Divergent series and physical observables: the case of Sudakov resummation

CERN, May 9th, 2008

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Plan of the talk:

1. A review of Sudakov resummation
2. Ambiguities in resummed results
3. The standard solution
4. A new prescription

Results from a long-standing collaboration with Stefano Forte and our students.

1. Sudakov resummation: a short review*

Physical cross sections are always inclusive over arbitrarily soft particles in the final state, because of finite detector resolution.

A crucial role in QCD: infrared divergences from virtual corrections are cancelled by radiation of undetected real gluons.

The finite left-over of these cancellations give large contributions if the tagged final state is forced to take most of the available energy (and sometimes even if it is not).

***S. Catani, hep-ph/9610413**

Schematically: $(1-z)\sqrt{s}$ **total energy carried by unobserved radiation. Virtual and real soft gluon corrections:**

$$\frac{dw_{\rm virtual}}{dz} = -2C\,\alpha_{\rm S}\,\delta(1-z)\,\int_0^{1-\epsilon}\frac{dy}{1-y}\,\log\frac{1}{1-y}$$

$$\frac{dw_{\rm real}}{dz} = +2C\,\alpha_{\rm S}\,\frac{1}{1-z}\,\log\frac{1}{1-z}\,\theta(1-z-\epsilon)$$

(because of the bremsstrahlung spectrum $d\omega/\omega$ **and the collinear spectrum** $d\theta/\theta$**).**

$$\frac{dw}{dz} = \frac{dw_{\rm virtual}}{dz} + \frac{dw_{\rm real}}{dz} = 2\,C\,\alpha_{\rm S}\,\left[\frac{1}{1-z}\,\log\frac{1}{1-z}\right]_+$$

Thus,

$$\int_x^1 dz\,\frac{dw}{dz} = -C\,\alpha_{\rm S}\,\log^2(1-x)$$

is the finite contribution of soft gluon emission to the cross section. As $x \to 1$ **in the final state, the phase space for real emission is suppressed, and the finite left-over becomes large.**

At order n, at most two powers of $\log(1-x)$ for each power of α_s appear in the perturbative coefficients:

$$C_n\,\alpha_\mathrm{s}^n = \alpha_\mathrm{s}^n \sum_{m=0}^{2n} c_{nm}\,\log^m(1-x) + \text{non singular terms}$$

The perturbative expansion becomes unreliable; logarithmically enhanced contributions must be resummed to all orders.

Logarithmic contributions are expected to be relevant when $x \to 1$.

However, resummation may have an impact even at smaller values of x, depending on the shape of parton densities.

Examples:

1. **lepton-nucleon scattering in the quasi-elastic limit:**

$$x = x_{\mathrm{Bj}} = \frac{Q^2}{2p \cdot q}, \qquad x \to 1$$

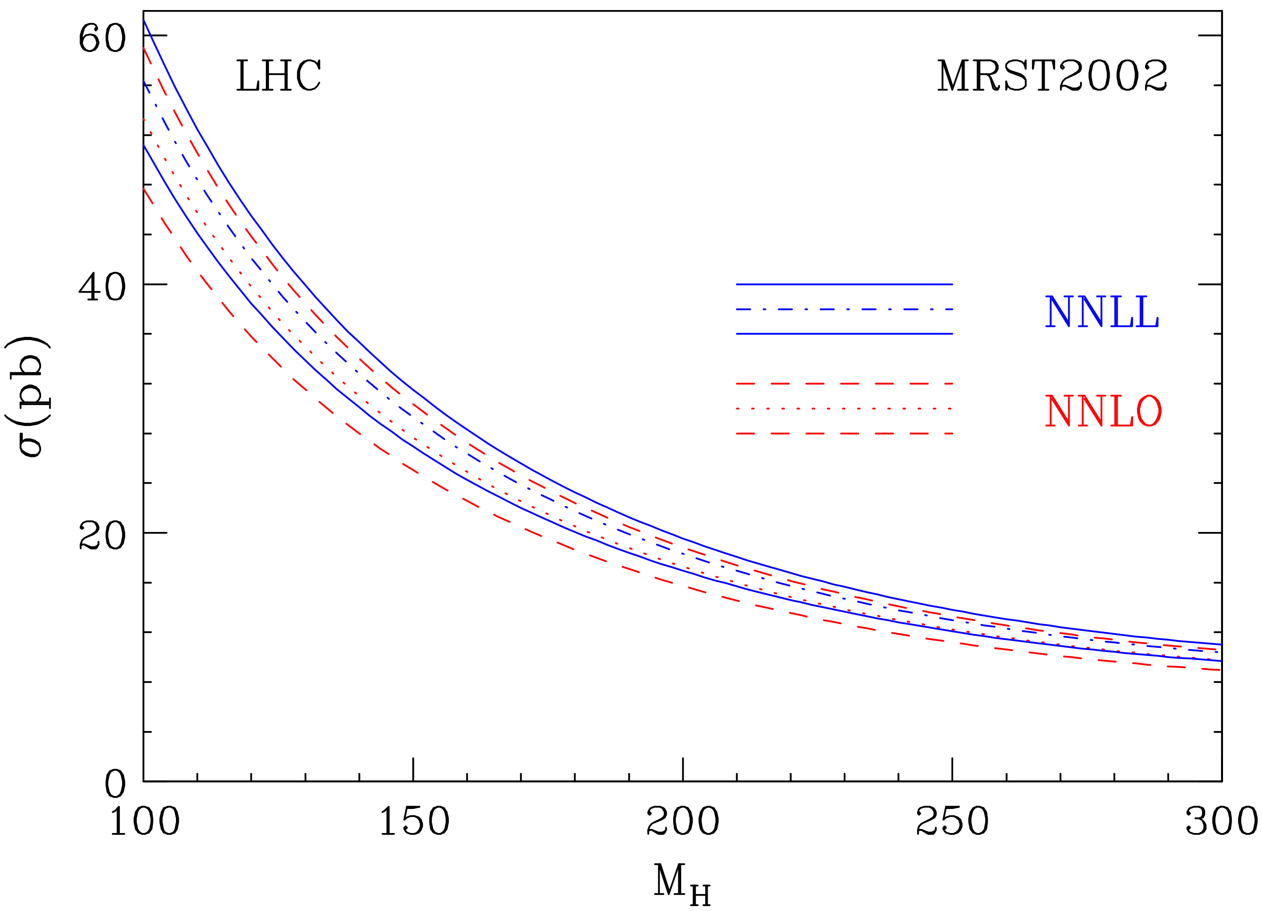
2. **the production of heavy systems close to threshold:**

