$H_b \to X_x\, l^+ l^-$ @ $q^2 > 14$ GeV^2

"$b \to s\, l^+ l^-$ Workshop"
Imperial College
2 April 2014

[hep-ph/9308246, hep-ph/9801456, 0707.1694, 0902.4446]

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Content

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I will talk about things I never worked on.

H_{eff} for $b \to s\, l^+ l^-$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[\sum_{i=1}^{8} C_i(\mu) Q_i + \frac{\alpha}{2\pi} \tilde{C}_9(\mu) (\bar{s}b)_{V-A} (\bar{l}l)_V + \frac{\alpha}{2\pi} \tilde{C}_{10} (\bar{s}b)_{V-A} (\bar{l}l)_A \right]$$

We know the Standard Model H_{eff} for $b \to s\, l^+ l^-$ including NNLO QCD + some QED and Electroweak corrections.

The relevant Wilson Coefficients C_7, C_{10} & C_9 are constrained from $B \to X_s \gamma$, $B_s \to \mu^+ \mu^-$ & $B \to K^{(*)} l^+ l^-$.

What can we learn from $B \to X_s\, l^+ l^-$, or $H_b \to X\, l^+ l^-$?

Inclusive decays are thought to be theoretically clean.

Differential Branching Fraction

For an inclusive quantity we use the optical theorem

$\text{Im}\, M(A \to A) \propto \Sigma_X \, \Gamma(A \to X)$

For B Physics: $\text{Im}\, M(B \to B)$

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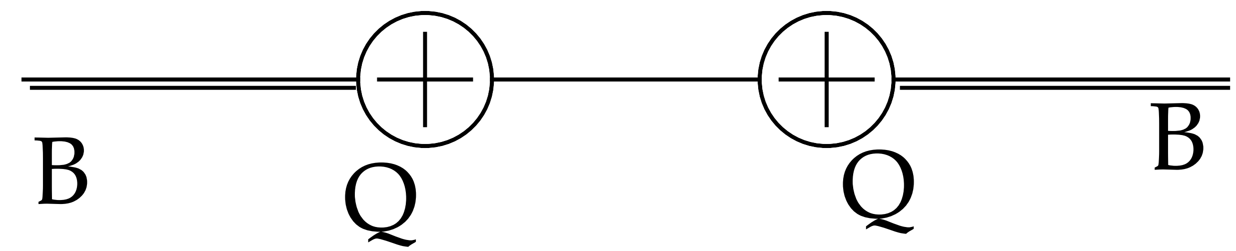

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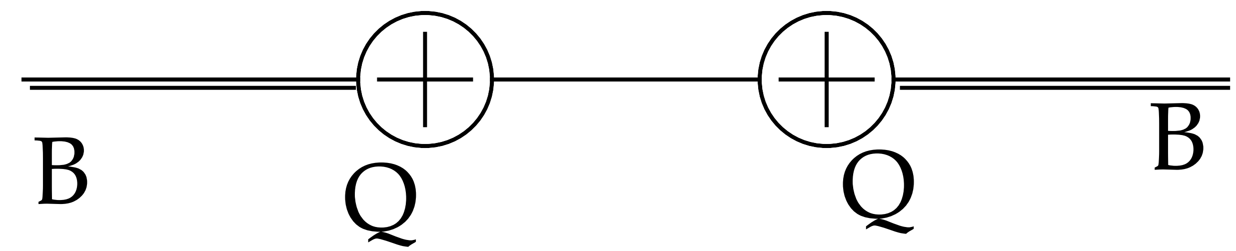

