KINEMATIC ENDPOINT SYMMETRIES IN $B \to K^{*}ll$ AND BEYOND

Hiller and RZ 1312.1923 to (JHEP)

CP³ Origins
Cosmology & Particle Physics

Schwinger on Feynman graphs:
“Sure, you can split it into different topological parts, but at the end you need to patch it together” …
… into amplitudes

CENTRE FOR THEORETICAL PHYSICS · Higgs

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2 April 2014 — Imperial College Workshop

Main Idea

- Q: what is an amplitude?
  A: a Lorentz invariant formed out of momenta and pol. vectors

$$\mathcal{A} = f(p_1, p_2, ..\omega_1, \omega_2, ..)$$

- Hence for kinematic situation where momenta are linearly dependent, the number of independent structure decreases

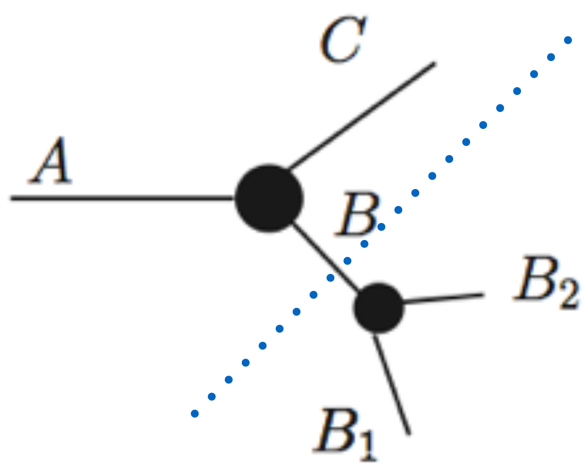
➠ symmetries of helicity amplitudes

- Situation arises when two decaying particles are at rest; so-called *kinematic endpoint* (highest q^2)

Upshot

- Assumption: i) Lorentz-invariance
  ii) neglect FSI between (ll) and ($K\pi$)-pairs
- Useful for fits (as unavoidable constraints)
  Hambrock, Hiller, Schacht and RZ PRD'14
- Experimental crosscheck (e.g. LHCb results)
- Reduction of parameters at cost of statistics
  (useful for resonance searches in gg ➡h ➡ 4l)

- Implementation of this idea for a decay topology …



is straightforward (sequential decay)

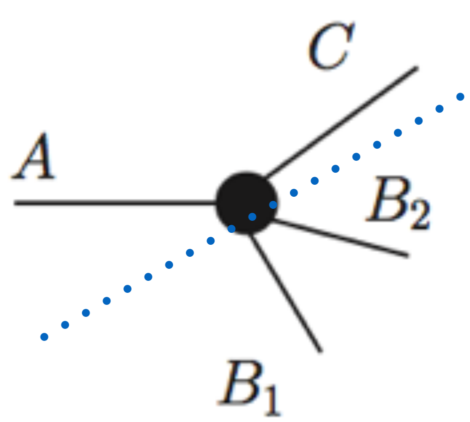
$\mathcal{A} \propto H^{A\to BC} H^{B\to B_1 B_2}$Wigner matrices Jacob Wick '59, ….

helicity conservation: $\lambda_A = \bar{\lambda}_B + \lambda_C$

factorisation

$B \to J/\Psi(\to \ell\ell)\phi(\to KK)$

$H \to Z(\to \ell\ell)Z^*(\to \ell\ell)$ also RZ 1309.7802



not so straightforward (this talk)

