OPE, duality, resonances

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b➔ $sl^+ l^-$ workshop

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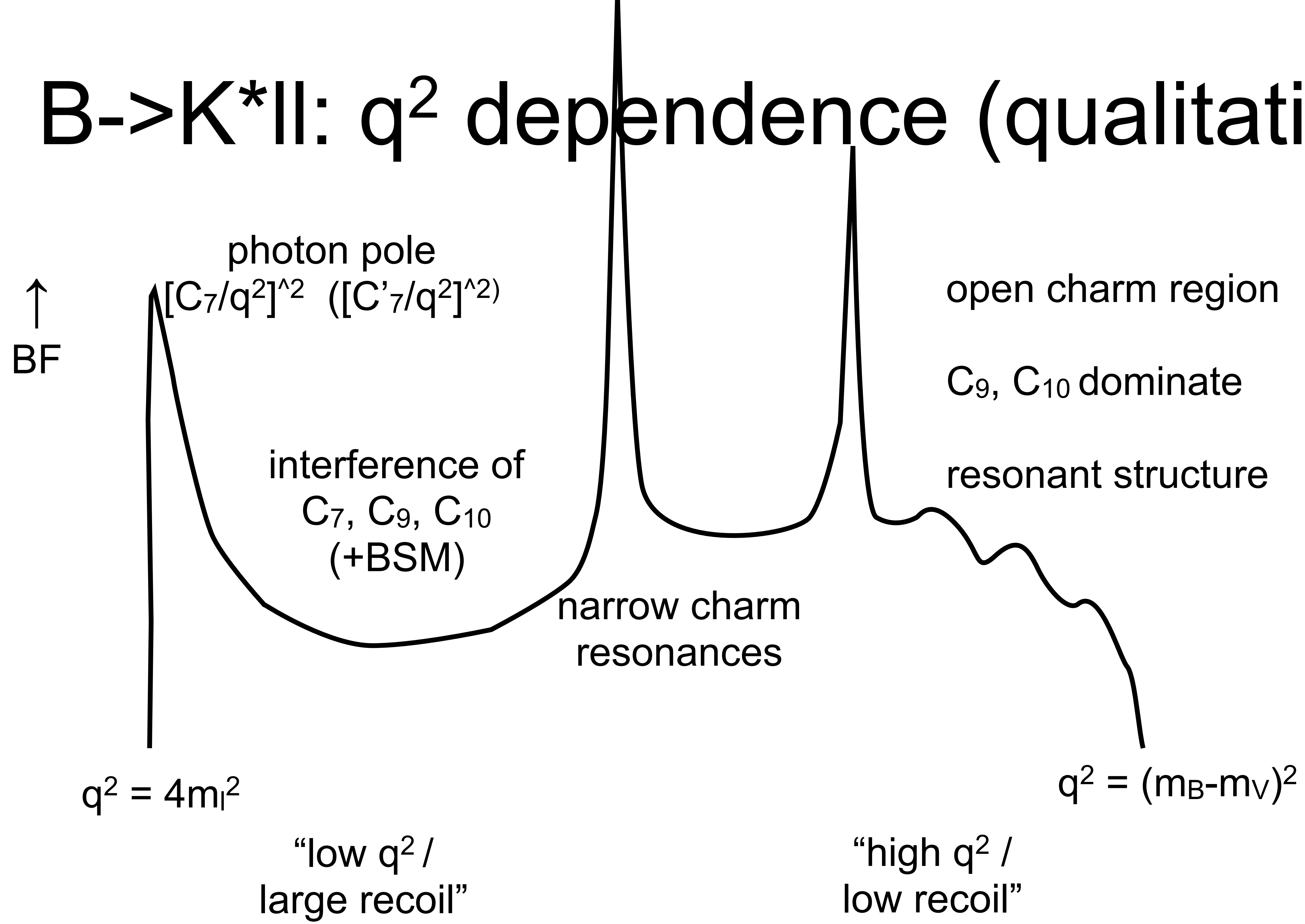
Outline

