

ISOLDE Nuclear Reaction and Nuclear Structure Course

Antonio M. Moro

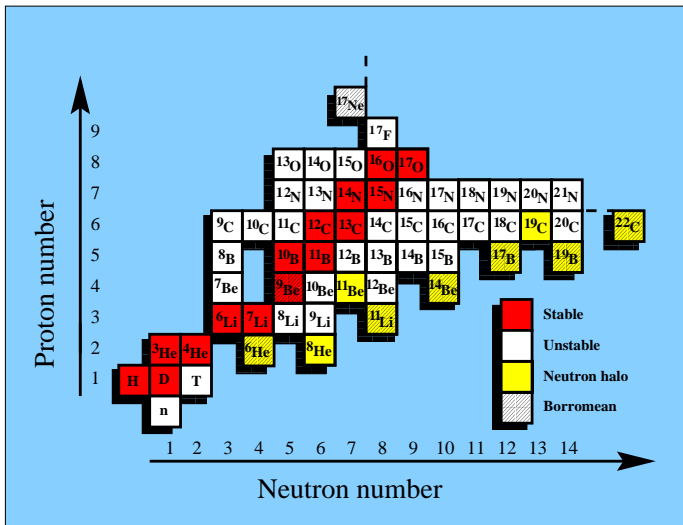


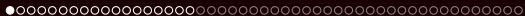
Universidad de Sevilla, Spain

April 25, 2014



Exotic nuclei, halo nuclei, and Borromean systems





Inclusion of breakup channels: the CDCC method

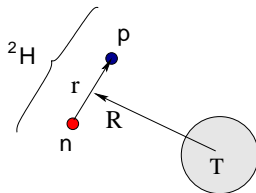
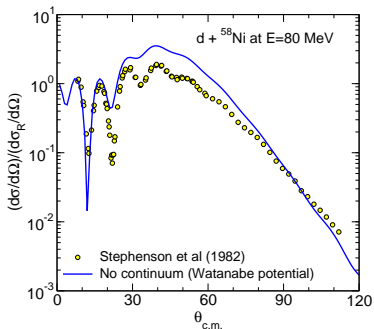
Extension of the CC method to include breakup channels

- Light exotic nuclei usually present a cluster structure (recall talk by B.Jonson in this course)
- To use the CC formalism, one needs to extend the method in order to:
 - ⇒ Describe the cluster (or single-particle) structure of light exotic nuclei
 - ⇒ Permit the inclusion of unbound (continuum) states (breakup channels)

Application of the CC method to weakly-bound systems

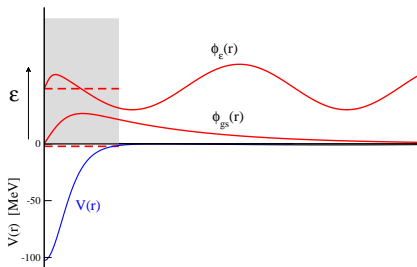
Example: Three-body calculation (p+n+⁵⁸Ni) with Watanabe potential:

$$V_{dt}(\mathbf{R}) = \int d\mathbf{r} \phi_{gs}(\mathbf{r}) \left\{ V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt}) \right\} \phi_{gs}(\mathbf{r})$$



☞ *Three-body calculations omitting breakup channels fail to describe the experimental data.*

Bound versus scattering states

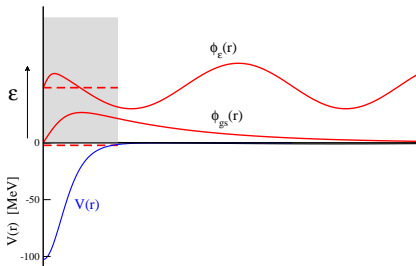


Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r} [Y_{\ell}(\hat{r}) \otimes \chi_s]_{jm}$$

$$\epsilon = \frac{\hbar^2 k^2}{2\mu}$$

Bound versus scattering states



Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r} [Y_{\ell}(\hat{r}) \otimes \chi_s]_{jm}$$

