ISOLDE Nuclear Reaction and Nuclear Structure Course

FRESCO examples

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Introduction

Fresco, Xfresco and Sfresco

- What is FRESCO?
  Program developed by Ian Thompson since 1983, to perform coupled-reaction channels calculations in nuclear physics.

- Some general features:
  - Multi-platform (Windows, Linux, Unix, VAX)
  - Treats many direct reaction models: elastic scattering (optical model), transfer, inelastic excitation to bound and unbound states, etc
  - Can be run in text mode and graphical mode (XFRESCO interface)
  - FRESCO and XFRESCO can be freely downloaded at http://www.fresco.org.uk/
  - SFRESCO: Extension of Fresco, to provide χ^2 searches of potential and coupling parameters.

Reminder of the OM

- Projectile-target interaction described by effective complex interaction $U(\mathbf{R})$:
- Relative motion wavefunction obtained from:

$$[T_{\mathbf{R}} + U(\mathbf{R}) - E_\alpha]\chi_0^{(+)}(\mathbf{K},\mathbf{R}) \qquad (E_\alpha = E - \varepsilon_\alpha = \frac{\hbar^2 K^2}{2\mu})$$

- Boundary condition:

$$\chi_0^{(+)}(\mathbf{K},\mathbf{R}) \rightarrow e^{i\mathbf{K}\cdot\mathbf{R}} + f(\theta)\frac{e^{iKR}}{R}$$

$f(\theta)$=scattering amplitude

Partial wave decomposition

- Central potential $U = U(R)$:

$$\chi_0^{(+)}(\mathbf{K},\mathbf{R}) = \frac{1}{KR}\sum_{\ell m} i^\ell (2\ell+1)\chi_\ell(K,R)P_\ell(\cos\theta)$$

- $\chi_\ell(K,R)$ obtained from:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{R^2} + U(R) - E_0\right]\chi_\ell(K,R) = 0.$$

- Boundary condition:

$$\chi_\ell(K,R) \to e^{i\sigma_\ell}\left[F_\ell(\eta,KR) + T_\ell H_\ell^{(+)}(\eta,KR)\right]$$
$$= (i/2)e^{i\sigma_\ell}\left[H_\ell^{(-)}(\eta,KR) - S_\ell H_\ell^{(+)}(\eta,KR)\right]$$

☞ $\sigma_\ell(\eta)$=Coulomb phase shift
☞ $F_\ell(\eta,KR)$=regular Coulomb wave
☞ $H_\ell^{(\pm)}(\eta,KR)$=outgoing/ingoing Coulomb wave

Numerical procedure

1. Fix a *matching radius*, R_m, such that $U(R_m) \approx 0$
2. Integrate $\chi_\ell(R)$ from $R = 0$ up to R_m, starting with the condition:

$$\lim_{R\to 0} \chi_\ell(K,R) = 0$$

3. At $R = R_m$ impose the boundary condition:

$$\chi_\ell(K,R) \to F_\ell(KR) + T_\ell H_\ell^{(+)}(KR)$$
$$= \frac{i}{2}[H_\ell^{(-)}(KR) - S_\ell H_\ell^{(+)}(KR)]$$

☞ *T_ℓ=transmission coefficient* *S_ℓ=reflection coefficient (S-matrix)*

4. Phase-shifts:

$$S_\ell = 1 + 2iT_\ell \equiv e^{i2\delta_\ell}$$

Scattering amplitude (Coulomb plus nuclear)

Total scattering amplitude:

$$f(\theta) = f_C(\theta) + \frac{1}{2iK}\sum_\ell (2\ell+1)e^{2i\sigma_\ell}(S_\ell - 1)P_\ell(\cos\theta)$$

☞ $f_C(\theta)$ is the amplitude for pure Coulomb:

$$\frac{d\sigma_R}{d\Omega} = |f_C(\theta)|^2 = \frac{\eta^2}{4K^2\sin^4(\frac{1}{2}\theta)} = \left(\frac{Z_pZ_te^2}{4E}\right)^2 \frac{1}{\sin^4(\frac{1}{2}\theta)}$$

Useful resources

- Fresco and Xfresco:
  fresco.org.uk
- TWOFNR (Surrey version by Jeff Tostevin)
  http://www.nuclear theory.net/NPG/code.htm
- TUNL Nuclear Data Project:
  http://www.tunl.duke.edu/NuclData/
- Nuclear Reaction Video project:
  http://nrv.jinr.ru/nrv/
- Lund database for masses, 1n,2n,1p,2p separation energies, Q-values
  http://nucleardata.nuclear.lu.se/database/

Optical model calculations with Fresco

Essential ingredients of an OM calculation:

- **Physical:**
  - Identify projectile and target (mass, spin, etc)
  - Incident energy
  - Parametrization of the optical potential
- **Numerical:**
  - Radial step for numerical integration (HCM in fresco)
  - Maximum radius R for integration (RMATCH)
  - Maximum angular momentum L. (JTMAX)

RMATCH and JTMAX are linked by: $kR_g\left(1-2\eta/kR_g\right)\approx L_g+1/2$
(L_g=grazing angular momentum)

