ISOLDE Nuclear Reaction and Nuclear Structure Course

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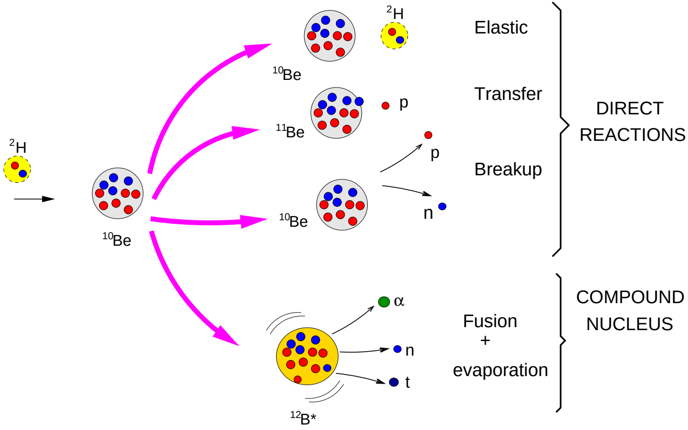
Direct and compound nucleus processes



Direct versus compound reactions

Direct: elastic, inelastic, transfer,. . .

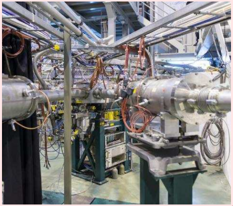
- only a few modes (degrees of freedom) involved
- small momentum transfer
- angular distribution asymmetric about $\pi/2$ (peaked forward)

Compound: complete, incomplete fusion.

- many degrees of freedom involved
- large amount of momentum transfer
- "loss of memory" $\Rightarrow$ almost symmetric distributions forward/backward

Linking theory with experiments: the cross section

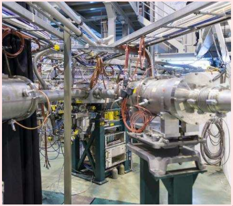
EXPERIMENT



THEORY
$(H\Psi = E\Psi)$

Linking theory with experiments: the cross section

EXPERIMENT

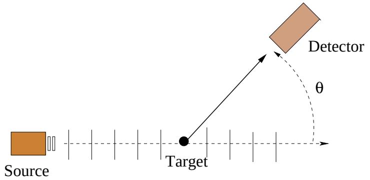


THEORY
$(H\Psi = E\Psi)$

CROSS SECTIONS

$$\frac{d\sigma}{d\Omega}, \frac{d\sigma}{dE}, etc$$

Experimental cross section



$$\Delta I = I_0 \, n_t \, \frac{d\sigma}{d\Omega} \Delta\Omega$$

- ΔI: detected particles per unit time in $\Delta\Omega$
- I_0: incident particles per unit time
- n_t: number of target nuclei per unit surface
- $\Delta\Omega$: solid angle of detector
- $d\sigma/d\Omega$: differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{\text{flux of scattered particles through } dA = r^2 d\Omega}{\text{incident flux}}$$

Model Hamiltonian

Full Hamiltonian

$$H = \hat{T}_{\mathbf{R}} + H_p(\xi_p) + H_t(\xi_t) + V(\mathbf{R}, \xi_p, \xi_t)$$

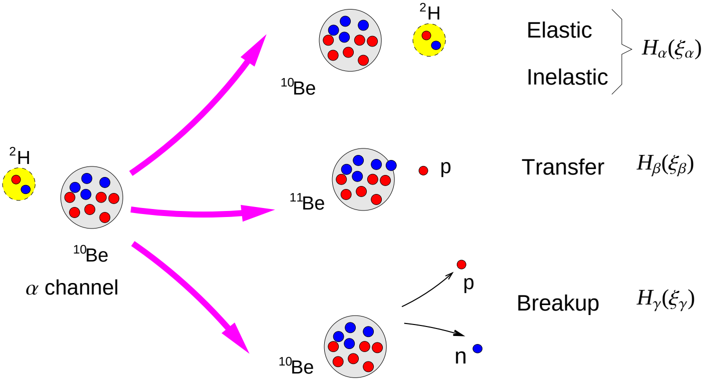
- $\hat{T}_{\mathbf{R}}$: proj.–target kinetic energy
- $H_p(\xi_p)$: projectile Hamiltonian
- $H_t(\xi_t)$: target Hamiltonian
- $V(\mathbf{R}, \xi_p, \xi_t)$: projectile–target interaction