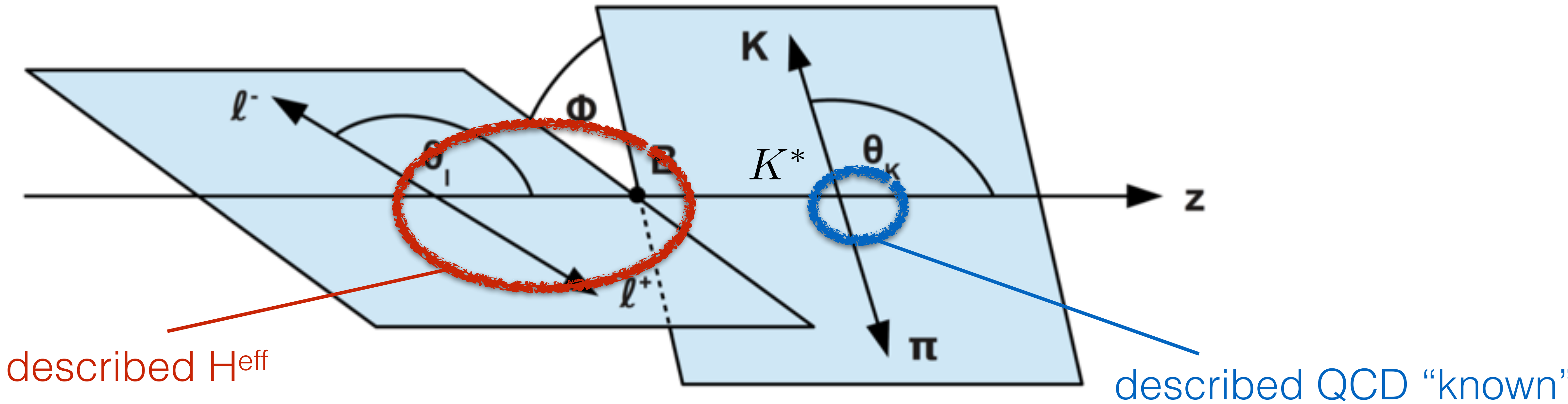
Hiller and RZ 1312.1923 (JHEP)

$B \to (\ell\ell)K^*(\to K\pi)$

$B \to (\to \ell\nu)D^*(\to D\pi)$

many others ... outside flavour physics

Final States



$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = (J_{1s} + J_{2s}\cos 2\theta_\ell + J_{6s}\cos\theta_\ell)\sin^2\theta_K$$
$$+ (J_{1c} + J_{2c}\cos 2\theta_\ell + J_{6c}\cos\theta_\ell)\cos^2\theta_K$$
$$+ (J_3\cos 2\phi + J_9\sin 2\phi)\sin^2\theta_K\sin^2\theta_\ell$$
$$+ (J_4\cos\phi + J_8\sin\phi)\sin 2\theta_K\sin 2\theta_\ell$$
$$+ (J_5\cos\phi + J_7\sin\phi)\sin 2\theta_K\sin\theta_\ell\,,$$

$J_i \propto H_a H_b^* \times \text{kinematics}$

for generic dim 6 Heff
12 "directions"

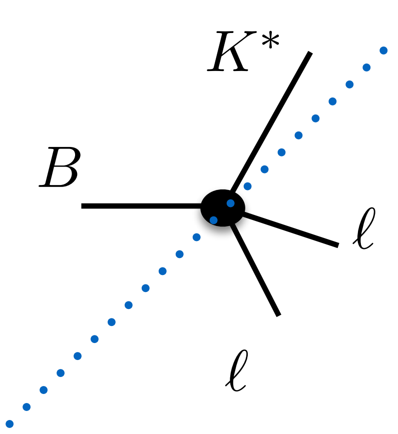
What is Heff?

$$\mathcal{H}_{\text{eff}} = -4\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i(\mu) O_i(\mu)$$

- complete set dim 6 operators*:

$$O_{S(P)} = \bar{s}_L b \, \bar{\ell}(\gamma_5)\ell \, , \quad O_{V(A)} = \bar{s}_L \gamma^{\mu} b \, \bar{\ell}\gamma_{\mu}(\gamma_5)\ell \, ,$$
$$O_T = \bar{s}_L \sigma^{\mu\nu} b \, \bar{\ell}\sigma_{\mu\nu}\ell \, , \quad O' = O|_{s_L \to s_R} \, .$$

- effective vertex



factorisation

not sequential 1→2
trick: insert complete!?
set of polarisation
vectors X-times

$$\sum_{\lambda,\lambda' \in \{t,\pm,0\}} \omega^{\mu}(\lambda)\omega^{*\nu}(\lambda') G_{\lambda\lambda'} = g^{\mu\nu}$$

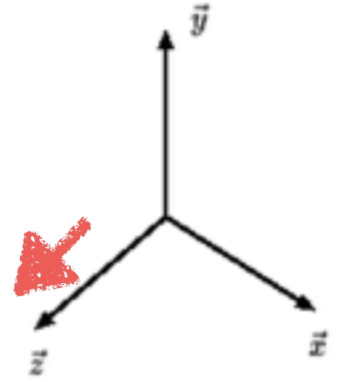
* photon emission absorbed into O_T

spin-1 polarisation vectors

- For $q^2 \neq 0$ three polarisation states span $\mathbf{R}^3$ but not $\mathbf{R}^{3,1}$

$$\vec{\omega}(0) = \vec{e}_z \;, \quad \sqrt{2}\vec{\omega}(\pm) = \vec{e}_x \pm i \vec{e}_y$$

$$\sum_{\lambda \in \{\pm, 0\}} \omega^\mu(\lambda) \omega^{*\nu}(\lambda') = -\left(g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right)$$



