# A new insight on the Anomaly 

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## $P_{1}$ and $P_{2}$ observables function of $A_{\perp}$ and $A_{\|}$amplitudes

- $\mathbf{P}_{1}$ : Proportional to $\left|A_{\perp}\right|^{2}-\left|A_{\|}\right|^{2}$
- Test the LH structure of SM and/or existence of RH currents that breaks $A_{\perp} \sim-A_{\|}$
- $\mathbf{P}_{2}$ : Proportional to $\operatorname{Re}\left(A_{i} A_{j}\right)$
- Zero of $P_{2}$ at the same position as the zero of $A_{F B}$
- $P_{2}$ is the clean version of $A_{F B}$. Their different normalizations offer different sensitivities.


- $P_{3}$ and $P_{6,8}^{\prime}$ are proportional to $\operatorname{Im} A_{i} A_{j}$ and small if there are no large phases. All are $<0.1$.
- $P_{i}^{C P}$ are all negligibly small if there is no New Physics in weak phases.
$P_{4}^{\prime}$ and $P_{5}^{\prime}$ observables function of $A_{\perp, \|}$ and also $A_{0}$ amplitudes
- $\mathbf{P}_{4,5}^{\prime}$ : Proportional to $\operatorname{Re}\left(A_{i} A_{j}\right)$
- $\left|P_{4,5}\right| \leq 1$ but $\left|P_{4,5}^{\prime}\right|$ can be $>1$.


In the large-recoil limit

$$
\begin{aligned}
A_{\perp, \|}^{L} & \propto\left[\mathcal{C}_{9}^{\mathrm{eff}}-\mathcal{C}_{10}+\frac{2 \hat{m}_{b}}{\hat{s}} \mathcal{C}_{7}^{\mathrm{eff}}\right] \xi_{\perp}\left(E_{K^{*}}\right) \quad A_{\perp, \|}^{R} \propto\left[\mathcal{C}_{9}^{\mathrm{eff}}+\mathcal{C}_{10}+\frac{2 \hat{m}_{b}}{\hat{s}} \mathcal{C}_{7}^{\mathrm{eff}}\right] \xi_{\perp}\left(E_{K^{*}}\right) \\
A_{0}^{L} & \propto\left[\mathcal{C}_{9}^{\mathrm{eff}}-\mathcal{C}_{10}+2 \hat{m}_{b} \mathcal{C}_{7}^{\mathrm{eff}}\right] \xi_{\|}\left(E_{K^{*}}\right) \quad A_{0}^{R} \propto\left[\mathcal{C}_{9}^{\mathrm{eff}}+\mathcal{C}_{10}+2 \hat{m}_{b} \mathcal{C}_{7}^{\mathrm{eff}}\right] \xi_{\| \|}\left(E_{K^{*}}\right)
\end{aligned}
$$

- In the SM $C_{9}^{S M} \sim-C_{10}^{S M}$, this cancellation strongly suppresses $A_{\perp, \|}^{R}$ above $4 \mathrm{Gev}^{2}: A_{\perp, \|}^{L} \gg A_{\perp, \|}^{R}$. This makes $P_{4} \rightarrow 1$ and $P_{5} \rightarrow-1$ for $q^{2} \rightarrow 8 \mathrm{GeV}^{2}$ quite fast BUT the fact that $\left|A_{\|}\right|>\left|A_{\perp}\right|$ and that $P_{4}^{\prime} \propto A_{0}^{L *} A_{\|}^{L}+A_{0}^{R} A_{\|}^{R *}$ and $P_{5}^{\prime} \propto A_{0}^{L *} A_{\perp}^{L}-A_{0}^{R} A_{\perp}^{R *}$ makes less efficient the convergence in the case of $P_{5}^{\prime}$.
- In presence of New Physics affecting only $C_{9}$ the cancellation $C_{9} \sim-C_{10}$ is less efective, consequently $A_{\perp, \|}^{R}$ is less suppressed and one should expect to see the effect of $C_{9} \neq C_{9}^{S M}$ in $P_{5}^{\prime}$.