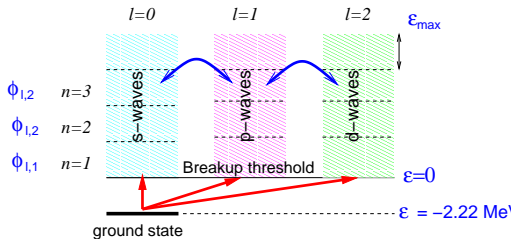
$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Unbound states are not suitable for CC calculations:

- They have a continuous (infinite) distribution in energy.
- Non-normalizable: $\langle u_{k,\ell sj}(r) | u_{k',\ell sj}(r) \rangle \propto \delta(k - k')$

SOLUTION \Rightarrow continuum discretization

Continuum discretization for deuteron scattering



- ⇒ Select a number of partial waves ($\ell = 0, \dots, \ell_{\max}$).
- ⇒ For each ℓ , set a maximum excitation energy ε_{\max} .
- ⇒ Divide the interval $\varepsilon = 0 - \varepsilon_{\max}$ in a set of sub-intervals (*bins*).
- ⇒ For each **bin**, calculate a representative wavefunction.

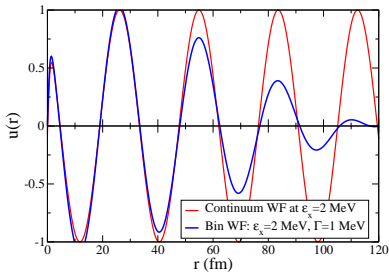
CDCC formalism: construction of the bin wavefunctions

Bin wavefunction:

$$\varphi_{\ell jm}^{[k_1, k_2]}(\mathbf{r}) = \frac{u_{\ell j}^{[k_1, k_2]}(r)}{r} [Y_{\ell}(\hat{r}) \otimes \chi_s]_{jm} \quad [k_1, k_2] = \text{bin interval}$$

$$u_{\ell sjm}^{[k_1, k_2]}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k, \ell sj}(r) dk$$

- k : linear momentum
- $u_{k, \ell sj}(r)$: scattering states (radial part)
- $w(k)$: weight function



CDCC formalism for deuteron scattering

- **Hamiltonian:** $H = T_{\mathbf{R}} + h_r(\mathbf{r}) + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- **Model wavefunction:**

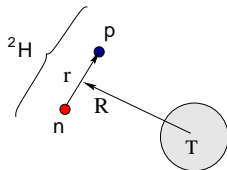
$$\Psi^{(+)}(\mathbf{R}, \mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^N \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$

- **Coupled equations:** $[H - E]\Psi(\mathbf{R}, \mathbf{r}) = 0$

$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})$$

- **Transition potentials:**

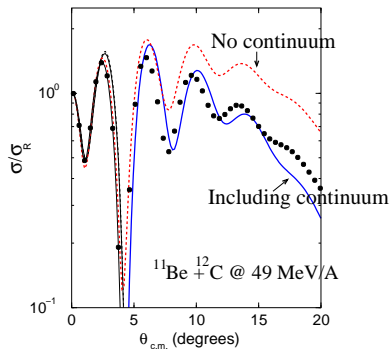
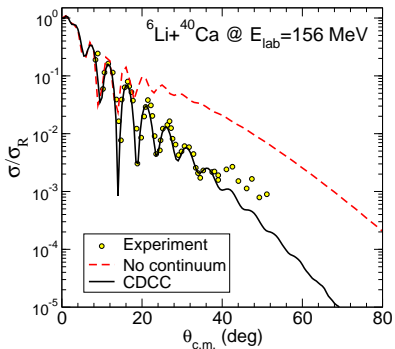
$$V_{n,n'}(\mathbf{R}) = \int d\mathbf{r} \phi_n(\mathbf{r})^* \left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2}) \right] \phi_{n'}(\mathbf{r})$$