OM example: ^{4}He+^{58}Ni

Input example:

```
4he58ni_e10.in: 4He + 58Ni elastic scattering Ecm=10.0 MeV
NAMELIST
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
       thmin=1.0 thmax=180.0 thinc=2.0
       smats=2 xstabl=1
       elab=10.7 /

&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28 nex=1 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
&partition /

&POT kp=1 at=58 rc=1.4 /
&POT kp=1 type=1
    p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /

&overlap /
&coupling /
```

OM example: ^{4}He+^{58}Ni

General variables

```
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
       thmin=1.00 thmax=180.00 thinc=2.00
       smats=2 xstabl=1
       elab=10.7 /
```

Mass partitions & states

```
&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28
          nex=1 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
&partition /
```

Potentials

```
&POT kp=1 itt=F at=58 rc=1.4 /
&POT kp=1 type=1
    p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /
```

OM example: ^{4}He+^{58}Ni

Essential input variables: FRESCO namelist

```
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
       thmin=1.00 thmax=180.00 thinc=2.00
       smats=2 xstabl=1
       elab=10.7 /
```

- `hcm`: step for integration of radial equations.
- `rmatch`: matching radius (for `R > RMATCH` asymptotic behaviour is assumed)
- `elab`: laboratory energy
- `jtmax`: maximum total angular momentum (projectile+target+relative)
- `smats`: trace variable
  `smats=2` → print elastic S-matrix
- `xstbl`: trace variable
  `xstbl=1` → print cross sections

OM example: ^{4}He+^{58}Ni

Essential input variables: partitions and states

```
&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28
           nex=1 /
```

- `namep` / `namet`: projectile / target name
- `massp` / `masst`: projectile / target mass (amu)
- `zp` / `zt`: projectile / target charge
- `nex`: number of (pairs) of states in this partition

```
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
```

- `jp` / `jt`: projectile / target spins
- `bandp` / `bandt`: projectile / target parities ($\pm$ 1)
- `cpot`: index of potential for this pair of states.

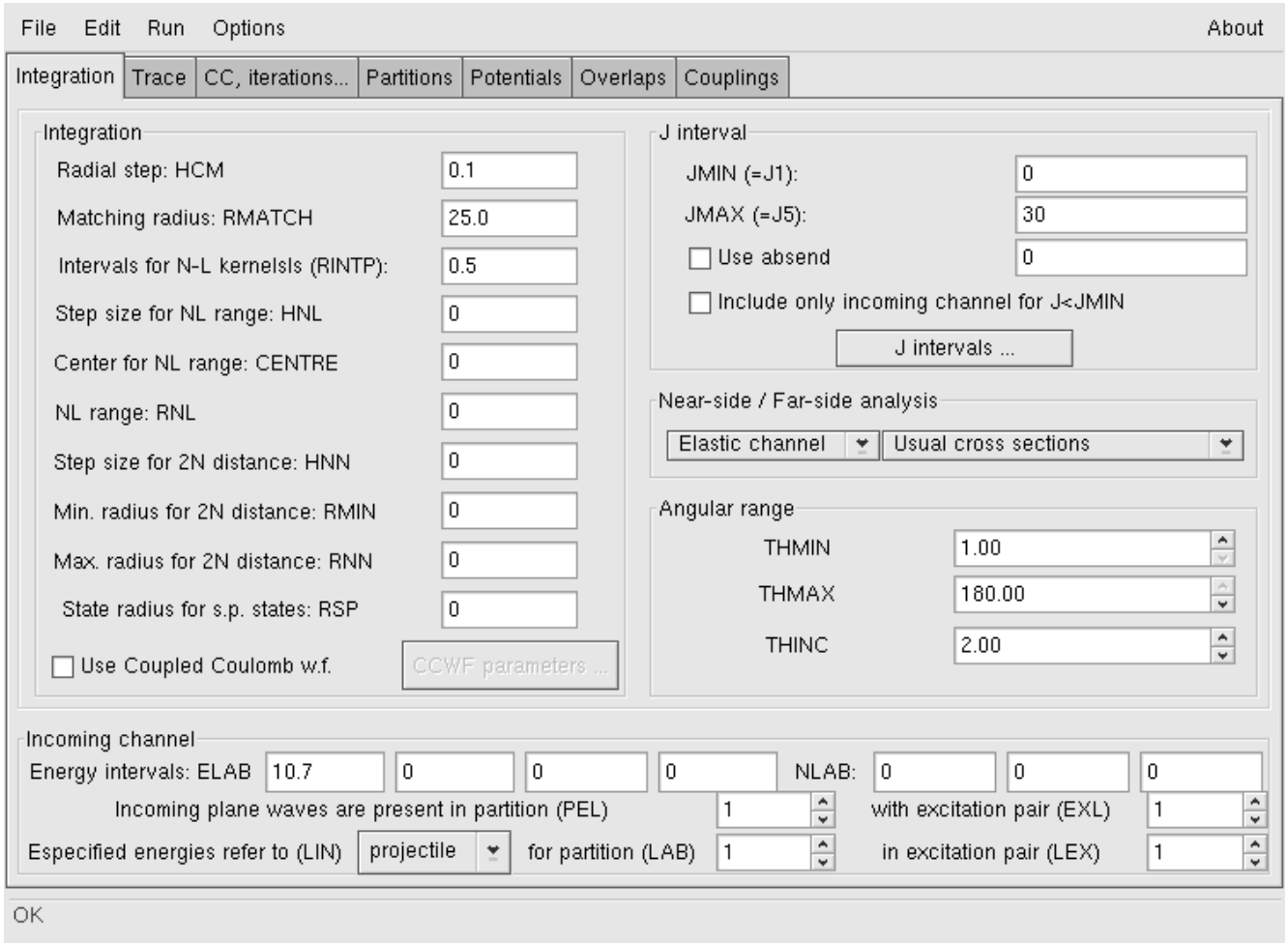
OM example: ^{4}He+^{58}Ni

```
&POT kp=1 type=0 ap=0 at=58 rc=1.4 /
&POT kp=1 type=1 shape=0
    p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /
```