Present bins: $[0.1,2],[2,4.3],[4.3,8.68],[1,6],[14.18,16],[16,19] \mathrm{GeV}^{2}$.

| Observable | Experiment | SM prediction | Pull |  |
| :---: | :---: | :---: | :---: | :---: |
| $\left\langle P_{1}\right\rangle$ | $-0.19_{-0.35}^{+0.40}$ | $0.007_{-0.044}^{+0.043}$ | -0.5 | (large error bars) |
| $\left\langle P_{1}\right\rangle_{[2,4.3]}$ | $-0.29_{-0.46}^{+0.65}$ | $-0.051_{-0.046}^{+0.046}$ | -0.4 |  |
| $\left\langle P_{1}\right\rangle_{[4.3,8.68]}$ | $0.36_{-0.31}^{+0.30}$ | $-0.117_{-0.052}^{+0.056}$ | +1.5 | ver value of the second and |
| $\left\langle P_{1}\right\rangle_{[1,6]}$ | $0.15_{-0.41}^{+0.39}$ | $-0.055_{-0.043}^{+0.041}$ | +0.5 | ird bins of $A_{\text {FB }}$ is consist |
| $\left\langle P_{2}\right\rangle_{[0.1,2]}$ | $0.03_{-0.15}^{+0.14}$ | $0.172_{-0.02}^{+0.020}$ | -1.0 | $2.9 \sigma(1.7 \sigma)$ deviation in cond (third) bin of $P_{2}$. |
| $\left\langle P_{2}\right\rangle_{[2,4.3]}$ | $0.50_{-0.07}^{+0.00}$ | $0.234_{-0.086}^{+0.060}$ | +2.9 |  |
| $\left\langle P_{2}\right\rangle_{[4.3,8.68]}$ | $-0.25_{-0.08}^{+0.07}$ | $-0.407_{-0.037}^{+0.049}$ | +1.7 |  |
| $\left\langle P_{2}\right\rangle_{[1,6]}$ | $0.33_{-0.12}^{+0.11}$ | $0.084_{-0.078}^{+0.060}$ | +1.8 | $A_{\mathrm{FB}}$ (same as the zero of |
| $\left\langle A_{\text {FB }}\right\rangle_{[0.1,2]}$ | $-0.02_{-0.13}^{+0.13}$ | $-0.136_{-0.048}^{+0.051}$ | +0.8 | Both effects can be |
| $\left\langle A_{\text {FB }}\right\rangle_{[2,4.3]}$ | $-0.20_{-0.08}^{+0.08}$ | $-0.081_{-0.069}^{+0.055}$ | -1.1 | accommodated with $\mathcal{C}_{7}^{\mathbb{N P}}<0$ |
| $\left\langle A_{\text {FB }}\right\rangle_{[4.3,8.68]}$ | $0.16_{-0.05}^{+0.06}$ | $0.220_{-0.113}^{+0.138}$ | -0.5 | and/or $\mathcal{C}_{9}^{\mathrm{NP}}<0$. |
| $\left\langle A_{\text {FB }}\right\rangle_{[1,6]}$ | $-0.17_{-0.06}^{+0.06}$ | $-0.035_{-0.034}^{+0.037}$ | -2.0 |  |


| Observable | Experiment | SM prediction | Pull |
| :---: | :---: | :---: | :---: |
| $\left\langle P_{4}^{\prime}\right\rangle_{[0.1,2]}$ | $0.00_{-0.52}^{+0.52}$ | $-0.342_{-0.026}^{+0.031}$ | +0.7 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[2,4.3]}$ | $0.74{ }_{-0.60}^{+0.54}$ | $0.569_{-0.063}^{+0.073}$ | +0.3 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[4.3,8.68]}$ | $1.18{ }_{-0.32}^{+0.26}$ | $1.003_{-0.032}^{+0.028}$ | +0.6 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[1,6]}$ | $0.58{ }_{-0.36}^{+0.32}$ | $0.555_{-0.058}^{+0.067}$ | +0.1 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[0.1,2]}$ | $0.45{ }_{-0.24}^{+0.21}$ | $0.533_{-0.041}^{+0.033}$ | -0.4 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[2,4.3]}$ | $0.29{ }_{-0.39}^{+0.40}$ | $-0.334_{-0.113}^{+0.097}$ | +1.6 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[4.3,8.68]}$ | $-0.19_{-0.16}^{+0.16}$ | $-0.872_{-0.041}^{+0.053}$ | +4.0 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[1,6]}$ | $0.21_{-0.21}^{+0.20}$ | $-0.349_{-0.100}^{+0.088}$ | +2.5 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[14.18,16]}$ | $-0.18_{-0.70}^{+0.54}$ | $1.161_{-0.332}^{+0.190}$ | -2.1 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[16,19]}$ | $0.70_{-0.52}^{+0.44}$ | $1.263_{-0.248}^{+0.119}$ | -1.1 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[14.18,16]}$ | $-0.79_{-0.22}^{+0.27}$ | $-0.779_{-0.363}^{+0.328}$ | +0.0 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[16,19]}$ | $-0.60{ }_{-0.18}^{+0.21}$ | $-0.601_{-0.367}^{+0.282}$ | +0.0 |