Application of the CDCC method: ${}^6\text{Li}$ and ${}^6\text{He}$ scattering

➡ The CDCC has been also applied to nuclei with a cluster structure:

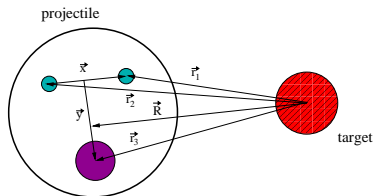
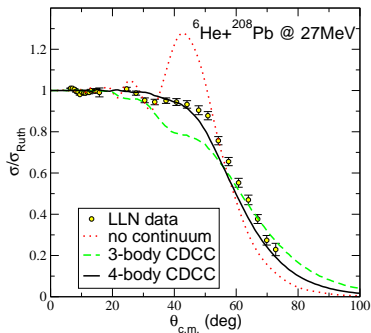
- ${}^6\text{Li} = \alpha + d$
- ${}^{11}\text{Be} = {}^{10}\text{Be} + n$





Extension to 3-body projectiles

Eg: ${}^6\text{He} = \alpha + n + n$

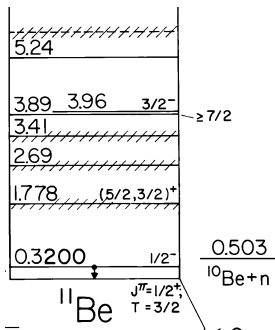


M. Rodríguez-Gallardo et al, PRC 77, 064609 (2008)

Exploring structures in the continuum

The continuum spectrum is not “homogeneous”; it contains in general energy regions with special structures:

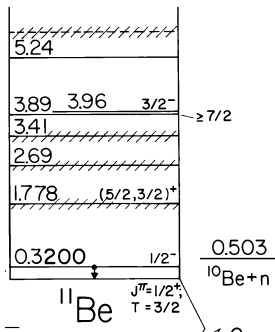
- Resonances
- Virtual states



Exploring structures in the continuum

The continuum spectrum is not “homogeneous”; it contains in general energy regions with special structures:

- Resonances
- Virtual states



→ *These structures may (or may not!) show up in reaction observables*

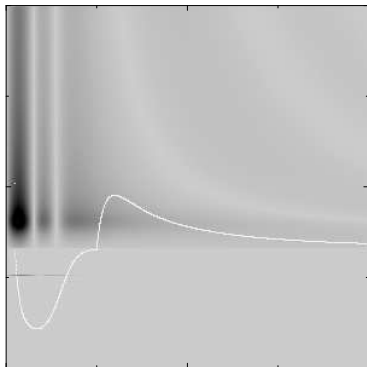


What is a resonance?

- It is a structure on the continuum which may, or may not, produce a maximum in the cross section, depending on the reaction mechanism and the phase space available.
- The resonance occurs in the range of energies for which the phase shift is close to $\pi/2$.
- In this range of energies, the continuum wavefunctions have a large probability of being in the radial range of the potential.
- The continuum wavefunctions are not square normalizable. However, a normalized “bin” of wavefunctions can be constructed to represent the resonance.

Distinctive features of a resonance

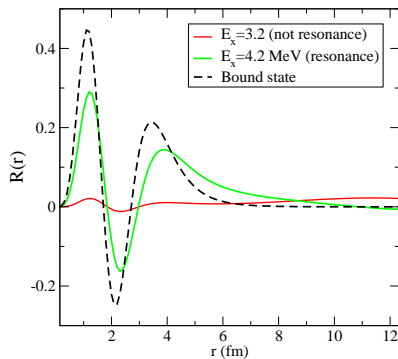
In the energy range of the resonance, the continuum wavefunctions have a large probability of being within the range of the potential.



