

ISOLDE Nuclear Reaction and Nuclear Structure Course

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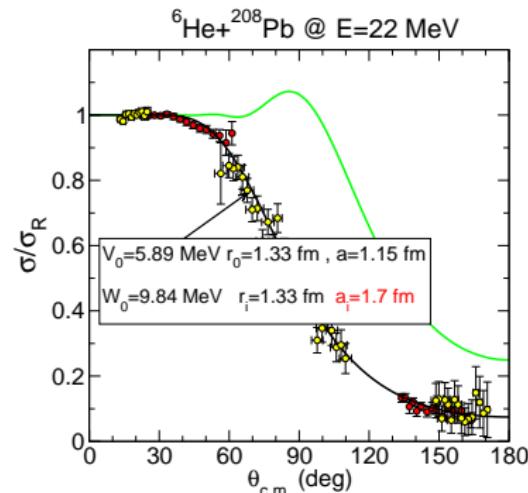
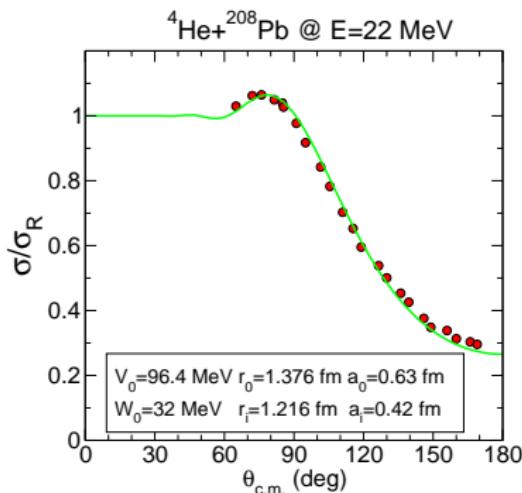
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1 Reactions with exotic nuclei

- The CDCC method
- Resonant breakup
- Coulomb dissociation experiments
- Radiative capture from Coulomb dissociation
- Knock-out reactions
- Transfer with exotic nuclei



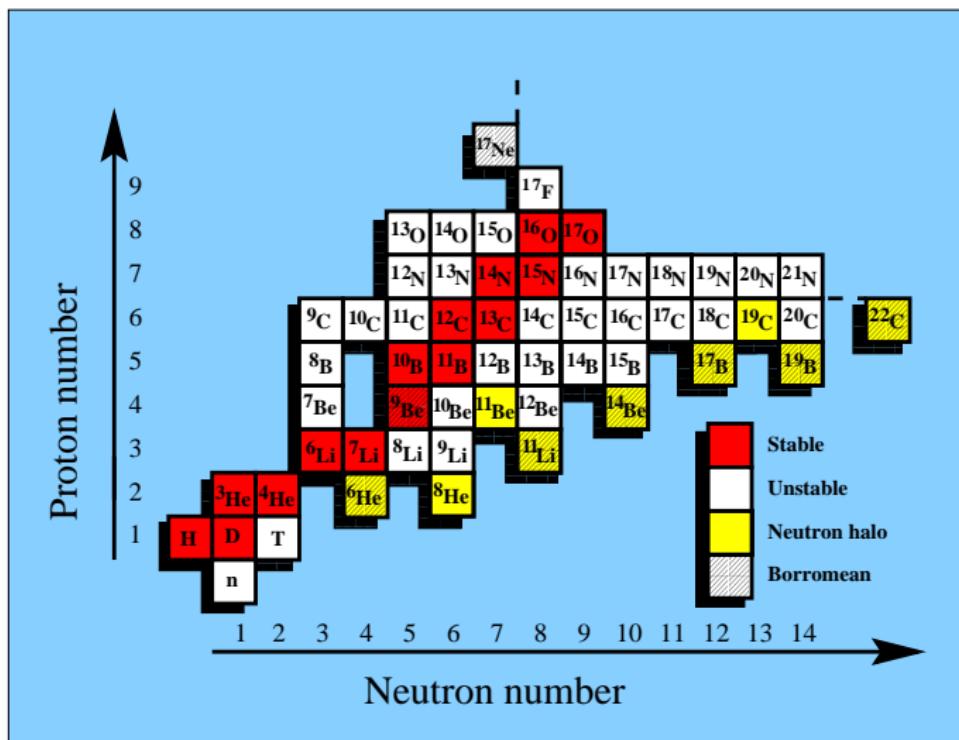
Evidences of the importance of coupling to breakup channels



- ${}^4\text{He} + {}^{208}\text{Pb}$ shows typical Fresnel pattern → *strong absorption*
- ${}^6\text{He} + {}^{208}\text{Pb}$ shows a prominent reduction in the elastic cross section due to the flux going to other channels (mainly break-up)
- ${}^6\text{He} + {}^{208}\text{Pb}$ requires a large imaginary diffuseness → *long-range absorption*

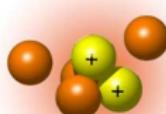


Exotic nuclei, halo nuclei, and Borromean systems



Light exotic nuclei: halo nuclei and Borromean systems

- **Radioactive nuclei:** they typically decay by β emission.
E.g.: ${}^6\text{He} \xrightarrow{\beta^-} {}^6\text{Li}$ ($\tau_{1/2} \approx 807 \text{ ms}$)
 - **Weakly bound:** typical separation energies are around 1 MeV or less.
 - **Spatially extended**
 - **Halo structure:** one or two weakly bound nucleons (typically neutrons) with a large probability of presence beyond the range of the potential.
 - **Borromean nuclei:** Three-body systems with no bound binary sub-systems.





Challenges found in the study of reactions with exotic beams

Experimentally:

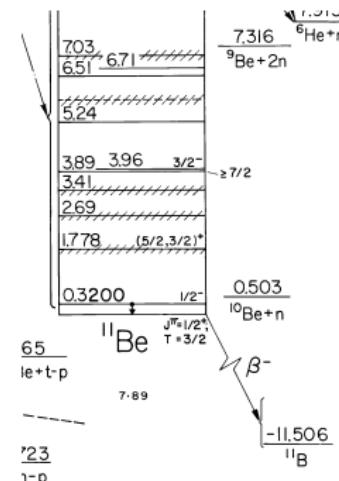
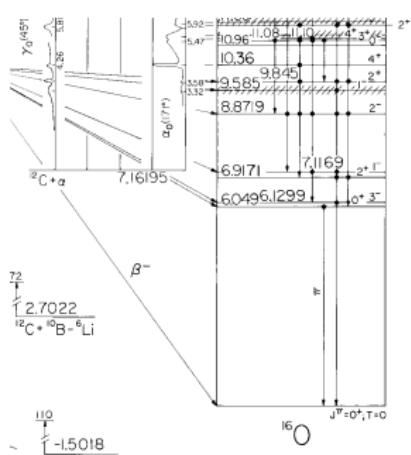
- Exotic nuclei are short-lived and difficult to produce. Beam intensities are typically small.

Theoretically:

- Exotic nuclei are easily broken up in nuclear collisions \Rightarrow coupling to the unbound states (continuum) plays an important role.
- Effective NN interactions, level schemes, etc are different from stable nuclei.
- Many exotic nuclei exhibit complicated cluster (few-body) structure.

Inelastic scattering of weakly bound nuclei

- Single-particle (or cluster) excitations become dominant.
 - Excitation to continuum states important.

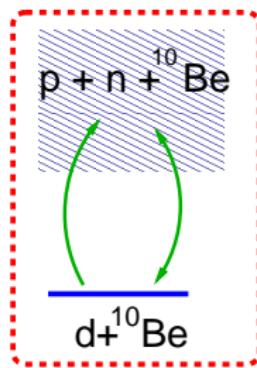


☞ Halo nuclei are weakly bound \Rightarrow coupling to continuum states becomes an important reaction channel



Inclusion of breakup channels: the CDCC method

DWBA modelspace



→ We want to include explicitly in the modelspace the breakup channels of the projectile or target, using a cluster model description of the projectile.

Reminder of the CC method (bound states)

We need to incorporate explicitly in the Hamiltonian the internal structure of the nucleus being excited (eg. **target**).

$$H = T_R + h(\xi) + V(\mathbf{R}, \xi)$$

- T_R : Kinetic energy for projectile-target relative motion.
 - $\{\xi\}$: Internal degrees of freedom of the target (depend on the model).
 - $h(\xi)$: Internal Hamiltonian of the target.

$$h(\xi)\phi_n(\xi) = \varepsilon_n\phi_n(\xi)$$

- $V(\mathbf{R}, \xi)$: Projectile-target interaction

CC model wavefunction

We expand the total wave function in a subset of internal states (the P space):

$$\Psi_{\text{model}}^{(+)}(\mathbf{R}, \xi) = \phi_0(\xi)\chi_0(\mathbf{R}) + \sum_{n>0} \phi_n(\xi)\chi_n(\mathbf{R})$$

Boundary conditions for the $\chi_n(\mathbf{R})$ (unknowns):

$$\chi_0^{(+)}(\mathbf{R}) \rightarrow e^{i\mathbf{K}_0 \cdot \mathbf{R}} + \textcolor{red}{f_{0,0}(\theta)} \frac{e^{iK_0 R}}{R} \quad \text{for n=0 (elastic)}$$

$$\chi_n^{(+)}(\mathbf{R}) \rightarrow f_{n,0}(\theta) \frac{e^{iK_n R}}{R} \quad \text{for } n > 0 \text{ (non-elastic)}$$



Calculation of $\chi_n^{(+)}(\mathbf{R})$: the coupled equations

- The model wavefunction must satisfy the Schrödinger equation:

$$[H - E]\Psi_{\text{model}}^{(+)}(\mathbf{R}, \xi) = 0$$

- Projecting onto the internal states one gets a system of coupled-equations for the functions $\{\chi_n(\mathbf{R})\}$:

$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})] \chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R}) \chi_{n'}(\mathbf{R})$$

- Coupling potentials:

$$V_{n,n'}(\mathbf{R}) = \int d\xi \phi_{n'}(\xi)^* V(\mathbf{R}, \xi) \phi_n(\xi)$$

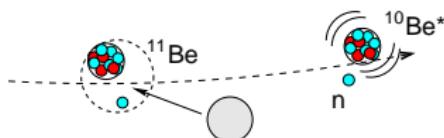
☞ $\phi_n(\xi)$ will depend on the structure model (collective, single-particle,etc).