- `kp`: index to identify this potential
- `ap, at`: projectile and target mass, for conversion from reduced to physical radii: $R = r(\mathrm{ap}^{1/3} + \mathrm{at}^{1/3})$
- `type, shape`: potential cathegory and shape: $\Rightarrow$
  - `type=0`: Coulomb potential
    `shape=0`: uniform charge sphere
  - `type=1`: volume nuclear potential
    `shape=0`: Woods-Saxon shape
- `rc`: reduced radius for charge distribution
- `p1,p2,p3`: V_0, r_0, a_0 (real part)
- `p4,p5,p6`: W_0, r_i, a_i (imaginary part)

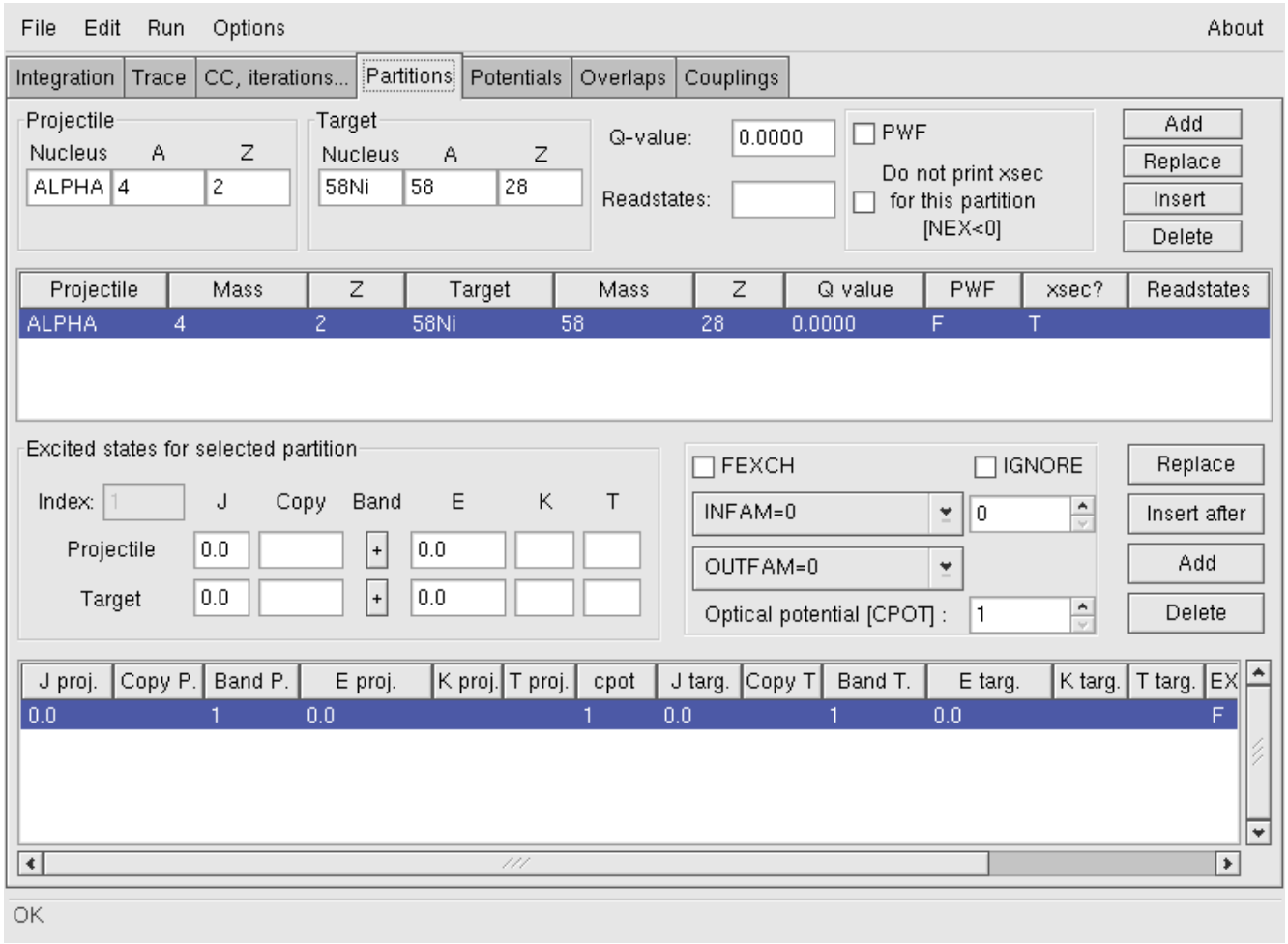
Xfresco interface

General variables:



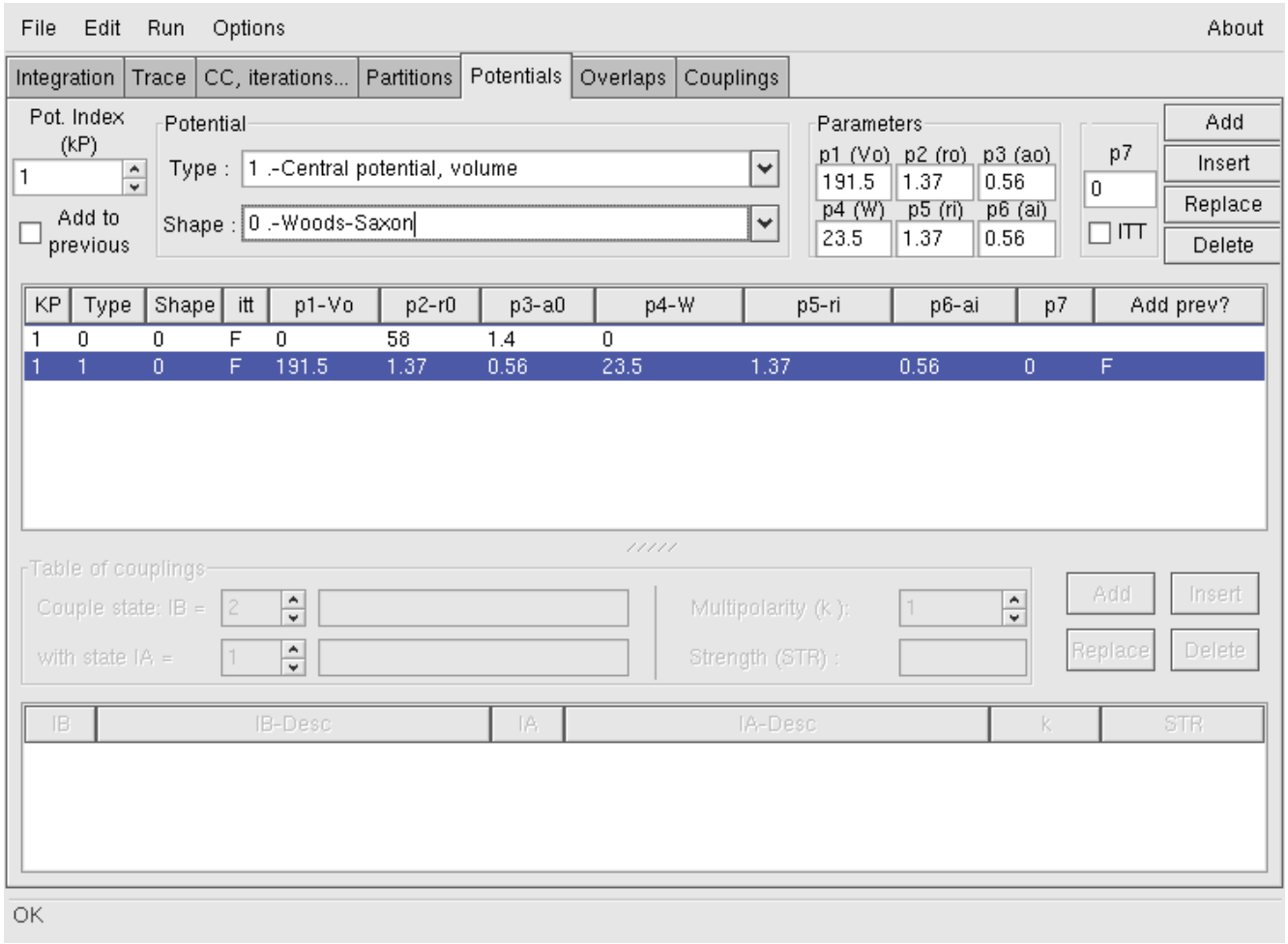
Optical model with XFRESCO

Partitions & states:



Optical model with XFRESCO

Potentials:



Useful output information in OM calculations

Useful output files:

- Main output file (stdout)
- `fort.201` : Elastic scattering angular distribution
  - `thmax > 0`: relative to Rutherford.
  - `thmax < 0`: absolute units (mb/sr).
- `fort.7`: Elastic S-matrix (real part, imaginary part, angular momentum)
- `fort.56`: Fusion (absorption), reaction and inelastic cross section for each angular momentum

