Comments on $B \to K^{(*)}$ form factors calculated from LCSRs

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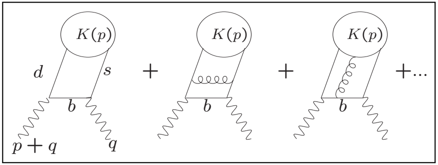
Questions to be discussed

- basic assumptions and input in the LCSR calculation
- use of other QCD sum rules
- the error budget; any tacit assumptions ?
- optimizing observables
- tasks for the future

mainly for $B \to K$ form factors

LCSR for $B \to K$ form factors: scheme of derivation

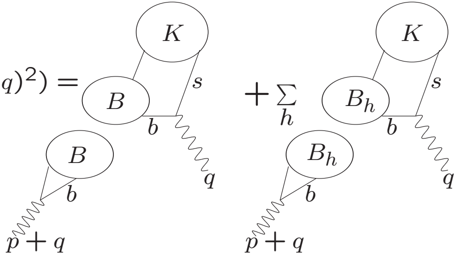


$\Leftarrow$ the correlation function

calculated in terms of
Operator Product Expansion
at $(p+q)^2, q^2 \ll m_b^2$

$\Downarrow$

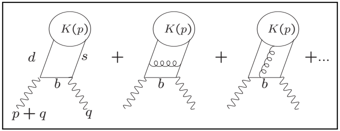
hadronic dispersion relation $\Big\}$ $F(q^2,(p+q)^2) =$



$f_B \; f^+_{BK}(q^2)$ $\qquad\qquad$ $\sum_{B_h} \to duality \; (s_0^B)$

Basic assumptions: OPE

- the correlation function   at fixed $q^2 \ll m_b^2$



$$[F(q^2,(p+q)^2)]_{OPE} =$$

$$= \sum_{t=2,3,4,..} \int_0^1 \mathcal{D}u \; T^{(t)}(\alpha_s, m_b, m_s; q^2, (p+q)^2, u, \mu)\, \varphi_K^{(t)}(u,\mu)$$

$$\Uparrow \qquad\qquad \Uparrow$$

{diagrams with b-propagator} $\otimes$ {kaon Distribution Amplitudes}

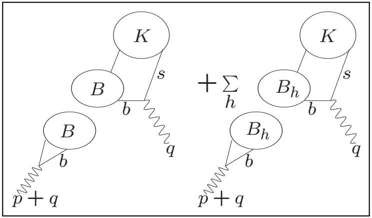
- kaon DA's, polynomial expansion:

$$\varphi_K^{(t)}(u,\mu) = f_K^{(t)}\{C_0(u) + \sum_{n=1} a_n^{(t)}(\mu) C_n(u)\}$$

- input for OPE
  - truncation level: $O(\alpha_s)$, $t \leq 4$, $n \leq 4$
  - parameters m_b, m_s, α_s, $f_K^{(t)}$, $a_n^{(t)}(\mu_0)$
  - variable scales: μ, $(p+q)^2 \to M^2 \sim m_b\chi$, $m_b \gg \chi \gg \Lambda_{QCD}$

Basic assumptions: hadronic dispersion relation

- hadronic dispersion relation
  (analyticity $\oplus$ unitarity in QFT)



$$[F(q^2,(p+q)^2)]_{OPE} = \frac{m_B^2 f_B f^+_{BK}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds \, \frac{\rho_h(s)}{s-(p+q)^2}$$

- quark-hadron "semilocal" duality

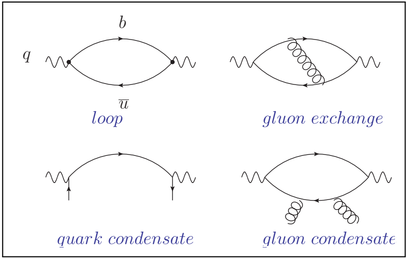
$$\Uparrow$$

$$\boxed{\int\limits_{(m_{B^*}+m_K)^2}^{\infty} ds \, \frac{\rho_h(s)}{s-(p+q)^2} = \int\limits_{s_0}^{\infty} ds \, \frac{[\mathrm{Im} F(q^2,s)]_{OPE}]}{s-(p+q)^2}}$$

- input:
  - f_B (2pt SR)
  - variable scale: $(p+q)^2 \to M^2 \sim m_b\chi$ $\to$ optimal interval of M^2
  - s_0 (determined by calculating m_B^2)