Extension of the CC method to include breakup channels

- Light exotic nuclei usually present a cluster structure (recall talk by B.Jonson in this course)
- To use the CC formalism, one needs to extend the method in order to:
 - ⇒ Describe the cluster (or single-particle) structure of light exotic nuclei
 - ⇒ Permit the inclusion of unbound (continuum) states (breakup channels)

CC method with a cluster model

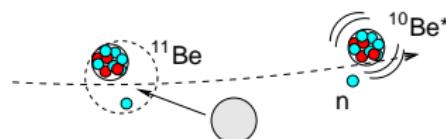
Microscopic approach



- ⇒ Start from (effective) NN interaction.
- ⇒ Complicated many-body scattering problem

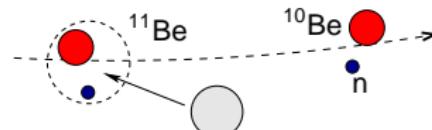
CC method with a cluster model

Microscopic approach



- ⇒ Start from (effective) NN interaction.
- ⇒ Complicated many-body scattering problem

Few-body approach



- ⇒ Inert target approximation
- ⇒ Projectile described with few-body model
- ⇒ Phenomenological NA interactions

Inelastic scattering in a few-body model

- Some nuclei allow a description in terms of two or more clusters:
 $d=p+n$, ${}^6\text{Li}=\alpha+d$, ${}^7\text{Li}=\alpha+{}^3\text{H}$.
- Projectile-target interaction:

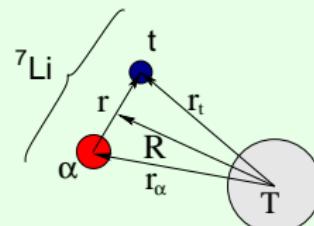
$$V(\mathbf{R}, \mathbf{r}) = U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)$$

Example: ${}^7\text{Li}=\alpha+t$

$$\mathbf{r}_\alpha = \mathbf{R} - \frac{m_t}{m_\alpha + m_t} \mathbf{r}; \quad \mathbf{r}_t = \mathbf{R} + \frac{m_\alpha}{m_\alpha + m_t} \mathbf{r}$$

Internal states:

$$[T_{\mathbf{r}} + V_{\alpha-t}(\mathbf{r}) - \varepsilon_n] \phi_n(\mathbf{r}) = 0$$

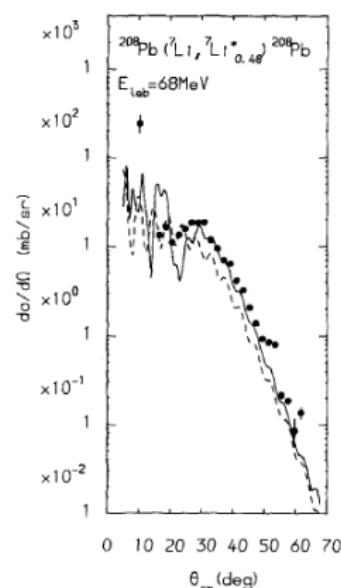
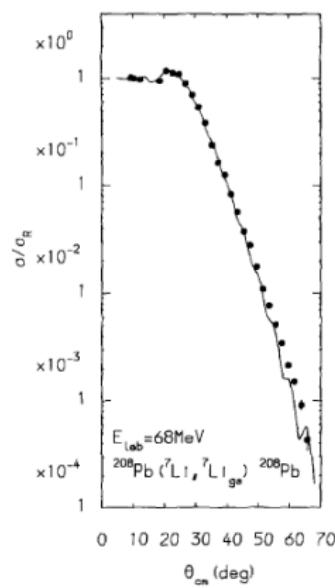
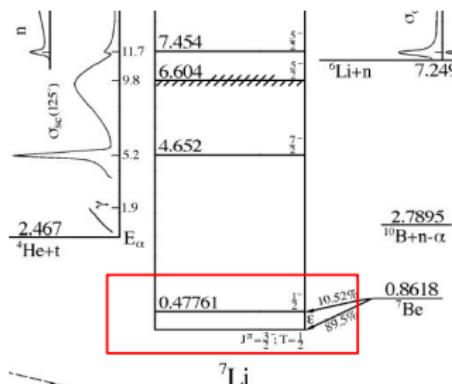


- Transition potentials:

$$V_{n,n'}(\mathbf{R}) = \int d\mathbf{r} \phi_n^*(\mathbf{r}) [U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)] \phi_{n'}(\mathbf{r})$$

Inelastic scattering: cluster model

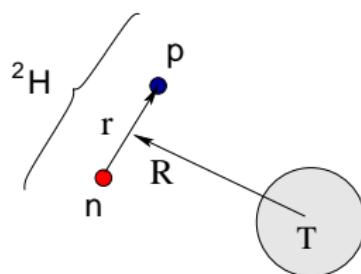
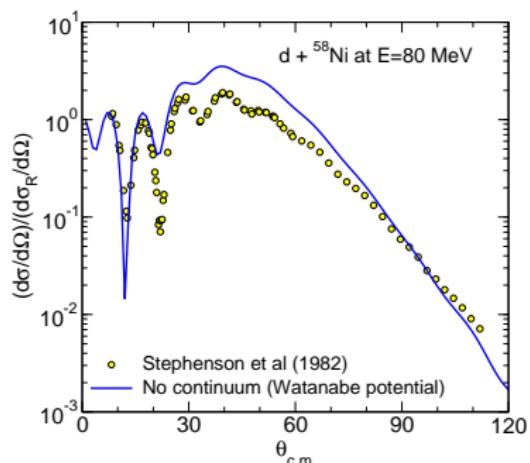
Example: ${}^7\text{Li}(\alpha+t) + {}^{208}\text{Pb}$ at 68 MeV (Phys. Lett. 139B (1984) 150):
 ⇒ CC calculation with 2 channels ($3/2^-$, $1/2^-$)



Application of the CC method to weakly-bound systems

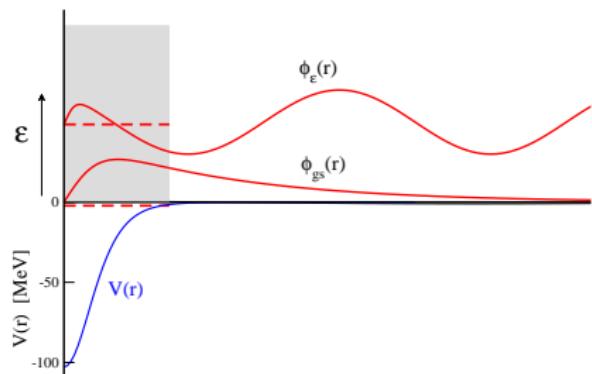
Example: Three-body calculation ($p + n + {}^{58}\text{Ni}$) with Watanabe potential:

$$V_{dt}(\mathbf{R}) = \int d\mathbf{r} \phi_{gs}(\mathbf{r}) \left\{ V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt}) \right\} \phi_{gs}(\mathbf{r})$$



☞ Three-body calculations omitting breakup channels fail to describe the experimental data.

Bound versus scattering states

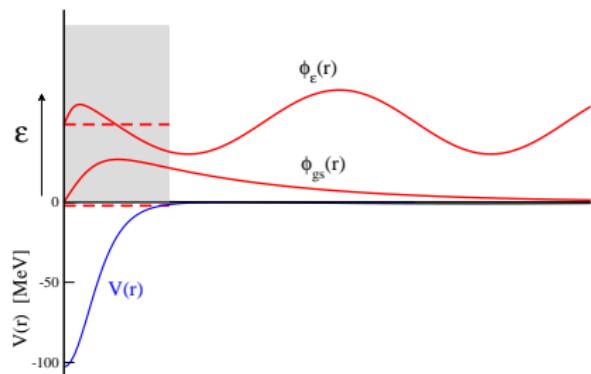


Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r} [Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Bound versus scattering states



Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r} [Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

$$\epsilon = \frac{\hbar^2 k^2}{2\mu}$$

Unbound states are not suitable for CC calculations:

- They have a continuous (infinite) distribution in energy.
- Non-normalizable: $\langle u_{k,\ell sj}(r) | u_{k',\ell sj}(r) \rangle \propto \delta(k - k')$

SOLUTION \Rightarrow continuum discretization



The role of the continuum in the scattering of weakly bound nuclei

- Continuum discretization method proposed by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)].

PHYSICAL REVIEW C

VOLUME 9, NUMBER 6

JUNE 1974

Effect of deuteron breakup on elastic deuteron-nucleus scattering

George H. Rawitscher*

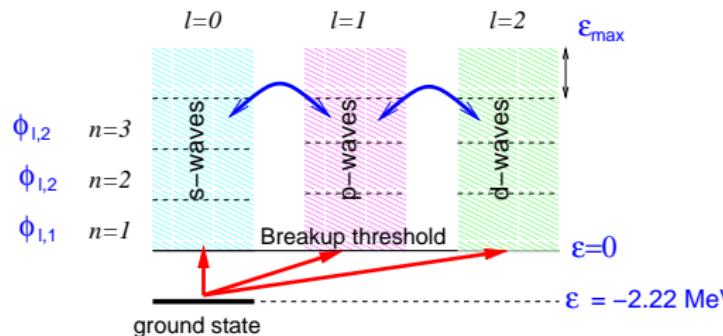
*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139,
and Department of Physics, University of Surrey, Guildford, Surrey, England*

(Received 1 October 1973; revised manuscript received 4 March 1974)

The properties of the transition matrix elements $V_{ab}(R)$ of the breakup potential V_N taken between states $\phi_a(\vec{r})$ and $\phi_b(r)$ are examined. Here $\phi_a(\vec{r})$ are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy ϵ_a are positive (continuum states) or negative (bound deuteron); $V_N(\vec{r}, \vec{R})$ is the sum of the phenomenological proton nucleus $V_{p-A}(|\vec{R} - \frac{1}{2}\vec{r}|)$ and neutron nucleus $V_{n-A}(|\vec{R} + \frac{1}{2}\vec{r}|)$ optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transi-

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

Continuum discretization for deuteron scattering



- ⇒ Select a number of partial waves ($\ell = 0, \dots, \ell_{\max}$).
- ⇒ For each ℓ , set a maximum excitation energy ϵ_{\max} .
- ⇒ Divide the interval $\epsilon = 0 - \epsilon_{\max}$ in a set of sub-intervals (*bins*).
- ⇒ For each *bin*, calculate a representative wavefunction.