## Definition of the anomaly:

- $\mathbf{P}_{\mathbf{5}}^{\prime}$ : There is a striking $4.0 \sigma(1.6 \sigma)$ deviation in the third (second) bin of $P_{5}^{\prime}$.

Consistent with large negative contributions in $\mathcal{C}_{7}^{\mathrm{NP}}$ and/or $\mathcal{C}_{9}^{\mathrm{NP}}$.

- $\mathbf{P}_{4}^{\prime}$ : in agreement with the SM , but within large uncertainties, and it has future potential to determine the sign of $\mathcal{C}_{10}^{\mathrm{NP}}$.
- $\mathbf{P}_{6}^{\prime}$ and $\mathbf{P}_{8}^{\prime}$ : show small deviations with respect to the SM, but such effect would require complex phases in $\mathcal{C}_{9}^{\mathrm{NP}}$ and/or $\mathcal{C}_{10}^{\mathrm{NP}}$.

Us: $(-0.19-(-0.872)) / \sqrt{0.16^{2}+0.053^{2}}=4.05$ and Exp: $(-0.19-(-0.872+0.053)) / \sqrt{0.16^{2}+0.053^{2}}=3.73$

## Our SM predictions+LHCb data



Figure: Experimental measurements and SM predictions for some $B \rightarrow K^{*} \mu^{+} \mu^{-}$observables. The black crosses are the experimental LHCb data. The blue band corresponds to the SM predictions for the differential quantities, whereas the purple boxes indicate the corresponding binned observables.

Result of our analysis (large+low recoil data+rad) if we allow all Wilson coefficients to vary freely:

| Coefficient | $1 \sigma$ | $2 \sigma$ | $3 \sigma$ |
| :--- | :---: | :---: | :---: |
| $\mathcal{C}_{7}^{\mathrm{NP}}$ | $[-0.05,-0.01]$ | $[-0.06,0.01]$ | $[-0.08,0.03]$ |
| $\mathcal{C}_{9}^{\mathrm{NP}}$ | $[-1.6,-0.9]$ | $[-1.8,-0.6]$ | $[-2.1,-0.2]$ |
| $\mathcal{C}_{10}^{\mathrm{NP}}$ | $[-\mathbf{0 . 4}, \mathbf{1} .0]$ | $[-1.2,2.0]$ | $[-2.0,3.0]$ |
| $\mathcal{C}_{7^{\prime}}^{\mathrm{NP}}$ | $[-\mathbf{0 . 0 4}, \mathbf{0 . 0 2 ]}$ | $[-0.09,0.06]$ | $[-0.14,0.10]$ |
| $\mathcal{C}_{9^{\prime}}^{\mathrm{NP}}$ | $[-0.2,0.8]$ | $[-0.8,1.4]$ | $[-1.2,1.8]$ |
| $\mathcal{C}_{10^{\prime}}^{\mathrm{NP}}$ | $[-\mathbf{0 . 4}, \mathbf{0 . 4}]$ | $[-1.0,0.8]$ | $[-1.4,1.2]$ |

Table: $68.3 \%(1 \sigma), 95.5 \%(2 \sigma)$ and $99.7 \%(3 \sigma)$ confidence intervals for the NP contributions to WC.

- This table tells you again that there is strong evidence for a $\mathcal{C}_{9}^{\mathrm{NP}}<\mathbf{0}$, preference for $\mathcal{C}_{7}^{\mathrm{NP}}<0$ and no clear-cut evidence for $\mathcal{C}_{10,71,91,10,}^{\mathrm{NP}} \neq 0$.
- This does not imply that they will be at the end zero but that present data does not point clearly for a positive or negative value.