Use of 2-point QCD sum rules

- vacuum-to-vacuum correlation function $\langle 0|\bar{b}\gamma_5 u(x), \bar{u}\gamma_5 b(0)|0\rangle$:



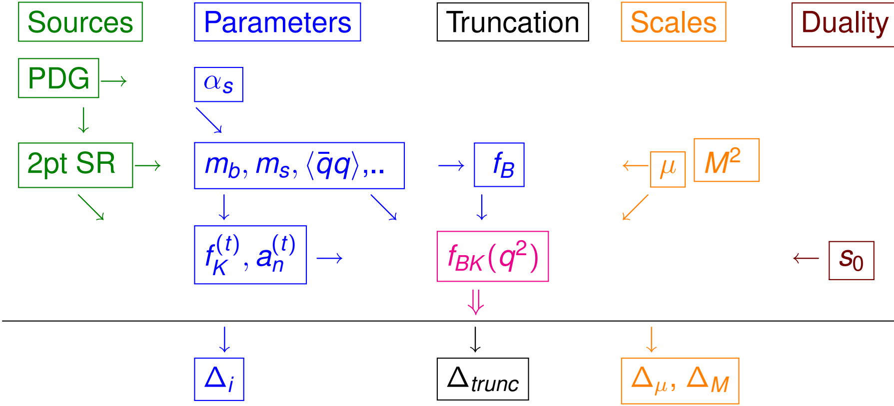
$$= \overbrace{\frac{\langle 0|\bar{b}\gamma_5 u|B\rangle\langle B|\bar{u}\gamma_5 b|0\rangle}{m_B^2 - q^2}}^{f_B^2} \underbrace{+ \sum_{B_h} \frac{\langle 0|\bar{b}\gamma_5 u|B_h\rangle\langle B_h|\bar{u}\gamma_5 b|0\rangle}{m_{B_h}^2 - q^2}}_{\text{quark-hadron duality}}$$

- input: $m_b, \alpha_s, \langle\bar{q}q\rangle$, ...
- other important 2pt sum rules providing the input:
  - $\langle 0|\bar{b}\gamma_\mu, b(x)\bar{b}\gamma_\mu b(0)|0\rangle$ saturated by Υ states

  $\Rightarrow$ non-lattice determination of m_b (in $\overline{MS}$ scheme)

  - various 2pt sum rules with kaon currents: $\Rightarrow m_s$, $f_K^{(t)}, a_n^{(t)}$

Summary on assumptions, input and error budget



- total uncertainty estimate:

$$\boxed{\Delta f_{BK}(q^2) = \sqrt{\sum_i \Delta_i^2 + \Delta_{trunc}^2 + \Delta_\mu^2 + \Delta_M^2}}$$

correlations so far neglected !

Current accuracy of $B \to K^{(*)}$ form factors

- $0 < q^2 \leq 12 - 14$ GeV^2 estimated uncertainties for $B \to \pi, K$ form factors amount to $\pm(12 - 15)\%$

- "systematic error" of quark-hadron duality approximation

(suppressed with Borel transformation, controlled by the m_B calculation)

- optimizing/reducing uncertainties:

ratios of form factors, slopes, asymmetries, bins of BRs in q^2

- LCSR's for $B \to K^*$ form factors, accuracy of the correlation function at the same level as for $B \to K$

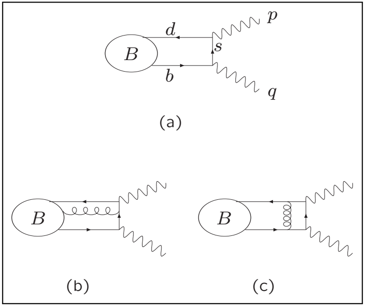
[P. Ball, V.Braun (1998), P.Ball, R. Zwicky (2004,...)]

ask Roman about detailed uncertainties

- $\Gamma_V = 0$ approximation (sort of "quenched")

  $\Rightarrow$ additional uncertainty

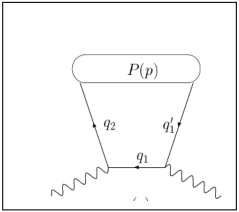
Use of other LCSRs and other observables