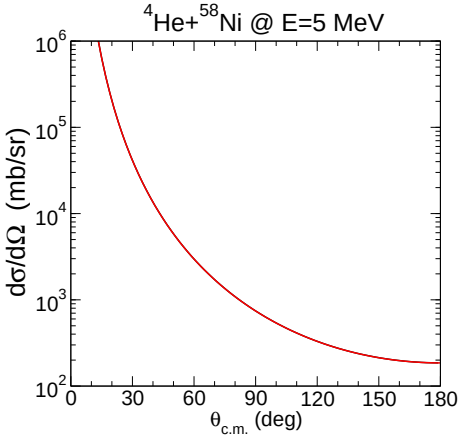
Elastic scattering: optical model

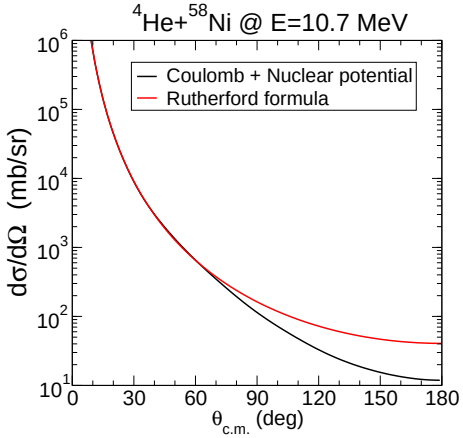
Dynamical effects: ^{4}He+^{58}Ni at E=5, 10.7, 25 and 50 MeV

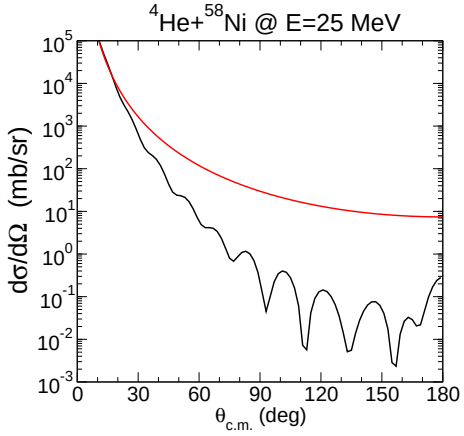
$E_{\rm lab}$ (MeV)	η	k (fm^{-1})	$\bar{\lambda} = 1/k$ (fm)	$2a_0$ (fm)
5	7.95	0.920	1.087	17.2
10.7	5.62	1.34	0.746	8.06
25	3.55	2.06	0.485	3.44
50	2.51	2.91	0.343	1.69

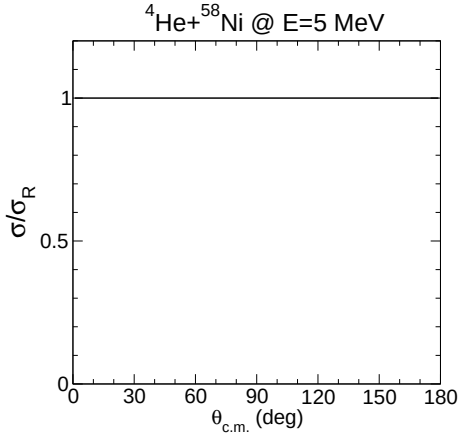
- $\eta \ggg 1$: Rutherford scattering: $\sigma(\theta) \propto 1/\sin^4(\theta/2)$
- $\eta \gg 1$: Fresnel scattering (rainbow)
- $\eta \leq 1$: Fraunhofer scattering (oscillatory behaviour):