## General case all WC free

In conclusion our pattern of [PRD88 (2013) 074002] obtained from an $\mathcal{H}_{\text {eff }}$ approach is

$$
\mathrm{C}_{9}^{N P} \sim[-1.6,-0.9], \quad \mathrm{C}_{7}^{N P} \sim[-0.05,-0.01], \quad \mathrm{C}_{9}^{\prime} \sim \pm \delta \quad \mathrm{C}_{10}, \mathrm{C}_{7,10}^{\prime} \sim \pm \epsilon
$$

where $\delta$ is small (at maximum half $\left|\mathbf{C}_{9}^{N P}\right|$ ) and $\epsilon$ is smaller.

A simplified version is $C_{9}^{N P}=-1.5$

Best fit points:

Large recoil: $\mathrm{C}_{9}^{\mathrm{NP}}=-1.5, C_{\text {7eff }}^{\mathrm{NP}}=-0.02$
Large recoil: $\mathrm{C}_{9}^{\mathrm{NP}}=-1.6, C_{7 \text { eff }}^{\mathrm{NP}}=-0.02, C_{10}^{\mathrm{NP}}>0, \mathbf{C}_{9}^{\mathrm{NP}}<\mathbf{0}, C_{7 \prime}^{\mathrm{NP}}>0, C_{10}^{\mathrm{NP}}<0$.
Large+Low: $C_{9}^{\mathrm{NP}}=-1.2, C_{7 \text { eff }}^{\mathrm{NP}}=-0.03, C_{10}^{\mathrm{NP}}>0, \mathbf{C}_{9,1}^{\mathrm{NP}}>\mathbf{0}, C_{7 \prime}^{\mathrm{NP}}<0, C_{10}^{\mathrm{NP}}<0$

# Can we test if the anomaly in $P_{5}^{\prime}$ is isolated? 

Important test for $3 \mathrm{fb}^{-1}$ data

## BUT

already now there are interesting hints...
[Egede, Hurth, JM, Ramon, Reece'10]
An important step forward was the identification of the symmetries of the distribution:
Transformation of amplitudes leaving distribution invariant.
Symmetries determine the minimal \# observables for each scenario:

$$
n_{\text {obs }}=2 n_{A}-n_{S}
$$

| Case | Coefficients | Amplitudes | Symmetries | Observables |
| :---: | :---: | :---: | :---: | :---: |
| $m_{\ell}=0, A_{S}=0$ | 11 | 6 | 4 | 8 |
| $m_{\ell}=0$ | 11 | 7 | 5 | $\mathbf{9}$ |
| $m_{\ell}>0, A_{S}=0$ | 11 | 7 | 4 | $\mathbf{1 0}$ |
| $m_{\ell}>0$ | 12 | 8 | 4 | $\mathbf{1 2}$ |

All symmetries (massive and scalars) were found explicitly later on.
[JM, Mescia, Ramon, Virto'12]
Symmetries $\Rightarrow$ \# of observables $\Rightarrow$ determine a basis: each angular observable constructed can be expressed in terms of this basis.

Let's review first the symmetry formalism for the massless angular distribution:

$$
\mathbf{n}_{\|}=\binom{A_{\|}^{L}}{A_{\|}^{R *}}, \quad \mathbf{n}_{\perp}=\binom{A_{\perp}^{L}}{-A_{\perp}^{R *}}, \quad \mathbf{n}_{0}=\binom{A_{0}^{L}}{A_{0}^{R *}} .
$$

All the coefficients $\boldsymbol{J}_{\mathbf{i}}$ can be expressed in terms of the products $\mathbf{n}_{\mathbf{i}}^{\dagger} \mathbf{n}_{\mathbf{j}}$ (example):

$$
J_{3}=\frac{1}{2}\left(\left|n_{\perp}\right|^{2}-\left|n_{\|}\right|^{2}\right), \quad J_{4}=\frac{1}{\sqrt{2}} \operatorname{Re}\left(n_{0}^{\dagger} n_{\|}\right), \quad J_{5}=\sqrt{2} \operatorname{Re}\left(n_{0}^{\dagger} n_{\perp}\right), \quad J_{9}=-\operatorname{lm}\left(n_{\perp}^{\dagger} n_{\|}\right)
$$