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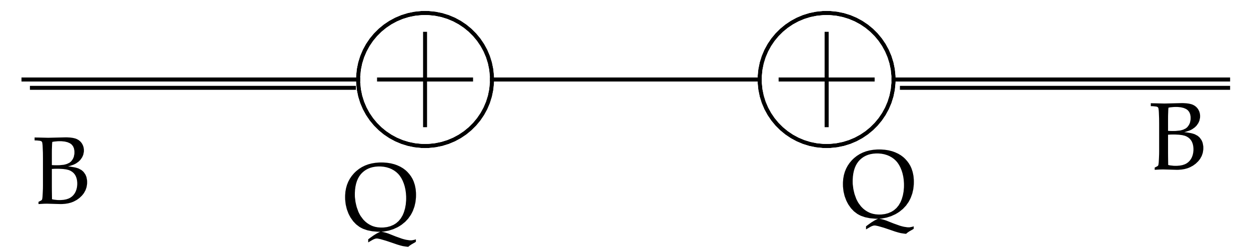
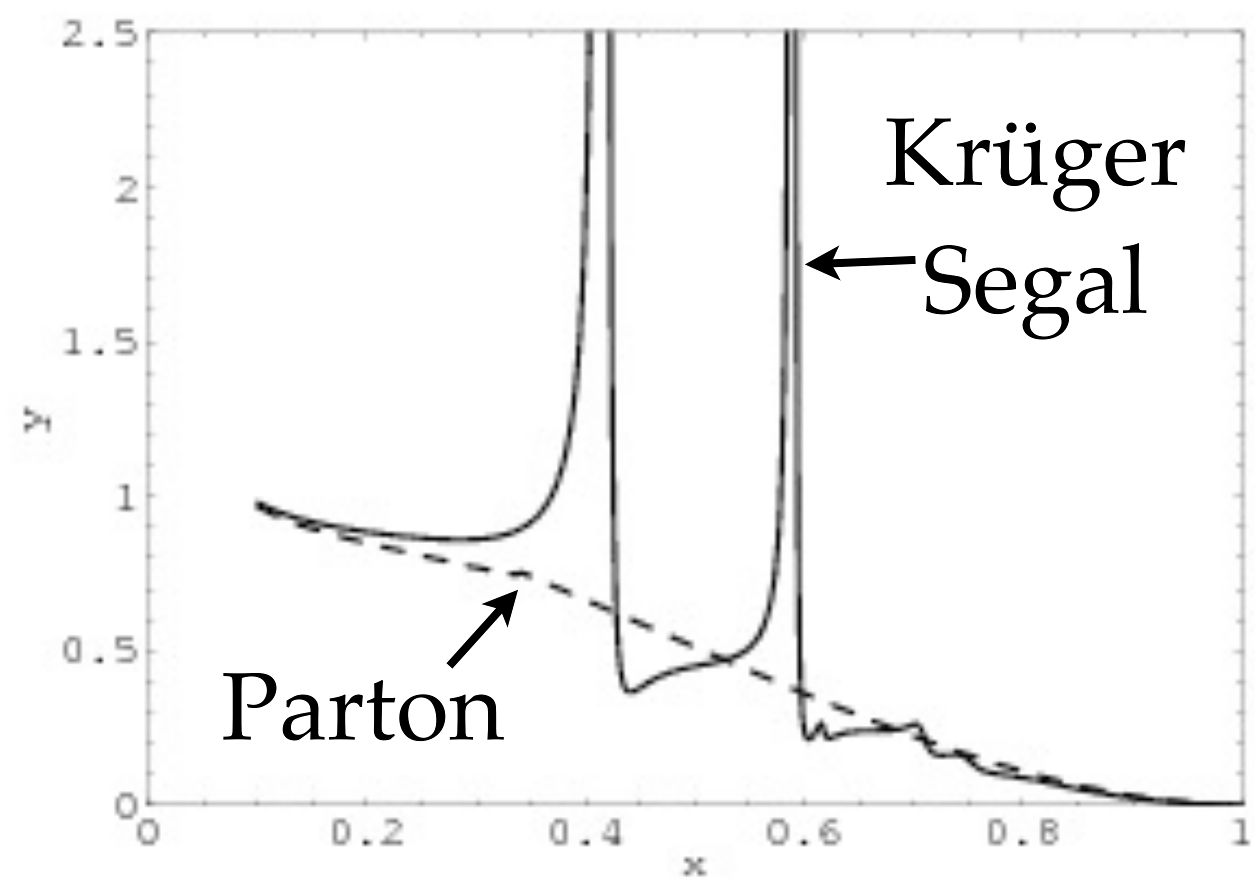
Γ(B → X) = Parton + Λ / m_b