- Trick: introduce (unphysical) timelike polarisation vector

$$\omega(t)_\mu = q_\mu / \sqrt{q^2}$$

$$\textstyle\sum_{\lambda, \lambda' \in \{t, \pm, 0\}} \omega^\mu(\lambda) \omega^{*\nu}(\lambda') G_{\lambda\lambda'} = g^{\mu\nu} \qquad G_{\lambda\lambda'} = \mathrm{diag}(1, -1, -1, -1)$$

- Generalised helicity conservation (hc)

What is hc? hc = azimuthal symmetry

$$\omega(t) \to \omega(t) \;, \quad \omega(0) \to \omega(0) \;, \quad \omega(\pm 1) \to e^{\mp i \phi} \omega(\pm 1) \;,$$

$$\lambda_A = \sum_{i=1}^{X} m(\lambda_{B_i}) + \bar{\lambda}_C \;, \quad m(t) = m(0) = 0 \;, \; m(\pm 1) = \pm 1 \;.$$

$$(1/2, 1/2)|_{SO(3,1)} \to (\mathbf{1} + \mathbf{3})_{SO(3)}$$

At Kinematic Endpoint

Observables finite as ratios.

$$\frac{d\Gamma}{d\kappa} \sim O(\kappa) \;, \quad \kappa = \text{velocity}$$

An example: O_V

$$O_{V(A)} = \bar{s}_L \gamma^{\mu} b \, \bar{\ell} \gamma_{\mu} (\gamma_5) \ell \; ,$$

- $O_V \Rightarrow$ one unphysical B-polarisation ghc: $\boldsymbol{\lambda \equiv \lambda_\gamma = \lambda_{K^*}}$

$$\mathcal{A}(B \to \gamma^*(\lambda_{\gamma^*}) K^*(\lambda_{K^*})) = \epsilon^*(\lambda_{\gamma^*})_\mu \eta^*(\lambda_{K^*})_\nu \, (x \, g^{\mu\nu} + y \, p^{\mu}_{\gamma^*} p^{\nu}_{K^*})$$

- *kinematic endpoint:* $q_\gamma \sim p_{K^*} \sim (1,0,0,0)$ and since

$$p \cdot \omega(p, \lambda) = 0$$

$$\mathcal{A}(B \to \gamma^*(\lambda_{\gamma^*}) K^*(\lambda_{K^*})) \propto \epsilon(\lambda_{\gamma^*}) \cdot \eta(\lambda_{K^*}) = \begin{cases} 1 & \lambda_{\gamma^*} = \bar{\lambda}_{K^*} = \pm \\ -1 & \lambda_{\gamma^*} = \lambda_{K^*} = 0 \\ 0 & \text{otherwise} \end{cases}$$

Clebsch-Gordan coefficeint

scalars $\left(O^{(\prime)}_{S,P}\right)$: $H = 0 \, ,$

vectors $\left(O^{(\prime)}_{V,A}\right)$: $H_0 = -H_+ = -H_- \, ,$ $\left[H_\parallel = -\sqrt{2} H_0 \, , H_\perp = 0\right] ,$

$H_t = 0 \, ,$

tensors $\left(O^{(\prime)}_{T}\right)$: $H_{+-} = -H_{+0} = -H_{-0} \, ,$ $\left[H^{\mathcal{T}}_\parallel = -\sqrt{2} H^{\mathcal{T}}_0 \, , H^{\mathcal{T}}_\perp = 0\right] ,$

$H_{t0} = -H_{t+} = -H_{t-}$ $\left[H^{\mathcal{T}_t}_\parallel = -\sqrt{2} H^{\mathcal{T}_t}_0 \, , H^{\mathcal{T}_t}_\perp = 0\right] ,$

prediction at endpoint

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = (J_{1s} + J_{2s}\cos 2\theta_\ell + J_{6s}\cos\theta_\ell)\sin^2\theta_K$$
$$+ (J_{1c} + J_{2c}\cos 2\theta_\ell + J_{6c}\cos\theta_\ell)\cos^2\theta_K$$
$$+ (J_3\cos 2\phi + J_9\sin 2\phi)\sin^2\theta_K\sin^2\theta_\ell$$
$$+ (J_4\cos\phi + J_8\sin\phi)\sin 2\theta_K\sin 2\theta_\ell$$
$$+ (J_5\cos\phi + J_7\sin\phi)\sin 2\theta_K\sin\theta_\ell\,,$$