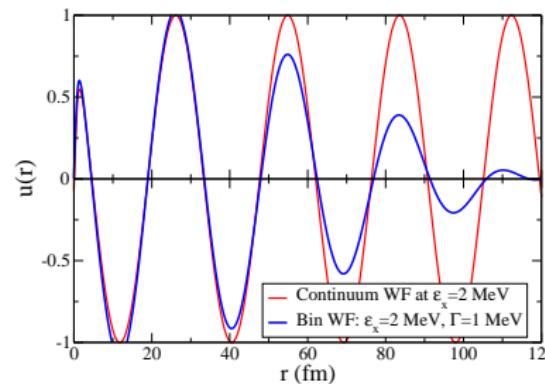
CDCC formalism: construction of the bin wavefunctions

Bin wavefunction:

$$\varphi_{\ell jm}^{[k_1, k_2]}(\mathbf{r}) = \frac{u_{\ell j}^{[k_1, k_2]}(r)}{r} [Y_\ell(\hat{r}) \otimes \chi_s]_{jm} \quad [k_1, k_2] = \text{bin interval}$$

$$u_{\ell sjm}^{[k_1, k_2]}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k, \ell sj}(r) dk$$

- k : linear momentum
- $u_{k, \ell sj}(r)$: scattering states (radial part)
- $w(k)$: weight function

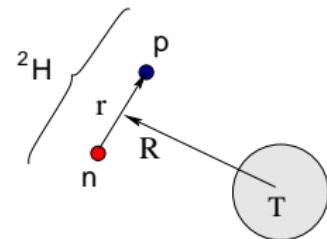


CDCC formalism for deuteron scattering

- Hamiltonian: $H = T_{\mathbf{R}} + h_r(\mathbf{r}) + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- Model wavefunction:

$$\Psi^{(+)}(\mathbf{R}, \mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^N \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$

- Coupled equations: $[H - E]\Psi(\mathbf{R}, \mathbf{r}) = 0$



$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})$$

- Transition potentials:

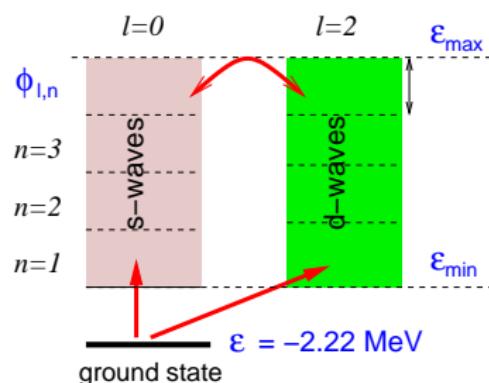
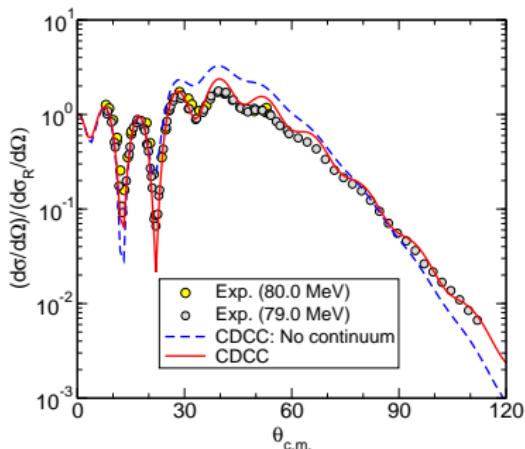
$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r} \phi_n(\mathbf{r})^* \left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2}) \right] \phi_{n'}(\mathbf{r})$$

Application of the CDCC formalism: d+ ^{58}Ni

Coupled-Channels + Continuum discretization



Continuum-Discretized Coupled-Channels (CDCC)

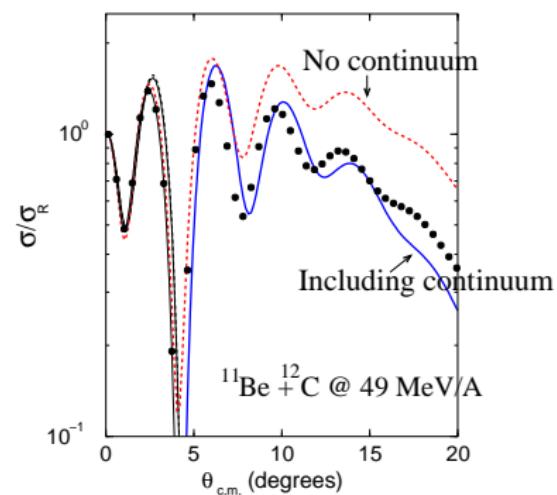
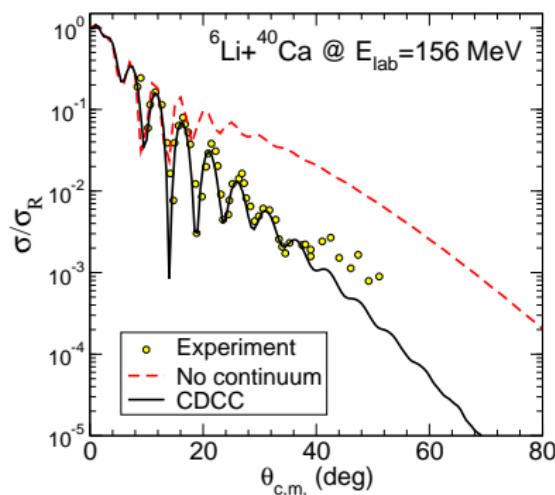


☞ Coupling to breakup channels has an important effect on the reaction dynamics

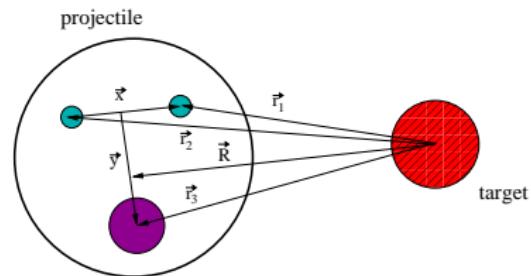
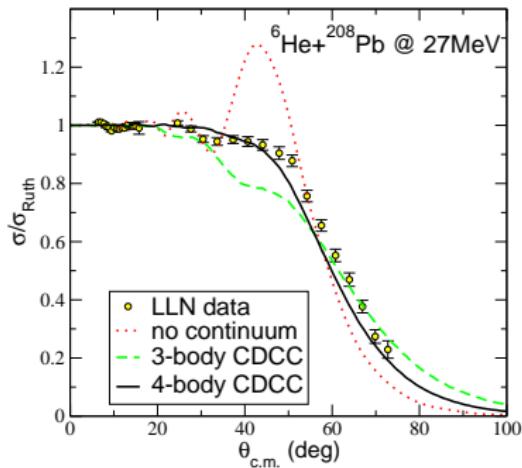
Application of the CDCC method: ^6Li and ^6He scattering

☞ The CDCC has been also applied to nuclei with a cluster structure:

- $^6\text{Li} = \alpha + d$
- $^{11}\text{Be} = ^{10}\text{Be} + n$



Extension to 3-body projectiles

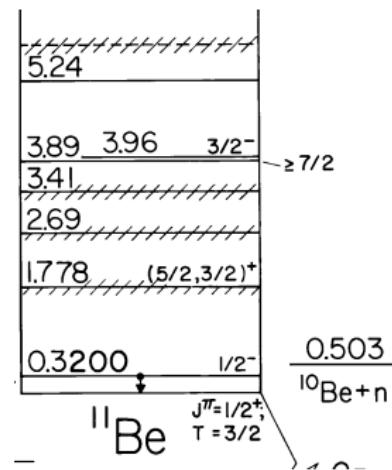
Eg: ${}^6\text{He} = \alpha + \text{n} + \text{n}$ 

M.Rodríguez-Gallardo et al, PRC 77, 064609
(2008)

Exploring structures in the continuum

The continuum spectrum is not “homogeneous”; it contains in general energy regions with special structures:

- Resonances
 - Virtual states

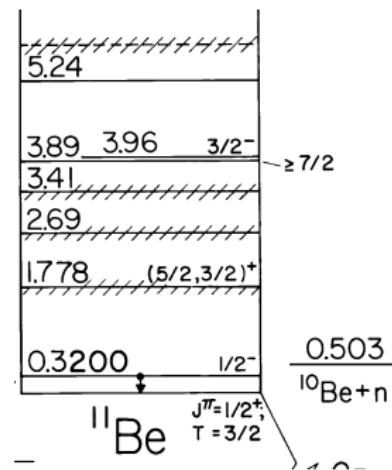




Exploring structures in the continuum

The continuum spectrum is not “homogeneous”; it contains in general energy regions with special structures:

- Resonances
- Virtual states



☞ These structures may (or may not!) show up in reaction observables

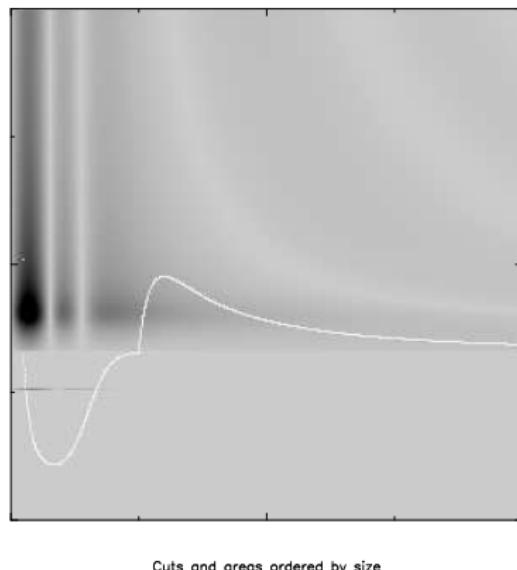


What is a resonance?

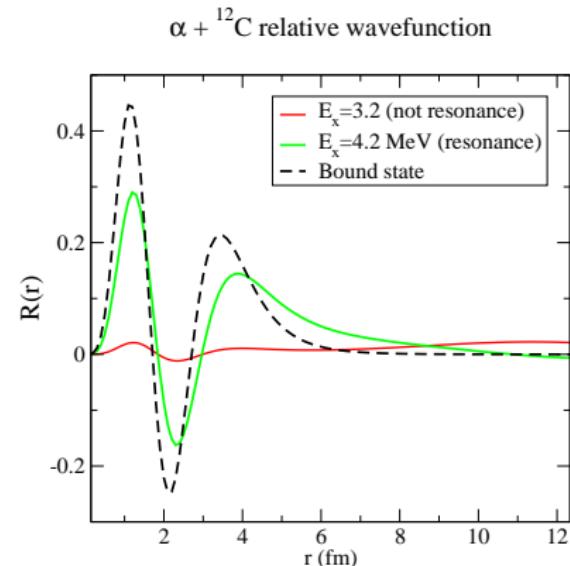
- It is a structure on the continuum which may, or may not, produce a maximum in the cross section, depending on the reaction mechanism and the phase space available.
- The resonance occurs in the range of energies for which the phase shift is close to $\pi/2$.
- In this range of energies, the continuum wavefunctions have a large probability of being in the radial range of the potential.
- The continuum wavefunctions are not square normalizable. However, a normalized “bin” of wavefunctions can be constructed to represent the resonance.

