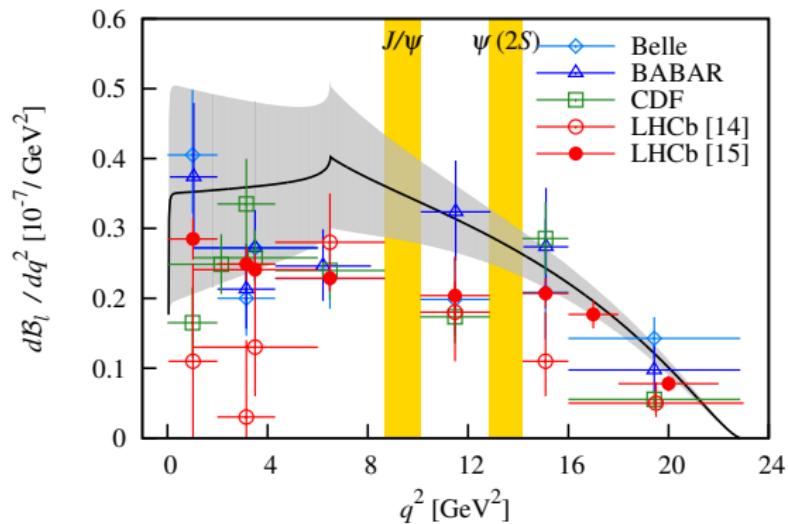
Current assumptions in lattice QCD calculations of $b \rightarrow s \ell^+ \ell^-$ processes

Stefan Meinel



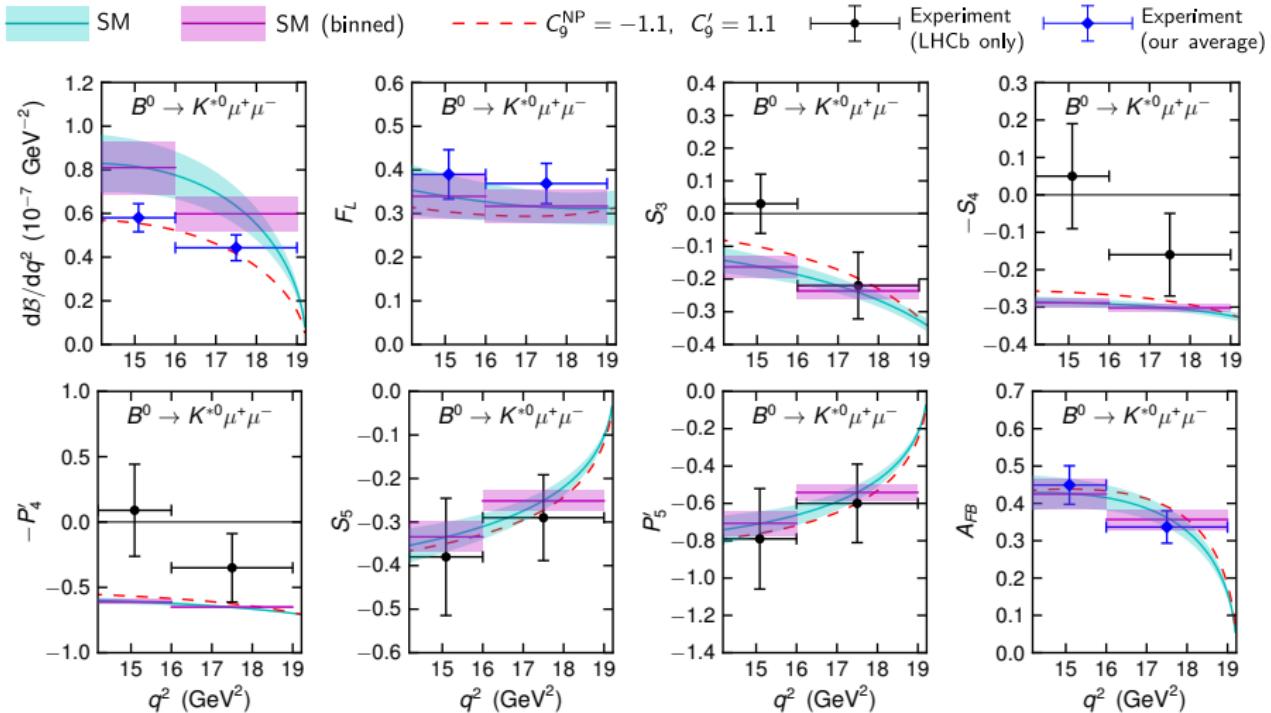
April 1, 2014

$B \rightarrow K\ell^+\ell^-$

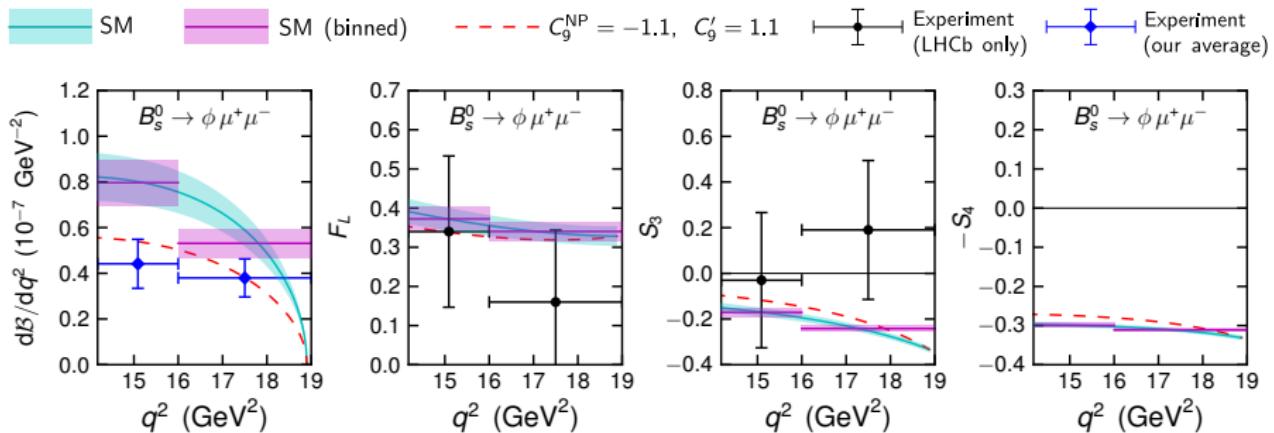


[C. Bouchard, G. P. Lepage, C. Monahan, H. Na, J. Shigemitsu, arXiv:1306.0434]

$$B^0 \rightarrow K^{*0} \mu^+ \mu^-$$

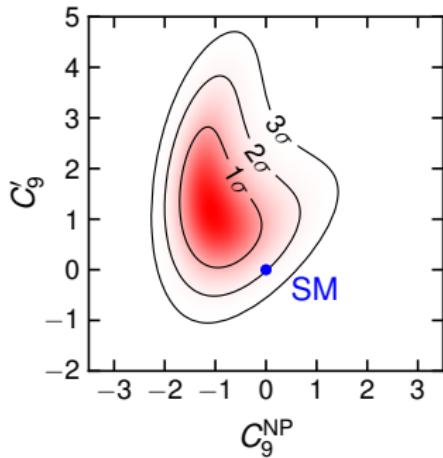


$B_s \rightarrow \phi \mu^+ \mu^-$



[R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, arXiv:1310.3887]

Fit of C_9 and C'_9 to $B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$



[R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, arXiv:1310.3887]

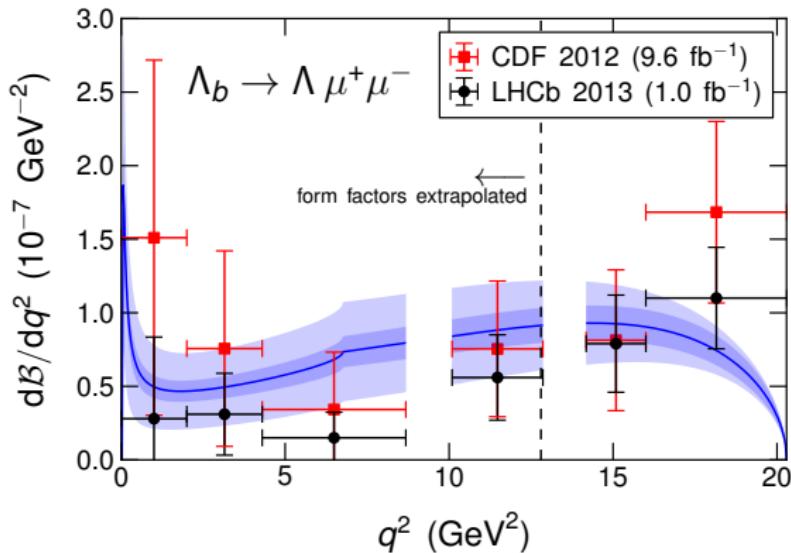
- This fit (high q^2 , lattice QCD):

$$C_9^{\text{NP}} = -1.1 \pm 0.5, \quad C'_9 = 1.1 \pm 0.9$$

- Altmannshofer and Straub (low q^2 , light-cone sum rules):

$$C_9^{\text{NP}} = -1.0 \pm 0.3, \quad C'_9 = 1.0 \pm 0.5$$

$$\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$$



[W. Detmold, C.-J. D. Lin, S. Meinel, M. Wingate, arXiv:1212.4827]