- 10 relation among 12 observables

$$J_{2s}(q^2_{\rm max}) = -J_{2c}(q^2_{\rm max})/2\,, \quad J_{1s}(q^2_{\rm max}) - J_{2s}(q^2_{\rm max})/3 = J_{1c}(q^2_{\rm max}) - J_{2c}(q^2_{\rm max})/3\,,$$
$$J_3(q^2_{\rm max}) = -J_4(q^2_{\rm max})\,, \quad J_{2c}(q^2_{\rm max}) = J_3(q^2_{\rm max})\,, \quad J_{5,6s,6c,7,8,9}(q^2_{\rm max}) = 0\,,$$

- A few examples

$$F_L = \frac{|H_0|^2}{|H_0|^2 + |H_+|^2 + |H_-|^2} = \frac{1}{3} \qquad \frac{d\Gamma}{\Gamma d\theta_{K,\ell}} = \text{constant}$$

no preferred direction

$$\frac{d\Gamma}{\Gamma d\phi} = (1 + r_\phi\cos(2\phi))\,, \quad r_\phi|_{\rm SM} = -1/3 + \mathcal{O}(m_\ell)$$

only example sensitive new physics (later...)

$$r_\phi|_{\rm BSM} = r_\phi|_{\rm SM} + \mathcal{O}\left(\frac{\text{tensor}}{\text{vector}}\right)$$

COMPARISON WITH LHCB AT 1FB^{-1}

- endpoint relations depend Lorentz-covariance only! whether BSM or approximations of various kind
  $\Rightarrow$ expect agree with experiment

- statistically in full agreement (best outcome!)

	F_L	S_3	$^aP_4'$	S_7	$^bP_5'/A_{\rm FB}$	$^bS_8/S_9$	$^aA_{\rm FB}$	P_5'	$^aS_8$	S_9
endpoint	1/3	-1/3	$\sqrt{2}$	0	$\sqrt{2}$	$-1/2$	$\hat{R}\kappa$	$\sqrt{2}\hat{R}\kappa$	$1/3\hat{I}\kappa$	$-2/3\hat{I}\kappa$
$B \to K^*$	0.38 ± 0.04	-0.22 ± 0.09	$0.70^{+0.44}_{-0.52}$	$0.15^{+0.16}_{-0.15}$	1.63 ± 0.57	-0.5 ± 2.2	-0.36 ± 0.04	$-0.60^{+0.21}_{-0.18}$	-0.03 ± 0.12	$0.06^{+0.11}_{-0.10}$
$B_s \to \phi$	$0.16^{+0.18}_{-0.12}$	$0.19^{+0.30}_{-0.31}$	–	–	–	–	–	–	–	–

exact endpoint prediction

world average (mostly LHCb) last bin
q^2 in [16,19]GeV^2 (expect 10-15% deviation)

IN THE VICINITY OF THE KINEMATIC ENDPOINT

i) universality O(velocity) ii) non-universality O(velocity^2).

kinematics

dynamics

κ-expansion

- endpoint "done deal" look vicinity - expand velocity of K*

$$\kappa = |\vec{p_{K^*}}| = \sqrt{\frac{\lambda(p_{K^*}^2, p_B^2, q^2)}{4p_B^2}} \,, \qquad \lambda(x,y,y) \equiv (x-(y+z)^2)(x-(y-z)^2) \,,$$

- parity covariance: $\text{HA} \propto C_+\kappa^n + C_-\kappa^{n\pm1} \,, \quad C_\pm \equiv C \pm C' \,,$
  NLO O(**κ**) universal opposite parity — O(**κ**²) depends dynamics

HA	H	H_t	$H_{0,\parallel}$	$H_\perp$	$H^{\mathcal{T}_t}_{0,\parallel}$	$H^{\mathcal{T}_t}_\perp$	$H^{\mathcal{T}}_{0,\parallel}$	$H^{\mathcal{T}}_\perp$
WCC	C_-	C_-	C_-	C_+	C_-	C_+	C_+	C_-
$\propto \kappa^n$	$\kappa + ..$	$\kappa + ..$	$\kappa^0 + ..$	$\kappa + ..$	$\kappa^0 + ..$	$\kappa + ..$	$\kappa^0 + ..$	$\kappa + ..$