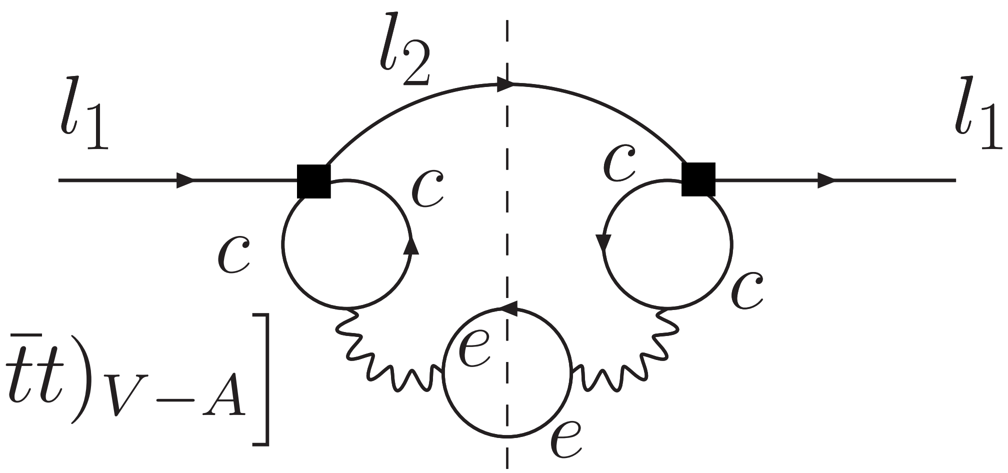




Quark-hadron duality not expected to hold

Discussed e.g. by BBNS [0902.4446] using a toy model

B → X_s l^+ l^- is not inclusive



$$\mathcal{H}_{eff} = \frac{G}{\sqrt{2}}\left[(\bar{l}_2 l_1)_{V-A}\,(\bar{c}c)_{V-A} - (\bar{l}_2 l_1)_{V-A}\,(\bar{t}t)_{V-A}\right]$$

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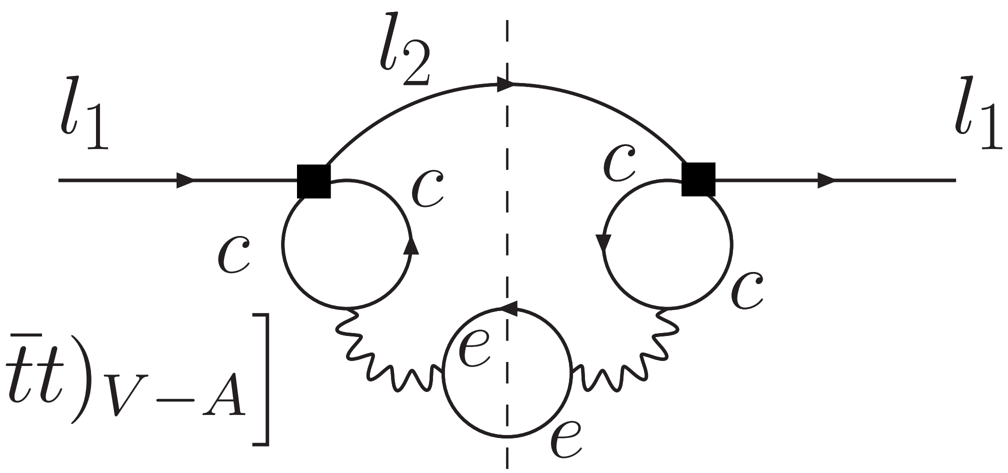