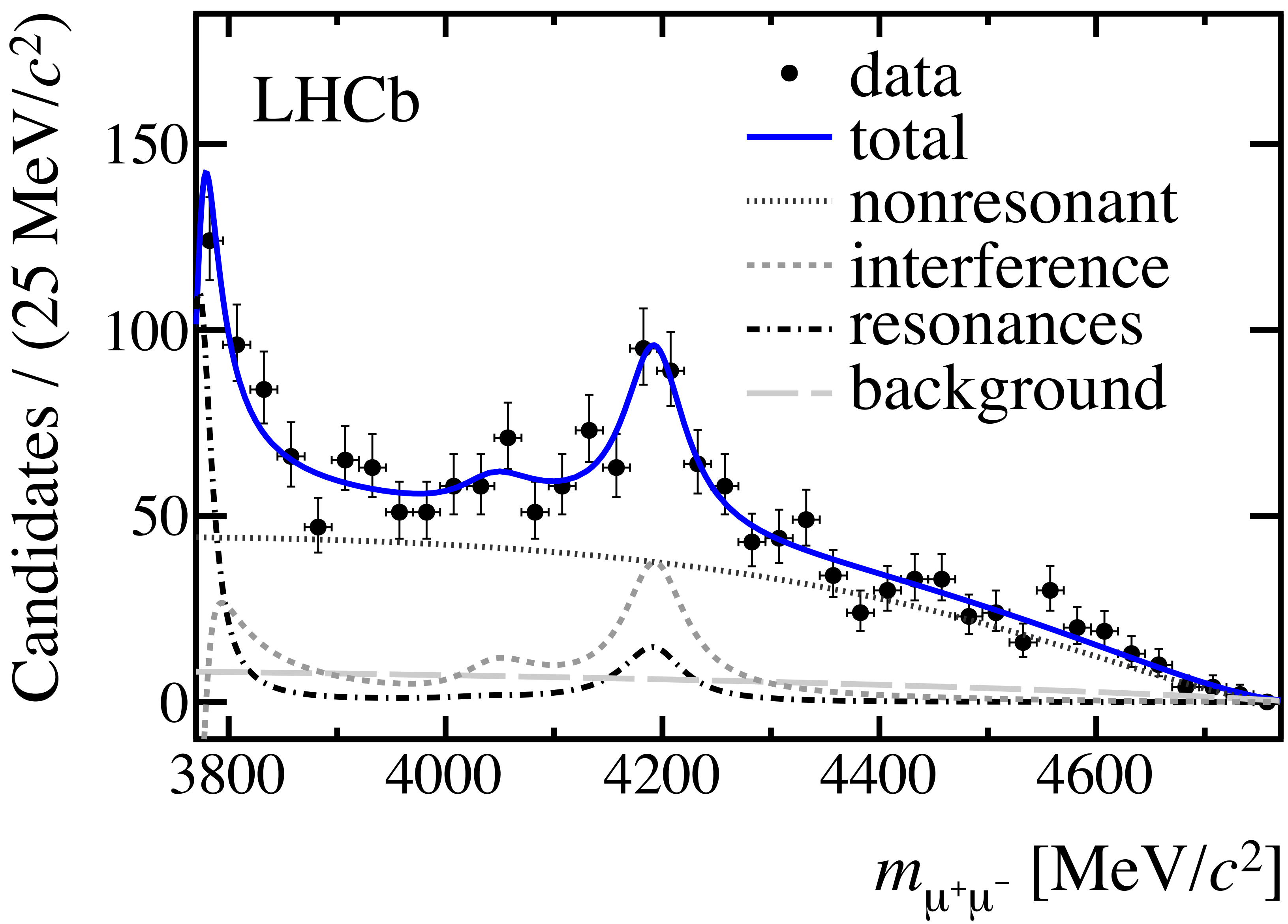
- OPE, parton-hadron duality, emergence of resonant structure
- Shifman model for e+ e- -> hadrons
- Application to B->K l l, B->K*ll
- What about duality violation below the charm threshold (“light resonances”)?

B->K*ll: q^2 dependence (qualitative)



Note - artist’s impression only.
LHCb has not yet published sufficiently fine binning to show the resonant features
Open charm resonances are however visible in published B->K l l data.

Resonant structure in B->Kll



OPE

Many physical processes involving a large mass or energy scale have an operator product expansion

$$\mathrm{Obs} = \sum_i C_i(\alpha_s)\langle f|O_i|i\rangle$$

$$C_i = 1 + r_{i1}\alpha_s + r_{i2}\alpha_s^2 + \cdots$$

short-distance physics
(often) (approximately)
perturbative

$$\langle f|O_i|i\rangle = a_i\left(\frac{\Lambda}{Q}\right)^{d_i}$$

long-distance physics
often essentially non-perturbative
(as in our examples)

It is generally believed that (in most cases)
- perturbation series for C_i factorially divergent -> ambiguous. The ambiguity behaves like a higher-dim matrix element [via renormalon poles in Borel transform]. (Eg quark pole mass.)