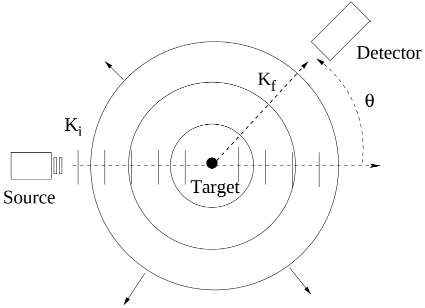
Scattering wavefunction

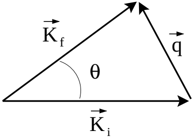
$$[H - E]\Psi = 0$$

Projectile and target internal Hamiltonians

- Full internal Hamiltonian: $H_\alpha(\xi_\alpha) \equiv H_p(\xi_p) + H_t(\xi_t)$
- Internal states: $[H_\alpha(\xi_\alpha) - \varepsilon_\alpha]\Phi_\alpha(\xi_\alpha) = 0 \quad \{\varepsilon_\alpha\}$ = excitation energies
- Non-elastic partitions will have different Hamiltonians: $H_\beta(\xi_\beta)$, $H_\gamma(\xi_\gamma)$, etc







Among the many mathematical solutions of $[H - E]\Psi = 0$ we are interested only in those behaving asymptotically as:

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \to \Phi_\alpha(\xi_\alpha) e^{i\mathbf{K}_\alpha \cdot \mathbf{R}_\alpha} + (\text{outgoing spherical waves in } \alpha, \beta, \ldots)$$

Scattering amplitude and cross sections

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \to \Phi_\alpha(\xi_\alpha) e^{i\mathbf{K}_\alpha\cdot\mathbf{R}_\alpha} + \Phi_\alpha(\xi_\alpha) f_{\alpha,\alpha}(\theta)\frac{e^{iK_\alpha R_\alpha}}{R_\alpha} \qquad \text{(elastic)}$$

$$+ \sum_{\alpha'\neq\alpha} \Phi_{\alpha'}(\xi_\alpha) f_{\alpha',\alpha}(\theta)\frac{e^{iK_{\alpha'} R_\alpha}}{R_\alpha} \qquad \text{(inelastic)}$$

$$+ \sum_{\beta} \Phi_{\beta}(\xi_\beta) f_{\beta,\alpha}(\theta)\frac{e^{iK_{\beta} R_\beta}}{R_\beta} \qquad \text{(transfer)}$$

☞ $f_{\beta,\alpha}$ is the scattering amplitude

Cross sections:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\alpha\to\beta} = \frac{K_\beta}{K_\alpha}\left|f_{\beta,\alpha}(\theta)\right|^2 \qquad E = \frac{\hbar^2 K_\alpha^2}{2\mu_\alpha} + \varepsilon_\alpha = \frac{\hbar^2 K_\beta^2}{2\mu_\beta} + \varepsilon_\beta$$

Defining our model space: Feshbach formalism

- Divide the full space into two groups: P and Q
  - P: channels of interest
  - Q: remaining channels
- Write $\Psi = \Psi_P + \Psi_Q$

$$(E - H_{PP})\Psi_P = H_{PQ}\Psi_Q$$
$$(E - H_{QQ})\Psi_Q = H_{QP}\Psi_P$$

$$(H_{PP} = PHP, H_{PQ} = PHQ, \text{etc})$$

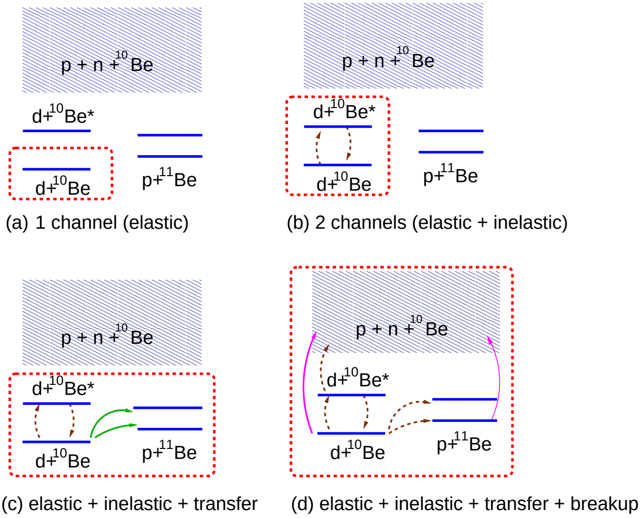
- Eliminate (formally) Ψ_Q:

$$\underbrace{\left[H_{PP} + H_{PQ}\frac{1}{E - H_{QQ} + i\epsilon}H_{QP}\right]}_{H_{\rm eff}}\Psi_P = E\Psi_P$$

- $H_{\rm eff}$ too complicated (complex, energy dependent, non-local) $\Rightarrow$ needs to be replaced by a simpler Hamiltonian:

$$H_{\rm eff} \longrightarrow H_{\rm model} \quad \text{(complex, energy dependent)}$$

Example: the d+^{10}Be case



Single-channel scattering: effective potential

- P space represents just the ground state of projectile and target
- Wavefunction:

$$\Psi = \underbrace{\Psi_P}_{\text{elastic}} + \underbrace{\Psi_Q}_{\text{non-elastic}}$$

- Schrodinger equation in modelspace:

$$[T + H_\alpha(\xi_\alpha) + \mathcal{V}]\,\Psi_P = E\Psi_P$$

$$\mathcal{V} = \underbrace{V_{PP}}_{\text{Bare interaction}} + \underbrace{V_{PQ}\frac{1}{E - H_{QQ} + i\epsilon}V_{QP}}_{\text{“Polarization” potential}}$$

- $\mathcal{V}$ too complicated $\Rightarrow$ use $\mathcal{V} \approx U(\mathbf{R})$ (complex)

Elastic scattering within the optical model

- Effective Hamiltonian:

$$H = T_{\mathbf{R}} + H_{\alpha}(\xi_{\alpha}) + U(\mathbf{R}) \qquad (U(\mathbf{R}) \text{ complex!})$$

- $U(\mathbf{R})$ independent of $\{\xi_{\alpha}\}$

$$\Psi^{(+)}_{\mathbf{K}}(\xi_{\alpha}, \mathbf{R}) = \Phi_0(\xi_{\alpha})\chi^{(+)}_0(\mathbf{K}, \mathbf{R})$$

- Schrödinger equation:

$$\boxed{[T_{\mathbf{R}} + U(\mathbf{R}) - E_{\alpha}]\chi^{(+)}_0(\mathbf{K}, \mathbf{R}) = 0} \qquad \left(E_{\alpha} = E - \varepsilon_{\alpha} = \frac{\hbar^2 K^2}{2\mu}\right)$$

- Boundary condition:

$$\chi^{(+)}_0(\mathbf{K}, \mathbf{R}) \rightarrow e^{i\mathbf{K}\cdot\mathbf{R}} + f(\theta)\frac{e^{iKR}}{R}$$

Partial wave decomposition

- For a central potential ($U(\mathbf{R}) = U(R)$):

$$\chi_0^{(+)}(\mathbf{K},\mathbf{R}) = \frac{1}{KR}\sum_{\ell m} i^\ell (2\ell+1)\chi_\ell(K,R)P_\ell(\cos\theta) \qquad (\theta = \hat{R}\cdot\hat{K})$$

- $\chi_\ell(K,R)$ obtained from:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{R^2} + U(R) - E_0\right]\chi_\ell(K,R) = 0.$$

- For $U(R) = 0$, $\chi_0^{(+)}(\mathbf{K},\mathbf{R})$ must reduce to the plane wave:

$$e^{i\mathbf{K}\cdot\mathbf{R}} = \frac{1}{KR}\sum_{\ell} i^\ell (2\ell+1)F_\ell(KR)P_\ell(\cos\theta)$$

$$F_\ell(KR) = (KR)j_\ell(KR) \rightarrow \sin(KR - \ell\pi/2)$$

⇨ So, for $U = 0 \Rightarrow \chi_\ell(K,R) = F_\ell(KR)$

Asymptotic solution for the case $U(R) \neq 0$

- For $R \gg \Rightarrow U(R) = 0$

$$\boxed{\chi_\ell(K,R) = AF_\ell(KR) + BG_\ell(KR)} \qquad G_\ell(KR) \rightarrow \cos(KR - \ell\pi/2)$$