Overview of methods/assumptions

	$B \rightarrow K\ell^+\ell^-$	$B \rightarrow K^*\mu^+\mu^-$, $B_s \rightarrow \phi\mu^+\mu^-$	$\Lambda_b \rightarrow \Lambda\ell^+\ell^-$
	arXiv:1306.0434	arXiv:1310.3887	arXiv:1212.4827
u, d, s -quark action	staggered	staggered	Domain Wall
b -quark action	NRQCD	NRQCD	static
Chiral extrapolation	$\text{HM}\chi\text{PT}$	linear	linear
Continuum extrapolation	✓	✗	✓
Extrap. to low q^2	z -expansion	(high q^2 only)	dipole model
Treatment of $O_{1\dots 6}, O_8$	1-loop	OPE, 2-loop	1-loop
Strong decay of final hadron	N/A	Narrow-width approx.	N/A

Outline

- 1 Local vs nonlocal matrix elements
- 2 More about arXiv:1306.0434 ($B \rightarrow K\ell^+\ell^-$)
- 3 More about arXiv:1310.3887 ($B \rightarrow K^*\mu^+\mu^-$, $B_s \rightarrow \phi\mu^+\mu^-$)
- 4 Modelling the effects of the $\psi(3770)$ and $\psi(4160)$
- 5 Strong decays of the K^* and ϕ
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Local vs nonlocal matrix elements

Decay amplitude:

$$\mathcal{M} = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[(\mathcal{A}_\mu + \mathcal{T}_\mu) \bar{u}_\ell \gamma^\mu v_\ell + \mathcal{B}_\mu \bar{u}_\ell \gamma^\mu \gamma_5 v_\ell \right],$$

● Local:

$$\mathcal{A}_\mu = -\frac{2m_b}{q^2} q^\nu \langle K | \bar{s} i\sigma_{\mu\nu} (C_7 P_R + C'_7 P_L) b | B \rangle + \langle K | \bar{s} \gamma_\mu (C_9 P_L + C'_9 P_R) b | B \rangle$$

$$\mathcal{B}_\mu = \langle K | \bar{s} \gamma_\mu (C_{10} P_L + C'_{10} P_R) b | B \rangle$$

● Nonlocal:

$$\mathcal{T}_\mu = \frac{-16i\pi^2}{q^2} \sum_{i=1\dots 6;8} C_i \int d^4x e^{iq \cdot x} \langle K | T O_i(0) j_\mu(x) | B \rangle$$

OPE, 2-loop

At high q^2 :

$$\begin{aligned} T_\mu &= -T_7(q^2) \frac{2m_b}{q^2} q^\nu \langle \bar{K}^* | \bar{s} i\sigma_{\mu\nu} P_R b | \bar{B} \rangle + T_9(q^2) \langle \bar{K}^* | \bar{s} \gamma_\mu P_L b | \bar{B} \rangle \\ &\quad + \mathcal{O}(\Lambda^2/m_b^2, m_c^4/q^4, \alpha_s \Lambda/m_b), \end{aligned}$$

$$\begin{aligned} T_7(q^2) &= -(1/3) \left[C_3 + 4 C_4/3 + 20 C_5 + 80 C_6/3 \right] \\ &\quad + \alpha_s/(4\pi) \left[(C_1 - 6 C_2) A(q^2) - C_8 F_8^{(7)}(q^2) \right], \end{aligned}$$

$$\begin{aligned} T_9(q^2) &= (4/3) \left[C_3 + (16/3) C_5 + (16/9) C_6 \right] \\ &\quad + h(0, q^2) \left[4 C_1/3 + C_2 + 11 C_3/2 - 2 C_4/3 + 52 C_5 - 32 C_6/3 \right] \\ &\quad + (8 m_c^2/q^2) \left[4 C_1/9 + C_2/3 + 2 C_3 + 20 C_5 \right] \\ &\quad - h(m_b, q^2) \left[7 C_3/2 + 2 C_4/3 + 38 C_5 + 32 C_6/3 \right] \\ &\quad + \alpha_s/(4\pi) \left[C_1 (B(q^2) + 4 C(q^2)) - 3 C_2 (2 B(q^2) - C(q^2)) - C_8 F_8^{(9)}(q^2) \right]. \end{aligned}$$

1-loop

$$T_7 = -(1/3) \left[C_3 + 4 C_4/3 + 20 C_5 + 80 C_6/3 \right],$$

$$\begin{aligned} T_9(q^2) &= (4/3) \left[C_3 + (16/3)C_5 + (16/9)C_6 \right] \\ &\quad - \frac{1}{2} h(0, q^2) \left[C_3 + 4C_4/3 + 16C_5 + 64C_6/3 \right] \\ &\quad + h(m_c, q^2) \left[4C_1/3 + C_2 + 6C_3 + 60C_5 \right] \\ &\quad - \frac{1}{2} h(m_b, q^2) \left[7C_3 + 4C_4/3 + 76C_5 + 64C_6/3 \right] \end{aligned}$$

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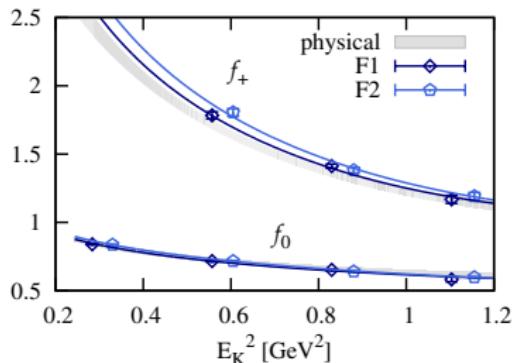
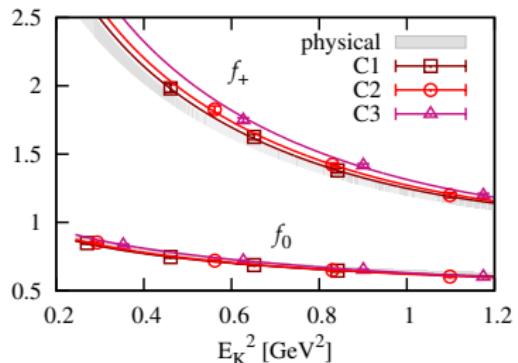
More about arXiv:1306.0434 ($B \rightarrow K\ell^+\ell^-$)

Lattice parameters:

Set	$L^3 \times T$	a (fm)	m_π (MeV)
C1	$24^3 \times 64$	≈ 0.12	≈ 260
C2	$20^3 \times 64$	≈ 0.12	≈ 370
C3	$20^3 \times 64$	≈ 0.12	≈ 520
F1	$28^3 \times 96$	≈ 0.09	≈ 340
F2	$28^3 \times 96$	≈ 0.09	≈ 480

More about arXiv:1306.0434 ($B \rightarrow K\ell^+\ell^-$)

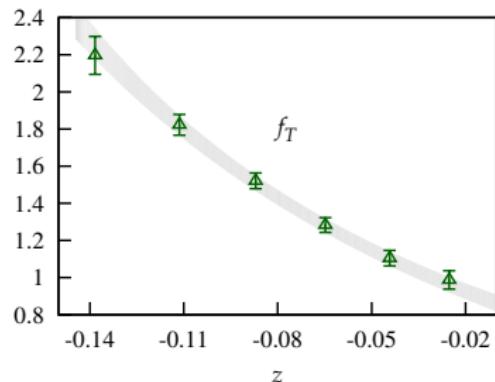
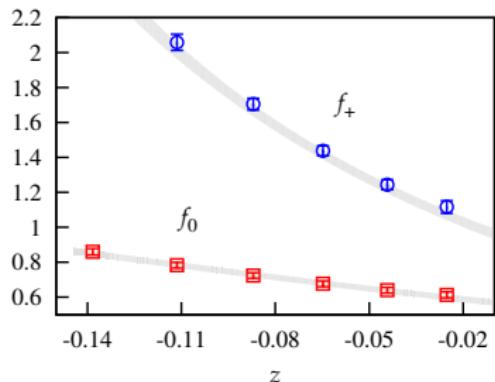
Data analysis step 1: chiral/continuum extrapolation using HM χ PT including a^2 terms and finite-V corrections



To estimate uncertainties, add NNLO terms with size limited by Bayesian constraints; also try removing highest E_K

More about arXiv:1306.0434 ($B \rightarrow K\ell^+\ell^-$)

Data analysis step 2: Generate synthetic data points from physical curves.
 Then extrapolate to low q^2 using z expansion



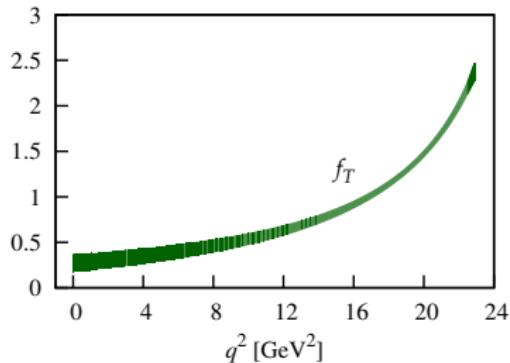
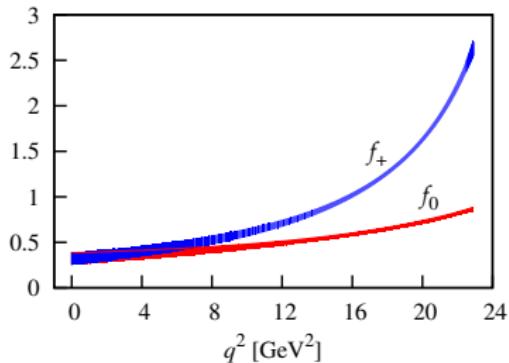
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

$$f_0(q^2) = \sum_{k=0}^K a_k^0 z(q^2)^k,$$

$$f_i(q^2) = \frac{1}{P_i(q^2)} \sum_{k=0}^{K-1} a_k^i \left[z(q^2)^k - (-1)^{k-K} \frac{k}{K} z(q^2)^K \right]$$