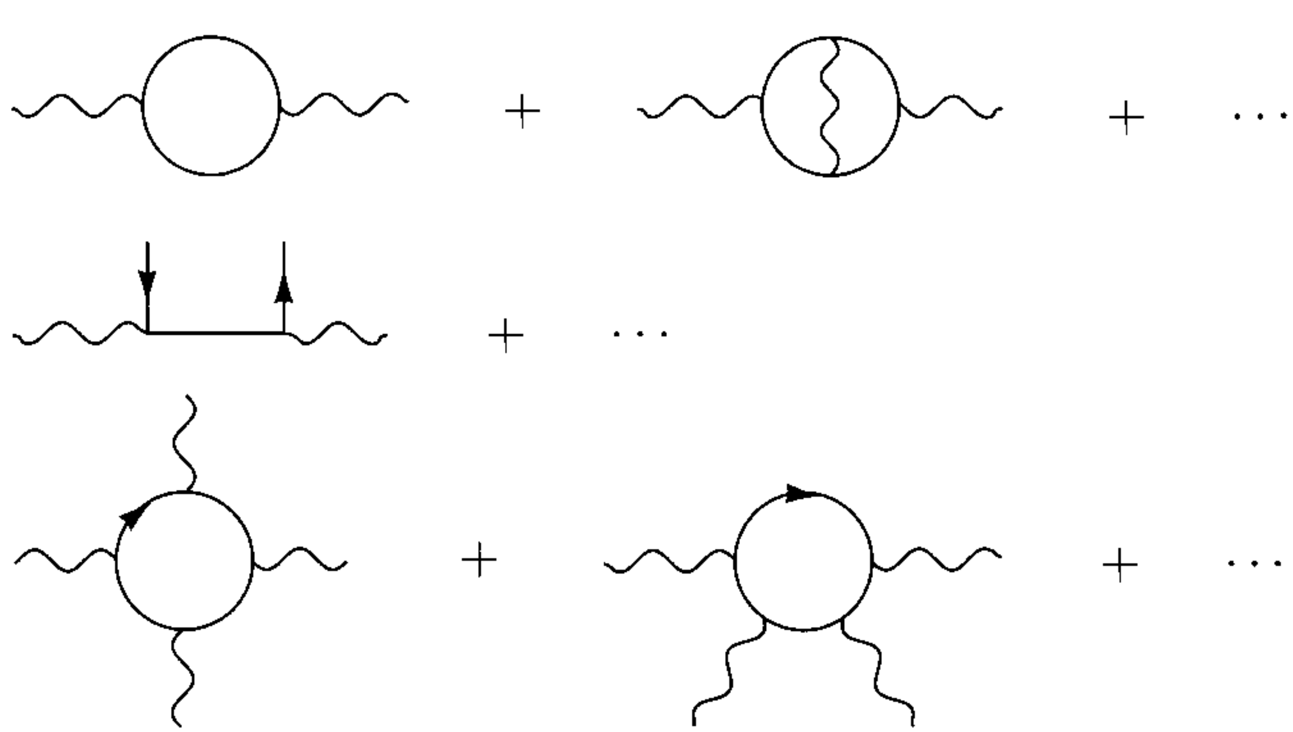
- OPE itself is factorially divergent -> ambiguous [via analyticity properties of amplitude.] Ambiguity behaves like $\exp(-C\,Q^2/\Lambda^2)$

't Hooft 1977
[Gross, Neveu, Lautrup, Mueller,...]
Shifman et al 1994 - 2000

Origin of resonant behaviour

Duality in e$^+$e$^-$ -> hadrons

$$\sigma(e^+e^- \to \mathrm{hadrons}) \propto \mathrm{Im}\,(-i)\int d^4x e^{-iqx}\langle 0|T(j_\mu(x)j^\mu(x)|0\rangle = 3q^2\Pi(-q^2)$$



$$\times \quad \langle 0|1|0\rangle$$

$$\times \quad \langle 0|\bar{q}q|0\rangle$$

$$\times \quad \langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle$$

[Fig Peskin & Schroeder]