Cuts and areas ordered by size

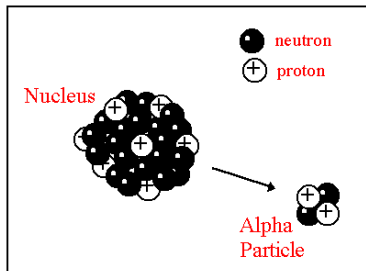
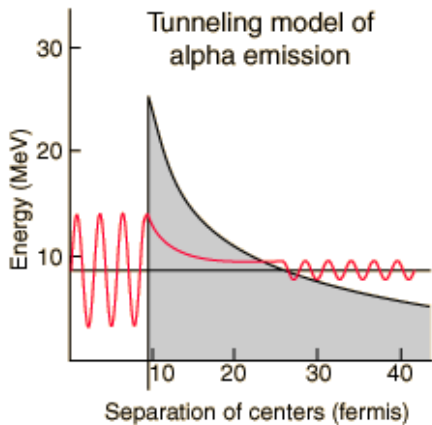
(Courtesy of C. Dasso)

$\alpha + {}^{12}\text{C}$ relative wavefunction

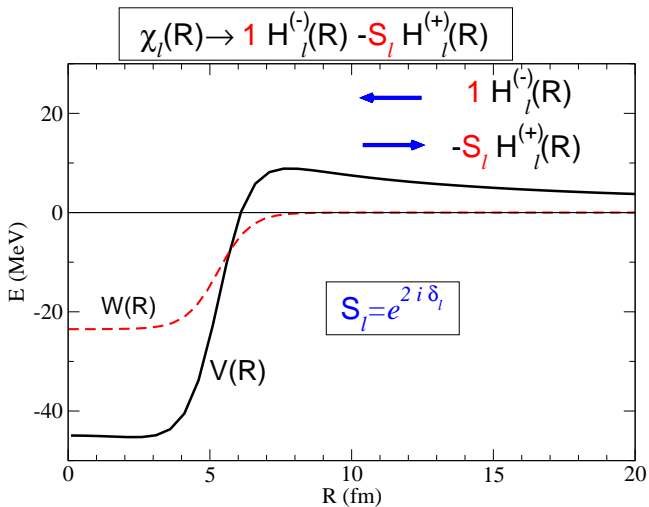


Distinctive features of a resonance

The decay of the resonance is also behind the α decay phenomenon:



Resonances and phase-shifts

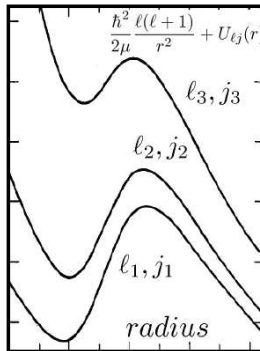
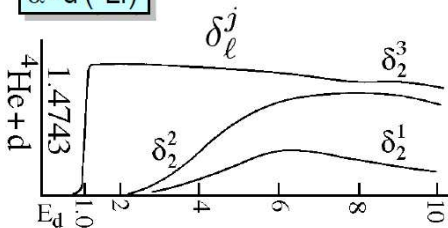


Resonances and phase-shifts

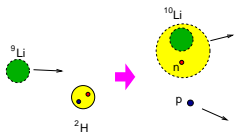
Potential pockets can lead to resonant behaviour – the system being able to be trapped in the pocket for some (lifetime) τ .

A signal is the rise of the phase shift through 90 degrees.