More about arXiv:1306.0434 ($B \rightarrow K\ell^+\ell^-$)

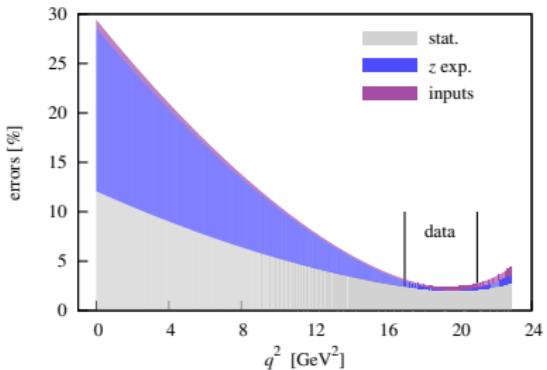
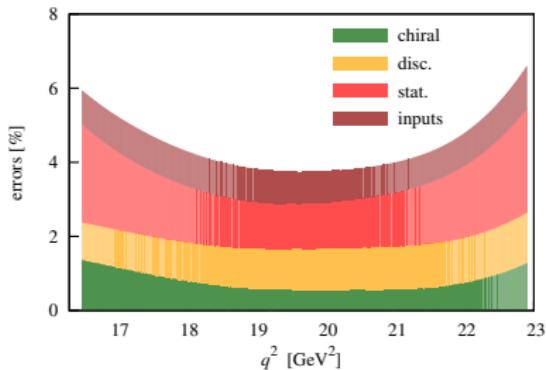
Data analysis step 2: Generate synthetic data points from physical curves.
Then extrapolate to low q^2 using z expansion



Main fits use $K = 3$. To estimate uncertainty, try $K = 2$ and $K = 4$. Also propagate uncertainties from changing the chiral/continuum fit functions.

More about arXiv:1306.0434 ($B \rightarrow K\ell^+\ell^-$)

Estimated form factor uncertainties (shown here: f_+):



+ 4% from missing higher-order terms in matching to $\overline{\text{MS}}$ scheme (done at one loop)

This assumes that higher-order terms are no larger than the included one-loop terms, which is itself a 4% correction.

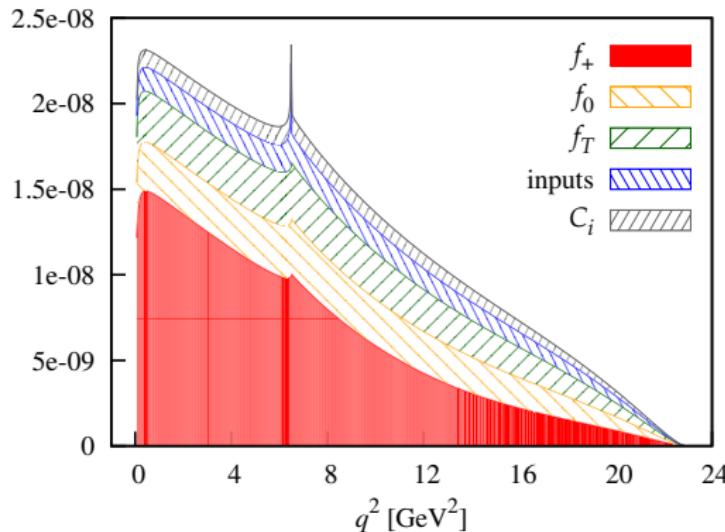
More about arXiv:1306.0434 ($B \rightarrow K\ell^+\ell^-$)

Assumptions in calculation of $B \rightarrow K\ell^+\ell^-$ observables (decay rate, flat term in angular distribution):

- Nonlocal matrix elements are included only at one loop. No uncertainty is assigned to them!
- Uncertainty of NNLO Wilson coefficients (taken from [Altmannshofer et al., arXiv:0811.1214]): 2%
- $\alpha_{e.m.}$ is evaluated at $\mu = m_Z$, giving $\alpha_{e.m.} = 128.957(20)$. No uncertainty is assigned to this scale choice
- observables are isospin-averaged

More about arXiv:1306.0434 ($B \rightarrow K\ell^+\ell^-$)

Estimated contributions to uncertainty in $B \rightarrow K\ell^+\ell^-$ decay rate:



Apparently dominated by the form factor uncertainties.

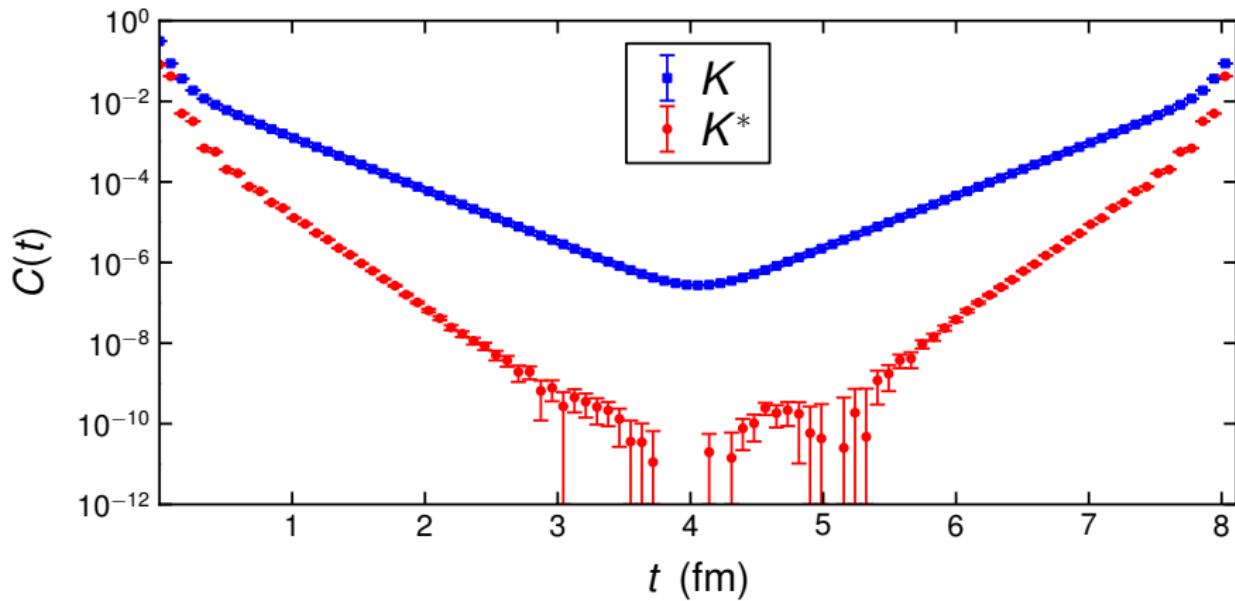
The large uncertainties due to the nonlocal matrix elements are missing.

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More about arXiv:1310.3887 ($B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$)

Statistical uncertainties: K vs K^* two-point function
using equal number of gauge configurations



More about arXiv:1310.3887 ($B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$)

→ **Statistical uncertainties will be dominant.**

We use fewer ensembles but higher statistics on each one:

Set	$L^3 \times T$	a (fm)	m_π (MeV)
c007	$24^3 \times 64$	0.1187(9)	313.4(2.4)
c02	$20^3 \times 64$	0.1183(9)	519.2(3.9)
f0062	$28^3 \times 96$	0.0846(6)	344.3(2.4)

More about arXiv:1310.3887 ($B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$)

We fit the lattice data using

$$F(q^2) = P_F(q^2) \left[(1 + c_{01} m_\pi^2) a_0 + a_1 z(q^2, t_0) \right],$$

where

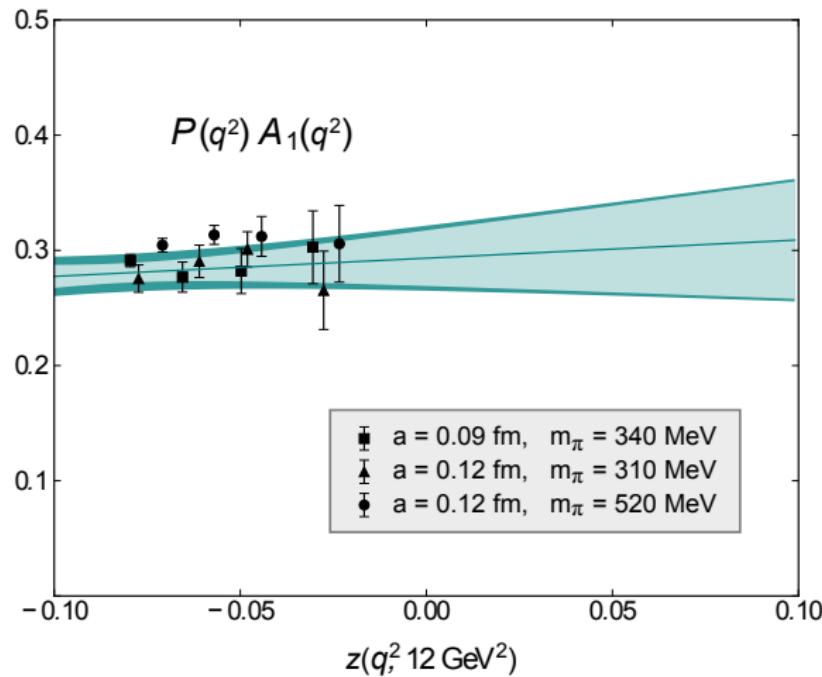
$$P_F(q^2) = \frac{1}{1 - q^2/m_{\text{pole}(F)}^2},$$

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_B + m_{K^*})^2, \quad t_0 = 12 \text{ GeV}$$