refinement of earlier result

- BSM-sensitive R consistent with SM according to LHCb

	${}^{a}A_{\rm FB}$	P_5'	${}^{a}S_8$	S_9
endpoint	$\hat{R}\kappa$	$\sqrt{2}\hat{R}\kappa$	$1/3\hat{I}\kappa$	$-2/3\hat{I}\kappa$
$B \to K^*$	-0.36 ± 0.04	$-0.60^{+0.21}_{-0.18}$	-0.03 ± 0.12	$0.06^{+0.11}_{-0.10}$

$$\hat{R}_{\rm fit} = (-0.67 \pm 0.07)\,{\rm GeV}^{-1}$$
$$\hat{R}_{\rm SM} = (-0.73^{+0.12}_{-0.13})\,{\rm GeV}^{-1}$$

$$|A|^2 \equiv |a_\parallel^L|^2 + |a_\parallel^R|^2 = \frac{1}{2}(|a_\parallel^V|^2 + |a_\parallel^A|^2)\,,$$
$$R \equiv {\rm Re}[a_\parallel^L a_\perp^{L*} - a_\parallel^R a_\perp^{R*}] = -\frac{1}{2}{\rm Re}[a_\parallel^V a_\perp^{A*} + a_\parallel^A a_\perp^{V*}]\,,$$
$$I \equiv {\rm Im}[a_\parallel^L a_\perp^{L*} + a_\parallel^R a_\perp^{R*}] = \frac{1}{2}{\rm Im}[a_\parallel^V a_\perp^{V*} + a_\parallel^A a_\perp^{A*}]\,,$$

$$A_{\rm FB} = \frac{J_{6s} - J_{6c}/2}{3J_{1s} - J_{2s}} = \hat{R}\kappa\,, \quad P_5' = \frac{J_5}{2\sqrt{-J_{2c}J_{2s}}} = \sqrt{2}\hat{R}\kappa\,,$$
$$S_8 = \frac{4/3 J_8}{3J_{1s} - J_{2s}} = \frac{1}{3}\hat{I}\kappa\,, \quad S_9 = \frac{4/3 J_9}{3J_{1s} - J_{2s}} = -\frac{2}{3}\hat{I}\kappa\,,$$

$$\hat{R} \equiv \frac{R}{|A|^2}\,, \qquad \hat{I} \equiv \frac{I}{|A|^2}\,.$$

Some more detail of a possible high q²-parameterisation

$$-\sqrt{2}H_0^x = \sqrt{q^2_{\rm max}/q^2}(a_0^x + b_0^x\kappa^2 + \frac{c_0}{q^2} + ..)(1 + \sum_r \Delta_0^{(r)}(q^2)) \;, \quad x = L, R \;,$$

$$H_\perp^x = \kappa(a_\perp^x + b_\perp^x\kappa^2 + \frac{c_\perp}{q^2} + ..)(1 + \sum_r \Delta_\perp^{(r)}(q^2)) \;,$$

$$H_\parallel^x = (a_\parallel^x + b_\parallel^x\kappa^2 + \frac{c_\parallel}{q^2} + ..)(1 + \sum_r \Delta_\parallel^{(r)}(q^2)) \;,$$

- Endpoint symmetry: $a_0^x = a_\parallel^x \;, \quad c_0 = c_\parallel \;, \quad \Delta_0^{(r)}(q^2_{\rm max}) = \Delta_\parallel^{(r)}(q^2_{\rm max}) \;,$
- photon couples V-like: $c^R = c^L,\ \Delta^R = \Delta^L$

EPILOGUE

- Endpoint symmetries based on Lorentz invariance should hold
- Endpoint symmetries useful
  - as experimental cross-check
  - for constraints on fits

  no freedom
- BSM searches in κ-expansion

  freedom
- Generally of help in eliminating invariants at cost of limiting statistics
  $\Rightarrow$ useful in other areas with $1 \to 3+..$ decays
- Derive symmetry properties of high-q^2 OPE

  freedom

THANKS FOR YOUR ATTENTION

BACKUP SLIDES

ADDITIONAL TOPICS

select

- threshold or κ-expansion (search BSM near endpoint)
- higher spin K-mesons (spin 2)
- high-q^2-OPE — charm resonances
- life of F_L for $S \to VV$ decays