A symmetry of the angular distribution will be a unitary transformation $n_{i} \rightarrow U n_{i}$

$$
n_{i}^{\prime}=U n_{i}=\left[\begin{array}{ll}
e^{i \phi_{\mathrm{L}}} & 0 \\
0 & e^{-i \phi_{\mathrm{R}}}
\end{array}\right]\left[\begin{array}{rr}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{array}\right]\left[\begin{array}{rr}
\cosh i \tilde{\theta} & -\sinh i \tilde{\theta} \\
-\sinh i \tilde{\theta} & \cosh i \tilde{\theta}
\end{array}\right] n_{i}
$$

$U$ defines the four symmetries of the massless angular distribution:

- two global phase transformations ( $\phi_{\mathrm{L}}$ and $\phi_{\mathrm{R}}$ ),
- a rotation $\theta$ among the real and imaginary components of the amplitudes independently
- another rotation $\tilde{\theta}$ that mixes real and imaginary components of the transversity amplitudes.

Solving the system of equations of $A_{\perp, \|, 0}$ in terms of $J_{i}$ (using three of the symmetries) we found:

$$
e^{i\left(\phi_{-}^{L}-\phi_{\perp}^{L}\right)}=\frac{2\left(2 J_{2 s}-J_{3}\right)\left(J_{5}+2 i J_{8}\right)-\left(2 J_{4}+i J_{7}\right)\left(J_{6 s}-2 i J_{9}\right)}{\sqrt{16 J_{2 s}^{2}-4 J_{3}^{2}-J_{6 s}^{2}-4 J_{9}^{2}} \sqrt{2 J_{1 c}\left(2 J_{2 s}-J_{3}\right)-4 J_{4}^{2}-J_{7}^{2}}},
$$

This equation is related to the freedom associated to the fourth unused symmetry transformation $\tilde{\theta}$. Imposing that its modulo is one we find:

$$
\begin{aligned}
J_{2 c}= & -2 \frac{\left(2 J_{2 s}+J_{3}\right)\left(4 J_{4}^{2}+\beta_{\ell}^{2} J_{7}^{2}\right)+\left(2 J_{2 s}-J_{3}\right)\left(\beta_{\ell}^{2} J_{5}^{2}+4 J_{8}^{2}\right)}{16 J_{2 s}^{2}-\left(4 J_{3}^{2}+\beta_{\ell}^{2} J_{6 s}^{2}+4 J_{9}^{2}\right)} \\
& +4 \frac{\beta_{\ell}^{2} J_{6 s}\left(J_{4} J_{5}+J_{7} J_{8}\right)+J_{9}\left(\beta_{\ell}^{2} J_{5} J_{7}-4 J_{4} J_{8}\right)}{16 J_{2 s}^{2}-\left(4 J_{3}^{2}+\beta_{\ell}^{2} J_{6 s}^{2}+4 J_{9}^{2}\right)},
\end{aligned}
$$

Indeed an identical equation can be written in terms of the $\overline{J_{i}}$.

This equation can be expressed in terms of $P_{i}$ and $P_{i}^{C P}$ observables to get:

$$
\bar{P}_{2}=+\frac{1}{2 \bar{k}_{1}}\left[\left(\bar{P}_{4}^{\prime} \bar{P}_{5}^{\prime}+\delta_{1}\right)+\frac{1}{\beta} \sqrt{\left(-1+\bar{P}_{1}+\bar{P}_{4}^{\prime 2}\right)\left(-1-\bar{P}_{1}+\beta^{2} \bar{P}_{5}^{\prime 2}\right)+\delta_{2}+\delta_{3} \bar{P}_{1}+\delta_{4} \bar{P}_{1}^{2}}\right]
$$

where

$$
\bar{P}_{i}=P_{i}+P_{i}^{C P} \quad \beta=\sqrt{1-4 m_{\ell}^{2} / s}
$$

The sign in front of the square root is taken "+" everywhere by comparison with exact result in SM, at low-recoil both solutions ( + and -) converge. (Plot with $\delta_{i} \rightarrow 0$ )


REMARK:

- This is an exact equation valid for any $q^{2}$ (low, large) and obtained from symmetries.
- It involves $6 P_{i}$ of the basis plus one redundant.