- A, B determined to give the physically known behaviour:

$$\begin{array}{lllll}
 & \chi_0^{(+)}(\mathbf{KR}) & \rightarrow & e^{i\mathbf{K}\cdot\mathbf{R}} & + \;\; f(\theta)\dfrac{e^{iKR}}{R} \\
 & \Downarrow & & \Downarrow & \;\;\;\;\; \Downarrow \\
U = 0 & \chi_\ell(KR) & \rightarrow & F_\ell(KR) & + \;\; 0 \\
U \neq 0 & \chi_\ell(KR) & \rightarrow & F_\ell(KR) & + \;\; T_\ell[G_\ell(KR) + iF_\ell(KR)]
\end{array}$$

$$G_\ell(KR) \pm iF_\ell(KR) \equiv H^{(\pm)}(KR) \rightarrow e^{\pm i(KR-\ell\pi/2)} \qquad \text{(outgoing/ingoing free solutions)}$$

Numerical procedure

1. Fix a *matching radius*, R_m, such that $U(R_m) \approx 0$
2. Integrate $\chi_\ell(R)$ from $R = 0$ up to R_m, starting with the condition:

$$\lim_{R\to 0} \chi_\ell(K,R) = 0$$

3. At $R = R_m$ impose the boundary condition:

$$\begin{aligned} \chi_\ell(K,R) &\to F_\ell(KR) + T_\ell H_\ell^{(+)}(KR) \\ &= \frac{i}{2}[H_\ell^{(-)}(KR) - S_\ell H_\ell^{(+)}(KR)] \end{aligned}$$

☞ T_ℓ=transmission coefficient S_ℓ=reflection coefficient (S-matrix)

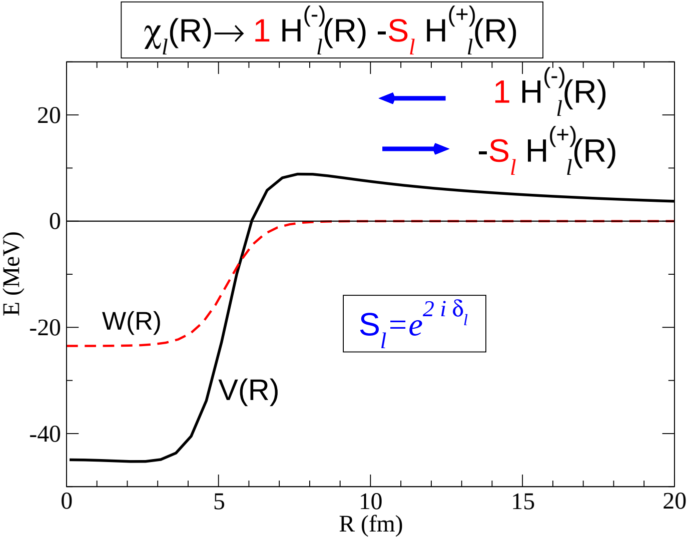
4. Phase-shifts:

$$\boxed{S_\ell = 1 + 2iT_\ell \equiv e^{i2\delta_\ell}}$$

The S-matrix and phase-shifts

- S_ℓ =coefficient of the outgoing wave for partial wave ℓ.
- $|S_\ell|^2$ is the *survival* probability for the partial wave ℓ:
  - U real $\Rightarrow |S_L| = 1 \Rightarrow \delta_\ell$ real
  - U complex $\Rightarrow |S_L| < 1 \Rightarrow \delta_\ell$ complex
- $U(R) = 0 \Rightarrow$ No scattering $\Rightarrow S_\ell = 1 \Rightarrow \delta_\ell = 0$
- For $\ell \gg \Rightarrow S_\ell \to 1$