$$x = \tau = \frac{Q^2}{s}, \qquad s \gtrsim Q^2$$

3. **the transverse momentum spectrum in the small-q_T region:**

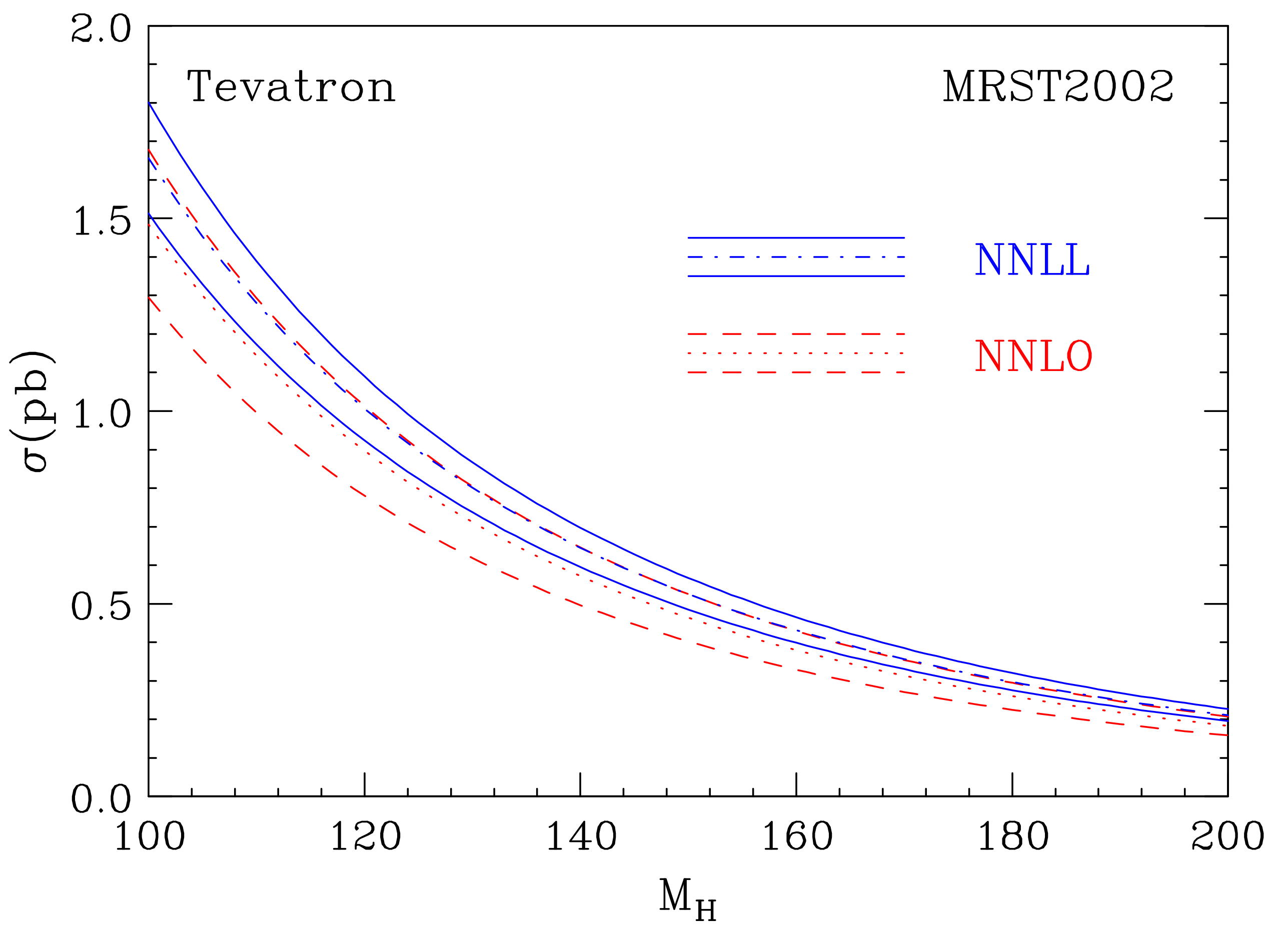
$$1 - x = \hat{q}_{\mathrm{T}}^2 = \frac{q_{\mathrm{T}}^2}{Q^2}, \qquad q_{\mathrm{T}}^2 \ll Q^2$$

Resummation can be performed. A few results in the next slides.