- LCSRs with *B* meson DAs
-an alternative method valid for all
$B \to P, V$ form factors

still large errors related to the B-meson DA,
absence of NLO $O(\alpha_s)$-corrections

- cross check of $f_K^{(t)}, a_n^{(t)}$ from LCSRs for
$D \to K$ form factors vs experiment



- LCSRs for the kaon
electromagnetic form factor
at large spacelike $q^2 = -Q^2$:

anticipating important constraints on kaon twist-2 DAs

(ongoing kaon electroproduction measurement at Jefferson Lab)

Tasks for the future

- improving LCSRs with B meson DAs

- $B \to \pi\pi(\rho, f_0), B \to K\pi(K^*, \kappa(0^+))$
form factors from LCSRs with 2-meson DAs

- OPE for $B \to \pi, K$:
  twist-2 complete NNLO;
  twist 3 NLO for nonasymptotic DAs;
  twist 5 LO; e.m. corrections to LCSRs

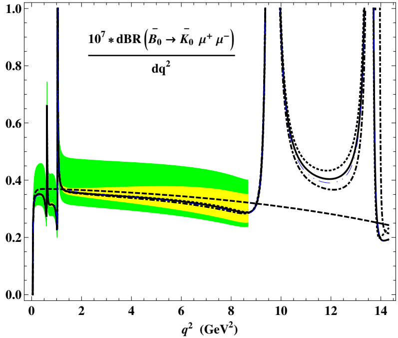
$d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2$ and bins

from [arXiv:1211.0234 [hep-ph]]

solid (dotted) lines - central input, default (alternative) parametrization for the dispersion integrals.

long-dashed line -the width calculated without nonlocal hadronic effects.

The green (yellow) shaded area indicates the uncertainties including (excluding) the one from the $B \to K$ FF normalization.



- our predicted *dBR* is somewhat lalrger than the in the LHCb paper 1403.8044 [hep-ex],
- tension due to the form factor $B \to K$?

LCSR agrees with the most recent HPQCD $B \to K$ FF

- isospin asymmetry is now in a better agreement with our expectations for SM

questions/comments please !

Backup Slides

Building up the OPE for $B \to \pi, K$ LCSRs

$$F(q^2,(p+q)^2) = \left(T_0^{(2)} + (\alpha_s/\pi)T_1^{(2)}\right) \otimes \varphi_K^{(2)}$$

$$+\frac{\mu_K}{m_b}\left(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}\right) \otimes \varphi_K^{(3)} + \frac{\delta_K^2}{m_b\chi}T^{(4)} \otimes \varphi_K^{(4)} + \ldots$$

$$\mu_K = m_K^2/(m_s+m_u), \quad m_b \gg \chi \gg \Lambda_{QCD}$$

- LO twist 2,3,4 $q\bar{q}$ and $q\bar{q}G$ terms
  [V.Belyaev, A.K., R.Rückl (1993); V.Braun, V.Belyaev, A.K., R.Rückl (1996)]
- NLO $O(\alpha_s)$ twist 2, (collinear factorization)
  [A.K., R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997);]
- NLO $O(\alpha_s)$ twist 3 (coll.factorization for asympt. DA)
  [P. Ball, R. Zwicky (2001); G.Duplancic,A.K.,B.Melic, Th.Mannel,N.Offen (2007)]
- part of NNLO $O(\alpha_s^2\beta_0)$ twist 2 *[A. Bharucha (2012)]*

$B_{(s)}$ and $D_{(s)}$ decay constants

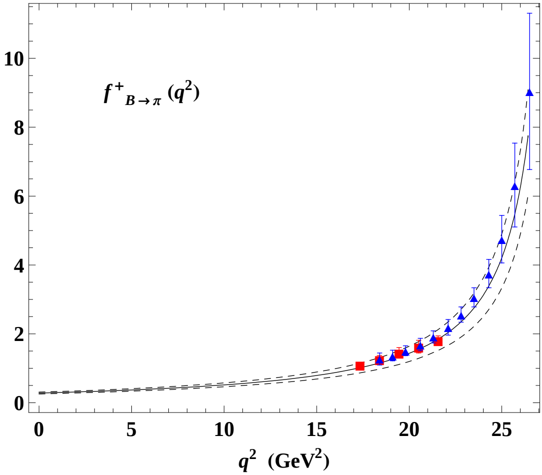
[P.Gelhausen, AK, A.A.Pivovarov, D.Rosenthal, 1305.5432 hep/ph]