High-q² OPE - charm resonances

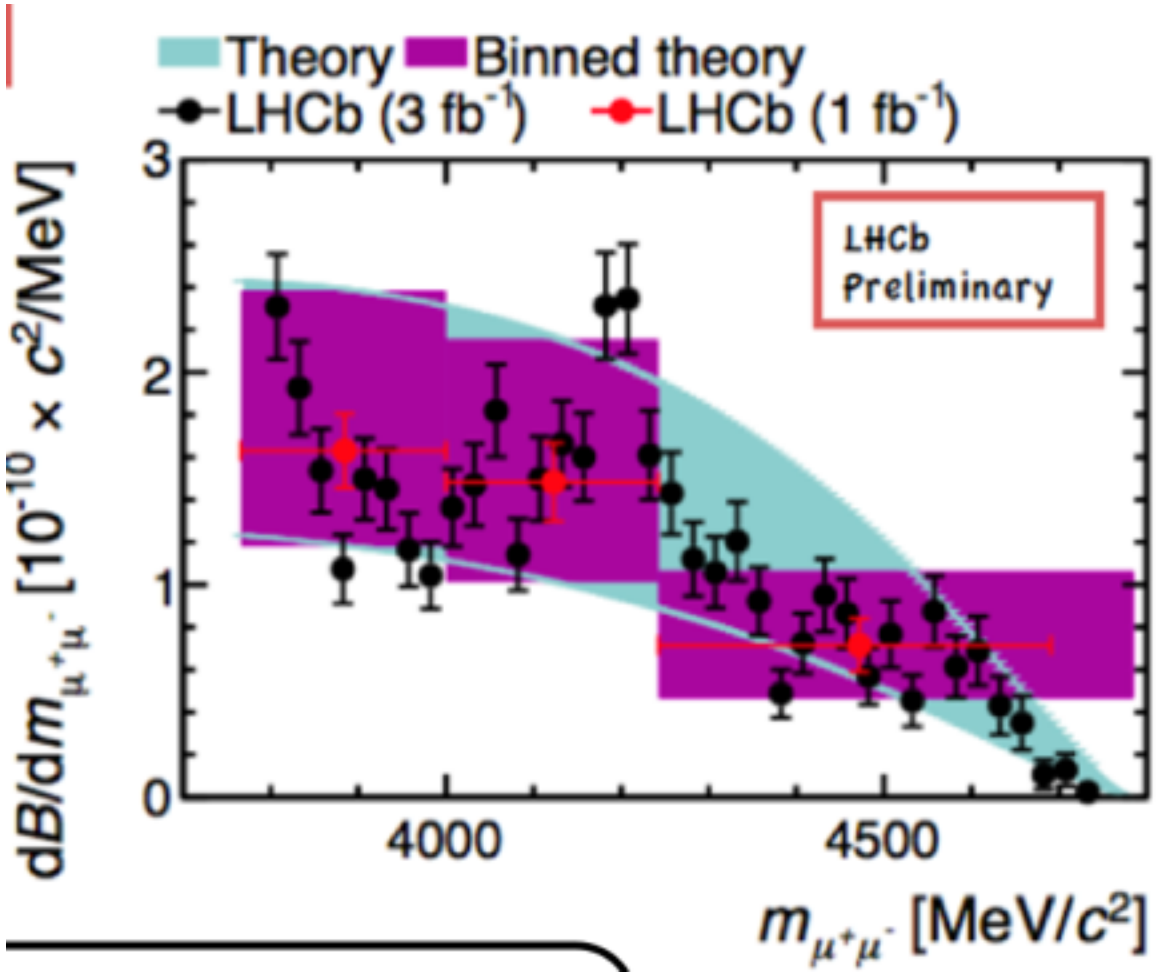
- Above J/Ψ,Ψ'
  OPE=short distance proposed

- There are further resonances
  known e^+e^- (same physics)
  different interference
  ⇒relies "local" quark hadron duality

2 comments

- 1 fb^{-1} works "ok" at 3 fb^{-1} not so great

- endpoint relations not violated "OPE",
  so initial "ok" looks even more dubious



Higher spin K-meson

- e.g. $B \to K_2 ll$ (spin 2) what's new?
- SM-basis e.g. O_V-operators hc: $\lambda_{K2} = \lambda_{ll} = \{0,1,-1\}$
  $\Rightarrow |\lambda_{K2}| = 2$ forbidden ("selection rule")
- Exact predictions but not uniform in $\theta_{K,l}$

$$H_{\bar{2}} : H_{\bar{1}} : H_0 : H_1 : H_2 = 0 : 1 : \frac{-2}{\sqrt{3}} : 1 : 0$$