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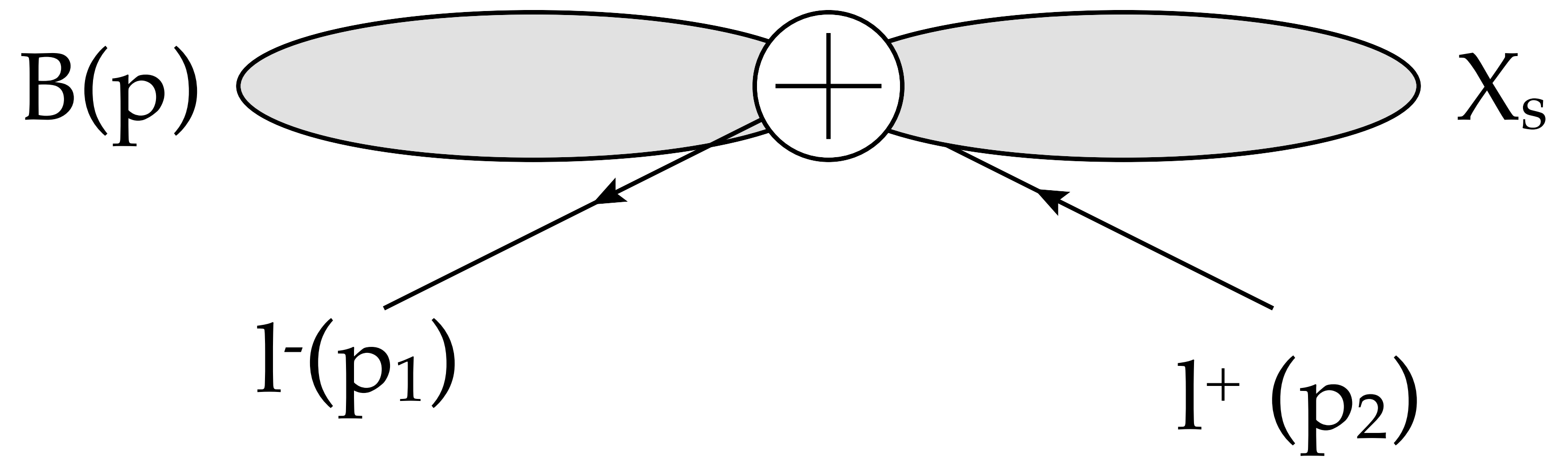
For B → X_s l^+ l^-

$$R_\psi \equiv \frac{\mathrm{B}(B \to X_s \psi \to X_s \, l^+ l^-)}{\mathrm{B}(B \to X_s \, l^+ l^-)_{\mathrm{SD}}} = \frac{512\pi^5 \kappa^2 \, a_2^2 \, (1-r)^2 \, (1+2r)}{9(\langle |C_9|^2 \rangle + |C_{10}|^2)} \times \frac{f_\psi^2}{m_b^2} \times \frac{f_\psi^2}{M_\psi \Gamma_\psi}$$

Gives $R_\Psi \approx 93$, also $R_\rho \approx [10\text{–}50] \times \dfrac{f_\rho^2}{m_b^2} \times \dfrac{f_\rho^2}{M_\rho \Gamma_\rho} \approx [0.007\text{–}0.036]$

What about higher Ψ resonances?