Decay constant	Lattice QCD [ref.]	this work
f_B[MeV]	196.9 ± 9.1 [1] 186 ± 4 [2]	207^{+17}_{-9}
f_{B_s}[MeV]	242.0 ± 10.0 [1] 224 ± 5 [2]	242^{+17}_{-12}
f_{B_s}/f_B	1.229 ± 0.026 [1] 1.205 ± 0.007 [2]	$1.17^{+0.04}_{-0.03}$
f_D[MeV]	218.9 ± 11.3 [1] 213 ± 4 [2]	201^{+12}_{-13}
f_{D_s}[MeV]	260.1 ± 10.8 [1] 248.0 ± 2.5 [2]	238^{+13}_{-23}
f_{D_s}/f_D	1.188 ± 0.025 [1] 1.164 ± 0.018 [2]	$1.15^{+0.04}_{-0.05}$

[1]-Fermilab/MILC, [2]-HPQCD

$B \to \pi$ form factor: LCSR vs lattice QCD

[A.K, Th.Mannel, N.Offen, Y-M. Wang (2011)]



$q^2 \leq 12$ GeV^2 -LCSR, $q^2 > 12$ GeV^2 - [HPQCD, FNAL/MILC]

$B \to K$ form factor: LCSR vs lattice QCD

- dashed: LCSR, central input

[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]

- solid: unitarity bounds for the z-transformed form factor,

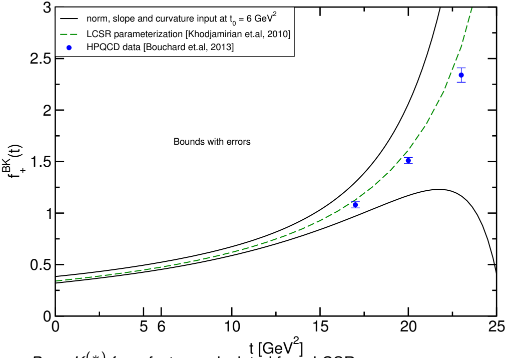
[L.Lellouch (1996); Th.Mannel,B.Postler(1998)]

(PRELIMINARY),
S.Imsong, AK, Th.Mannel,
work in progress

input for the bounds :

$f^{+}_{BK}(q^2 = 6.0\ \text{GeV}^2)$

$\oplus$ slope $\oplus$ curvature



$B \to K, K^{(*)}$ form factors from LCSR's

[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]

form factor	$F^i_{BK^{(*)}}(0)$	b^i_1	$B_s(J^P)$	input at $q^2 < 12$ GeV^2
f^+_{BK}	$0.34^{+0.05}_{-0.02}$	$-2.1^{+0.9}_{-1.6}$	$B_s^*(1^-)$	
f^0_{BK}	$0.34^{+0.05}_{-0.02}$	$-4.3^{+0.8}_{-0.9}$	no pole	LCSR
f^T_{BK}	$0.39^{+0.05}_{-0.03}$	$-2.2^{+1.0}_{-2.00}$	$B_s^*(1^-)$	with K DA's
V^{BK^*}	$0.36^{+0.23}_{-0.12}$	$-4.8^{+0.8}_{-0.4}$	$B_s^*(1^-)$	
$A_1^{BK^*}$	$0.25^{+0.16}_{-0.10}$	$0.34^{+0.86}_{-0.80}$	$B_s(1^+)$	
$A_2^{BK^*}$	$0.23^{+0.19}_{-0.10}$	$-0.85^{+2.88}_{-1.35}$	$B_s(1^+)$	LCSR
$A_0^{BK^*}$	$0.29^{+0.10}_{-0.07}$	$-18.2^{+1.3}_{-3.0}$	$B_s(0^-)$	with B DA's
$T_1^{BK^*}$	$0.31^{+0.18}_{-0.10}$	$-4.6^{+0.81}_{-0.41}$	$B_s^*(1^-)$	
$T_2^{BK^*}$	$0.31^{+0.18}_{-0.10}$	$-3.2^{+2.1}_{-2.2}$	$B_s(1^+)$	
$T_3^{BK^*}$	$0.22^{+0.17}_{-0.10}$	$-10.3^{+2.5}_{-3.1}$	$B_s(1^+)$	

correlations between normalization & slope out of the scope