Diagram calculation & OPE justified for spacelike q^2<0

The q^2>0 “physical” result is **defined** through analytic continuation (in practice, dispersion relations)

remainder becomes oscillatory, ~ sin(c E)/E^(power): resonances

Shifman model of duality violation

Chibisov, Dikeman, Shifman, Uraltsev 1996
Blok, Shifman, Zhang 1997
Beylich, Buchalla, Feldmann 2011

$$\Delta(q^2) = -\frac{N}{6}\frac{1}{1-b/\pi}\left[\psi(z+1)+\gamma\right]$$

$$\Delta(q^2) \equiv 2\pi^2\ \left(\Pi(q^2)-\Pi(0)\right)$$

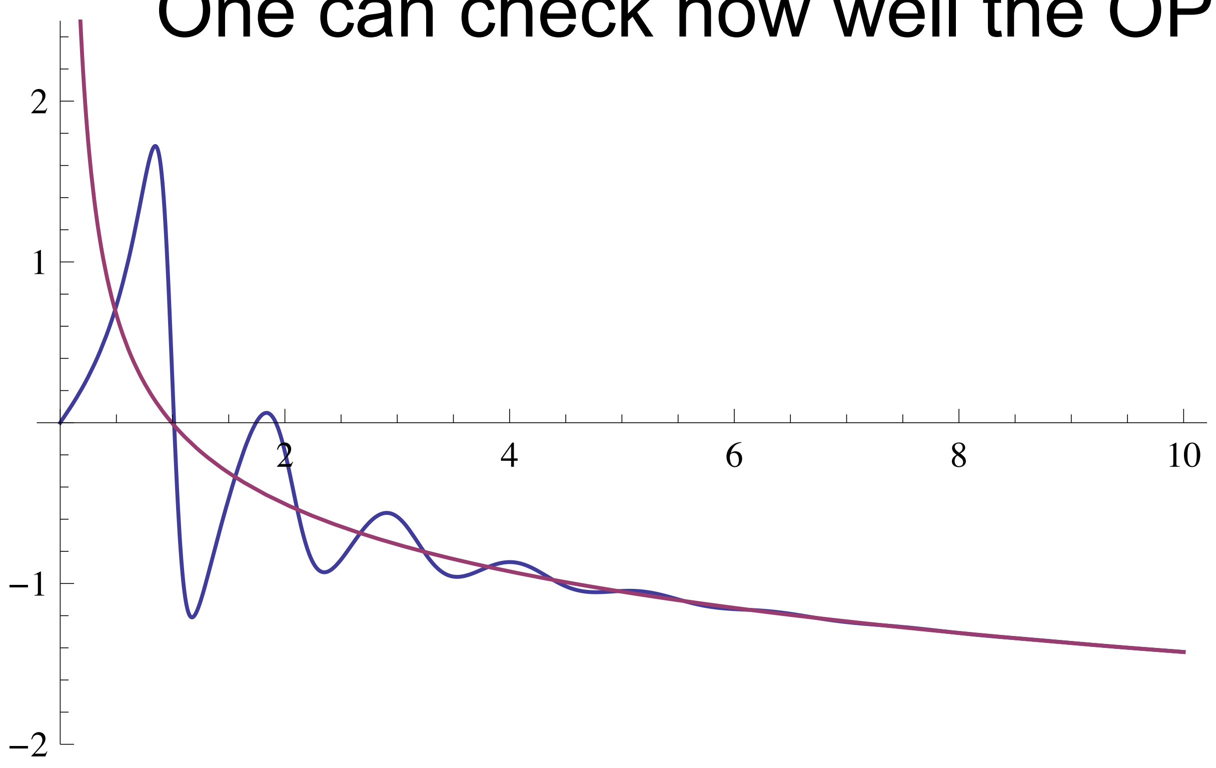
$$z = (-r-i\epsilon)^{1-b/\pi} \qquad r = \frac{q^2}{\lambda^2}$$

$$b \equiv B/N = \Gamma_n/M_n$$

Is a model of the current-current correlator with massless quarks

The entire correlator is modelled (not just a remainder term)

One can check how well the OPE approximates it.