Distinctive features of a resonance

In the energy range of the resonance, the continuum wavefunctions have a large probability of being within the range of the potential.

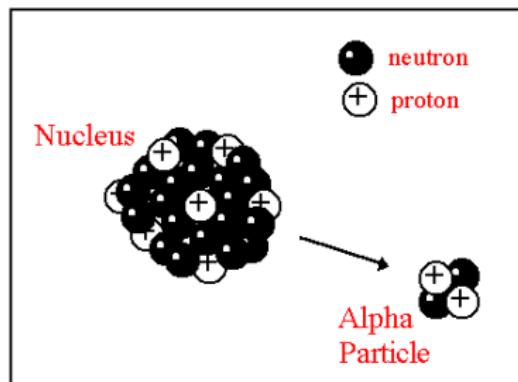
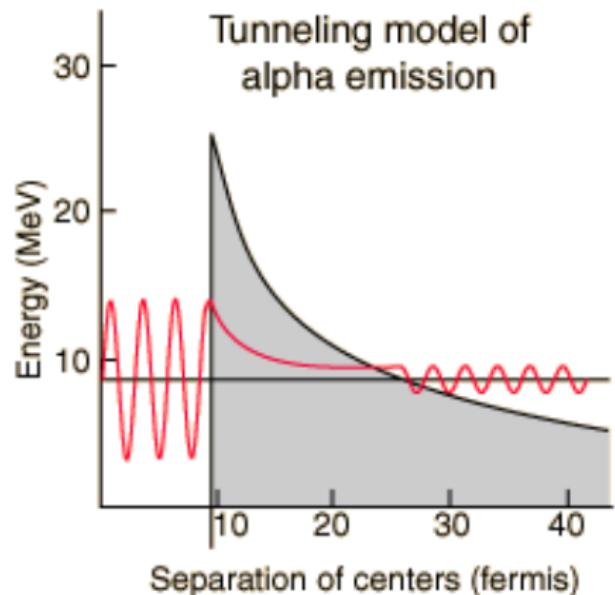


(Courtesy of C. Dasso)



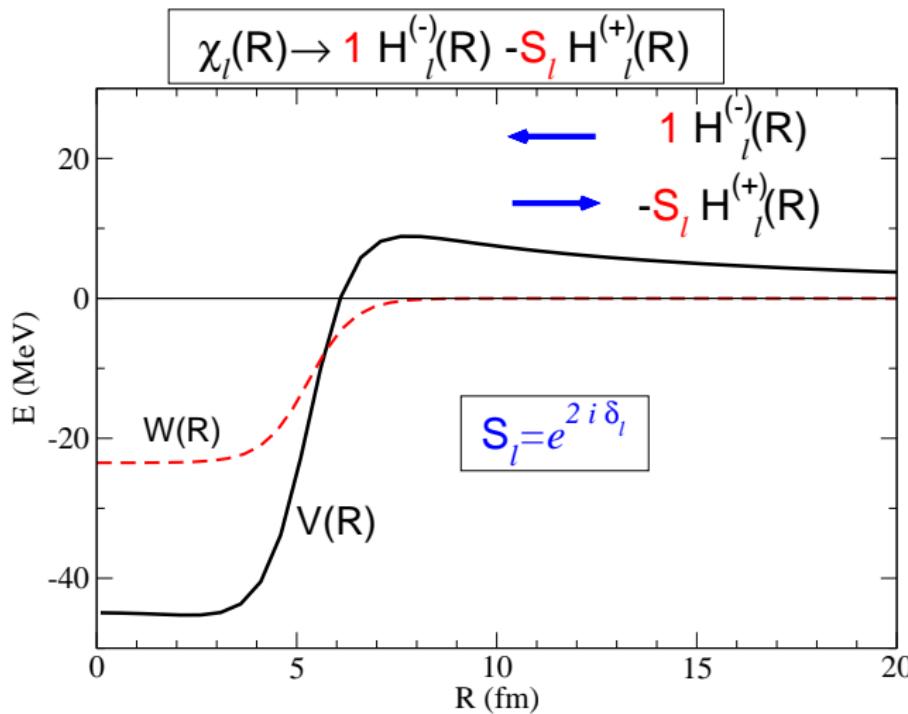
Distinctive features of a resonance

The decay of the resonance is also behind the α decay phenomenon:





Resonances and phase-shifts

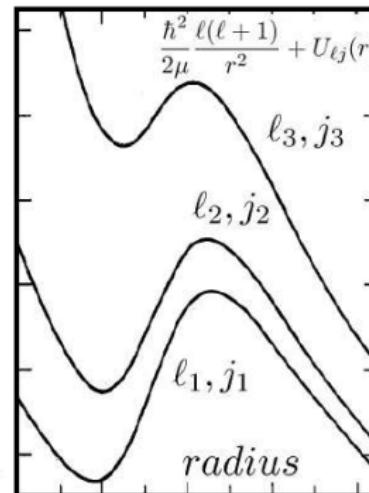
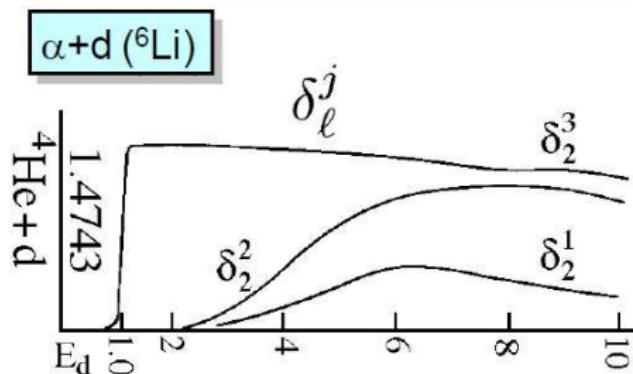




Resonances and phase-shifts

Potential pockets can lead to resonant behaviour – the system being able to be trapped in the pocket for some (life)time τ .

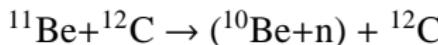
A signal is the rise of the phase shift through 90 degrees.



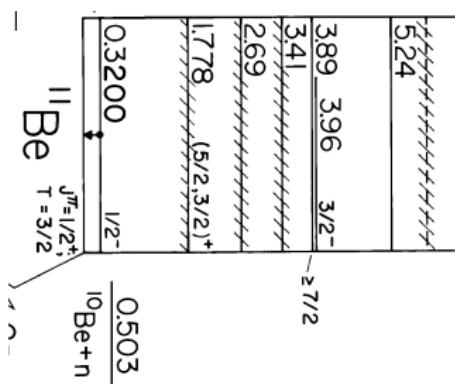
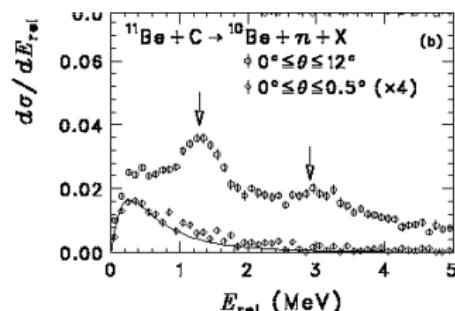
Potential parameters should describe any known resonances



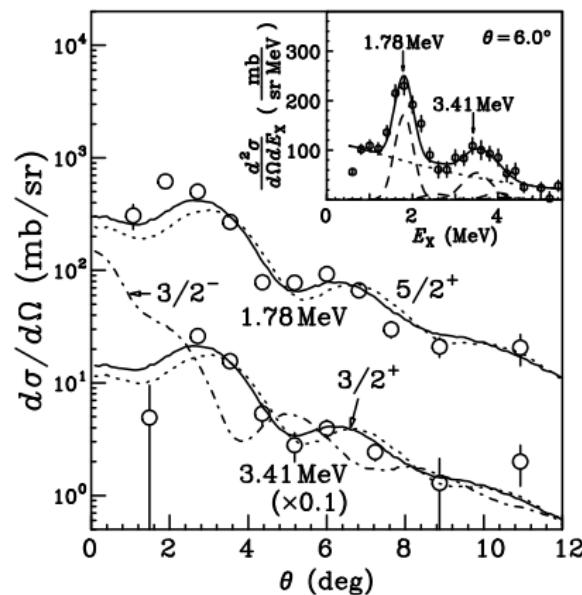
Populating resonances by “inelastic scattering”: $^{11}\text{Be} + ^{12}\text{C}$

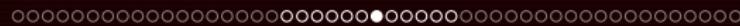


Fukuda et al, Phys. Rev. C70 (2004) 054606

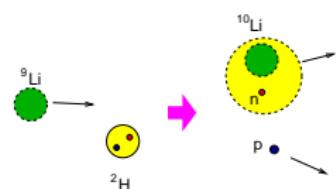


DWBA calculations

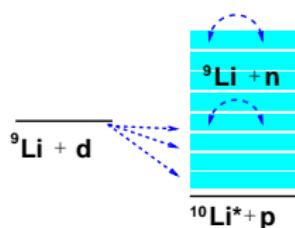




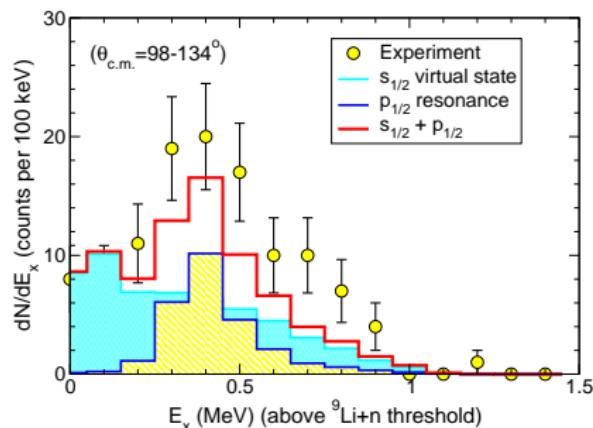
Populating resonances via transfer reactions ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}^*$