The S-matrix and phase-shifts



The scattering amplitude

- Replace the asymptotic $\chi_\ell(K,R)$ in the general expansion:

$$\chi_0^{(+)}(\mathbf{K},\mathbf{R}) \rightarrow \frac{1}{KR}\sum_\ell i^\ell(2\ell+1)\left\{F_\ell(KR) + T_\ell H_\ell^{(+)}(KR)\right\}P_\ell(\cos\theta)$$
$$= e^{i\mathbf{K}\cdot\mathbf{R}} + \frac{1}{K}\sum_\ell(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\frac{e^{iKR}}{R}$$

- The scattering amplitude is the coefficient of e^{iKR}/R:

$$f(\theta) = \frac{1}{K}\sum_\ell(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)$$
$$= \frac{1}{2iK}\sum_\ell(2\ell+1)(S_\ell-1)P_\ell(\cos\theta).$$

- Elastic cross section:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2.$$

Coulomb plus nuclear case

Radial equation:

$$\left[\frac{d^2}{dR^2} + K^2 - \frac{2\eta K}{R} + \frac{2\mu}{\hbar^2}U(R) + \frac{\ell(\ell+1)}{R^2}\right]\chi_\ell(K,R) = 0$$

$$\eta = \frac{Z_p Z_t e^2}{\hbar v} = \frac{Z_p Z_t e^2 \mu}{\hbar^2 K}$$

(Sommerfeld parameter)

Asymptotic condition:

$$\chi^{(+)}(\mathbf{K},\mathbf{R}) \to e^{i[\mathbf{K}\cdot\mathbf{R}+\eta\log(kR-\mathbf{K}\cdot\mathbf{R})]} + f(\theta)\frac{e^{i(KR-\eta\log 2KR)}}{R}$$

$$\chi_\ell(K,R) \to e^{i\sigma_\ell}\left[F_\ell(\eta,KR) + T_\ell H_\ell^{(+)}(\eta,KR)\right]$$
$$= (i/2)e^{i\sigma_\ell}\left[H_\ell^{(-)}(\eta,KR) - S_\ell H_\ell^{(+)}(\eta,KR)\right]$$

- ☞ $\sigma_\ell(\eta)$=Coulomb phase shift
- ☞ $F_\ell(\eta,KR)$=regular Coulomb wave
- ☞ $H_\ell^{(\pm)}(\eta,KR)$=outgoing/ingoing Coulomb wave

Coulomb plus nuclear case: scattering amplitude

Total scattering amplitude:

$$f(\theta) = f_C(\theta) + \frac{1}{2iK} \sum_{\ell} (2\ell + 1) e^{2i\sigma_\ell} (S_\ell - 1) P_\ell(\cos\theta)$$

☞ $f_C(\theta)$ is the amplitude for pure Coulomb:

$$\frac{d\sigma_R}{d\Omega} = |f_C(\theta)|^2 = \frac{\eta^2}{4K^2 \sin^4(\frac{1}{2}\theta)} = \left(\frac{Z_p Z_t e^2}{4E}\right)^2 \frac{1}{\sin^4(\frac{1}{2}\theta)}$$

Integrated cross sections

- Total elastic cross section (uncharged particles!)

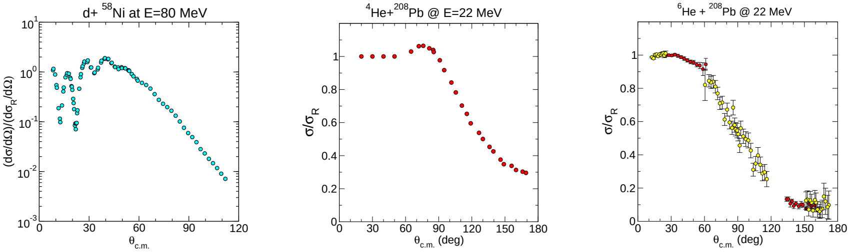
$$\sigma_{el} = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{\pi}{K^2} \sum_{\ell} (2\ell + 1)|1 - S_\ell|^2$$

- Total reaction cross section (loss of flux from elastic channel)

$$\sigma_{reac} = \frac{\pi}{K^2} \sum_{\ell} (2\ell + 1)(1 - |S_\ell|^2) = \frac{\pi}{K^2} \sum_{\ell} (2\ell + 1)|T_\ell|^2$$

Elastic scattering

What can we learn from the analysis of the elastic cross section?