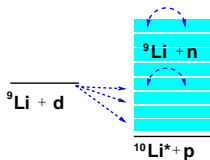
$$\alpha+d \text{ (} {}^6\text{Li)}_1$$



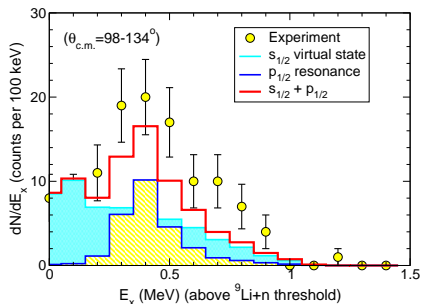
Potential parameters should describe any known resonances

Populating resonances via transfer reactions ${}^9\text{Li}(d,p){}^{10}\text{Li}^*$ 

CCBA with unbound states

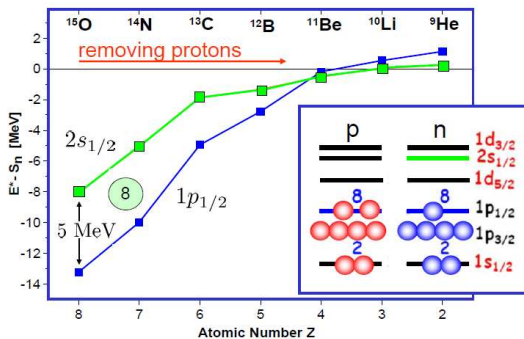
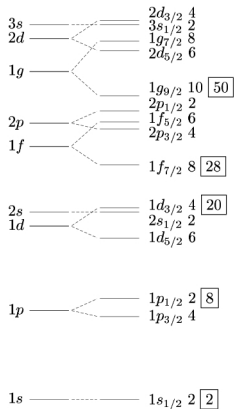


HP.Jeppesen et al, PLB642 (2006) 449



Shell evolution with neutron/proton asymmetry

Shell-evolution for N=7 isotones

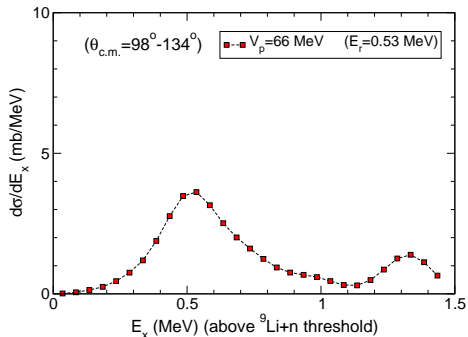
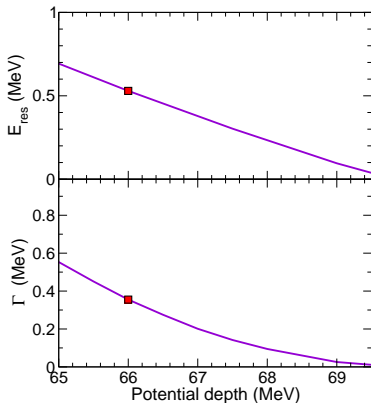


From: P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci 53 (2003) 219

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ caseStructure: $p_{1/2}$ resonance

Reaction

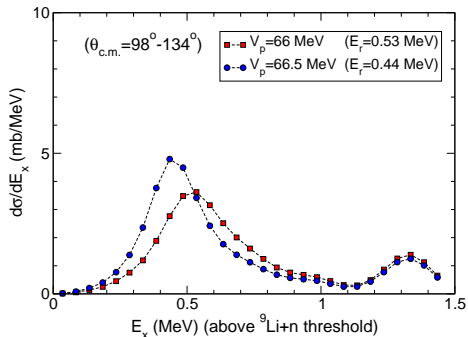
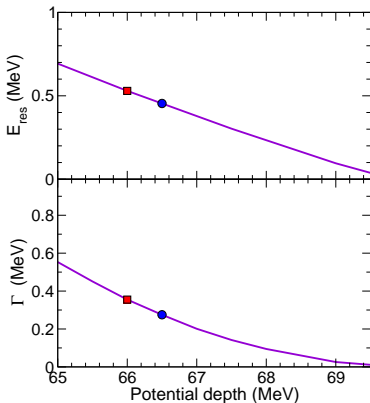
$$V_{nc}^{(p)}(r) = -V_p \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$