CCBA with unbound states



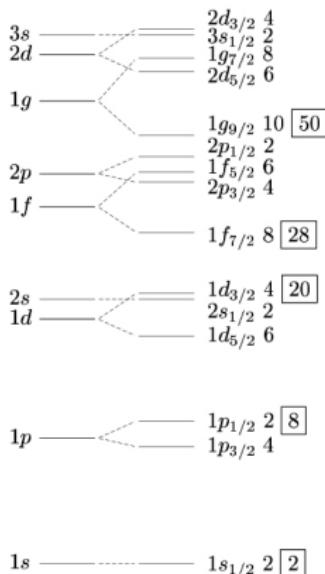
HP.Jeppesen et al, PLB642 (2006) 449





Shell evolution with neutron/proton asymmetry

Shell-evolution for N=7 isotones



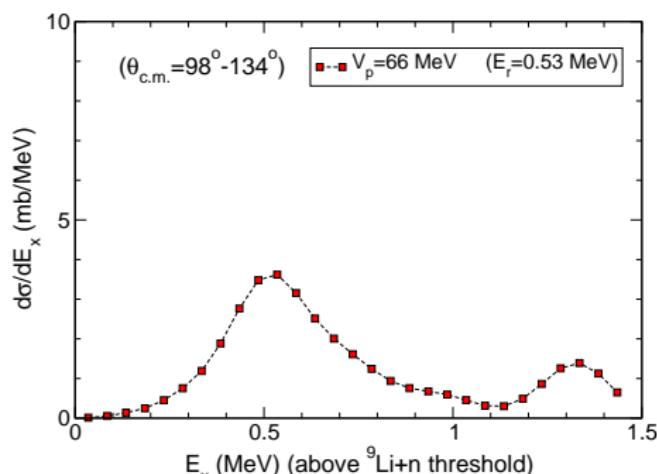
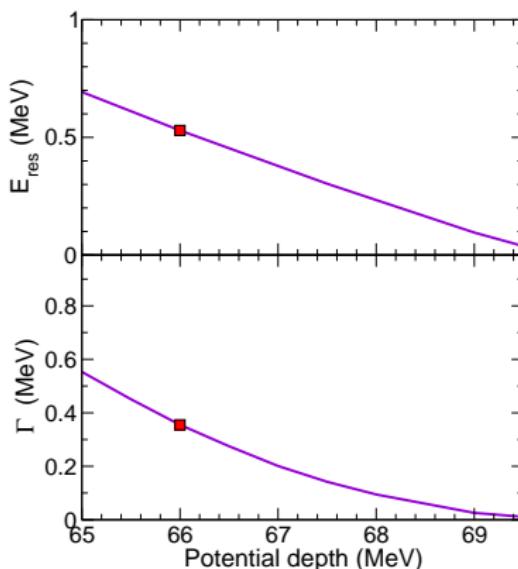


Spectroscopy to unbound states: ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}$ case

Structure: $p_{1/2}$ resonance

Reaction

$$V_{nc}^{(p)}(r) = -V_p \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$



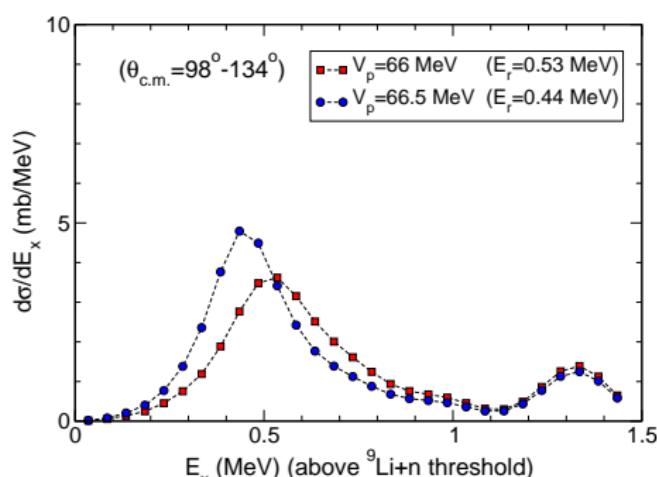
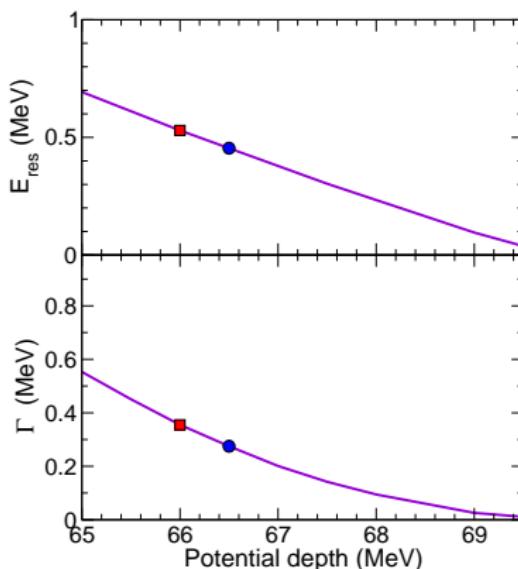


Spectroscopy to unbound states: ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}$ case

Structure: $p_{1/2}$ resonance

Reaction

$$V_{nc}^{(p)}(r) = -V_p \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$



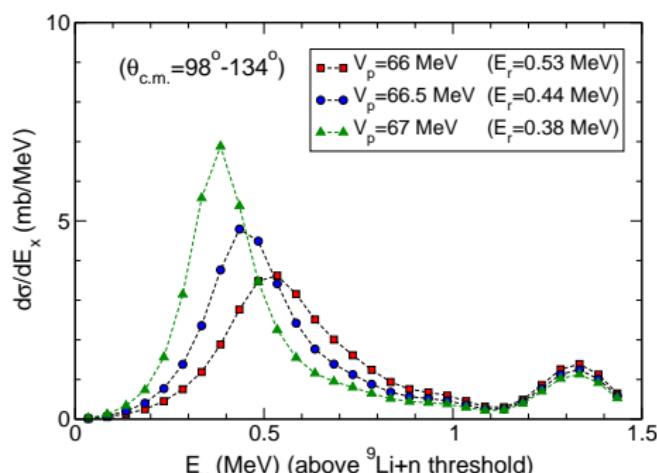
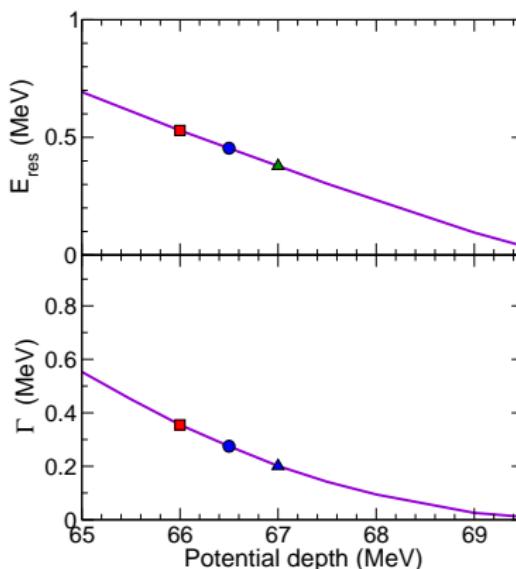


Spectroscopy to unbound states: ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}$ case

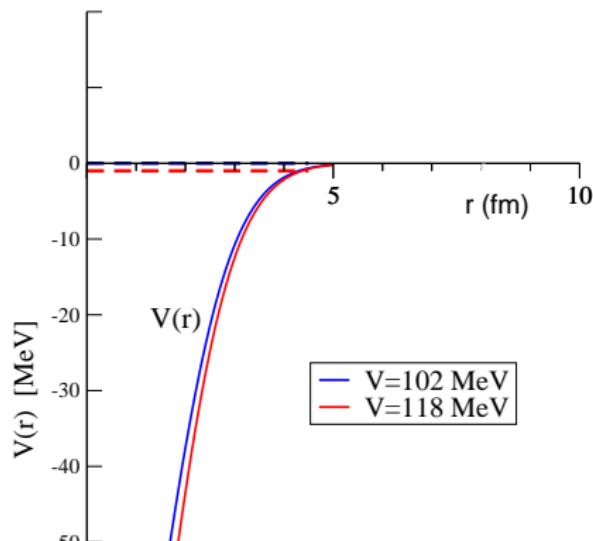
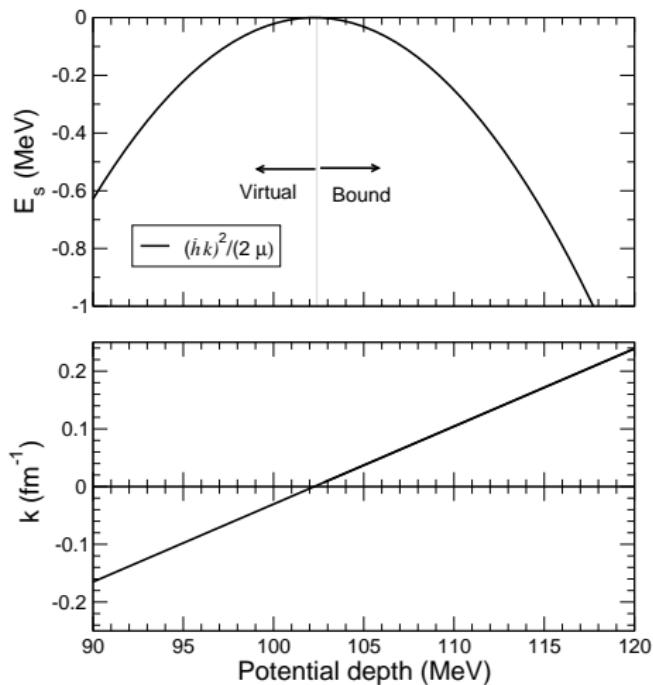
Structure: $p_{1/2}$ resonance

Reaction

$$V_{nc}^{(p)}(r) = -V_p \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$

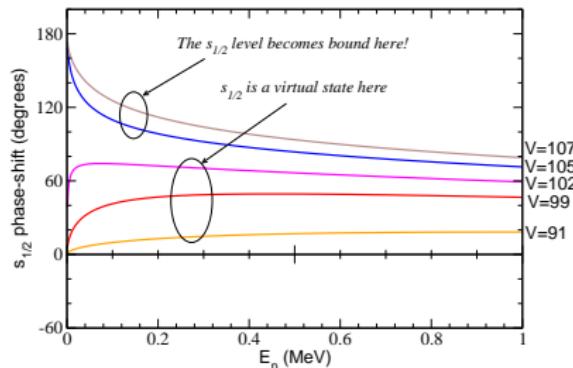


Appearance of a virtual state in $^{10}\text{Li} = ^9\text{Li} + \text{n}$:



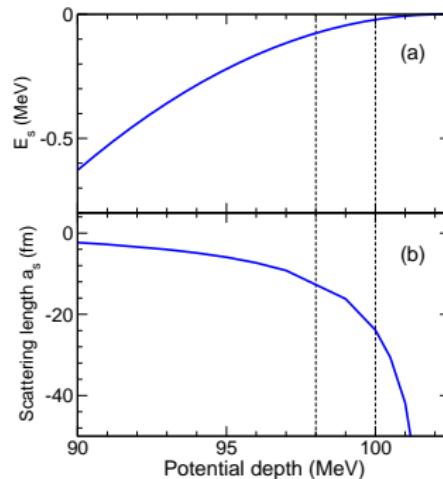


Virtual state in ${}^{10}\text{Li}$



Scattering length:

$$a_s = - \lim_{k \rightarrow 0} \tan \frac{\delta(k)}{k}$$

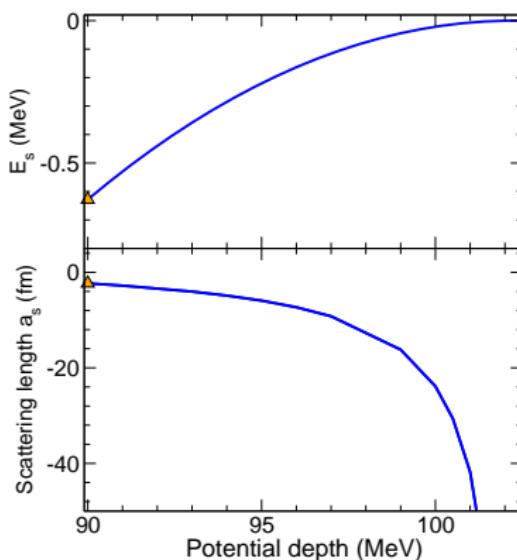




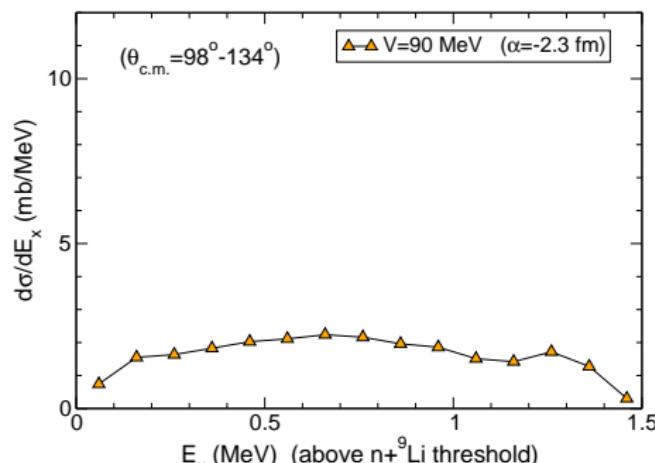
Spectroscopy to unbound states: ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}$ case

Structure: $s_{1/2}$ virtual state

$$V_{nc}^{(s)}(r) = -V_s \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$



Reaction

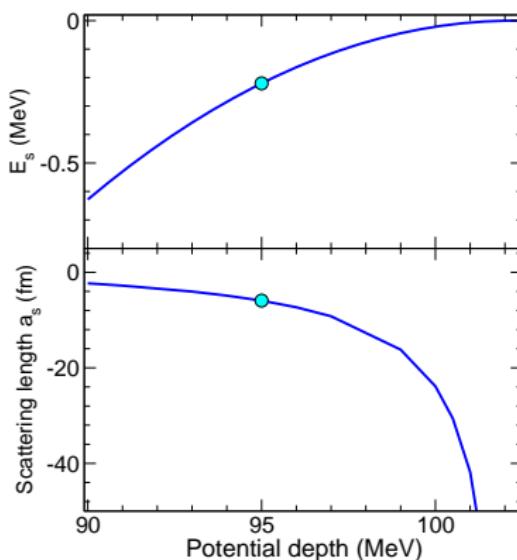




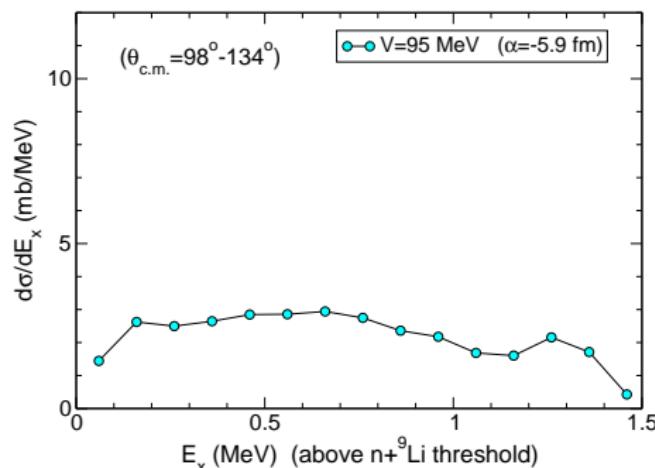
Spectroscopy to unbound states: ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}$ case

Structure: $s_{1/2}$ virtual state

$$V_{nc}^{(s)}(r) = -V_s \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$



Reaction

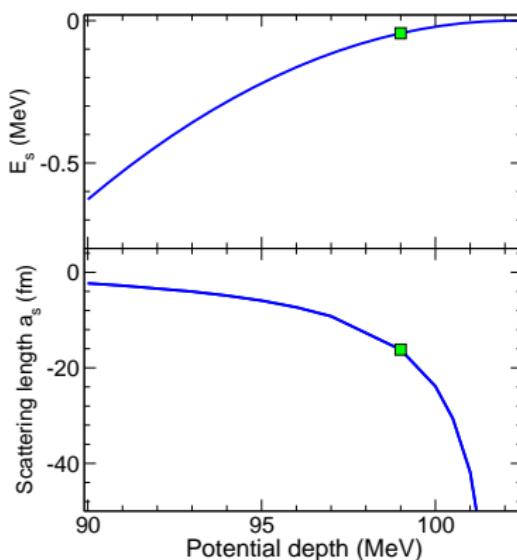




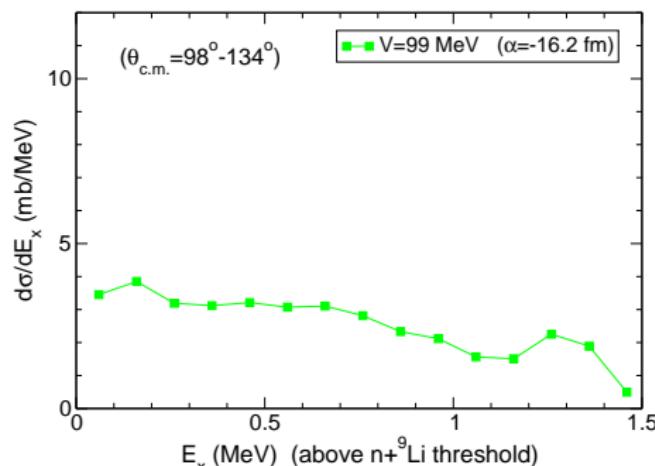
Spectroscopy to unbound states: ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}$ case

Structure: $s_{1/2}$ virtual state

$$V_{nc}^{(s)}(r) = -V_s \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$



Reaction

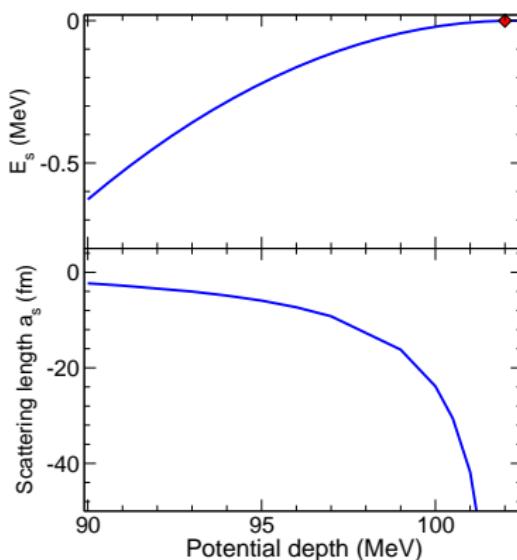




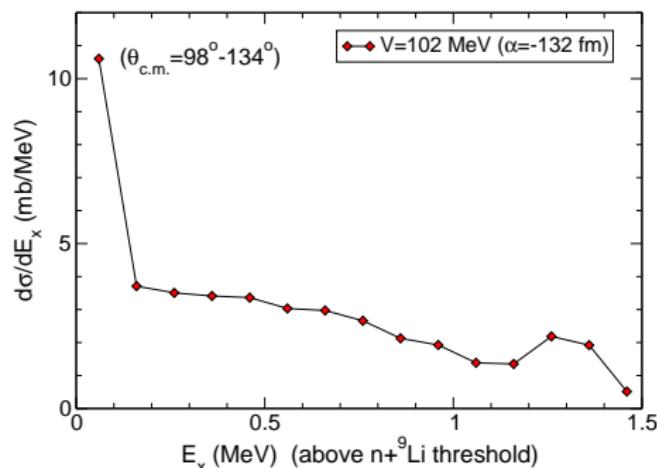
Spectroscopy to unbound states: ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}$ case

Structure: $s_{1/2}$ virtual state

$$V_{nc}^{(s)}(r) = -V_s \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$



Reaction



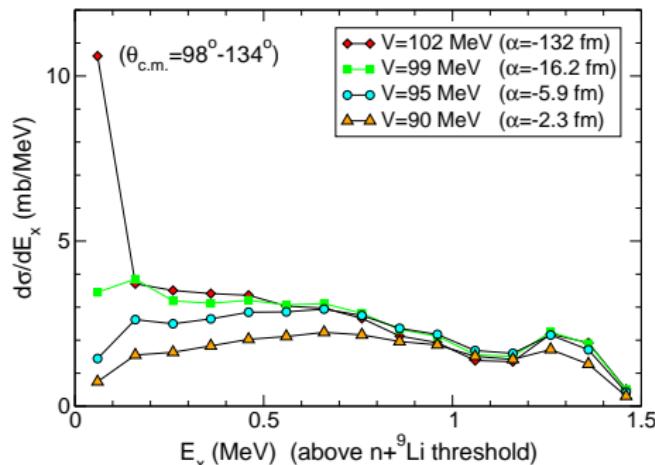
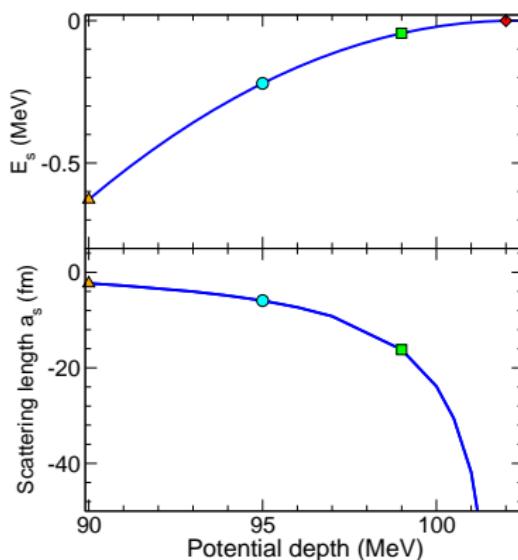