B → X_s l^+ l^- Ignoring Charm Resonances

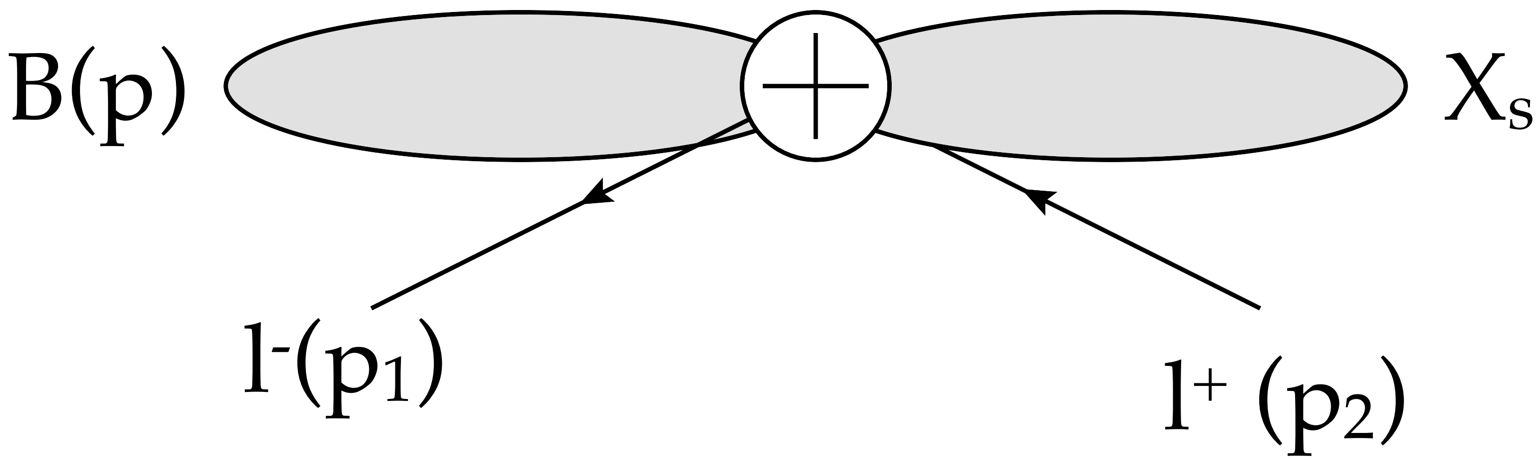


$$\frac{d\Gamma(\bar{B} \to X_s l^+ l^-)}{dx\ dy\ ds} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{ts}^* V_{tb}|^2 \frac{\alpha^2}{4\pi^2}\frac{3}{4\pi m_b^2}\frac{m_b}{M_B} \times$$
$$\times \Big[L_{\mu\nu}^S \Big\{ \Big(|\tilde{C}_9^{eff}|^2 + |\tilde{C}_{10}|^2 \Big)\ W_9^{\mu\nu} + 4m_b^2 |C_7|^2\ W_7^{\mu\nu} + 4m_b \mathrm{Re}\ C_7 \tilde{C}_9^{eff*}\ W_{97}^{\mu\nu} \Big\}$$
$$+ L_{\mu\nu}^A \Big\{ 2\mathrm{Re}\ \tilde{C}_9^{eff*} \tilde{C}_{10}\ W_9^{\mu\nu} + 4m_b \mathrm{Re}\ C_7 \tilde{C}_{10}^*\ W_{97}^{\mu\nu} \Big\} \Big] \ .$$

$$L_{\mu\nu}^S = p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu} - g_{\mu\nu} p_1 \cdot p_2$$

$$W_9^{\mu\nu} = 2\mathrm{Im} \left(i \int d^4x e^{-iq\cdot x} \langle B | T\, j_9^{\dagger\mu}(x) j_9^{\nu}(0) | B \rangle \right)$$

$B \to X_s\, l^+\, l^-$ Ignoring Charm Resonances



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$$\times \Big[L_{\mu\nu}^S \Big\{ \Big(|\tilde{C}_9^{eff}|^2 + |\tilde{C}_{10}|^2 \Big)\ W_9^{\mu\nu} + 4m_b^2 |C_7|^2\ W_7^{\mu\nu} + 4m_b \mathrm{Re}\ C_7 \tilde{C}_9^{eff*}\ W_{97}^{\mu\nu} \Big\}$$

$$+ L_{\mu\nu}^A \Big\{ 2\mathrm{Re}\ \tilde{C}_9^{eff*} \tilde{C}_{10}\ W_9^{\mu\nu} + 4m_b \mathrm{Re}\ C_7 \tilde{C}_{10}^*\ W_{97}^{\mu\nu} \Big\} \Big] \ .$$

$$\boxed{L_{\mu\nu}^S} = p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu} - g_{\mu\nu} p_1 \cdot p_2$$