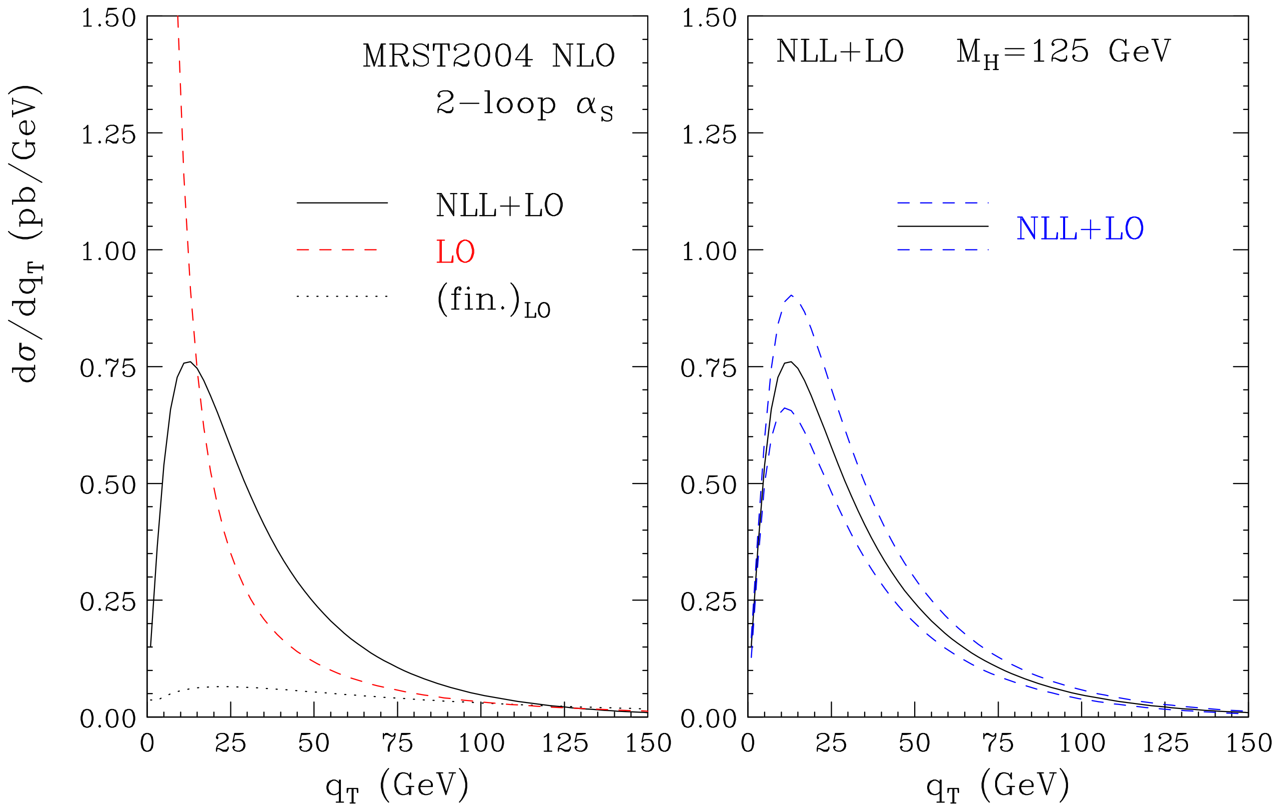
Higgs production at the LHC.

Catani, De Florian, Grazzini, Nason, JHEP 0307(2003)028, arXiv:hep-ph/0306211v1



Higgs production at the Tevatron.

Catani, De Florian, Grazzini, Nason, JHEP 0307(2003)028, arXiv:hep-ph/0306211v1.

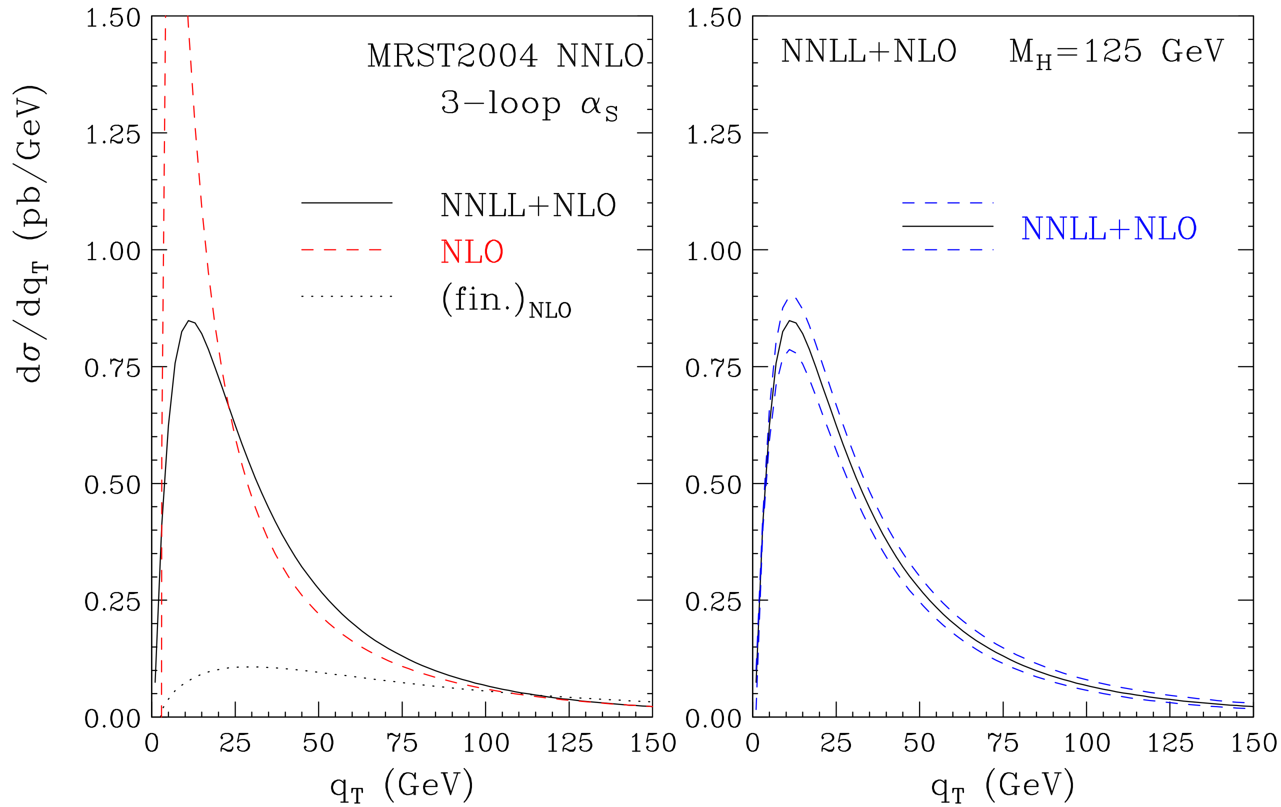


The q_T spectrum of Higgs production at the LHC

Left: NLL+LO compared with the LO spectrum

Right: uncertainty band from scale variations.