Spectroscopy to unbound states: ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}$ case

Structure: $s_{1/2}$ virtual state

Reaction

$$V_{nc}^{(s)}(r) = -V_s \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$

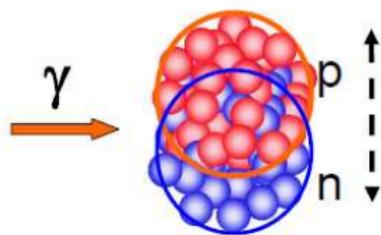




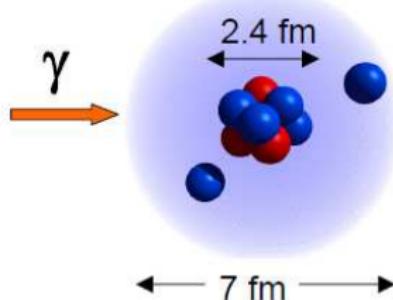
Coulomb dissociation



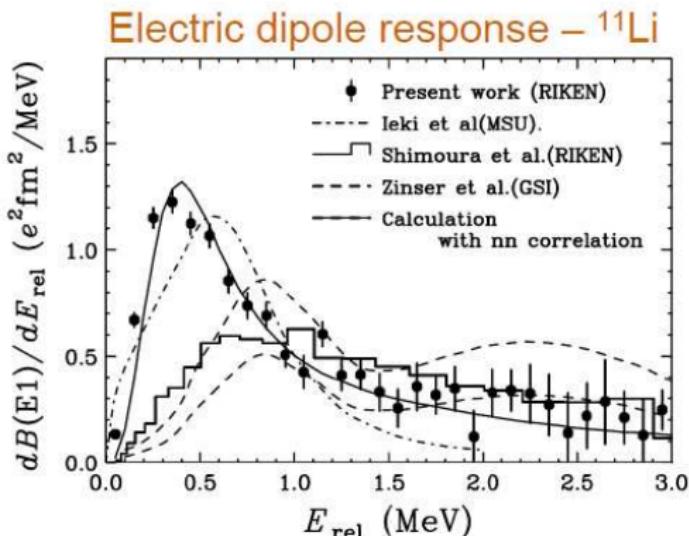
Strong response to electric fields



(Giant) electric Dipole excitation \rightarrow 10-20 MeV



${}^6\text{He}$ (2n, 1 MeV)	${}^{11}\text{Be}$ (1n, 0.5 MeV)
${}^{11}\text{Li}$ (2n, 0.5 MeV)	${}^{14}\text{Be}$ (2n, ~1 MeV)



T. Nakamura et al., PRL (2006) in press



Application to Coulomb dissociation of halo nuclei

- First-order semiclassical cross section for a $0 \rightarrow n$ excitation:

$$\left(\frac{d\sigma}{d\Omega} \right)_{0 \rightarrow n} = \left(\frac{Z_t e^2}{\hbar v} \right)^2 \frac{B(E\lambda, 0 \rightarrow n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta, \xi)$$

- Halo nuclei are weakly bound \Rightarrow excitation occurs to unbound (continuum) states

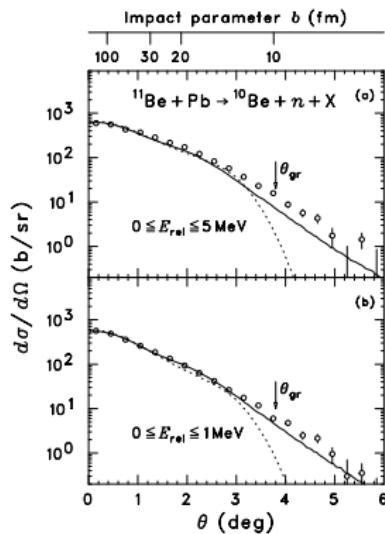
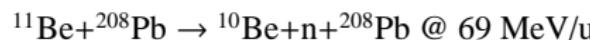
$$\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v} \right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE} \frac{df_\lambda(\theta, \xi)}{d\Omega}$$



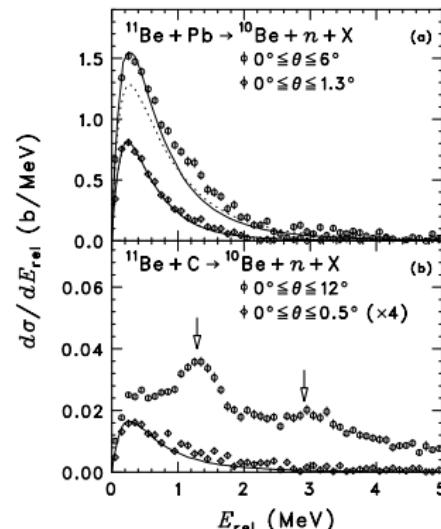
Application a CD of 1n-halo nuclei

- Halo nuclei are weakly bound \Rightarrow the systems are easily polarized in strong electric field (large E1 response)
- Large Coulomb dissociation probability with heavy targets
- At small-angles (large impact parameters) the dissociation is Coulomb dominated and hence can be used to extract the $E1$ transition probability using:

$$\frac{d\sigma}{dE_x}(\theta \ll) \propto \frac{dB(E1)}{dE_x}$$

Example: ^{11}Be dissociation

Fukuda, 2004





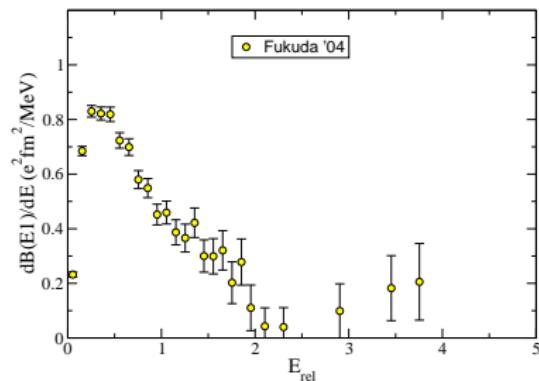
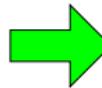
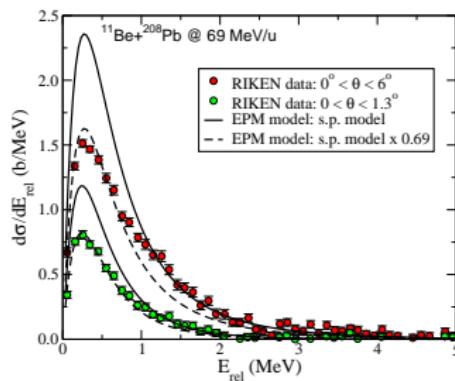
Core excitation in Coulomb breakup: $B(E1)$ response of ^{11}Be

$B(E1)$ extracted in a model-dependent way \Rightarrow compare directly cross sections

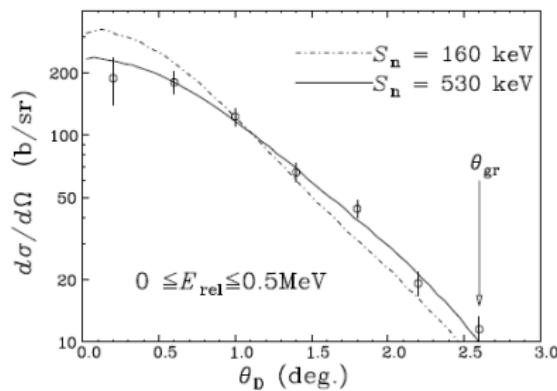
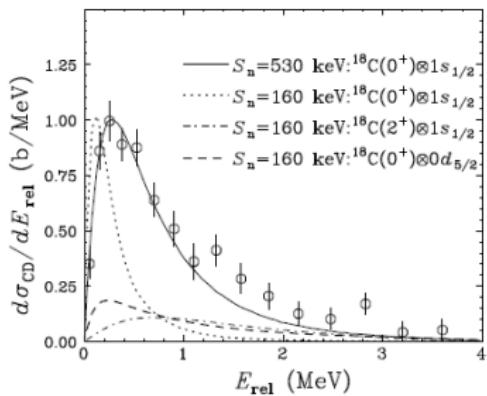
$$\left(\frac{d\sigma}{d\Omega}\right)_{0 \rightarrow n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \rightarrow n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta, \xi)$$

(Semiclassical 1st order)

Eg: $^{11}\text{Be} + ^{208}\text{Pb}$ at RIKEN *Fukuda et al, PRC70, 054606 (2004)*



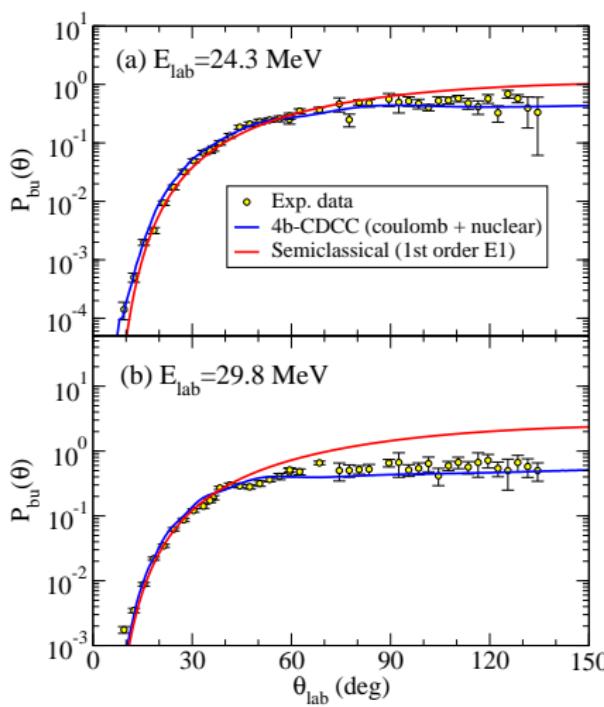
$^{11}\text{Be} + ^{208}\text{Pb}$ at RIKEN



Nakamura et al, Phys. Rev. Lett. 83 1112 (1999)

Comparison of 1st order with full quantum-mechanical calculation

Eg: $^{11}\text{Li} + ^{208}\text{Pb}$ at Coulomb barrier energies



- ⇒ $E_{lab} \sim V_b \Rightarrow$ Coulomb important
 - ⇒ At small angles, breakup dominated by E1 Coulomb

J.P. Fernandez-Garcia et al, PRL110, 142701(2013)