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OPE up to $O(\Lambda^2 / m_b^2)$

Neglecting C_7 we only need the OPE for W_9

$$\frac{3}{4\pi m_b M_B}\int dxdy\; L^S_{\mu\nu} W_9^{\mu\nu} \;=\; \left(1+\frac{\lambda_1}{2m_b^2}\right)(1-s)^2(1+2s) + \frac{3\lambda_2}{2m_b^2}(1-15s^2+10s^3)\ ,$$

[Ali et. al. `97] – see also [hep-ph/9801456]

$$\lambda_1 = \frac{\langle B|\bar{h}(iD)^2 h|B\rangle}{2M_B}\ , \qquad \lambda_2 = \frac{1}{6}\frac{\langle B|\bar{h} g\sigma\cdot G h|B\rangle}{2M_B} = \frac{M^2_{B^*}-M^2_B}{4}$$

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For $s \to 1$ we have [Ali et. al. `97]:

$(1 - s)^2 (1 + 2 s) \to 0$ and $(1 - 15 s^2 + 10 s^3) \to -4$

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Breakdown of the OPE for high q^2 region

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Interpolation to medium q^2 region

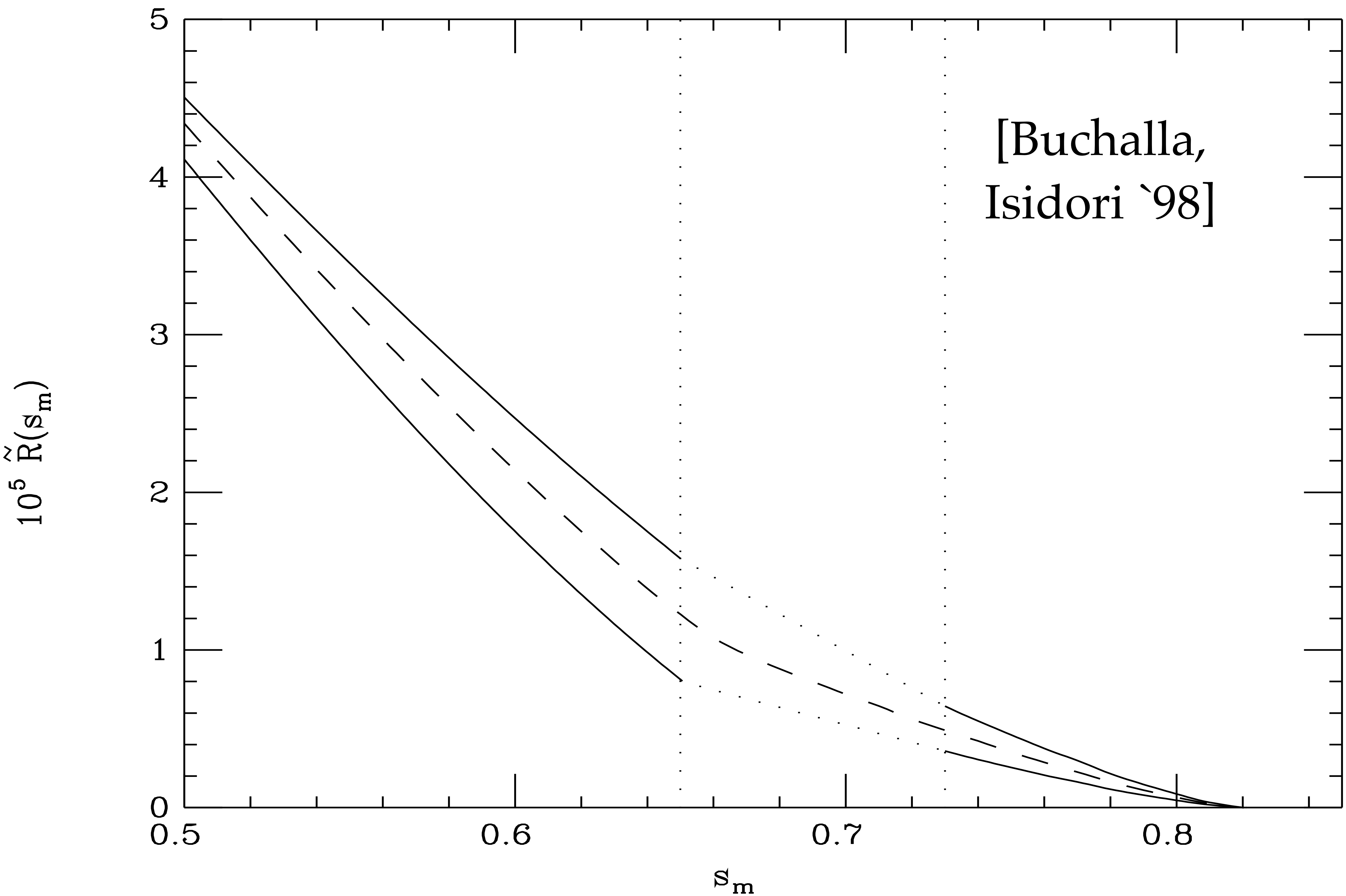

Figure 1: The dilepton invariant mass spectrum $(dB(\bar{B} \to X_s l^+ l^-)/ds_m)/B(\bar{B} \to X_c e\nu) \equiv \tilde{R}(s_m)$ as a function of $s_m = q^2/M_B^2$. For $s_m < 0.65$ the NLO partonic calculation, including $1/m_b^2$ effects, is used. There the lower, mid-