Phenomenological optical model

Effective potential: $U(R) = U_{\text{nuc}}(R) + U_{\text{coul}}(R)$

- Coulomb potential: charge sphere distribution

$$U_c(R) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R_c}\left(3 - \frac{R^2}{R_c^2}\right) & \text{if } R \leq R_c \\ \frac{Z_1 Z_2 e^2}{R} & \text{if } R \geq R_c \end{cases}$$

- Nuclear potential (complex): Woods-Saxon parametrization

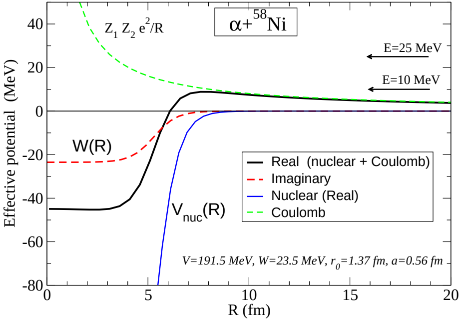
$$U_{\text{nuc}}(R) = V(r) + iW(r) = -\frac{V_0}{1 + \exp\left(\frac{R-R_0}{a_0}\right)} - i\,\frac{W_0}{1 + \exp\left(\frac{R-R_i}{a_i}\right)}$$

Typically: $R_0 = r_0(A_p^{1/3} + A_t^{1/3})$

- r_0=reduced radius ($r_0 \sim 1.1 - 1.4$ fm)
- A_p,A_t: projectile, target atomic numbers

Effective potential: ^{4}He+^{58}Ni example

Effective potential: $U(R) = U_{nuc}(R) + U_{coul}(R)$



☞ The maximum of $V_{\rm nuc}(R) + V_C(R)$ defines the Coulomb barrier. Approximately:

$$E_b \simeq \frac{Z_p Z_t e^2}{1.44(A_p^{1/3} + A_t^{1/3})} = \frac{Z_p Z_t}{(A_p^{1/3} + A_t^{1/3})} \quad [\text{MeV}]$$

Grazing radius and angular momentum

- Grazing collisions are those for which $b \approx R_1 + R_2 = R_g$
- Relation with angular momentum:
  1. Only nuclear: $bK = \sqrt{\ell(\ell+1)} \simeq \ell + 1/2$

     $$\boxed{KR_g \simeq \ell_g + 1/2} \qquad (\ell_g = \text{grazing angular momentum})$$

  2. Nuclear + Coulomb:

     $$\boxed{KR_g\left(1 - 2\eta/KR_g\right) \approx \ell_g + 1/2}$$

⇨ As E increases, so does the number of partial waves involved

⇨ Peripherical processes (inelastic, transfer) occur mainly around $\ell \sim \ell_g$

Elastic scattering: phenomenology

☞ Depending on the bombarding energy E and the charges of the interacting nuclei, we observe different types of elastic scattering.

☞ For medium/heavy systems, this can be characterized in terms of the Coulomb (or Sommerfeld) parameter:

$$\eta = \frac{Z_p Z_t e^2}{4\pi\epsilon_0 \hbar v}$$

- E well above the Coulomb barrier ($\eta \lesssim 1$) $\Rightarrow$ Fraunhofer scattering
- E around the Coulomb barrier ($\eta \gg 1$) $\Rightarrow$ Fresnel scattering
- E well below the Coulomb barrier ($\eta \ggg 1$) $\Rightarrow$ Rutherford scattering

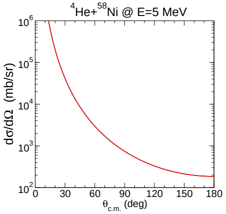
Elastic scattering: optical model

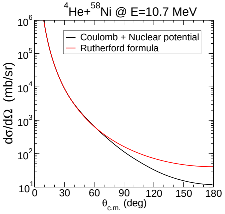
Dynamical effects: ^{4}**He+**^{58}**Ni at E=5, 10.7, 25 and 50 MeV**

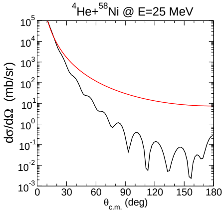
E_{lab} (MeV)	η	K (fm^{-1})	$\lambda\!\!\!^{-} = 1/K$ (fm)	$2a_0$ (fm)
5	7.95	0.920	1.087	17.2
10.7	5.62	1.34	0.746	8.06
25	3.55	2.06	0.485	3.44
50	2.51	2.91	0.343	1.69