An identical equation can be written in terms of $\hat{P}_{i}=P_{i}-P_{i}^{C P}$, substituting $\bar{P}_{i} \rightarrow \hat{P}_{i}$ everywhere. More importantly all terms inside the $\delta_{i}$ are strongly suppressed (by small strong and weak phases):

$$
\delta_{i} \sim \mathcal{O}\left(\left(\operatorname{Im} A_{i}\right)^{2}, 1-\bar{k}_{1}\right) \quad \text { and } \quad \bar{k}_{1}=1+F_{L}^{C P} / F_{L}
$$

Hypothesis: No New Physics in weak phases entering Wilson coefficients and not scalars/tensors. Both hypothesis can be tested, measuring $P_{i}^{C P}$ and $S_{1}$.

To an excellent approximation we have:
$P_{2}=\frac{1}{2}\left[P_{4}^{\prime} P_{5}^{\prime}+\frac{1}{\beta} \sqrt{\left(-1+P_{1}+P_{4}^{\prime 2}\right)\left(-1-P_{1}+\beta^{2} P_{5}^{\prime 2}\right)}\right]$
This equation can be used in binned form if:

- Observables are nearly constant inside the bin
- Or the size of the bin is very small.

We correct for this by $\left\langle P_{2}\right\rangle \rightarrow\left\langle P_{2}\right\rangle+\Delta_{\text {exact-relation }}^{\mathrm{NP}}$ where $\Delta_{\text {exact-relation }}^{\mathrm{NP}}$ is order $10^{-2}$ except for [0.1-2] bin and [1-6] bin.


Figure: Green: SM exact, dashed inside approximation, Red: NP $C_{9}^{N P}=-1.5$ exact, dashed inside approximation

The striking consequence of this equation is that it allows you to use data to predict the impact of the anomaly in $P_{5}^{\prime}$ in a completely different observable: $P_{2}$

- The terms $\delta_{i}$ has been computed in the SM and in presence of New Physics [constrained range]

$$
\begin{aligned}
& -0.1 \leq C_{7}^{N P} \leq 0.1,-2 \leq C_{9}^{N P} \leq 0,-1 \leq C_{10}^{N P} \leq 1 \\
& -0.1 \leq C_{7}^{\prime} \leq 0.1, \quad-2 \leq C_{9}^{\prime} \leq 2, \quad-1 \leq C_{10}^{\prime} \leq 1
\end{aligned}
$$

being always bounded within $10^{-1}-10^{-2}$.

|  | $\delta_{1}$ | $\delta_{2}$ | $\delta_{3}$ | $\delta_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\|S M\|$ | $\lesssim 0.01$ | $\lesssim 0.03$ | $\lesssim 0.01$ | $\lesssim 0.01$ |
| NP | $[-0.03,0.01]$ | $[-0.09,0.01]$ | $[-0.04,0.04]$ | $[-0.03,0.02]$ |

The smaller the size of the bin the smaller the error

| Point | $[0.1-2]^{*}$ | $[2-4.3]$ | $[4.3-8.68]$ | $[1-6]^{*}$ | $[1-2]$ | $[2-3]$ | $[3-4]$ | $[4-5]$ | $[5-6]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta_{\text {exact-relation }}^{\mathrm{SM}}$ | -0.14 | -0.06 | -0.03 | -0.21 | -0.02 | -0.02 | -0.01 | -0.01 | -0.01 |
| $\Delta_{\text {exact-relation }}^{\mathrm{SM}}$ upper |  |  |  |  |  |  |  |  |  |
| $\Delta_{\text {exact-relation }}^{\mathrm{NP}}$ | -0.07 | -0.02 | -0.02 | -0.08 | +0.00 | +0.00 | +0.00 | +0.00 | +0.01 |
|  | -0.23 | -0.10 | -0.09 | -0.28 | -0.07 | -0.04 | -0.03 | -0.02 | -0.04 |
| $\Delta_{\text {exact-relation }}^{\mathrm{CNP}}=-1.5$ | -0.11 | -0.04 | -0.04 | -0.16 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |

Imposing that the square root is well defined one finds:

$$
P_{5}^{\prime 2}-1 \leq P_{1} \leq 1-P_{4}^{\prime 2}
$$

- Indeed this is an exact bound that could be alternatively obtained from

$$
\left|P_{4}\right|=\left|P_{4}^{\prime}\right| / \sqrt{1-P_{1}} \leq 1 \quad \text { and } \quad\left|P_{5}\right|=\left|P_{5}^{\prime}\right| / \sqrt{1+P_{1}} \leq 1
$$

$\left|P_{4,5}\right| \leq 1$ comes from the geometrical interpretation of those observables in terms of $n_{i}$.