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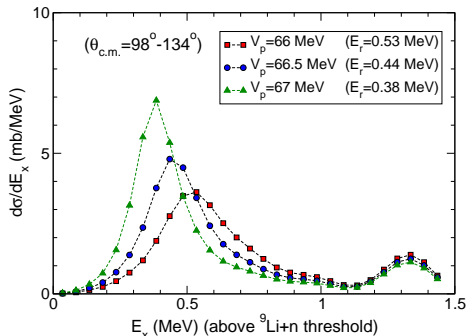
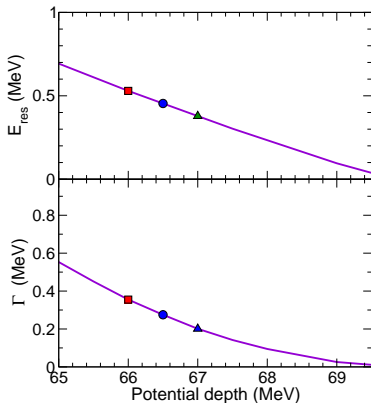
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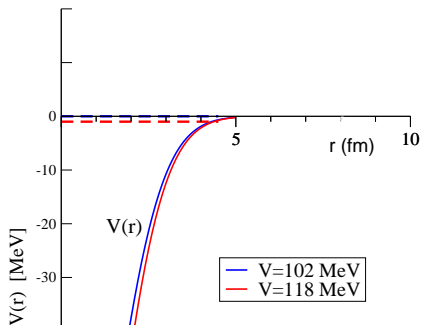
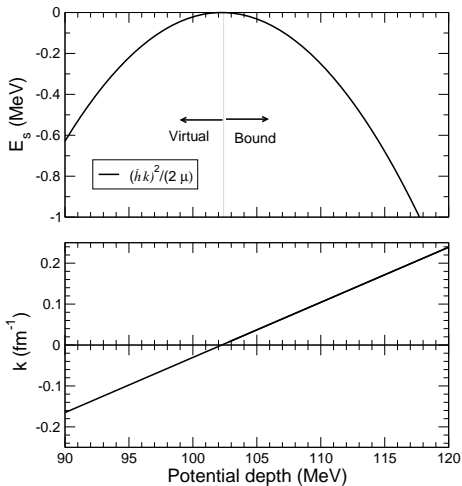
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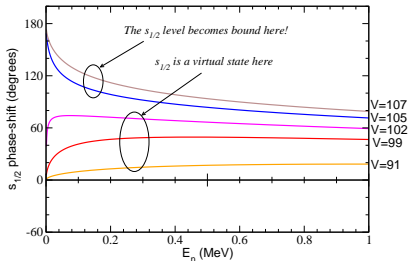
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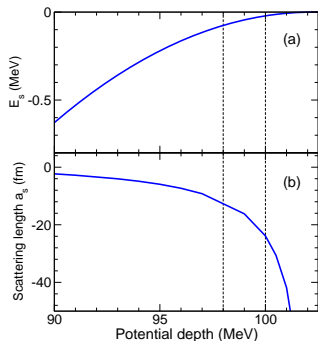
Appearance of a virtual state in $^{10}\text{Li} = ^9\text{Li} + n$:



Virtual state in ^{10}Li 

Scattering length:

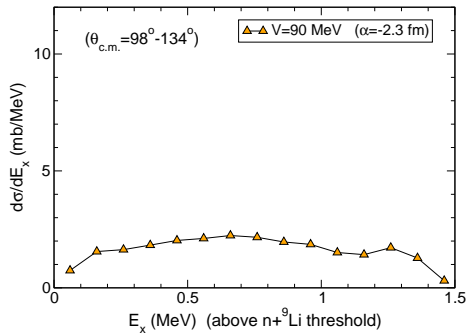
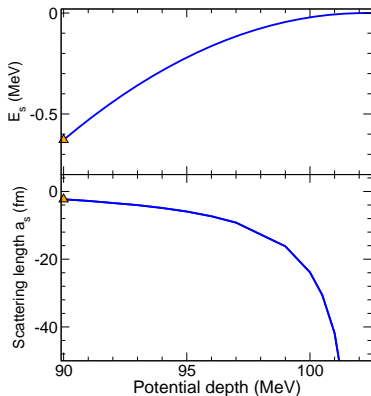
$$a_s = - \lim_{k \rightarrow 0} \tan \frac{\delta(k)}{k}$$



Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ caseStructure: $s_{1/2}$ virtual state

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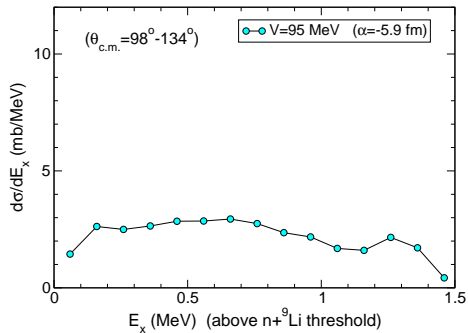
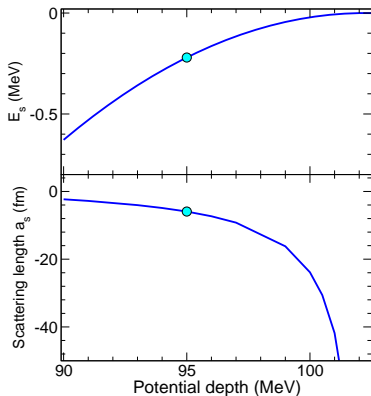
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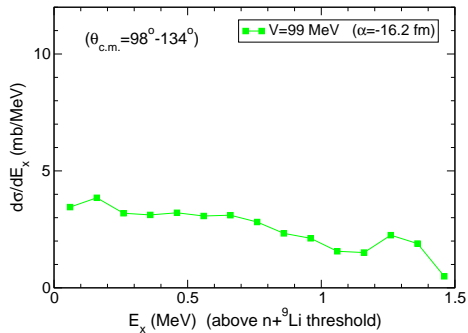
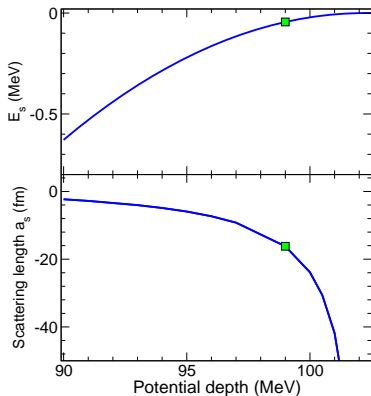
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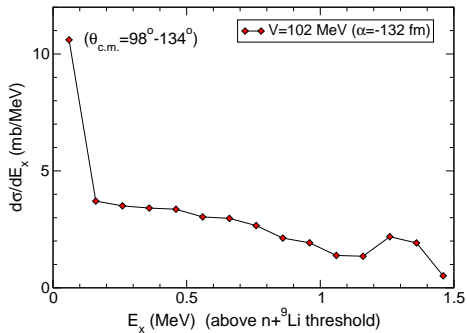
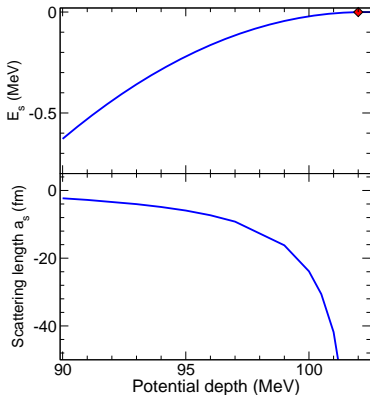
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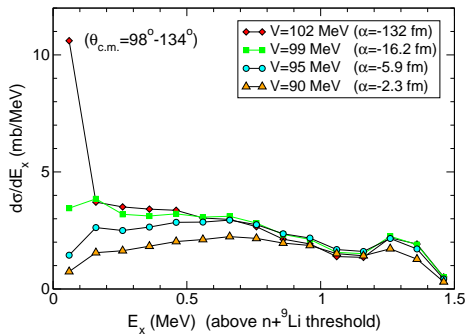
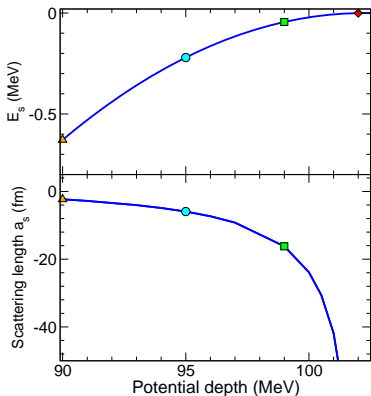
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$$V_{nc}^{(s)}(r) = -V_s \exp(-r^2/a^2) \quad (a = 2 \text{ fm})$$