Bozzi, Catani, de Florian, Grazzini, NPB737(2006)73, hep-ph/0508068



The q_T spectrum of Higgs production at the LHC

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Resummed cross sections: a schematic derivation

Typical expression of an observable in QCD (eg, Drell-Yan cross section):

$$\begin{aligned}\sigma(x,Q^2) &= \int_0^1 dz \int_0^1 dy\, \mathcal{L}(y,Q^2)\, \hat\sigma(z,\alpha_{\mathrm{S}}(Q^2))\, \delta(yz-x) \\ &= \int_x^1 \frac{dy}{y}\, \mathcal{L}(y,Q^2)\, \hat\sigma\left(\frac{x}{y},\alpha_{\mathrm{S}}(Q^2)\right)\end{aligned}$$

where the δ function forces energy conservation.

The function $\mathcal{L}(y,Q^2)$ is a parton luminosity, e.g.

$$\mathcal{L}(y,Q^2) = \int_y^1 \frac{dy'}{y'}\, f_1(y',Q^2)\, f_2\left(\frac{y}{y'},Q^2\right)$$

in hadron-hadron collisions.

A useful technique: Mellin transformation

$$f(N)=\int_0^1 dx\, x^{N-1}\, f(x); \qquad f(x)=\frac{1}{2\pi i}\int_{\bar{N}-i\infty}^{\bar{N}+i\infty} dN\, x^{-N}\, f(N)$$

(a Laplace transform with $x=e^{-t}$).

- **well defined and analytic in the half-plane $\mathrm{Re\,N}>N_0$ if $f(x)$ is at most as singular as x^{-N_0}**
- **In the space of the Mellin-conjugate variable N, convolution products are turned into ordinary products:**

$$\sigma(N,Q^2)=\mathcal{L}(N,Q^2)\,\hat{\sigma}(N,\alpha_{\mathrm{S}}(Q^2))$$

- **The region $x\to 1$ is mapped in the region $N\to\infty$. More precisely,**

$$\int_0^1 dx\, x^{N-1}\left[\frac{\log^k(1-x)}{1-x}\right]_+ = \frac{1}{k+1}\,\log^{k+1}\frac{1}{N}+\ldots$$

A strong motivation for the use of Mellin transform: phase space factorization. We have

$$\hat{\sigma}(z,\alpha_{\rm S}) = \delta(1-z) + \sum_{n=1}^{\infty} \int_0^1 dz_1 \dots dz_n \, \frac{dw_n(z_1,\dots,z_n)}{dz_1 \dots dz_n} \, \Theta_{PS}(z;z_1,\dots,z_n)$$

The multi-gluon emission probability factorizes in the soft limit,

$$\frac{dw_n(z_1,\dots,z_n)}{dz_1 \dots dz_n} \simeq \frac{1}{n!} \prod_{i=1}^{n} \frac{dw(z_i)}{dz_i}$$

but the phase space does not, unless one goes to Mellin moments:

$$\Theta_{PS}(z;z_1,\dots,z_n) = \delta(z - z_1 \dots z_n) \;\rightarrow\; z_1^{N-1} \dots z_n^{N-1}$$

Hence

$$\hat{\sigma}(N,\alpha_{\rm S}) = \exp \int_0^1 dz \, z^{N-1} \, \frac{dw}{dz}$$

Using

$$\frac{dw}{dz} = 2\,C\,\alpha_{\rm S}\,\left[\frac{1}{1-z}\,\log\frac{1}{1-z}\right]_+$$

we find

$$\int_0^1 dz\, z^{N-1}\,\left[\frac{\log(1-z)}{1-z}\right]_+ = \frac{1}{2}\,\log^2 N + O(\log N)$$

and therefore

$$\hat{\sigma}(N,\alpha_{\rm S}) = \exp\left(-2C\alpha_{\rm S}\log^2 N + O(\log N)\right)$$

Strictly valid in QED; in QCD, complication arises because of gluon emission from gluon lines and because of color structure, but the essential features remain the same.

Extension to QCD

QCD corrections essentially amount to the replacement

$$\alpha_{\mathrm{S}} \to \alpha_{\mathrm{S}}(Q^2(1-z))$$

The running coupling can then be expanded in powers of $\alpha(Q^2)$

$$\alpha_{\mathrm{S}}(Q^2(1-z)) = \frac{\alpha(Q^2)}{1+\alpha(Q^2)\beta_0 \log(1-z)} = \alpha(Q^2)\sum_{n=0}^{\infty}(-\alpha(Q^2)\beta_0)^n \log^n(1-z)$$

and the expansion integrated term by term. One gets

$$\hat{\sigma}(N, \alpha_{\mathrm{S}}) = \exp[\log N\, g_1(\alpha_{\mathrm{S}} \log N) + g_2(\alpha_{\mathrm{S}} \log N) + \alpha_{\mathrm{S}}\, g_3(\alpha_{\mathrm{S}} \log N) + \ldots]$$

which defines an improved expansion (in powers of α_{S} **with** $\alpha_{\mathrm{S}} \log N$ **fixed) for** $\log \hat{\sigma}(N, \alpha_{\mathrm{S}})$**.**

2. Ambiguities in resummed results

A difficulty immediately arises. Consider, for simplicity, the quantity

$$\gamma(N,\alpha_S) = \frac{\partial \log \hat\sigma(N,\alpha_S)}{\log Q^2}.$$

(but the same considerations apply to $\hat\sigma(N,\alpha_S)$ itself). To leading log approximation, one finds

$$\gamma_{LL}(\alpha_S(Q^2),N) = g_1 \int_1^N \frac{dn}{n}\,\alpha_S(Q^2/n) = -\frac{g_1}{\beta_0}\log\left(1+\beta_0\alpha_S(Q^2)\log\frac{1}{N}\right)$$

which has a branch cut on the real positive axis for

$$N \geq N_L \equiv e^{\frac{1}{\beta_0\alpha_S(Q^2)}}.$$

because of the Landau singularity.

Its inverse Mellin transform does not exist.