- Wait, how is disorientation resolved? $H_\lambda(K_2) \sim O(\boldsymbol{\kappa})$
  there is a preferred direction! $H_\lambda(K^*) \sim O(\boldsymbol{\kappa}^0)$

Life of F_L for S→VV

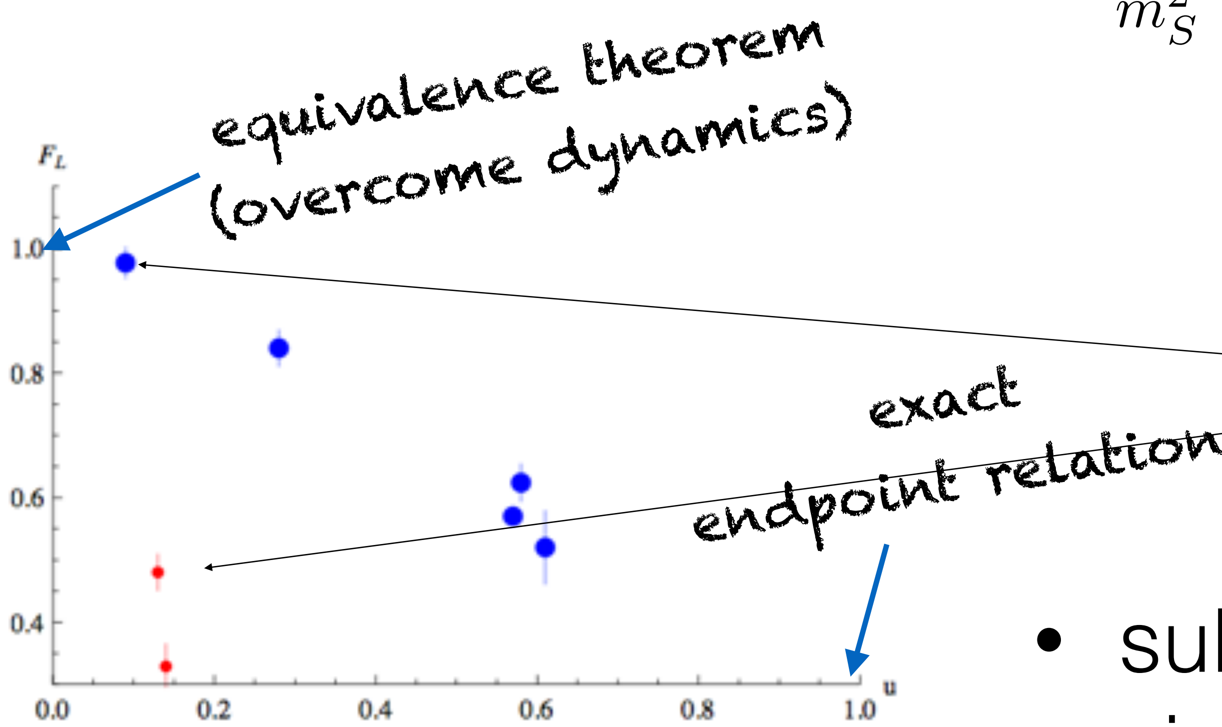
$$F_L = \frac{|H_0|^2}{|H_0|^2 + |H_+|^2 + |H_-|^2}$$

- Are endpoint relations valid non-leptonic decay ("factorisation")
If observed via $S \to V(\to S_1 S_2) V'(\to S_3 S_4)$ mostly yes ….

- $S \to VV'$ fixed $\boldsymbol{\kappa}_V$, generically not endpoint configuration!