Radiative capture



Relation to radiative capture

Radiative capture: $b + c \rightarrow a + \gamma$

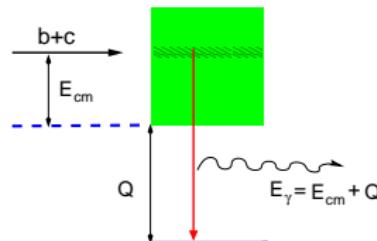
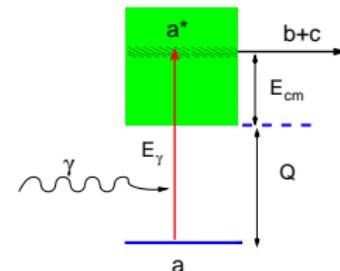


Photo-absorption: $a + \gamma \rightarrow b + c$



⇒ Related by detailed balance:

$$\sigma_{E\lambda}^{(rc)} = \frac{2(2J_a + 1)}{(2J_b + 1)(2J_c + 1)} \frac{k_\gamma^2}{k^2} \sigma_{E\lambda}^{(phot)} \quad (\hbar k_\gamma = E_\gamma/c)$$

⇒ Astrophysical S-factor:

$$S(E_{c.m.}) = E_{c.m.} \sigma_{E\lambda}^{(rc)} \exp[2\pi\eta(E_{c.m.})]$$



Photo-absorption cross section: virtual photon description

⇒ Photo-absorption (not proven here): $\gamma + a \rightarrow b + c$

$$\sigma_{E\lambda}^{\text{photo}} = \frac{(2\pi)^3(\lambda + 1)}{\lambda[(2\lambda + 1)!!]^2} \left(\frac{E_\gamma}{\hbar c} \right)^{2\lambda-1} \frac{dB(E\lambda)}{dE}$$

⇒ 1st order Coulomb breakup cross section in terms of photo-absorption:

$$\frac{d\sigma(E\lambda)}{d\Omega dE_\gamma} = \frac{1}{E_\gamma} \frac{dn_{E\lambda}}{d\Omega} \sigma_{E\lambda}^{\text{photo}}$$

(Equivalent Photon Method)

with the virtual photon number

$$\frac{dn_{E\lambda}}{d\Omega} = Z_t^2 \alpha \frac{\lambda[(2\lambda + 1)!!]^2}{(2\pi)^3(\lambda + 1)} \xi^{2(1-\lambda)} \left(\frac{c}{v} \right)^{2\lambda} \frac{df_{E\lambda}}{d\Omega}$$

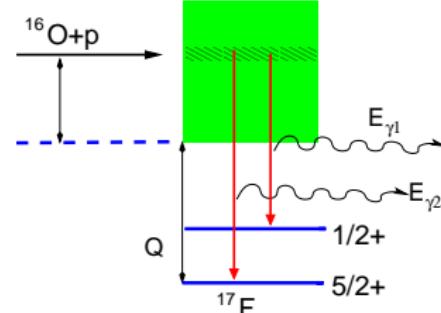
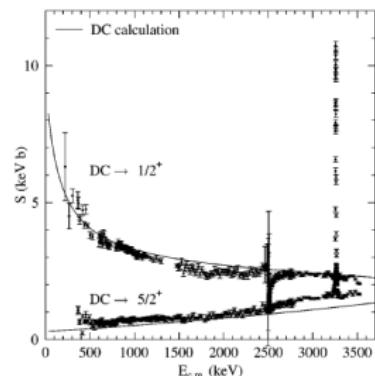
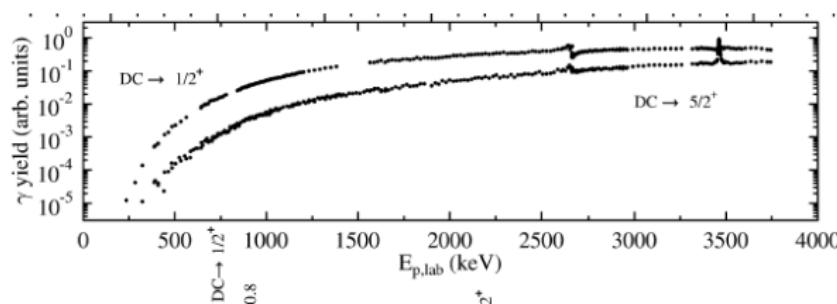
Radiative capture from Coulomb dissociation experiments

- ☞ Capture reactions have typically small cross sections
- ☞ Use breakup (Coulomb dissociation) reactions:

$$\frac{d\sigma}{d\Omega dE_{c.m.}} \rightarrow \sigma_{E\lambda}^{(\text{phot})} \rightarrow \sigma_{E\lambda}^{(rc)} \rightarrow S(E_{\text{c.m.}})$$

Example: $p + {}^{16}\text{O} \rightarrow {}^{17}\text{F} + \gamma$

Morlock, PRL79, 3837 (1997)

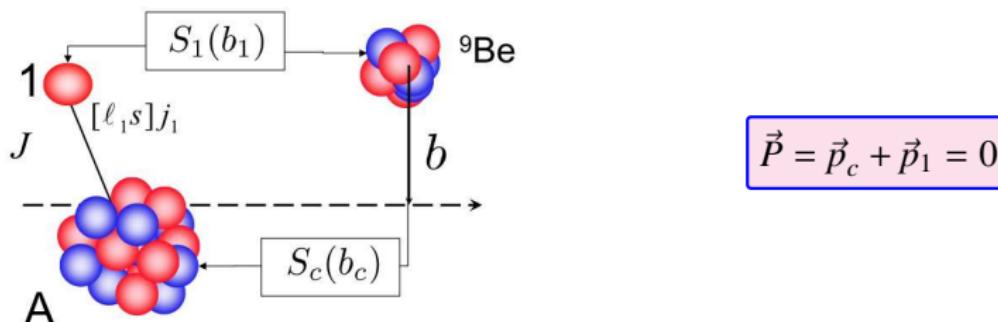




Knock-out reactions

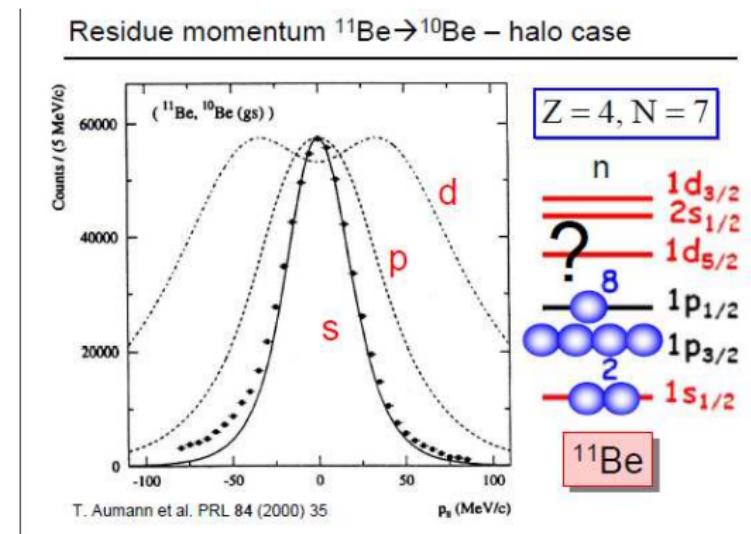
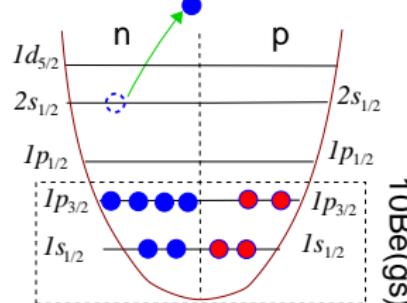
Spectroscopic from momentum distributions

- Fast-moving projectile on a (typically) light target.
- One nucleon suddenly removed (absorbed) due to its interaction with the target.
- The remaining nucleons remain unchanged and is detected.
- The momentum of the core is traced back to that of the removed nucleon because in the rest frame of the projectile $\vec{P} = 0$

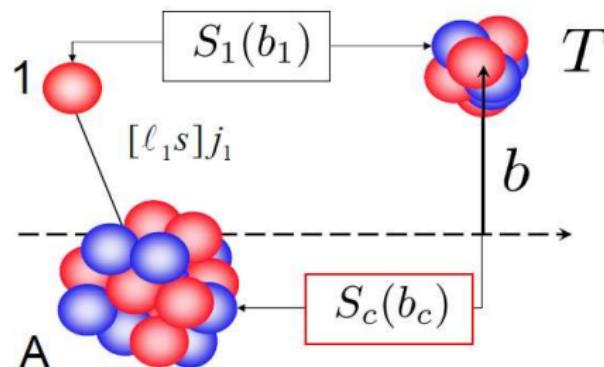


Angular momentum sensitivity of momentum distributions

- The shape is determined by the orbital angular momentum ℓ .
- The magnitude is determined by the amount of $s_{1/2}$ (spectroscopic factor)



Stripping cross section within a semiclassical (eikonal) theory



$$\sigma_{\text{strip}} = \int d\mathbf{b} \langle \phi_0 | |\mathbf{S}_c|^2 (1 - |\mathbf{S}_1|^2) | \phi_0 \rangle$$

- $|S_c(b_c)|^2$ =probability of survival of the core.
 - $1 - |S_1(b_1)|^2$ =probability of absorption of the neutron.



Transfer reactions with exotic nuclei



$^1\text{H}(^{11}\text{Be}, ^{10}\text{Be})^2\text{H}$ example

$$|^{11}\text{Be}\rangle = \textcolor{violet}{a} |^{10}\text{Be}(0^+) \otimes \nu 2s_{1/2}\rangle + \textcolor{violet}{b} |^{10}\text{Be}(2^+) \otimes \nu 1d_{5/2}\rangle + \dots$$

⇒ In DWBA:

$$\sigma(0^+) \propto |\textcolor{violet}{a}|^2; \quad \sigma(2^+) \propto |\textcolor{violet}{b}|^2$$



$^1\text{H}(^{11}\text{Be}, ^{10}\text{Be})^2\text{H}$ example

$$|^{11}\text{Be}\rangle = \textcolor{magenta}{a} |^{10}\text{Be}(0^+) \otimes \nu 2s_{1/2}\rangle + \textcolor{magenta}{b} |^{10}\text{Be}(2^+) \otimes \nu 1d_{5/2}\rangle + \dots$$

⇒ In DWBA:

$$\sigma(0^+) \propto |\textcolor{magenta}{a}|^2; \quad \sigma(2^+) \propto |\textcolor{magenta}{b}|^2$$

Fortier et al, PLB461, 22 (1999)