- The new upper bound is very stringent for the [4.3.8.68] bin, cutting most of the space for a positive $P_{1}: P_{1}^{[4.3,8.68]}<0.33$
- The lower bound is particularly relevant for the $[16,19]$ bin of $P_{1}: P_{1}^{[16,19]}>-0.68$.

Implication II: At the position of the zero $q_{0}^{2}$ of $P_{2}$ (same as $A_{F B}$ ) the following relation holds:

$$
\left.\left[P_{4}^{2}+P_{5}^{2}\right]\right|_{q^{2}=q_{0}^{2}}=1 \quad \text { or }\left.\quad\left[P_{4}^{\prime 2}+P_{5}^{\prime 2}\right]\right|_{q^{2}=q_{0}^{2}}=1-\eta\left(q_{0}^{2}\right)
$$

where

$$
\eta\left(q_{0}^{2}\right)=P_{1}^{2}+\left.P_{1}\left(P_{4}^{\prime 2}-P_{5}^{\prime 2}\right)\right|_{q^{2}=q_{0}^{2}}
$$

SM Zero of $A_{F B}: q_{0}^{2 S M}=3.95 \pm 0.38$ (our), $3.90 \pm 0.12$ (Buras'08), $2.9 \pm 0.3$ (Khodjamirian'10) $\mathrm{GeV}^{2}$ Experimental LHCb data: $q_{0}^{2 L H C b}=4.9 \pm 0.9 \mathrm{GeV}^{2}$



Assume that a future precise measurement of the zero confirms $q_{0}^{2 e x p} \sim 4.9 \mathrm{GeV}^{2}$ with small error.
If $P_{4}^{\prime} \sim 1$ and $P_{1} \geq 0$ at $q_{0}^{2}=4.9 \mathrm{GeV}^{2}$ (like present data seems to suggest) then one should find $P_{1}\left(q_{0}^{2}\right) \leq 1-P_{4}^{\prime 2} \sim 0$, $\eta\left(q_{0}^{2}\right) \sim 0$ and $\mathrm{P}_{5}^{\prime}\left(\mathrm{q}_{0}^{2}\right) \sim 0$
(notice that in SM $\left.P_{5}^{\prime}\left(q_{0}^{2}\right)=-0.75\right)$

A precise measurement of $q_{0}^{2}$ (zero of $A_{F B}$ ) outside the $S M$ region would serve as an indirect confirmation of the anomaly

Implication III: We can establish a new relation between the anomaly bin in $P_{5}^{\prime}$ and $P_{2}$ :

$$
\left\langle P_{2}\right\rangle=\frac{1}{2}\left[\left\langle P_{4}^{\prime}\right\rangle\left\langle P_{5}^{\prime}\right\rangle+\sqrt{\left(-1+\left\langle P_{1}\right\rangle+\left\langle P_{4}^{\prime}\right\rangle^{2}\right)\left(-1-\left\langle P_{1}\right\rangle+\left\langle P_{5}^{\prime}\right\rangle^{2}\right)}\right]+\Delta_{\text {exact }}^{b i n}
$$

where $\Delta_{\text {exact }}^{b i n}=-0.04$ for NP best fit point at 2 nd and 3 rd bin, while $\Delta_{\text {exact }}^{b i n}=-0.01$ for $1 \mathrm{GeV}^{2}$ size.


GRAY band: SM prediction. BLUE cross: Measured value of $P_{2}$ RED rectangle: $C_{9}^{N P}=-1.5 \mathrm{NP}$ prediction.
Green cross is $\left\langle P_{2}\right\rangle$ obtained from combining data of $\left\langle P_{4,5}^{\prime}\right\rangle$, $\left\langle P_{1}\right\rangle$, considering asymmetric errors and bound on $P_{1}$

- Bin [2,4.3]: LHCb data: $+0.50_{-0.07}^{+0}$,Relation: $+0.46_{-0.19}^{+0}$
$0.2 \sigma$ measured (blue cross) versus relation (green cross)
- $\operatorname{Bin}[4.3,8.68]:$ LHCb data: $-0.25_{-0.08}^{+0.07}$, Relation: $+0.10_{-0.13}^{+0.13}$
$2.4 \sigma$ measured (blue cross) versus relation (green cross), 1.9 $\sigma$ from relation to NP best fit point (red box), $3.6 \sigma$ from relation to SM .