- There is no χ PT for vector mesons \rightarrow we assume linear $m_{u,d}$ dependence
- Because of the larger statistical uncertainties, we cannot resolve a nonzero lattice-spacing dependence of the form factors
- To describe the lattice data region, first order in z expansion is sufficient. We do not use the form factors at low q^2
- Version 3 (to be posted) includes a 4th parameter $c_{01,s}$ to describe m_s dependence

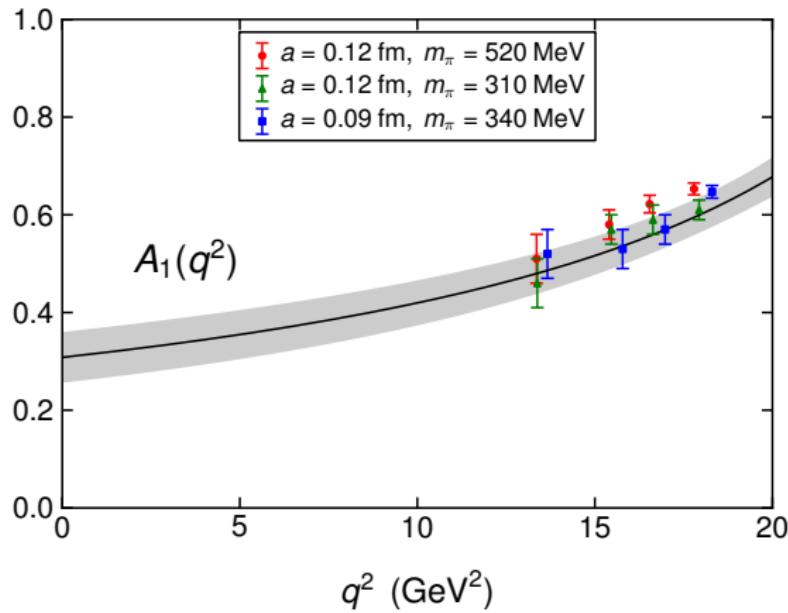
More about arXiv:1310.3887 ($B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$)

For example A_1 :



More about arXiv:1310.3887 ($B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$)

For example A_1 :



More about arXiv:1310.3887 ($B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$)

Estimates of systematic uncertainties in form factors:

- Missing $\mathcal{O}(\alpha_s^2)$ terms in matching: 4%
- Missing $\mathcal{O}(\alpha_s \Lambda/m_b)$ terms in matching: 2%
- Missing $\mathcal{O}(\Lambda^2/m_b^2)$ terms in matching: 1%
- Too heavy m_s : 2% – this is eliminated in version 3 (to be posted)

No estimates of discretization errors and chiral-extrapolation errors, but the results show no significant dependence on lattice spacing, and only mild quark-mass dependence.

No estimates of q^2 -extrapolation errors – that's ok because we only use our form factors in the region where we have lattice data

More about arXiv:1310.3887 ($B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$)

Assumptions in calculation of $B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$ observables:

- Nonlocal matrix elements are included using [two-loop OPE](#). Included an extra [5%](#) systematic uncertainty in vector amplitude ($\mathcal{A}_\mu + \mathcal{T}_\mu$) due to truncation of OPE / duality violations
- Uncertainty of NNLO Wilson coefficients (taken from [Altmannshofer et al., arXiv:0811.1214]): [2%](#)
- $\alpha_{\text{e.m.}}$ is evaluated at $\mu = m_b$, which minimizes higher-order electroweak corrections (at least for inclusive decay [C. Bobeth et al., hep-ph/0312090])
- Narrow-width approximation (next slide)

More about arXiv:1310.3887 ($B \rightarrow K^* \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$)

Narrow-width approximation [Krüger et al., hep-ph/9907386]:

$$\langle K^-(p_K) \pi^+(p_\pi) | J^\mu | \bar{B}(p) \rangle \approx -D_{K^*}(p'^2) \left[p_K^\nu - p_\pi^\nu - \frac{M_K^2 - M_\pi^2}{p'^2} (p_K^\nu + p_\pi^\nu) \right] \textcolor{magenta}{A}_{\nu\mu}$$

where $D_{K^*}(p'^2)$ is defined through

$$|D_{K^*}(p'^2)|^2 = \frac{48\pi^2}{\beta^3 M_{K^*}^2} \delta(k^2 - M_{K^*}^2)$$

and $\textcolor{magenta}{A}_{\nu\mu}$ is defined through

$$\langle \bar{K}^*(p', \varepsilon) | J^\mu | \bar{B}(p) \rangle = \varepsilon^{*\nu} \textcolor{magenta}{A}_{\nu\mu}.$$

Corresponds to pure P -wave decay $K^* \rightarrow K \pi$.

S -wave pollution is thought to be negligible at high q^2

[D. Bećirević and A. Tayduganov, arXiv:1207.4004]

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Modelling the effects of the $\psi(3770)$ and $\psi(4160)$

Replace

$$T_9(q^2) \longrightarrow T_9(q^2) + \frac{(\textcolor{red}{R}_1 + i\textcolor{violet}{I}_1)m_1^2}{q^2 - m_1^2 + im_1\Gamma_1} + \frac{(\textcolor{red}{R}_2 + i\textcolor{violet}{I}_2)m_2^2}{q^2 - m_2^2 + im_2\Gamma_2}$$

with

$$m_1 = 3773 \text{ MeV},$$

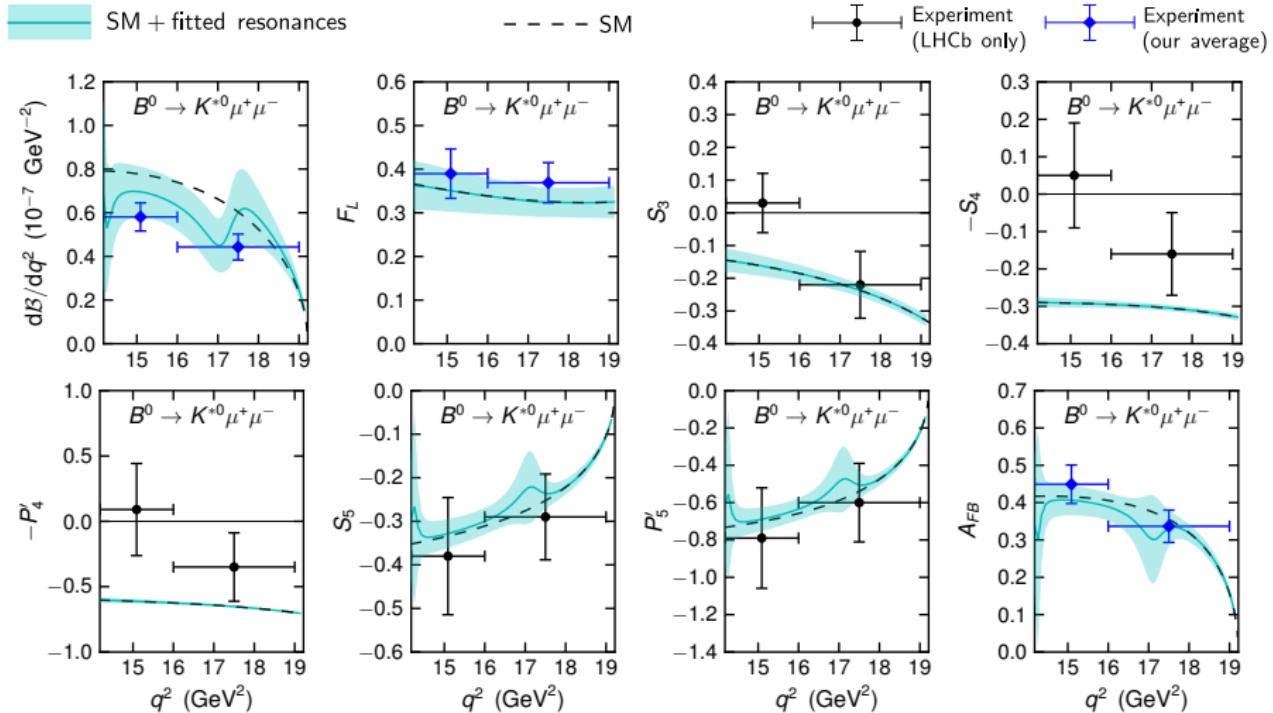
$$\Gamma_1 = 27 \text{ MeV},$$

$$m_2 = 4153 \text{ MeV},$$

$$\Gamma_2 = 103 \text{ MeV}.$$

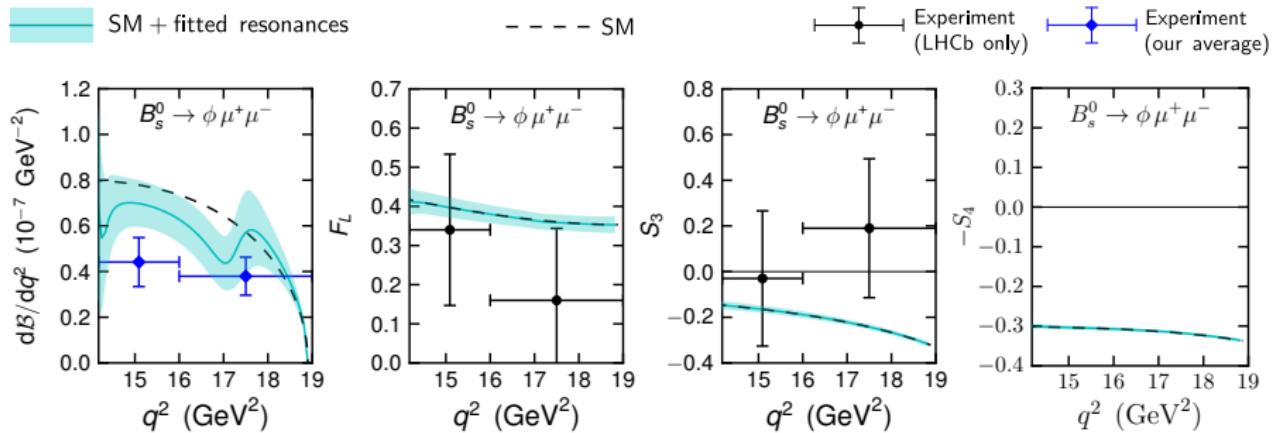
Now set $C_9^{\text{NP}} = 0$, $C'_9 = 0$ and fit instead $\textcolor{red}{R}_1$, $\textcolor{violet}{I}_1$, $\textcolor{red}{R}_2$, $\textcolor{violet}{I}_2$ to the experimental data
(assuming that these couplings are the same
for $B \rightarrow K^*\mu^+\mu^-$ and $B_s \rightarrow \phi\mu^+\mu^-$).