One possible way out: expand $\gamma_{\mathrm{LL}}(\alpha_{\mathrm{S}}(Q^2), N)$ in powers of $\alpha_{\mathrm{S}}(Q^2)$ and take the term-by-term inverse Mellin transform:

$$P_{\mathrm{LL}}(\alpha_{\mathrm{S}}(Q^2), z) = -\frac{g_1}{\beta_0} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \beta_0^k \alpha_{\mathrm{S}}^k(Q^2) \, \frac{1}{2\pi i} \int_{\bar{N}-i\infty}^{\bar{N}+i\infty} dN \, z^{-N} \, \log^k \frac{1}{N}$$

but the series is divergent! (otherwise, we could interchange the sum over k and the integral over N, but

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \beta_0^k \alpha_{\mathrm{S}}^k(Q^2) \, \log^k \frac{1}{N}$$

is only convergent for

$$\left| \beta_0 \alpha_{\mathrm{S}}(Q^2) \log \frac{1}{N} \right| < 1,$$

while the integral in N on the path $\mathrm{Re}\, N = \bar{N}$ involves values of N outside this range).

A second possible way out: take the inverse Mellin transform of each $\log^k N$ term at the relevant (leading, next-to-leading...) logarithmic level the perturbative series converges. For example, to leading log accuracy one has

$$\frac{1}{2\pi i}\int_{\bar{N}-i\infty}^{\bar{N}+i\infty} dN\, z^{-N}\, \log^k \frac{1}{N} = k\left[\frac{\log^{k-1}(1-z)}{1-z}\right]_+ + \mathrm{NLL}$$

The series now is convergent (for $z < z_L = 1 - e^{-\frac{1}{\beta_0 \alpha_S(Q^2)}}$) to

$$P_{\mathrm{LLx}}(\alpha_S(Q^2), z) = -g_1\left[\frac{1}{1-z}\frac{\alpha_S(Q^2)}{1+\beta_0\alpha_S(Q^2)\log(1-z)}\right]_+ = -g_1\left[\frac{\alpha_S(Q^2(1-z))}{1-z}\right]_+$$

which is singular at the Landau pole $z = z_L$.

Completely analogous considerations hold in the case of the resummation of large logarithms of q_T^2/Q^2 in the small-q_T region of the spectrum.

In this case

$$\text{Mellin tr.} \quad \int_0^1 dz\, z^{N-1}\, f(z) \rightarrow \text{Fourier tr.} \quad \frac{1}{2\pi}\int d\mathbf{b}\, \mathbf{e}^{-\mathbf{i b}\cdot\mathbf{q}_T}$$

In the space of the Fourier-conjugate variable b, the $\mathbf{q}_T$ conservation δ function factorizes:

$$\int d^2 q_T\, \delta(\mathbf{q}_T - \mathbf{q}_T^{\mathbf{1}} - \ldots - \mathbf{q}_T^{\mathbf{n}})\, \mathbf{e}^{\mathbf{i b}\cdot\mathbf{q}_T} = \prod_{\mathbf{i}=\mathbf{1}}^{\mathbf{n}} \exp(\mathbf{i b}\cdot\mathbf{q}_T^{\mathbf{i}})$$

The resummed cross section in b space has no inverse Fourier transform, again because of the Landau pole of the running coupling.

3. Proposed solutions

An idea of S. Catani, M. Mangano, P. Nason and L. Trentadue*: the minimal prescription. A very simple recipe: just take

$$\sigma(x,Q^2) = \frac{1}{2\pi i}\int_{N_{\rm MP}-i\infty}^{N_{\rm MP}+i\infty} dN\, x^{-N}\,\mathcal{L}(N,Q^2)\,\hat{\sigma}(N,\alpha_{\rm S}(Q^2))$$

with $N_{\rm MP} < N_L$.

This is not a true inverse Mellin: the integrand is not analytical in any right half-plane, because of the branch cut due to the Landau pole.

*[**NPB 478(1996)273, hep-ph/9604351**]

However:

- **it is well defined for all values of x**
- **it is an asymptotic sum of the original, divergent perturbative expansion**
- **the difference between the original series, truncated at the best-approximation term, and the minimal prescription, is suppressed more strongly than any power of Λ^2/Q^2.**
- **it has a few other nice features, not to be discussed here.**

[All the results previously shown have been obtained using the minimal prescription technique.]

A closer look at the minimal prescription:

$$\begin{aligned}\sigma(x,Q^2) &= \frac{1}{2\pi i}\int_{N_{\mathrm{MP}}-i\infty}^{N_{\mathrm{MP}}+i\infty} dN\, x^{-N}\,\hat\sigma(N,\alpha_{\mathrm{S}}(Q^2))\,\int_0^1 dy\, y^{N-1}\,\mathcal{L}(y,Q^2)\\ &= \int_0^1 \frac{dy}{y}\,\mathcal{L}(y,Q^2)\,\hat\sigma\left(\frac{x}{y},\alpha_{\mathrm{S}}(Q^2)\right)\end{aligned}$$

Looks like a convolution, but the integration region $0 \le y \le x$ cannot be excluded: indeed

$$\hat\sigma(z,\alpha_{\mathrm{S}}(Q^2)) = \frac{1}{2\pi i}\int_{N_{\mathrm{MP}}-i\infty}^{N_{\mathrm{MP}}+i\infty} dN\, z^{-N}\,\hat\sigma(N,\alpha_{\mathrm{S}}(Q^2))$$

does not vanish for $z > 1$ because of the Landau cut.

This is reflected in a difficulty in the numerical implementation of the minimal prescription formula: $\hat{\sigma}(x/y, \alpha_{\mathrm{S}})$ oscillates in the region $y \sim x$, where the luminosity is smooth, and large cancellations take place.

On the other hand, $\mathcal{L}(N, Q^2)$ is typically not available.

Also, from a conceptual point of view, one may object to adopting a parton cross section which violates kinematical constraints.

All these problems are addressed by CMNT in the original paper, and can be overcome by suitable techniques, but it is interesting to explore different possibilities.

4. A different approach

Consider a generic quantity Σ, resummed in N space and expanded in powers of $\log N$:

$$\Sigma(\alpha(Q^2), L) = \sum_{k=1}^{\infty} h_k(\alpha(Q^2))L^k; \qquad L \equiv \bar{\alpha} \log \frac{1}{N}; \quad \bar{\alpha} = a\beta_0\alpha(Q^2); \quad a = 1, 2$$

The series is convergent for $|L| < 1$, because of the Landau pole at $L = 1$. The expansion can be Mellin-inverted term by term, but the result is a divergent series.