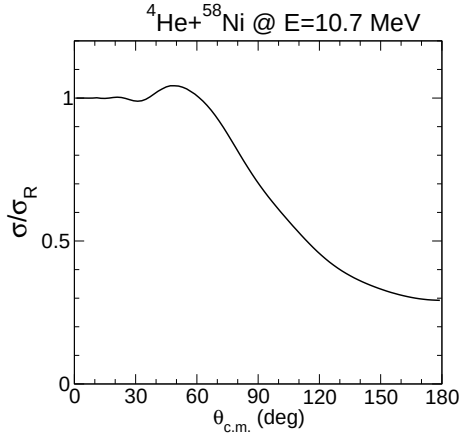
Elastic scattering: energy dependence

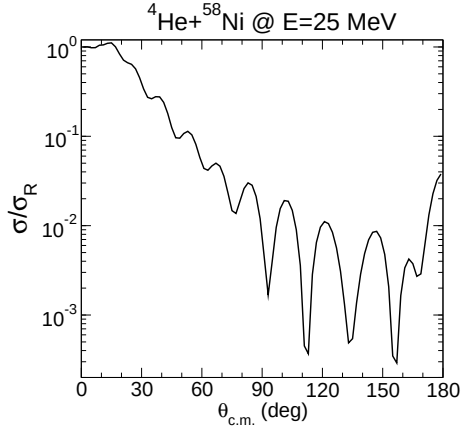








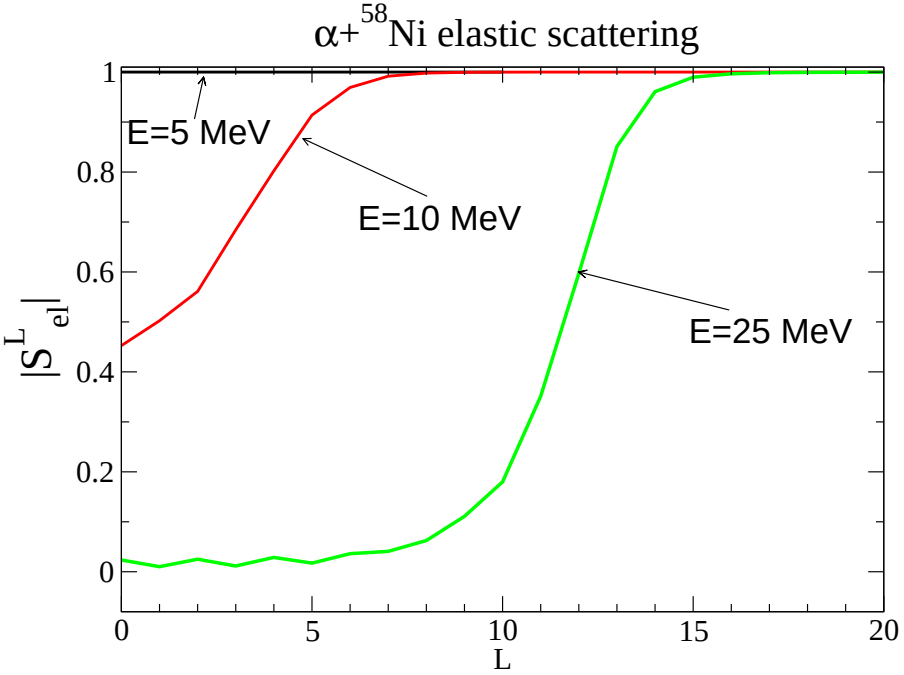


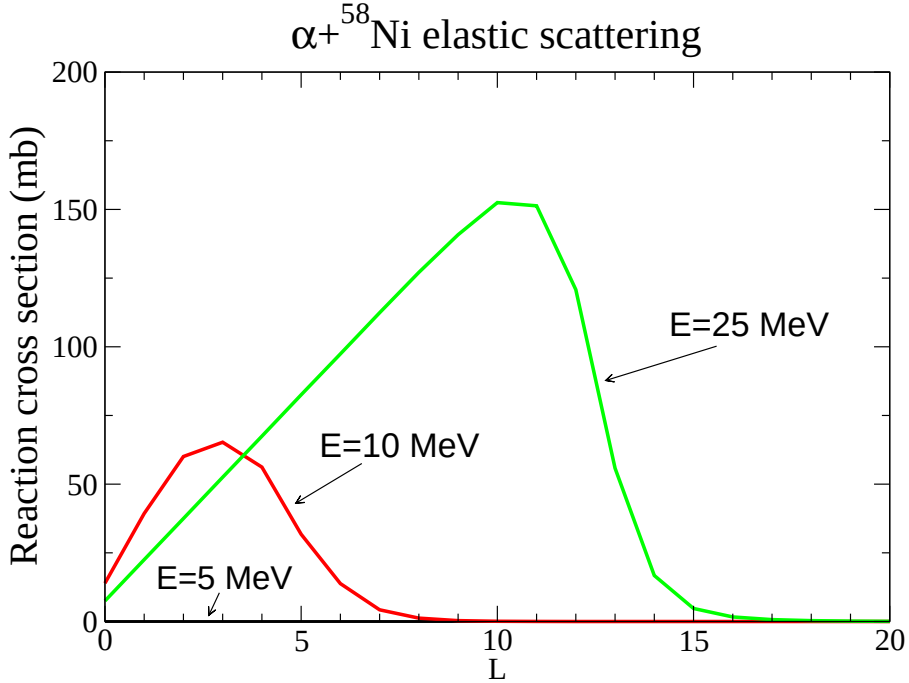


Rutherford scattering **Fresnel** **Fraunhöfer**

Elastic scattering: S-matrix elements

Elastic (nuclear) S-matrix (`fort.7`): $\chi_L^{el}(r) = I_L(r) - S_{el}^L O_L(r)$





$$kR_g\left(1 - 2\eta/kR_g\right) \approx L_g + 1/2$$

$\Rightarrow$ the number of partial waves required for convergence grows approximately as $\sqrt{E}$