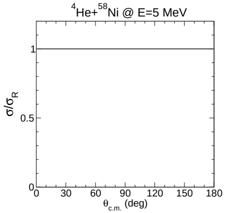
- $\eta \ggg 1$: Rutherford scattering: $\sigma(\theta) \propto 1/\sin^4(\theta/2)$
- $\eta \gg 1$: Fresnel scattering (rainbow)
- $\eta \leq 1$: Fraunhofer scattering (oscillatory behaviour):

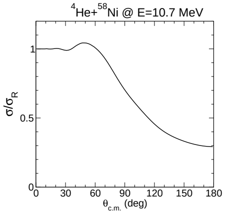
Elastic scattering: energy dependence

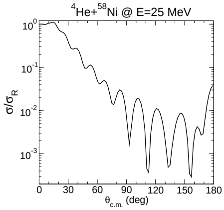








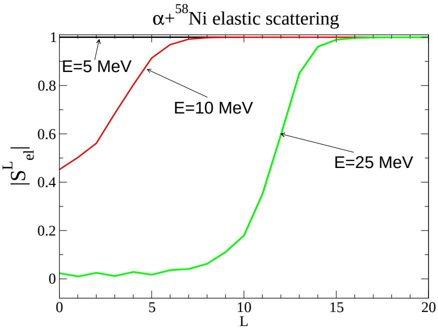


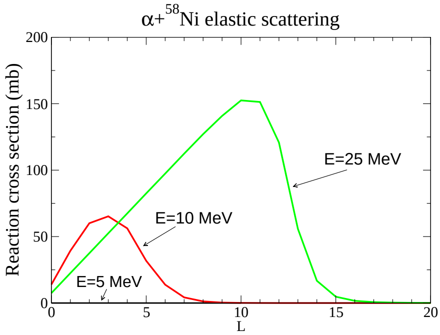


Rutherford scattering **Fresnel** **Fraunhöfer**

Elastic scattering: S-matrix elements

Elastic (nuclear) S-matrix : $\chi_\ell(K,R) = H_\ell^{(-)}(R) - S_\ell H_\ell^{(+)}(R)$



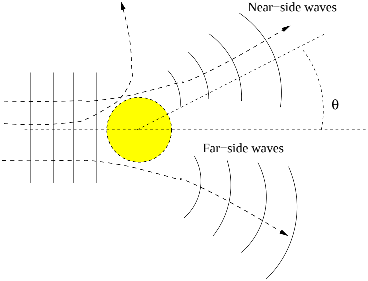


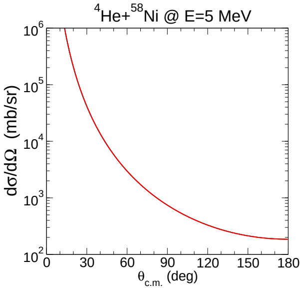
$$KR_g\left(1 - 2\eta/KR_g\right) \approx \ell_g + 1/2$$

⇒ the number of partial waves required for convergence grows approximately as $\sqrt{E}$

Elastic scattering: phenomenology

Rutherford scattering





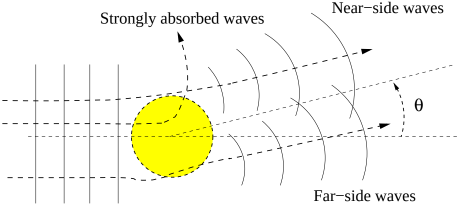
- Purely Coulomb potential ($\eta \ggg 1$)
- Bombarding energy well below the Coulomb barrier
- Obeys Rutherford law:

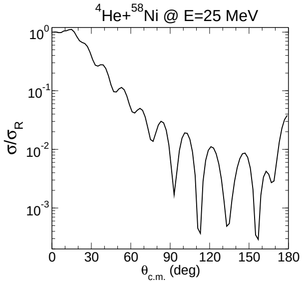
$$\frac{d\sigma}{d\Omega} = \frac{Z_p Z_t e^2}{4E} \frac{1}{\sin^4(\theta/2)}$$

.