- Assign measure

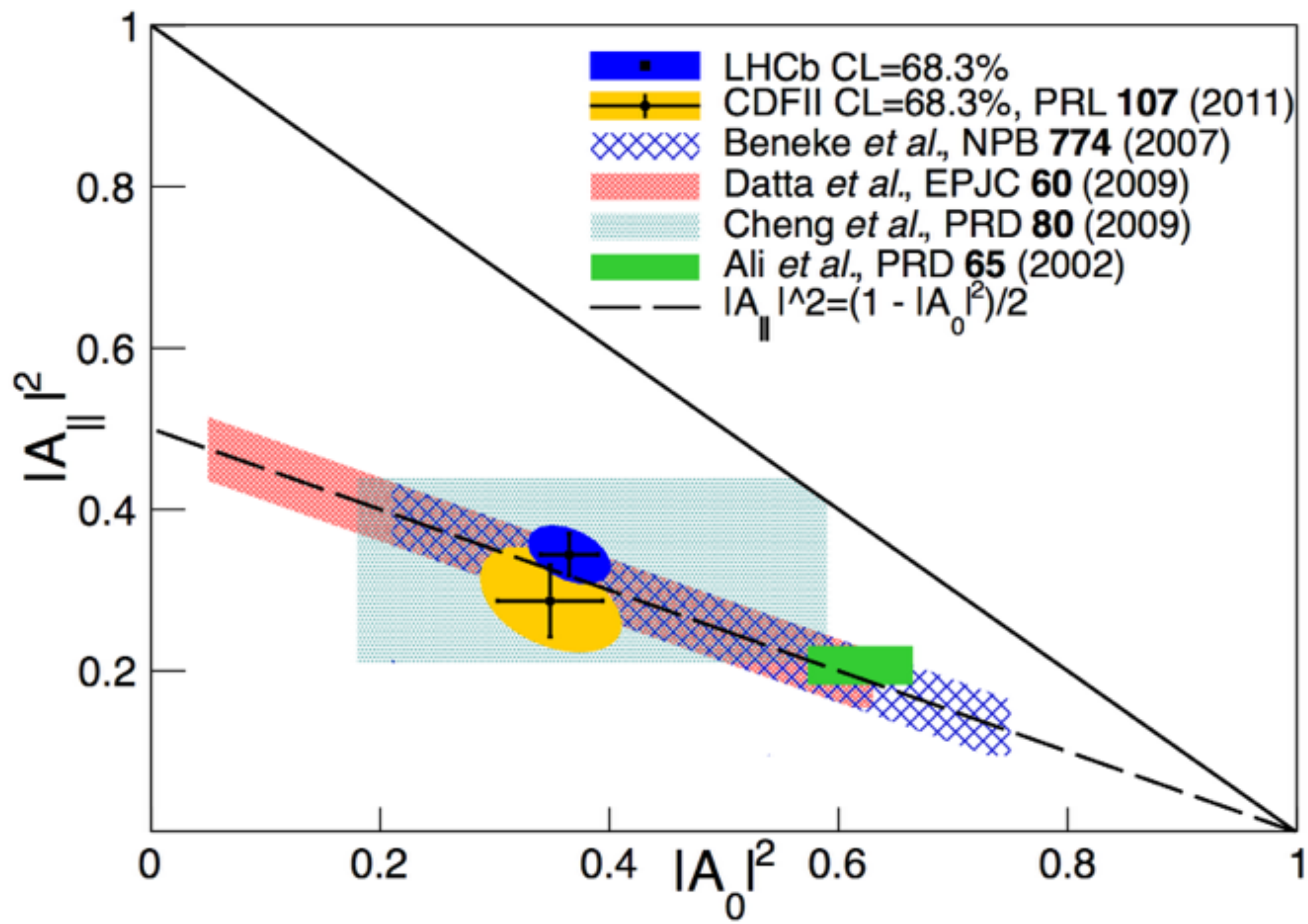
$$u = \frac{(m_V + m_{V'})^2}{m_S^2} = \begin{cases} 1 & \text{endpoint} \\ 0 & \text{fully relativistic} \end{cases}$$



$$F_L(B^0 \to \rho^+ \rho^-)_{u=0.09} = 0.977 \pm 0.026$$
$$F_L(B^0 \to \phi K^{*0})_{u=0.13} = 0.480 \pm 0.030$$

- subdominant weak annihilation
sizeable & endpt-divergent QCDF
large uncertainty - inconclusive

$B_s^0 \to \phi\phi$



$f(q^2)$ form factor

sum rule (end) $f(q^2)_{M^2} = \int_{m_b^2}^{s_0} ds\, \rho(s,q^2)\, e^{\frac{m_B^2 - s}{M^2}}$

M^2 = Borel parameter

$\langle x \rangle \equiv \int_{m_b^2}^{s_0} ds\, \rho(s)\, x\, e^{\frac{m_B^2 - s}{M^2}}$

i) $m_B^2 = \frac{\langle s \rangle}{\langle 1 \rangle}$ impose

self-consistent as best of all cases $\rho(s) \propto \delta(s - m_B^2)$

ii) $f(q^2)_{M^2}$ independent of M^2 => extremize

$$0 = \frac{d}{d\, 1/M^2} \ln f = \frac{m_B^2 \langle 1 \rangle - \langle s \rangle}{\langle 1 \rangle}$$

note ; $\langle 1 \rangle = f$

▷ procedure i) & ii) equivalent

LONG DISTANCE PHYSICS LCSR VS QCDF