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where $s = q^2/m_b^2$, and $1/(1-x)_+ = \lim_{\epsilon\to0}[\theta(1-x-\epsilon)/(1-x) + \delta(1-x-\epsilon)\ln\epsilon]$. For $B\to X_s\ell^+\ell^-$ [9, 14, 15, 21]

$$\begin{aligned}\frac{\mathrm{d}\Gamma_s}{\mathrm{d}q^2} = \frac{\Gamma_0}{2}\, m_b^3 \Bigg\{ &(\mathcal{C}_9^2+\mathcal{C}_{10}^2)\Bigg[(1-s)^2(1+2s)\,(2+\hat\lambda_1) + 3(1-15s^2+10s^3)\,(\hat\lambda_2-\hat\rho_2) + \frac{37+24s+33s^2+10s^3}{3}\,\hat\rho_1\Bigg] \\ &+ 4\,\mathcal{C}_7\,\mathcal{C}_9\Big[3(1-s)^2\,(2+\hat\lambda_1) - 3(5+6s-7s^2)\,(\hat\lambda_2-\hat\rho_2) + (13+14s-3s^2)\hat\rho_1\Big] \\ &+ \frac{4\,\mathcal{C}_7^2}{s}\Bigg[(1-s)^2(2+s)\,(2+\hat\lambda_1) - 3(6+3s-5s^3)(\hat\lambda_2-\hat\rho_2) + \frac{-22+33s+24s^2+5s^3}{3}\,\hat\rho_1\Bigg] \\ &- \big[(\mathcal{C}_9+2\mathcal{C}_7)^2+\mathcal{C}_{10}^2\big]\Bigg[\frac{16}{(1-s)_+}\,\hat\rho_1 + 8\,\delta(1-s)\,(\hat\rho_1+\hat f_s)\Bigg]\Bigg\}\,, \qquad (6\end{aligned}$$