“true” (model) correlator (blue) oscillates around its OPE (red)

resonance amplitude dies off at large q^2

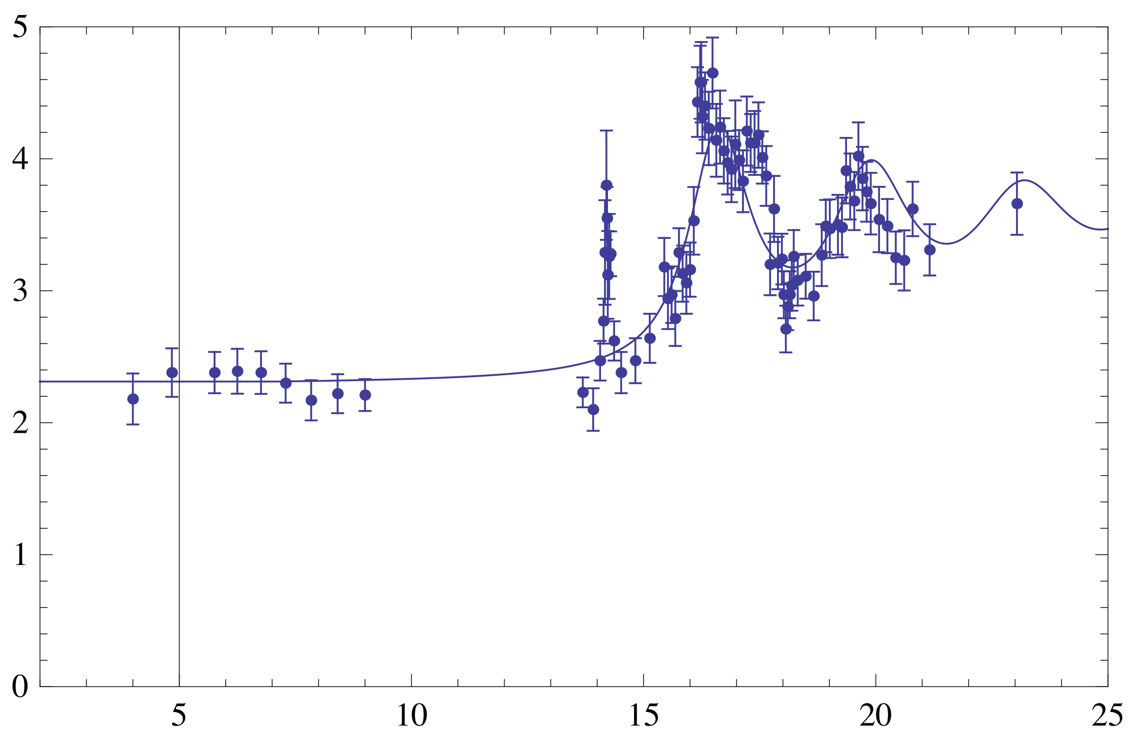
Duality violation: charm

Beylich, Buchalla, Feldmann 2011

Adapt Shifman model to include open-charm resonances

$$R = R_{\rm light} - \frac{4}{3}\,\frac{1}{(1-b/\pi)\,\pi}\,{\rm Im}\,\psi(3+z)\,, \qquad z = \left(-\frac{q^2-4m_c^2+i\epsilon}{\lambda^2}\right)^{1-b/\pi}$$

Resonances at $q^2 = n\lambda^2 + 4m_c^2 \ (n = 3, 4, 5, \ldots)$



fit to BES-II data

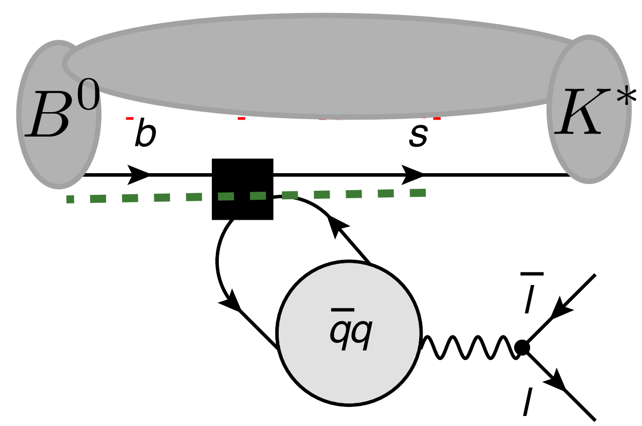
Application to B->K(K*)ll

Grinstein, Pirjol 2004
Beylich, Buchalla, Feldmann 2011

In naive factorisation, the charm loop contribution is proportional to the same hadronic 2-point correlator as the R-ratio

$$H^c = a_2 \, (\bar{s}b)_{V-A} (\bar{c}c)_{V-A}$$

a_2 = 0.3 fudge factor to model violation of naive factorisation (q^2-independent)