Modelling the effects of the $\psi(3770)$ and $\psi(4160)$



PRELIMINARY

Modelling the effects of the $\psi(3770)$ and $\psi(4160)$



PRELIMINARY

Modelling the effects of the $\psi(3770)$ and $\psi(4160)$

The fit gives

$$\begin{aligned} R_1 &= -0.005 \pm 0.020 \\ I_1 &= -0.011 \pm 0.021 \\ R_2 &= 0.040 \pm 0.039 \\ I_2 &= 0.032 \pm 0.021 \end{aligned}$$

and has

$$\Delta\chi^2 = 2.4$$

For comparison, the fit of C_9^{NP} and C'_9 (without $c\bar{c}$ resonances) achieves

$$\Delta\chi^2 = 5.7$$

Modelling the effects of the $\psi(3770)$ and $\psi(4160)$

Now set

$$R_1 = 0 \pm 0.02,$$

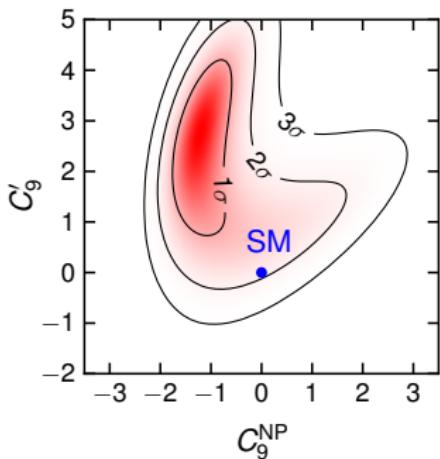
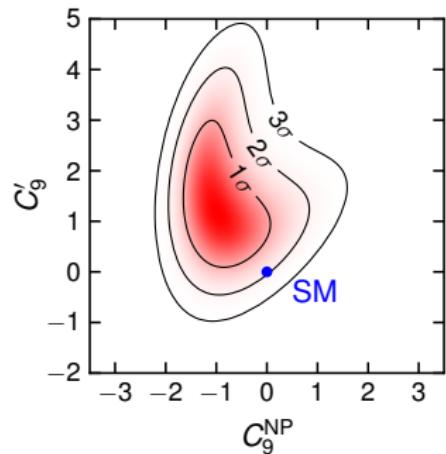
$$I_1 = 0 \pm 0.02,$$

$$R_2 = 0 \pm 0.04,$$

$$I_2 = 0 \pm 0.03$$

and perform fit of C_9^{NP} and C'_9

Modelling the effects of the $\psi(3770)$ and $\psi(4160)$



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Predicted L -dependence of spectrum

For single ρ -like resonance at zero momentum:

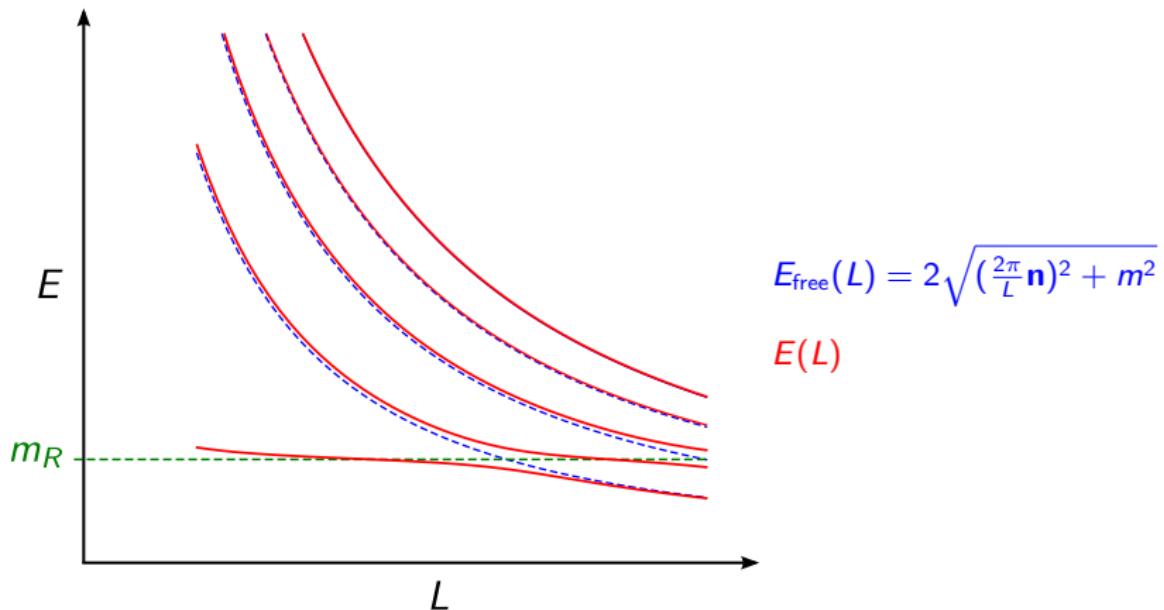
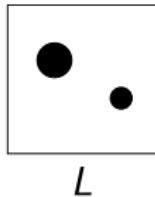


Figure based on [D. Mohler, arXiv:1211.6163]

Lüscher formula

Elastic scattering phase shifts from finite-volume energy shifts:



Define $p_*(L)$ through

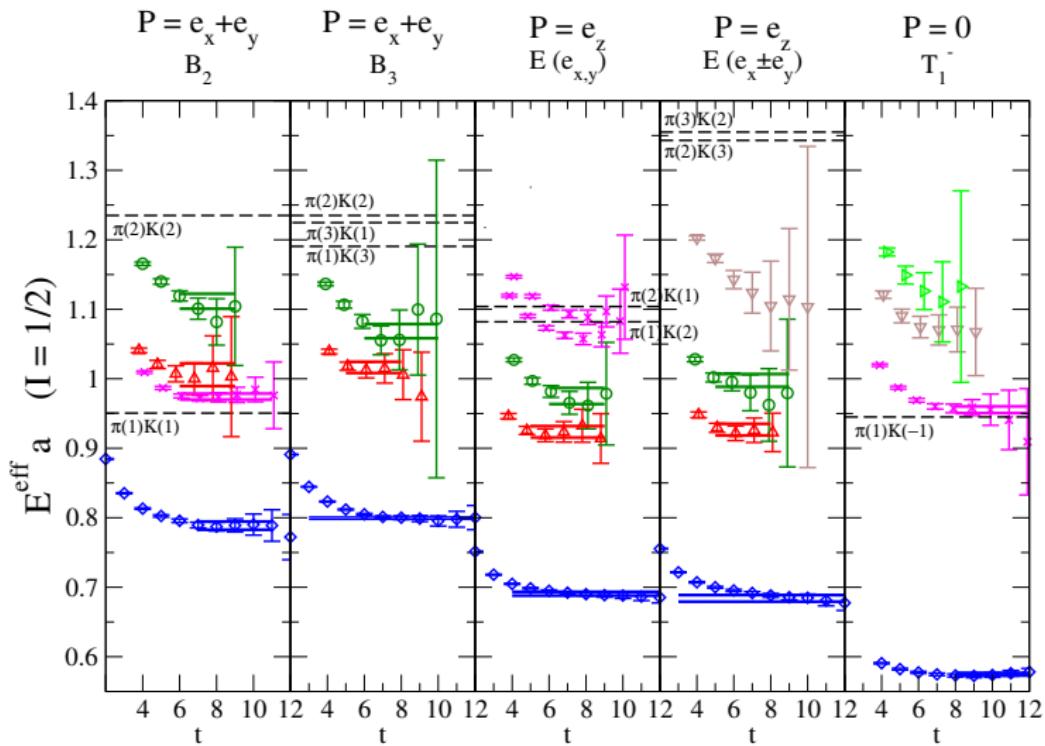
$$E(L) = \sqrt{p_*^2 + m_\pi^2} + \sqrt{p_*^2 + m_K^2}$$

Lüscher's formula relates p^* to the scattering phase shift:

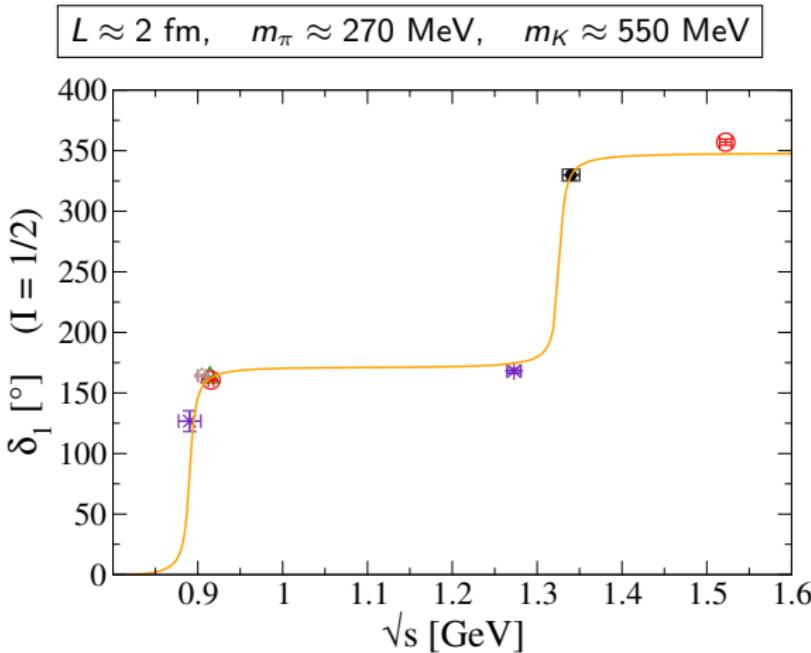
$$\delta(p^*) = f_{\text{Lüscher}}(p^*, L)$$

$|I = 1/2| P\text{-wave } K\pi \text{ scattering in lattice QCD}$

$L \approx 2 \text{ fm}, \quad m_\pi \approx 270 \text{ MeV}, \quad m_K \approx 550 \text{ MeV}$



$I = 1/2$ P -wave $K\pi$ scattering in lattice QCD

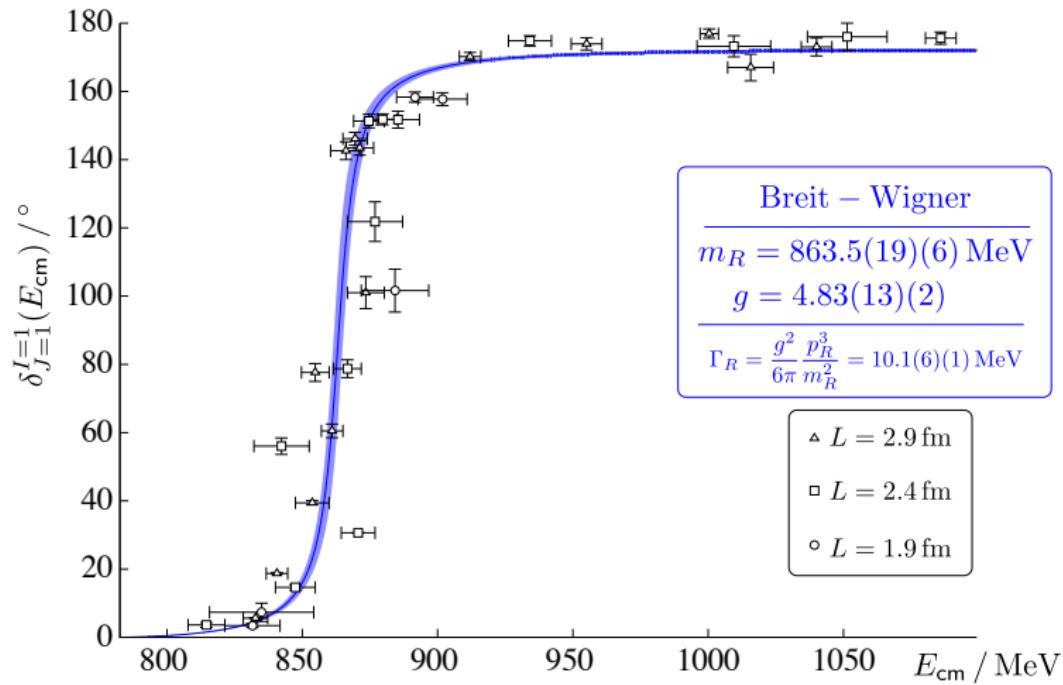


$$\Rightarrow g_{K^* K\pi} = 5.7 \pm 1.6 \quad (\text{experiment : } 5.72 \pm 0.06)$$

[S. Prelovsek, L. Leskovec, C. B. Lang, D. Mohler, arXiv:1307.0736]

$J = 1$ P -wave $\pi\pi$ scattering in lattice QCD

$m_\pi \approx 400$ MeV



Matrix elements: resonance or two-hadron?

Two possibilities:

- Aim to compute

$$\langle K(p_K)\pi(p_\pi)|\bar{s}\Gamma b|B(p_B)\rangle$$

on the lattice, using a generalization of the Lellouch-Lüscher approach

- Aim to compute “resonance matrix elements”, which are defined in the continuum through analytic continuation of the $B \rightarrow K^*$ three-point function to complex momentum

$$p_{K^*}^2 = m_{K^*}^2 - \Gamma_{K^*}^2/4 - i m_{K^*} \Gamma_{K^*}$$

[V. Bernard, D. Hoja, U.-G. Meißner, A. Rusetsky, arXiv:1205.4642]

Outline

- 1 Local vs nonlocal matrix elements
- 2 More about arXiv:1306.0434 ($B \rightarrow K\ell^+\ell^-$)
- 3 More about arXiv:1310.3887 ($B \rightarrow K^*\mu^+\mu^-$, $B_s \rightarrow \phi\mu^+\mu^-$)
- 4 Modelling the effects of the $\psi(3770)$ and $\psi(4160)$
- 5 Strong decays of the K^* and ϕ
- 6 More about arXiv:1212.4827 ($\Lambda_b \rightarrow \Lambda\ell^+\ell^-$)
- 7 $\Lambda_b \rightarrow \Lambda$ form factors with relativistic b quarks

More about arXiv:1212.4827 ($\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$)

- Leading-order (static) heavy-quark effective theory for b quark. Only 2 form factors in this limit:

$$\langle \Lambda | \bar{s} \Gamma Q | \Lambda_Q \rangle = \bar{u}_\Lambda [F_1 + \gamma F_2] \Gamma u_{\Lambda_Q}$$

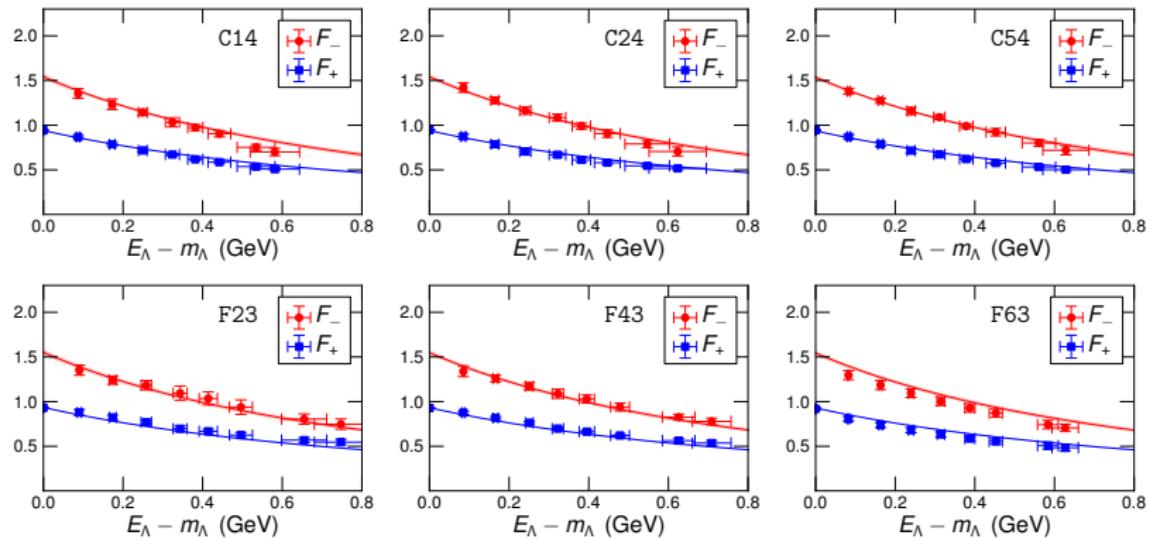
We use the combinations

$$F_+ = F_1 + F_2, \quad F_- = F_1 - F_2$$

- Domain-wall action for u, d, s quarks

Set	$L^3 \times T$	$am_{u,d}^{(\text{sea})}$	$am_s^{(\text{sea})}$	a (fm)	$am_{u,d}^{(\text{val})}$	$am_s^{(\text{val})}$	$m_\pi^{(\text{vv})}$ (MeV)
C14	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.001	0.04	245(4)
C24	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.002	0.04	270(4)
C54	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.005	0.04	336(5)
C53	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.005	0.03	336(5)
F23	$32^3 \times 64$	0.004	0.03	0.0849(12)	0.002	0.03	227(3)
F43	$32^3 \times 64$	0.004	0.03	0.0849(12)	0.004	0.03	295(4)
F63	$32^3 \times 64$	0.006	0.03	0.0848(17)	0.006	0.03	352(7)

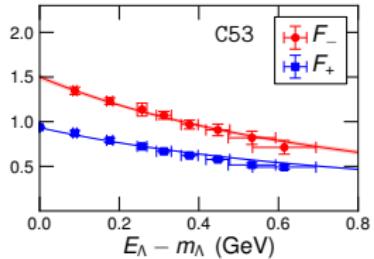
More about arXiv:1212.4827 ($\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$)



Dipole fit model:

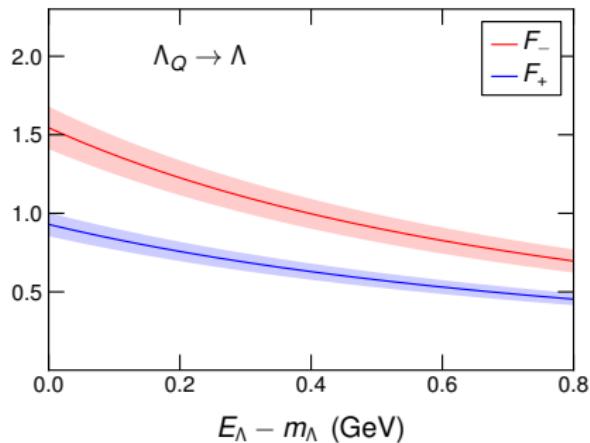
$$F_{\pm} = \frac{Y_{\pm}}{(\tilde{X}_{\pm} + E_{\Lambda} - m_{\Lambda})^2} [1 + d_{\pm}(aE_{\Lambda})^2],$$

$$\begin{aligned} \tilde{X}_{\pm} &= X_{\pm} + c_{l,\pm} [(m_{\pi})^2 - (m_{\pi}^{\text{phys}})^2] \\ &\quad + c_{s,\pm} [(m_{\eta_s})^2 - (m_{\eta_s}^{\text{phys}})^2] \end{aligned}$$



More about arXiv:1212.4827 ($\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$)

$\Lambda_Q \rightarrow \Lambda$ form factors at $m_\pi = m_\pi^{\text{phys}}$, $m_{\eta_s} = m_{\eta_s}^{\text{phys}}$, $a = 0$:

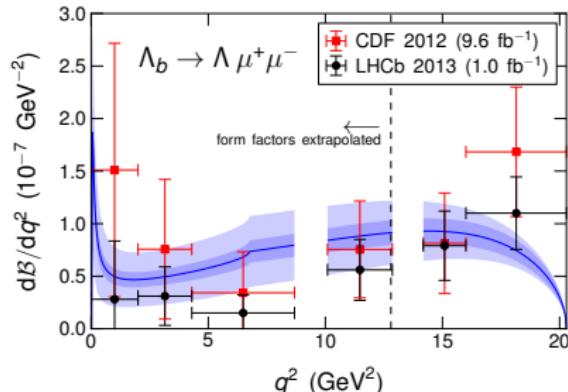


Error bands shown here include the following estimates of systematic uncertainties:

- renormalization: $\sim 6\%$
- finite volume: $\sim 3\%$
- chiral extrapolation: $\sim 3\%$
- continuum extrapolation: $\sim 2\%$

More about arXiv:1212.4827 ($\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$)

$\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ differential branching fraction



- Inner error band from statistical and systematic uncertainty in F_+ , F_-
- Outer error band includes $\frac{\sqrt{|\mathbf{p}'|^2 + \Lambda_{\text{QCD}}^2}}{m_b}$ uncertainty from static approximation
- no uncertainty estimates for:** Wilson coefficients, nonlocal matrix elements one-loop only, extrapolation to low q^2

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$\Lambda_b \rightarrow \Lambda$ form factors with relativistic b quarks

[S. Meinel, arXiv:1401.2685 and work in progress]

- for b quark, replace static action by “relativistic” heavy quark action
[RBC/UKQCD Collaboration, arXiv:1206.2554]
- “mostly nonperturbative” renormalization of heavy-light currents
- compute full set of 10 relativistic form factors

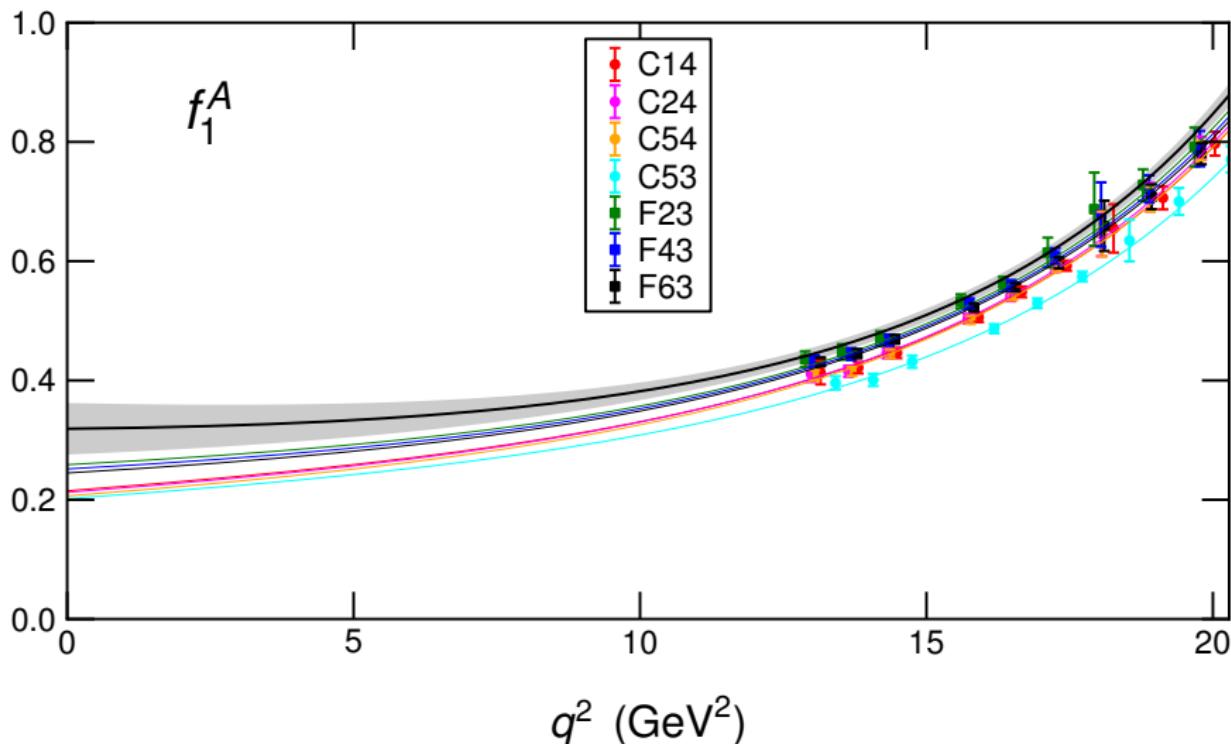
$$\begin{aligned}\langle \Lambda | \bar{s} \gamma^\mu b | \Lambda_b \rangle &= \bar{u}_\Lambda [f_1^V \gamma^\mu - f_2^V i\sigma^{\mu\nu} q_\nu / m_{\Lambda_b} + f_3^V q^\mu / m_{\Lambda_b}] u_{\Lambda_b}, \\ \langle \Lambda | \bar{s} \gamma^\mu \gamma_5 b | \Lambda_b \rangle &= \bar{u}_\Lambda [f_1^A \gamma^\mu - f_2^A i\sigma^{\mu\nu} q_\nu / m_{\Lambda_b} + f_3^A q^\mu / m_{\Lambda_b}] \gamma_5 u_{\Lambda_b}, \\ \langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu b | \Lambda_b \rangle &= \bar{u}_\Lambda [f_1^{TV} (\gamma^\mu q^2 - q^\mu \not{q}) / m_{\Lambda_b} - f_2^{TV} i\sigma^{\mu\nu} q_\nu] u_{\Lambda_b}, \\ \langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu \gamma_5 b | \Lambda_b \rangle &= \bar{u}_\Lambda [f_1^{TA} (\gamma^\mu q^2 - q^\mu \not{q}) / m_{\Lambda_b} - f_2^{TA} i\sigma^{\mu\nu} q_\nu] \gamma_5 u_{\Lambda_b}\end{aligned}$$

- will publish updated predictions for decay rate and **angular observables** of $\Lambda_b \rightarrow \Lambda(\rightarrow p \pi^-) \mu^+ \mu^-$

$\Lambda_b \rightarrow \Lambda$ form factors with relativistic b quarks

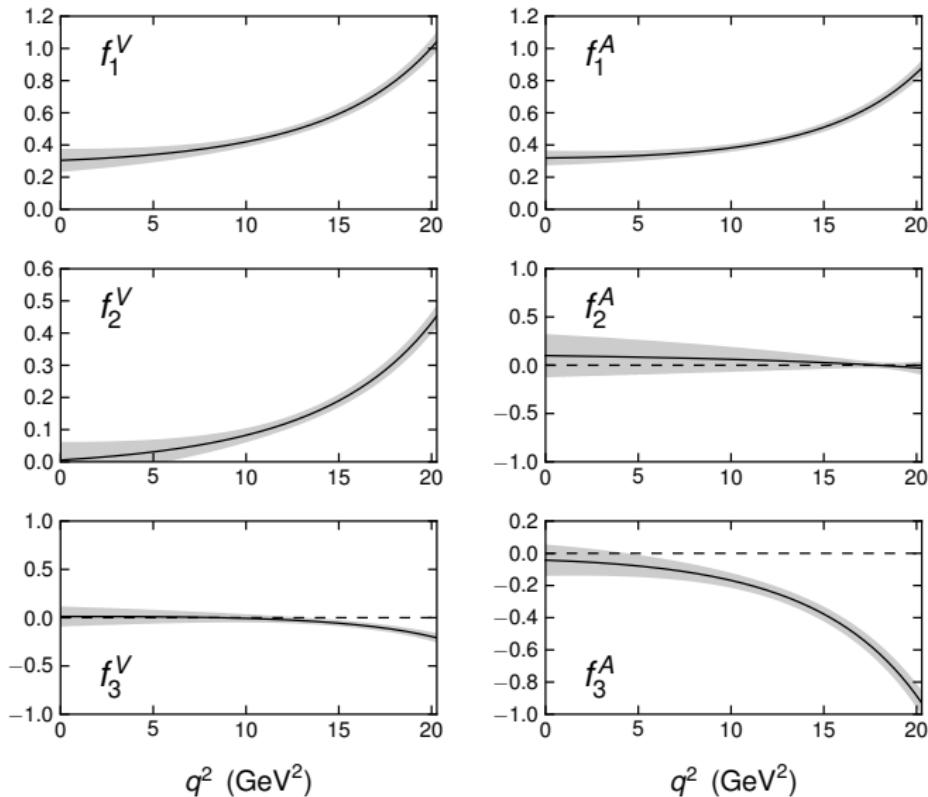
Chiral/continuum extrapolation using modified z expansion (to order z^2)