Optical Model fits with SFRESCO

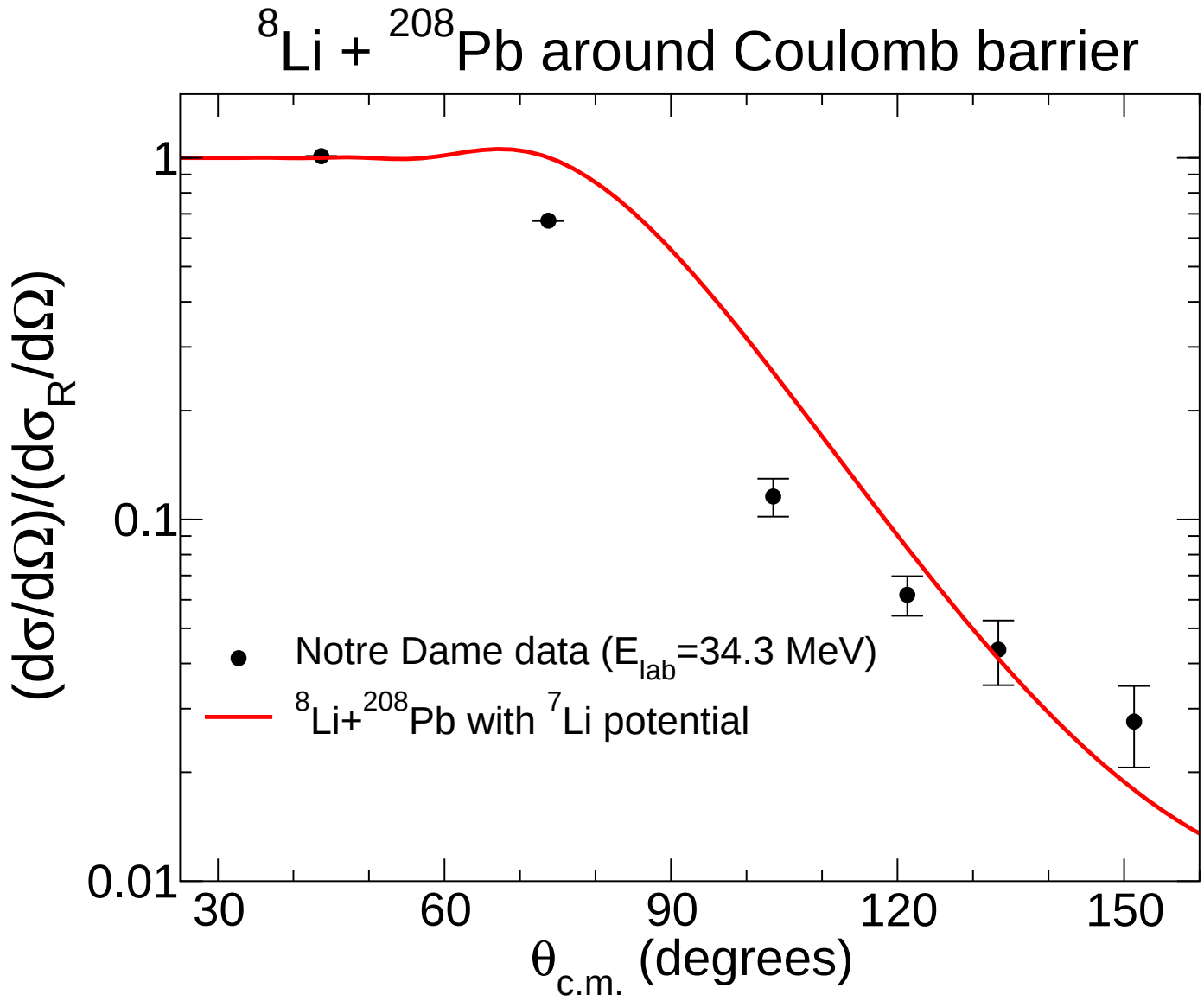
SFRESCO: Can be used together with FRESCO to do determine automatically optical model parameters by means of a χ^2 analysis of experimental angular distribution.

We need 3 input files:

1. FRESCO input file: `li8pb_e34.in`
2. MINUIT input file: `sfresco.in`
3. SEARCH input file: `search.in`

sfresco.in $\Longrightarrow$ search.in $\Longrightarrow$ `li8pb_e34.in`

^{8}Li+^{208}Pb OM before fit



FRESCO input file (before fit)

```
li8pb_e34.in
NAMELIST
&FRESCO hcm=0.05 rmatch=40.0 jtmax=60
       thmin=5.00 thmax=160.00 thinc=2.00
       smats=2 xstabl=1
       elab= 34.404 /

&PARTITION namep='Li-8' massp=8 zp=3 namet='Pb-208'
       masst=208 zt=82 qval=0.0000 pwf=T nex=1 /
&STATES jp=2.0 bandp=1 ep=0.000 cpot=1 jt=0.0
       bandt=1 et=0.000 fexch=F /

&partition /

&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=1 itt=F p1=15.4 p2=1.3 p3=0.65 p4=13.2 p5=1.3 p6=0.65 /
&pot /
```

Performing fits with SFRESCO:

1.- FRESCO input file: `li8pb_e34.in` (previous slide)

2.- MUNUIT input file: `sfresco.in`

```
search.in <---- file with search parameters
min
fix
migrad
end
q
show
plot
```

Performing fits with SFRESCO (continued):

3.- SEARCH input file: `search.in`

```
'li8pb_e34.in' 'li8pb_e34.out' 2 1

&variable kind=1 name='V' kp=1 pline=2 col=1 valmin=5.0 valmax=150.0 step=0.2/
&variable kind=1 name='W' kp=1 pline=2 col=4 valmin=5.0 valmax=100.0 step=0.2 /

&data type=0 iscale=2 idir=1 lab=F abserr=T/
43.7     1.01026  0.014
73.76    0.67003  0.014
103.537  0.11577  0.01394
121.296  0.06194  0.00778
133.351  0.04369  0.00888
151.332  0.02763  0.00701
&
```

```
sfresco < sfresco.in > sfresco.out
```

SEARCH input file (continued):

```
'li8pb_e34.in' 'li8pb_e34.out' 2 1
```

input_file, output_file, nvariables, ndatasets

```
&variable kind=1 name='V' kp=1 pline=2 col=1 valmin=5.0 valmax=150.0 /
&variable kind=1 name='W' kp=1 pline=2 col=4 valmin=5.0 valmax=100.0 /
```

- kind: type of variable (1=potential)
- kp: potential index
- pline=2: potential component
- col: column (identifies parameter within component)
- valmin-valmax: constraints for this parameter

SEARCH input file (continued):

```
&data type=0 iscale=2 idir=1 abserr=T/
43.7     1.01026  0.014
73.76    0.67003  0.014
(...)
 &
```