Is it possible to sum it using the Borel technique?

S. Forte, J. Rojo, M. Ubiali, GR, PLB635(2006)313, hep-ph/0601048

R. Abbate, S. Forte, GR, PLB657(2007)55, arXiv:0707.2452

Consider a generic power series, not necessarily convergent in the Cauchy sense:

$$f(\bar{\alpha}) = \sum_{k=1}^{\infty} f_k \, \bar{\alpha}^k$$

and define its Borel transform

$$\hat{f}(w) = \sum_{k=1}^{\infty} f_k \, \frac{w^{k-1}}{(k-1)!}$$

Because of the factor $(k-1)!$, the Borel transformed series $\hat{f}(w)$ has much better convergence properties. An inverse transformation exists, since

$$\int_0^{+\infty} dw \, e^{-\frac{w}{\bar{\alpha}}} \, w^{k-1} = (k-1)! \, \bar{\alpha}^k \rightarrow f_B(\bar{\alpha}) = \int_0^{+\infty} dw \, e^{-\frac{w}{\bar{\alpha}}} \, \hat{f}(w)$$

Various cases:

1. **The original series is convergent in the usual sense. Then $f_B(\bar{\alpha}) = f(\bar{\alpha})$, but the Borel sum may enlarge the convergence region. Example:**

$$f(\bar{\alpha}) = \sum_{k=1}^{\infty} \bar{\alpha}^k = \frac{\bar{\alpha}}{1-\bar{\alpha}}, \qquad |\bar{\alpha}| < 1$$

$$f_B(\bar{\alpha}) = \int_0^{+\infty} dw\, e^{-\frac{w}{\bar{\alpha}}}\, e^w = \frac{\bar{\alpha}}{1-\bar{\alpha}} \qquad \mathrm{Re}\, \bar{\alpha} < 1$$

2. **The original series is divergent, but the Borel sum exists:**

$$f(\bar{\alpha}) = \sum_{k=1}^{\infty} (-1)^{k-1}\, (k-1)!\, \bar{\alpha}^k \qquad \hat{f}(w) = \frac{1}{1+w}$$

$$f_B(\bar{\alpha}) = \int_0^{+\infty} dw\, e^{-\frac{w}{\bar{\alpha}}}\, \frac{1}{1+w} < \infty \qquad \mathrm{Re}\, \bar{\alpha} > 0$$

3. The original series is divergent, its Borel Transform exists, but it has a singularity in the range of the inversion integral:

$$f(\bar{\alpha}) = \sum_{k=1}^{\infty} (k-1)!\, \bar{\alpha}^k \qquad \hat{f}(w) = \frac{1}{1-w}$$

$$f_B(\bar{\alpha}) = \int_0^{+\infty} dw\, e^{-\frac{w}{\bar{\alpha}}} \, \frac{1}{1-w}$$

This is the case e.g. of renormalons.

Back to

$$\Sigma(\alpha_{\rm S}, L) = \sum_{k=1}^{\infty} h_k\, L^k; \qquad L = \bar{\alpha}\,\log\frac{1}{N}$$

To log accuracy,

$$\frac{1}{2\pi i}\int_{\overline{N}-i\infty}^{\overline{N}+i\infty} dN x^{-N}\,\ln^k\frac{1}{N} = \left[\frac{1}{1-z}\sum_{n=1}^{k}\binom{k-1}{n-1} k\,\Delta^{(n-1)}(1)\,\log^{k-n}(1-z)\right]_+$$

where $\Delta(z) = 1/\Gamma(z)$. Thus

$$\Sigma(\alpha(Q^2), z) = \left[\frac{R(z)}{1-z}\right]_+$$

$$R(z) = \sum_{k=1}^{\infty} k\,h_k \bar{\alpha}^k \sum_{n=1}^{k}\binom{k-1}{n-1}\Delta^{(n-1)}(1)\,\ell^{k-n}; \quad \ell = \log(1-z)$$

which is divergent.

We now change the summation order

$$R(z) = \sum_{n=1}^{\infty} \Delta^{(n-1)}(1) \sum_{k=n}^{\infty} \binom{k-1}{n-1} k\, h_k\, \bar{\alpha}^k\, \ell^{k-n}$$

and take the Borel tranform with respect to $\bar{\alpha}$:

$$\hat{R}(z,w) = \sum_{n=1}^{\infty} \Delta^{(n-1)}(1) \sum_{k=n}^{\infty} \binom{k-1}{n-1} k\, h_k\, \frac{w^{k-1}}{(k-1)!}\, \ell^{k-n}$$

which can be shown to be convergent. The inversion integral however,

$$R_B(z) = \int_0^{+\infty} dw\, e^{-\frac{w}{\bar{\alpha}}}\, \hat{R}(z,w)$$

is divergent at $+\infty$.

We cut off the integral at $w = C$:

$$
\begin{aligned}
R_B(z) &= \int_0^C dw\, e^{-\frac{w}{\bar\alpha}}\, \hat{R}(z,w) \\
&= \sum_{n=1}^{\infty} \Delta^{(n-1)}(1) \sum_{k=n}^{\infty} \binom{k-1}{n-1} k\, h_k\, \frac{\gamma(k; C/\bar\alpha)}{(k-1)!}\, \bar\alpha^k\, \ell^{k-n}
\end{aligned}
$$

where γ is the truncated Γ function

$$
\gamma(k;u) = \int_0^u dt\, e^{-t}\, t^{k-1} = (k-1)! \left(1 - e^{-u} \sum_{m=0}^{k-1} \frac{u^m}{m!}\right)
$$

Therefore

$$
R_B(z) = R(z) - R_{ht}(z)
$$

where

$$
R_{ht}(z) = e^{-\frac{C}{\bar\alpha}} \sum_{n=1}^{\infty} \Delta^{(n-1)}(1) \sum_{k=n}^{\infty} \binom{k-1}{n-1} k\, h_k \sum_{m=0}^{k-1} \frac{1}{m!} \left(\frac{C}{\bar\alpha}\right)^m \bar\alpha^k\, \ell^{k-n}.
$$

Remarks:

- **$R(z)$ is an asymptotic expansion of $R_B(z)$. Indeed, $R_{ht} \sim e^{-\frac{1}{\alpha}}$ vanishes faster than any power of $\alpha(Q^2)$ as $\alpha(Q^2) \to 0$; therefore $R_B(z) - R(z)|_N$ is of order $\alpha(Q^2)^{N+1}$.**

- $$\alpha(Q^2) \simeq \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda^2}} \to e^{-\frac{C}{\alpha}} \simeq \left(\frac{\Lambda^2}{Q^2}\right)^{C/a} ; \quad a = 1, 2$$

  Cutting off the Borel inversion integral at $w = C$ amounts to including a twist-t contribution $R_{ht}(z)$, with

  $$t = 2 + \frac{2C}{a}.$$

  The divergence of the higher twist term then cancels that of the divergent series, leading to a finite result.