Model **not** used by Beylich et al
to “improve” prediction, but rather to estimate duality violation

To estimate it, they separate out an oscillating part from the model, bin this from q^2_0 ~ 15 GeV^2 to q^2_{max}

Two contributions: linear (interference), oscillations largely cancel
quadratic (additive), no cancellation

Estimate 1.5% + 0.75% uncertainty on B->Kll rate from DV

-> Can LHCb fit their data to this 3-parameter model (inc a_2) ?

Remarks

1) The Shifman (et al) ansatz for the correlator satisfies various constraints from QCD:

- reproduces hadronic tau decays & R data (with charm fix)

- has resonances behaving as expected based on large-N/QCD-string arguments (masses, widths)

- has the correct OPE, in particular the hadronic states it implicitly sums over are such that the correct leading-log running of the EM coupling is reproduced

2) It is a simple model and not rigorous

3) There is no rigorous theory of duality violation.

Given 2) and 3), 1) is pretty good

Remarks

4) The duality violating piece is another C9-like contribution with all consequences, eg able to shift the zero crossings in FB asymmetries (ie I don't see that this cancels out).

5) LHCb might be able to do their own estimate of duality violation (or give input to it, by fitting back to the Shifman model as extended by Beylich et al, or other theoretical models).

In my opinion, please do **not** cut out prominent features; the rest of the signal will then undershoot the OPE result. If the precise value of q^2_0 has a strong impact on results, this suggests a more sizable uncertainty on the OPE prediction.

One could perhaps increase q^2_0 to move deeper into the duality regime, where DV is less pronounced, but then an updated theoretical calculation should estimate the error.

Light-quark resonances

Some resonant behaviour should be seen in the low-q^2 region.

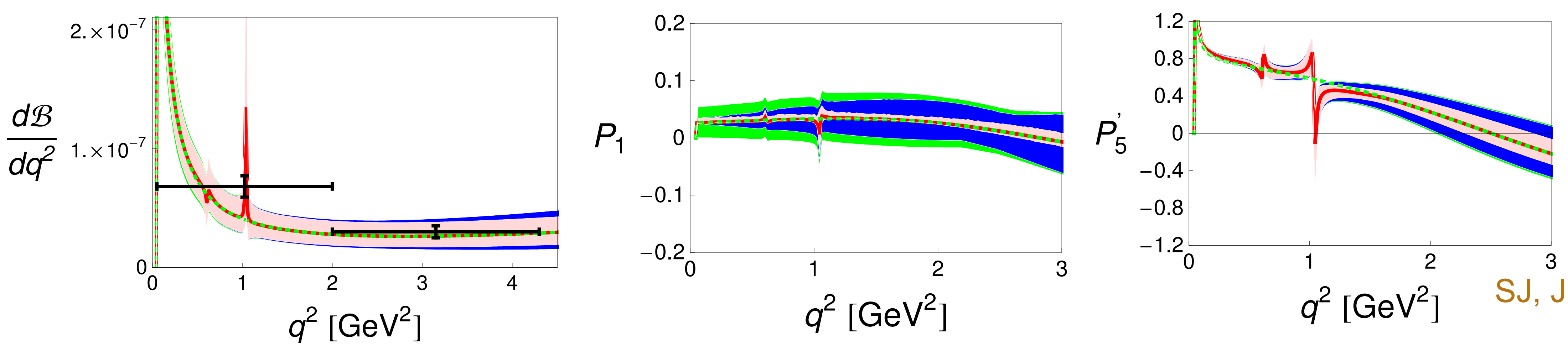
Differently to high-q^2, there is no OPE and the picture is less clear.

Still expect duality violation relative to the QCDF result.

Under the naive-factorisation assumption, one does have expressions in terms of local form factors and one could use the Shifman model to estimate the errors - will be tiny after binning.

I would not cut out resonances but smear (integrate) over them.

Another simple way to model the (presumably) most conspicous resonances is to compare vector-meson-dominance to the corresponding subset of QCDF contributions for a DV estimate



SJ, J Martin Camalich

DV etc references

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