Coulomb dissociation

Application to Coulomb dissociation of halo nuclei

- First-order semiclassical cross section for a $0 \rightarrow n$ excitation:

$$\left(\frac{d\sigma}{d\Omega}\right)_{0 \rightarrow n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \rightarrow n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta, \xi)$$

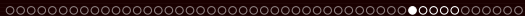
- Halo nuclei are weakly bound \Rightarrow excitation occurs to unbound (continuum) states

$$\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE} \frac{df_\lambda(\theta, \xi)}{d\Omega}$$

Application a CD of 1n-halo nuclei

- Halo nuclei are weakly bound \Rightarrow the systems are easily polarized in strong electric field (large E1 response)
- Large Coulomb dissociation probability with heavy targets
- At small-angles (large impact parameters) the dissociation is Coulomb dominated and hence can be used to extract the $E1$ transition probability using:

$$\frac{d\sigma}{dE_x}(\theta \ll) \propto \frac{dB(E1)}{dE_x}$$



Radiative capture

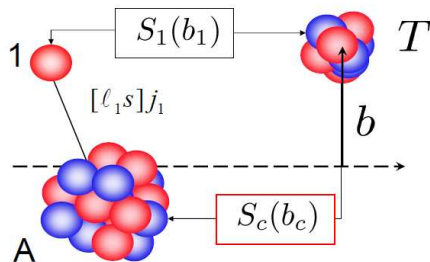
Radiative capture from Coulomb dissociation experiments

- ☞ Capture reactions have typically small cross sections
- ☞ Use breakup (Coulomb dissociation) reactions:

$$\frac{d\sigma}{d\Omega dE_{c.m.}} \rightarrow \sigma_{E\lambda}^{(\text{phot})} \rightarrow \sigma_{E\lambda}^{(rc)} \rightarrow S(E_{c.m.})$$

Knock-out reactions

Stripping cross section within a semiclassical (eikonal) theory



$$\sigma_{\text{strip}} = \int \mathbf{d}\mathbf{b} \langle \phi_0 || S_c|^2 (1 - |S_1|^2) | \phi_0 \rangle$$

- $|S_c(b_c)|^2$ = probability of survival of the core.
- $1 - |S_1(b_1)|^2$ = probability of absorption of the neutron.

Transfer reactions with exotic nuclei

${}^1\text{H}({}^{11}\text{Be}, {}^{10}\text{Be}){}^2\text{H}$ example

$$|{}^{11}\text{Be}\rangle = a |{}^{10}\text{Be}(0^+) \otimes \nu 2s_{1/2}\rangle + b |{}^{10}\text{Be}(2^+) \otimes \nu 1d_{5/2}\rangle + \dots$$

⇒ In DWBA:

$$\sigma(0^+) \propto |a|^2; \quad \sigma(2^+) \propto |b|^2$$



$^1\text{H}(^{11}\text{Be}, ^{10}\text{Be})^2\text{H}$ example

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Fortier et al, PLB461, 22 (1999)