- **The value of C is arbitrary. Dependence on C very mild below the Landau pole.**
- **The choice $C = a$ is minimal: it corresponds to the inclusion of a twist-four term, i.e. a term of the first subleading twist.**
- **Arbitrarily large values of C can be chosen (with some care in the numerical implementation).**

Numerical implementation

Our result

$$\hat{R}(z,w) = \sum_{n=1}^{\infty} \frac{\Delta^{(n-1)}(1)}{(n-1)!} \sum_{k=n}^{\infty} k\, h_k\, w^{k-1} \frac{1}{(k-n)!} \ell^{k-n}$$

is not very useful in this form. But both series can be summed:

$$\frac{1}{(k-n)!} = \frac{1}{2\pi i} \oint_H d\xi\, e^{\xi}\, \xi^{-(k-n)-1}$$

where H is any closed contour around the origin $\xi = 0$. So

$$\hat{R}(z,w) = \frac{1}{2\pi i} \oint_H \frac{d\xi}{\xi}\, e^{\ell\xi} \sum_{n=1}^{\infty} \frac{\Delta^{(n-1)}(1)}{(n-1)!} \xi^{n-1} \sum_{k=n}^{\infty} k\, h_k \left(\frac{w}{\xi}\right)^{k-1}$$

Now,

- **terms with $k=1,\ldots,n-1$ would give zero after ξ integration;**
- **the Taylor expansion of Δ has convergence radius ∞;**
- **the Taylor expansion of Σ has convergence radius 1**

So finally

$$\begin{aligned}
\hat{R}(z,w) &= \frac{1}{2\pi i}\oint_H d\xi\, e^{\ell\xi}\,\Delta(1+\xi)\,\frac{d}{dw}\Sigma\left(\frac{w}{\xi}\right); \quad |\xi|>w \text{ on } H \\
R(z) &= \int_0^C dw\, e^{-\frac{w}{\bar{\alpha}}}\,\hat{R}(z,w) \\
&= \frac{1}{2\pi i}\oint_H d\xi\, e^{\ell\xi}\,\Delta(1+\xi)\left[e^{-\frac{C}{\bar{\alpha}}}\,\Sigma\left(\frac{C}{\xi}\right)+\frac{1}{\bar{\alpha}}\int_0^C dw\, e^{-\frac{w}{\bar{\alpha}}}\,\Sigma\left(\frac{w}{\xi}\right)\right]
\end{aligned}$$

which is explicitly written in terms of the function Σ.

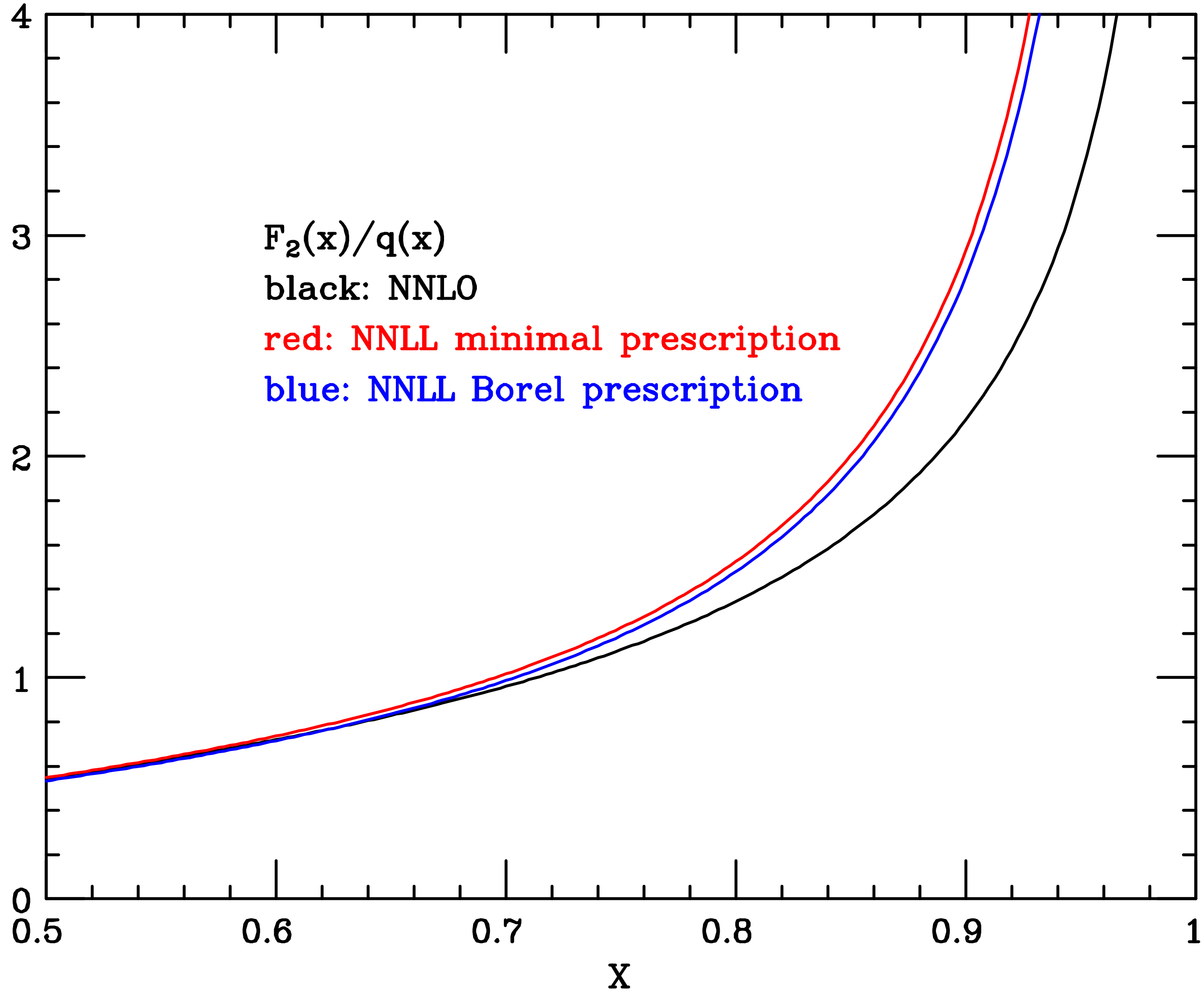
The resummed (Mellin-transformed) cross section $\Sigma(\alpha_{\rm S}, L)$ has a branch cut in the complex plane L in