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$$+ 4\,\mathcal{C}_7\,\mathcal{C}_9\Big[3(1-s)^2\,(2+\hat\lambda_1) - 3(5+6s-7s^2)\,(\hat\lambda_2-\hat\rho_2) + (13+14s-3s^2)\hat\rho_1\Big]$$
$$+ \frac{4\,\mathcal{C}_7^2}{s}\bigg[(1-s)^2(2+s)\,(2+\hat\lambda_1) - 3(6+3s-5s^3)(\hat\lambda_2-\hat\rho_2) + \frac{-22+33s+24s^2+5s^3}{3}\,\hat\rho_1\bigg]$$
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Non-perturbative uncertainties will cancel in the ratio
Tackmann et. al. see [0707.1694]

$$\frac{\displaystyle\int_{q_0^2}^{m_B^2} \frac{\mathrm{d}\Gamma(B \to X_s \ell^+ \ell^-)}{\mathrm{d}q^2}}{\displaystyle\int_{q_0^2}^{m_B^2} \frac{\mathrm{d}\Gamma(B \to X_u \ell \bar{\nu})}{\mathrm{d}q^2}} = \frac{|V_{tb} V_{ts}^*|^2}{|V_{ub}|^2} \frac{\alpha_{\mathrm{em}}^2}{8\pi^2} \mathcal{R}(q_0^2)$$

Numbers

Normalising to $B \to X_u \, l^+ \, \upsilon$ with with the same q^2 cut Ligeti and Tackmann find [0707.1694]

$$\begin{aligned}\mathcal{R}(14\,\mathrm{GeV}^2) &= \mathcal{C}_9^2 + \mathcal{C}_{10}^2 + 4.79\,\mathcal{C}_7^2 + 4.31\,\mathcal{C}_7\,\mathcal{C}_9 \\ &\quad + 1.06\,\mathcal{C}_9 + 2.24\,\mathcal{C}_7 + 0.95\,, \\ \mathcal{R}(15\,\mathrm{GeV}^2) &= \mathcal{C}_9^2 + \mathcal{C}_{10}^2 + 4.27\,\mathcal{C}_7^2 + 4.10\,\mathcal{C}_7\,\mathcal{C}_9 \\ &\quad + 0.97\,\mathcal{C}_9 + 1.91\,\mathcal{C}_7 + 0.93\,.\end{aligned}$$

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&\quad + 0.97\,\mathcal{C}_9 + 1.91\,\mathcal{C}_7 + 0.93\,.
\end{aligned}
$$

With small theory uncertainties:

$$
\begin{aligned}
\mathcal{R}(14\,\mathrm{GeV}^2) = 35.55\,\big(1 \;&\pm\; 0.046_{[f_{u,s}]} \pm 0.012_{[\lambda_2,\rho_1]} \\
&\pm\; 0.054_{[\mu]} \pm 0.030_{[\mathcal{C}_i]}\big)\,, \\
\mathcal{R}(15\,\mathrm{GeV}^2) = 35.42\,\big(1 \;&\pm\; 0.065_{[f_{u,s}]} \pm 0.016_{[\lambda_2,\rho_1]} \\
&\pm\; 0.051_{[\mu]} \pm 0.030_{[\mathcal{C}_i]}\big)\,.
\end{aligned}
$$

Questions

But only if we normalise to the corresponding $b \rightarrow u$ semileptonic decay with the same q^2 cut.

Would the required normalisation channels be present?

Nearly equal mixture of Λ_b, B_s, B^+ & B^0 in the initial state.

Also: The breakdown of the OPE is different in
$B \rightarrow X_u\, l^+\, \upsilon$ and $B \rightarrow X_u\, l^+\, l^-$, also $m_s \neq m_u$

Low q^2 and Conclusions

The OPE works much better at low q^2, but experimentally not accessible at LHCb?

Cuts introduce sensitivity to shape function – similar to $B \to X_s \gamma$.

Yet, the cuts could be removed – at least to some extent – at a Super Flavour Factory. What about the $b \to c\,(\to s\, l^+ \upsilon)\, l^- \bar{\upsilon}$

My personal view: If we want to learn something about short distance physics the better region is low q^2 .