Extremely simplified where $P_{4}^{\prime} \sim 1$ (if $P_{1} \sim 0$ ): $P_{2} \sim \frac{1}{2} P_{5}^{\prime}$


It is not surprising that the second bin in $P_{2}$ fits perfectly, while the third bin in $P_{2}$ goes on the right direction but does not fit perfectly.

Reason It is very difficult to get excellent agreement with the third bin of $P_{5}^{\prime}$ inside a global fit.

- Our large recoil best fit point gives $\left\langle P_{5}^{\prime}\right\rangle_{[4.3,8.68]}=-0.49$ and reduces tension with data at $1.8 \sigma$ (from $4 \sigma$ in SM ): $C_{9}^{\prime}<0$ is strongly favored by this bin.
- The best fit point with $C_{9}^{N P}=-1.5$ gives $\left\langle P_{5}^{\prime}\right\rangle_{[4.3,8.68]}=-0.61$.
- Any analysis with $C_{9}^{\prime}>0$ provides a much worst disagreement with data in this bin.

Most plausible scenario: Third bin in $P_{5}^{\prime}$ will go down (reducing distance with SM) while third bin in $P_{2}$ might go up (enlarging distance with SM): Global picture much more consistent.


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The first low-recoil bin $[14.18,16]$ can also be tested using this equation

LHCb data on $P_{2}$ in this bin gives: $-\mathbf{0 . 5 0}_{-0.00}^{+0.03}$
LHCb data on $P_{4}^{\prime}, P_{1}, P_{5}^{\prime}$ implies that $P_{2}$ should be: $+\mathbf{0 . 5 0}_{-0.27}^{+0}$ (if + ) or $-\mathbf{0 . 5 0}_{+0}^{+0.33}$ (if -)

- This shows a discrepancy of $3.7 \sigma$ if + solution is taken
- Or agreement if - solution is chosen

However both solutions + and - should give same result at low-recoil

Conclusion: The measurement of this first low recoil bin is probably exhibiting a statistical fluctuation or signaling a problem at low recoil.

## ALTERNATIVELY Full fit of the angular distribution with a small dataset

Under the assumption of real Wilson coefficients one has

- Free parameters $F_{L}, P_{1}, P_{4,5}^{\prime}$.
- $P_{2}$ is a function of the other observables and $P_{6,8}^{\prime}$ are set to zero.


Figure: Residual distribution of $P_{5}^{\prime}$ when fitting with 100 events. The fit of a gaussian distribution is superimosed.

We find testing this fit for values around the measured values: convergence and unbiased pulls with as little as 50 events per bin. Gaussian pulls are obtained with only 100 events.
This opens the possibility to perform a full angular fit analysis with small bins in $q^{2}$

The main hypothesis (real WC) can be tested measuring $P_{i}^{C P}$.

- We have addressed, using symmetries, the question:

Is the anomaly in $P_{5}^{\prime}$ isolated?

- The anomaly in $P_{5}^{\prime}$ should also appear in $P_{2}$ in a specific way: The interesting result is that the deviation observed in $P_{2}$ in this same bin goes in the direction predicted by the anomaly.
- The higher position of the zero of $A_{F B}$ the smaller the value of $P_{5}^{\prime}$ at this point (for a $P_{4}^{\prime}$ SM-like)
- A strong upper and lower bound on $P_{1}: P_{5}^{\prime 2}-1 \leq P_{1} \leq 1-P_{4}^{\prime 2}$
- The first low-recoil bin of $P_{4}^{\prime}$ exhibits a $3.7 \sigma$ tension between the measured and obtained value using "+" solution, pointing possibly to a statistical fluctuation or a low-recoil problem.
- The obtained relation among $P_{2}, P_{4,5}^{\prime}, P_{1}$ opens the possibility to perform now a full angular fit with a reduced number of events.


## BACK-UP slides