PRELIMINARY



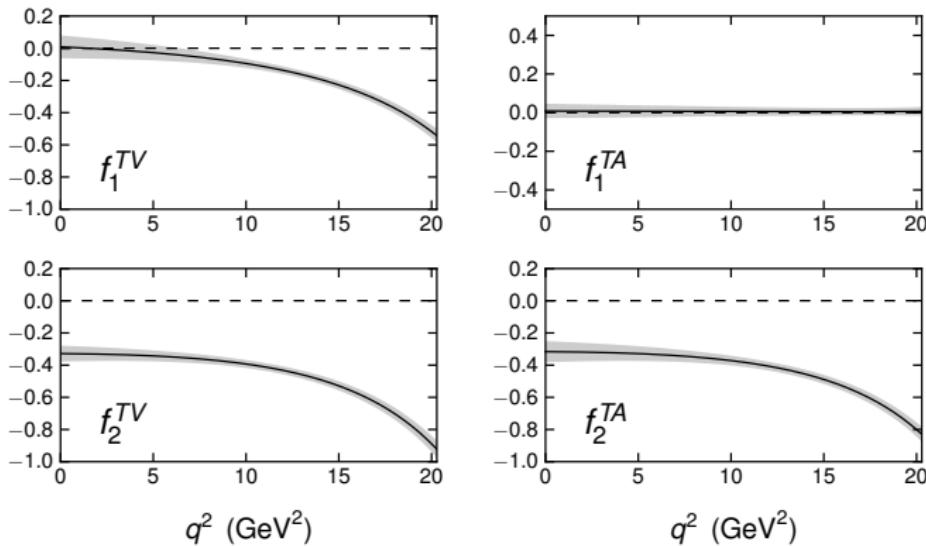
$\Lambda_b \rightarrow \Lambda$ form factors with relativistic b quarks

PRELIMINARY



$\Lambda_b \rightarrow \Lambda$ form factors with relativistic b quarks

PRELIMINARY



Extra slides

Lattice calculation of the local matrix elements

Interpolating fields; $b \rightarrow s$ current:

$$\begin{aligned}\Phi^{(B)} &= \bar{d} \gamma_5 b \\ \Phi_\mu^{(K^*)} &= \bar{d} \gamma_\mu s \\ J_\Gamma &= Z_\Gamma \bar{s} \Gamma b\end{aligned}$$

Three-point and two-point functions:

$$\begin{aligned}C_\mu^{(B \rightarrow K^*)} &= \sum_y \sum_z e^{-i\mathbf{p}' \cdot (\mathbf{x}-\mathbf{y})} e^{-i\mathbf{p} \cdot (\mathbf{y}-\mathbf{z})} \left\langle \Phi_\mu^{(K^*)}(x) \ J_\Gamma(y) \ \Phi^{(B)\dagger}(z) \right\rangle_{U,\psi,\bar{\psi}} \\ C^{(B)} &= \sum_x e^{-i\mathbf{p}' \cdot (\mathbf{x}-\mathbf{z})} \left\langle \Phi^{(B)}(x) \ \Phi^{(B)\dagger}(z) \right\rangle_{U,\psi,\bar{\psi}} \\ C_{\mu\nu}^{(K^*)} &= \sum_x e^{-i\mathbf{p} \cdot (\mathbf{x}-\mathbf{z})} \left\langle \Phi_\mu^{(K^*)}(x) \ \Phi_\nu^{(K^*)\dagger}(z) \right\rangle_{U,\psi,\bar{\psi}}\end{aligned}$$

$$\langle O \rangle_{U,\psi,\bar{\psi}} \equiv \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \ O \ e^{-(S_G + S_F)}$$

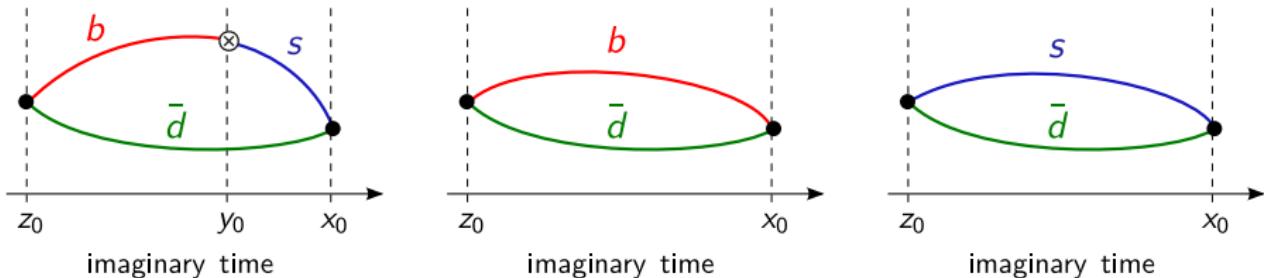
Lattice calculation of the local matrix elements

After performing the path integral over fermions:

$$\begin{aligned} C_\mu^{(B \rightarrow K^*)} &= \sum_y \sum_z e^{-i\mathbf{p}' \cdot (\mathbf{x}-\mathbf{y})} e^{-i\mathbf{p} \cdot (\mathbf{y}-\mathbf{z})} \left\langle \text{Tr} [\gamma_\mu G_s(x, y) \Gamma_J G_b(y, z) \gamma_5 G_d(z, x)] \right\rangle \\ C^{(B)} &= \sum_x e^{-i\mathbf{p}' \cdot (\mathbf{x}-\mathbf{z})} \left\langle \text{Tr} [\gamma_5 G_d(x, z) \gamma_5 G_b(z, x)] \right\rangle \\ C_{\mu\nu}^{(K^*)} &= \sum_x e^{-i\mathbf{p} \cdot (\mathbf{x}-\mathbf{z})} \left\langle \text{Tr} [\gamma_\mu G_d(x, z) \gamma_\nu G_s(z, x)] \right\rangle \end{aligned}$$

where

$$\langle O \rangle \equiv \frac{1}{Z} \int \mathcal{D}U \ O \ \det[D_F] \ e^{-S_G} \quad (\text{done numerically})$$



Lattice calculation of the local matrix elements

Spectral decomposition for large $|x_0 - y_0|$ and $|y_0 - z_0|$:

$$C_{\mu}^{(B \rightarrow K^*)} = \frac{1}{2E_{K^*}} \frac{1}{2E_B} \sum_{K^* \text{ spin}} \langle 0 | \Phi_{\mu}^{(K^*)} | K^* \rangle \langle K^* | \bar{s} \Gamma b | B \rangle \langle B | \Phi^{(B)\dagger} | 0 \rangle \\ \times e^{-E_{K^*}|x_0 - y_0|} e^{-E_B|y_0 - z_0|}$$

$$C^{(B)} = \frac{1}{2E_B} \langle 0 | \Phi^{(B)} | B \rangle \langle B | \Phi^{(B)\dagger} | 0 \rangle e^{-E_B|x_0 - y_0|}$$

$$C_{\mu\nu}^{(K^*)} = \frac{1}{2E_{K^*}} \sum_{K^* \text{ spin}} \langle 0 | \Phi_{\mu}^{(K^*)} | K^* \rangle \langle K^* | \Phi_{\nu}^{(K^*)\dagger} | 0 \rangle e^{-E_{K^*}|x_0 - y_0|}$$

$B \rightarrow K^*$ form factor definitions

$$\langle K^*(p', \varepsilon) | \bar{s} \gamma^\mu b | B(p) \rangle = \frac{2i \textcolor{red}{V}}{M_B + M_{K^*}} \epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* p'_\rho p_\sigma,$$

$$\begin{aligned} \langle K^*(p', \varepsilon) | \bar{s} \gamma^\mu \gamma_5 b | B(p) \rangle &= 2M_{K^*} \textcolor{red}{A}_0 \frac{\varepsilon^* \cdot q}{q^2} q^\mu \\ &\quad + (M_B + M_{K^*}) \textcolor{red}{A}_1 \left[\varepsilon^{*\mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right] \\ &\quad - \textcolor{red}{A}_2 \frac{\varepsilon^* \cdot q}{M_B + M_{K^*}} \left[p^\mu + p'^\mu - \frac{M_B^2 - M_{K^*}^2}{q^2} q^\mu \right], \end{aligned}$$

$$q^\nu \langle K^*(p', \varepsilon) | \bar{s} \sigma_{\mu\nu} b | B(p) \rangle = 4 \textcolor{red}{T}_1 \epsilon_{\mu\rho\kappa\sigma} \varepsilon^{*\rho} p^\kappa p'^\sigma,$$

$$\begin{aligned} q^\nu \langle K^*(p', \varepsilon) | \bar{s} \sigma_{\mu\nu} \gamma_5 b | B(p) \rangle &= -2i \textcolor{red}{T}_2 \left[\varepsilon_\mu^* (M_B^2 - M_{K^*}^2) - (\varepsilon^* \cdot q)(p + p')_\mu \right] \\ &\quad - 2i \textcolor{red}{T}_3 (\varepsilon^* \cdot q) \left[q_\mu - \frac{q^2}{M_B^2 - M_{K^*}^2} (p + p')_\mu \right]. \end{aligned}$$