$$-\infty < \mathrm{Re\,L} \leq -1; \qquad \mathrm{Im}\, L = 0$$

which is mapped into

$$-w \leq \mathrm{Re}\,\xi \leq 0; \qquad \mathrm{Im}\,\xi = 0$$

for $\Sigma(\alpha_{\rm S}, w/\xi)$. The contour H must be chosen so that it encloses the cut, and therefore is pushed to large negative values of $\mathrm{Re}\,\xi$ as $w \to +\infty$. In that region, $\Delta(1+\xi)$ oscillates with factorially growing amplitude, and the w integral does not converge.



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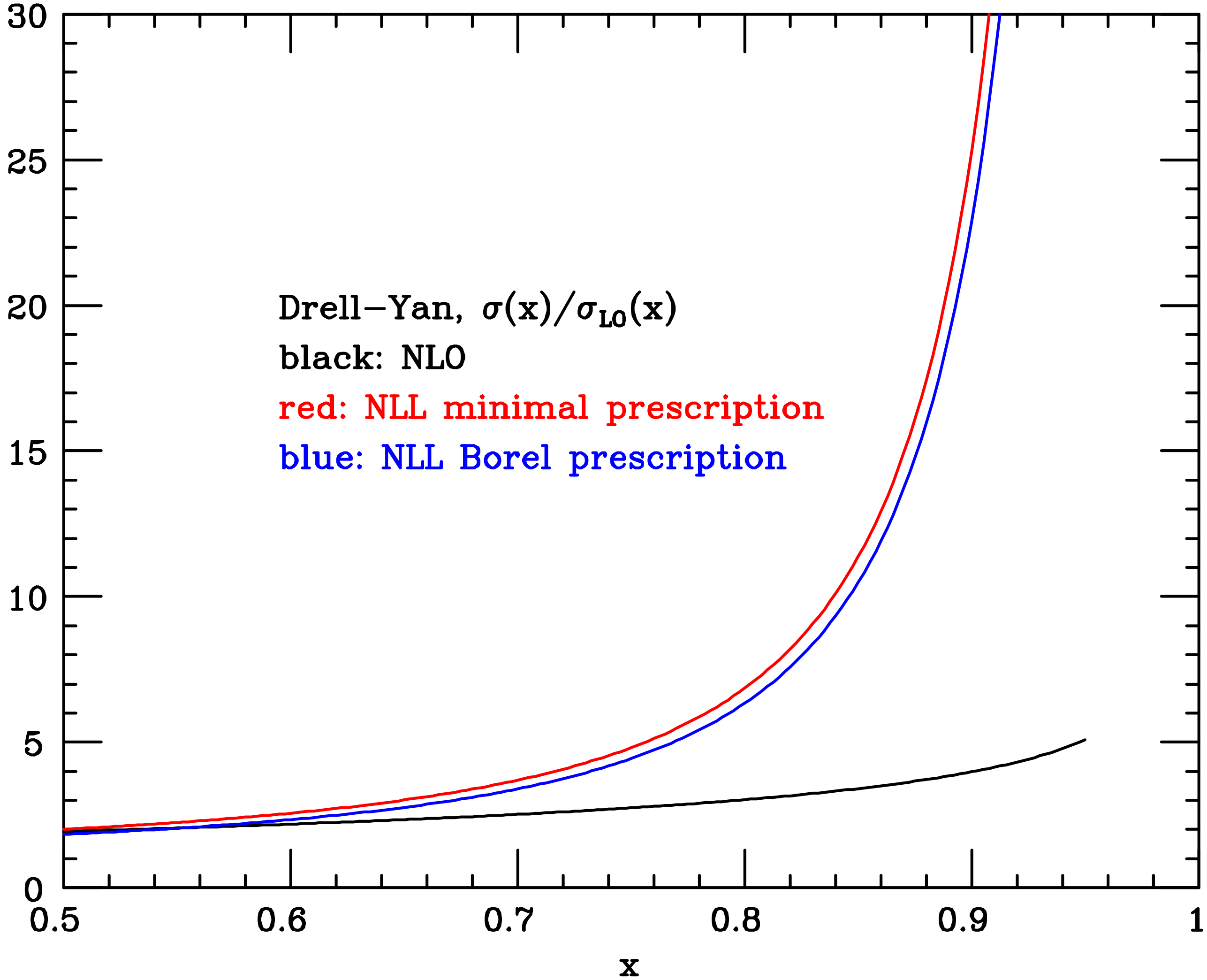
R. Abbate, S. Forte, GR, PLB657(2007)55, arXiv:0707.2452



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S. Forte, GR, in preparation

Outlook

- There are ambiguities in the computation of observables from resummed quantities, to be ascribed to the non-convergence of the perturbative expansion.
- A prescription based on Borel sum and twist expansion can be given, with some advantages on other, widely employed techniques.
- Future work: more realistic studies; extension to less inclusive observables, such as transverse momentum spectrum* or rapidity distributions†

* M. Bonvini, S. Forte, GR, in preparation

† S. Forte, A. Vicini, GR, in preparation