Elastic scattering: phenomenology

FRAUNHOFER SCATTERING:

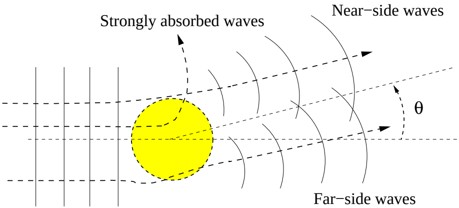


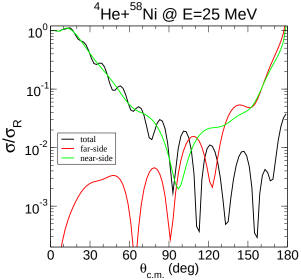


- Bombarding energy well above Coulomb barrier
- Coulomb weak ($\eta \lesssim 1$)
- Nearside/farside interference pattern (difracction)

Elastic scattering: phenomenology

FRAUNHOFER SCATTERING:





- Bombarding energy well above Coulomb barrier
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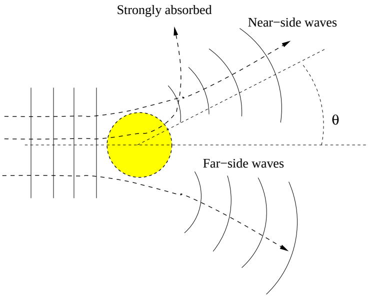
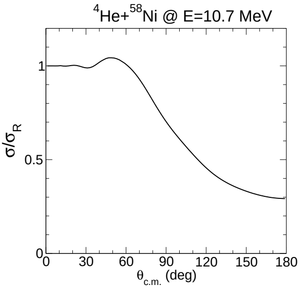
- Nearside/farside interference pattern (difracction)

Elastic scattering: phenomenology

FRESNEL SCATTERING:

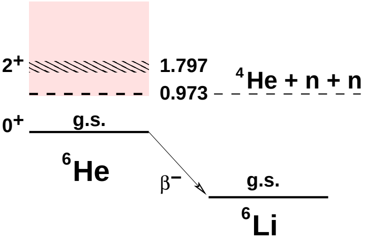


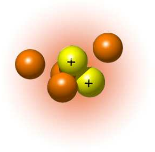


- Bombarding energy around or near the Coulomb barrier
- Coulomb strong ($\eta \gg 1$)
- 'Illuminated' region $\Rightarrow$ interference pattern (near-side/far-side)
  'Shadow' region $\Rightarrow$ strong absorption

Halo and Borromean nuclei: the ^{6}He case

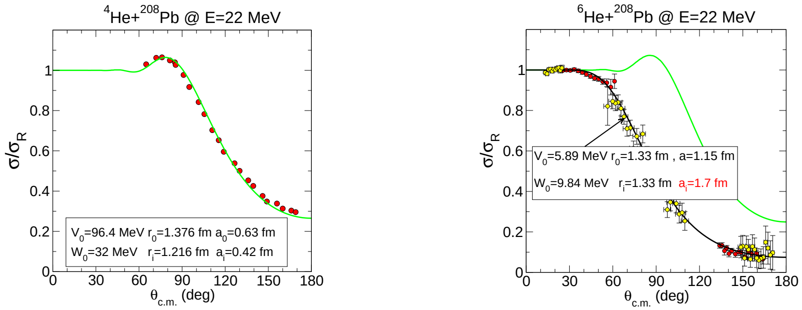
- Radioactive:
  $^6\text{He} \xrightarrow{\beta^-} {}^6\text{Li} \quad (t_{1/2} \simeq 807 \text{ ms})$
- Weakly bound:
  $\epsilon_b = -0.973$ MeV
- Neutron halo
- Borromean system:
  n-n and α-n unbound
- $\sim$ 3 body system:
  α almost inert





Normal versus halo nuclei

How does the halo structure affect the elastic scatterig?



- ^{4}He+^{208}Pb shows typical Fresnel pattern and “standard” optical model parameters
- ^{6}He+^{208}Pb shows a prominent reduction in the elastic cross section, suggesting that part of the incident flux goes to non-elastic channels (eg. breakup)

  Understanding and disentangling these non-elastic channels requires going beyond the optical model (eg. coupled-channels method ⇒ next lecture)