- `type`: type of observable (0= angular distribution for fixed energy)
- `iscale`: data units for absolute scale (2=mb/sr)
- `idir`: scale (1=ratio to Rutherford)
- `abserr`: specified errors are absolute (T) or relative (F).

sfresco.out

```
 Var  1=V        value   15.400000
 Var  2=W        value   13.200000

Total ChiSq/N = 78.8745 from   78.874
 (...)

PARAMETER CORRELATION COEFFICIENTS
    NO. GLOBAL    1     2
     1  0.62638 1.000-0.626
     2  0.62638 -0.626 1.000
 (...)

 Var  1=V        value   12.440562, step 0.2000, error 4.4317
 Var  2=W        value   60.305833, step 0.2000, error 4.9913

  Angle  Datum     Abs. error Theory      Chi
  43.700 1.0103   0.14000E-01 0.99683      0.9199
  73.760 0.67003  0.14000E-01 0.66383      0.1962
 103.537 0.11577  0.13940E-01 0.14595      4.6878
 121.296 0.61940E-01 0.77800E-02 0.59023E-01 0.1406
 133.351 0.43690E-01 0.88800E-02 0.34596E-01 1.0489
 151.332 0.27630E-01 0.70100E-02 0.18740E-01 1.6082

Total ChiSq/N =  1.4336 from    1.434
(...)
```

FRESCO input file (after fit)

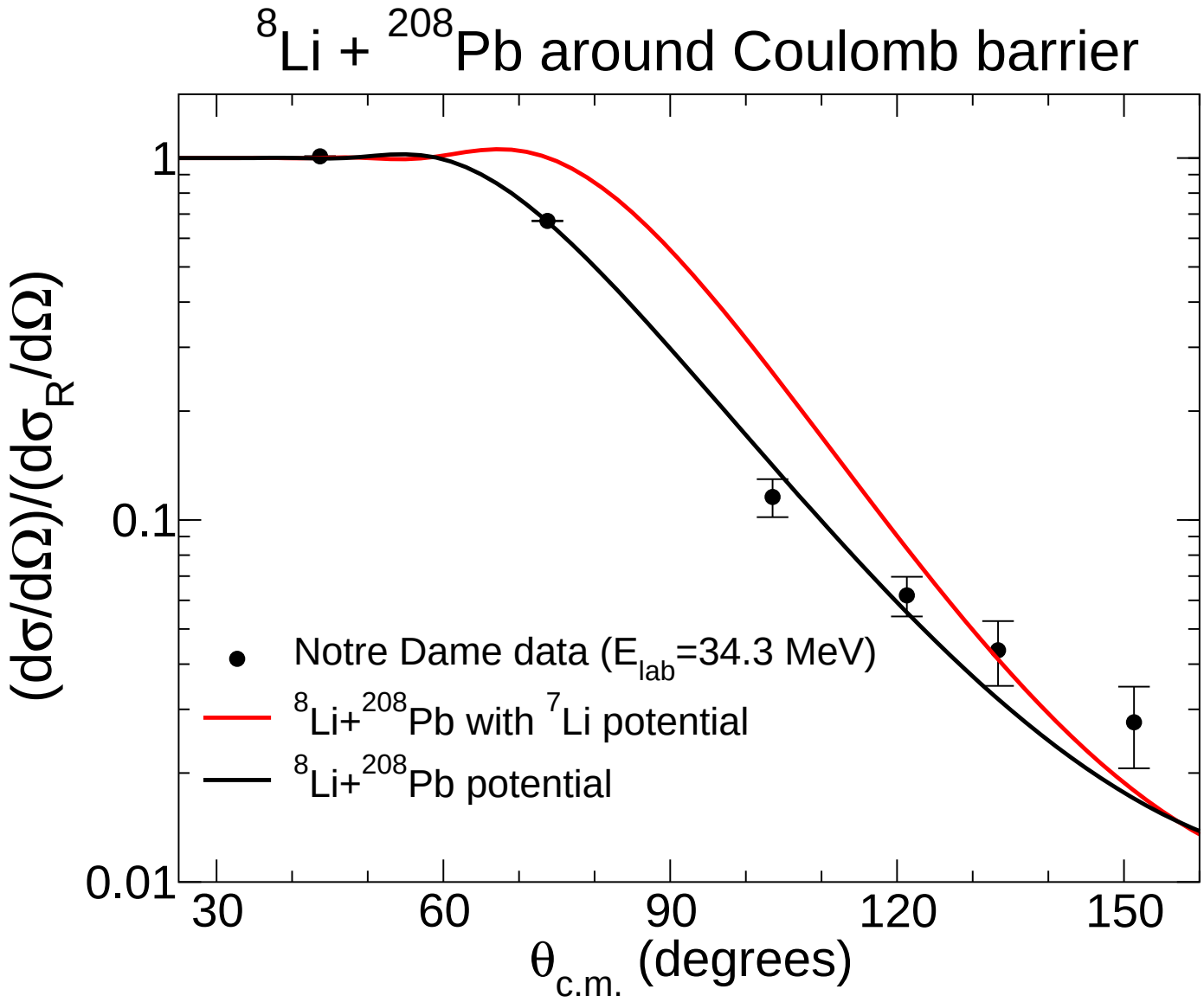
```
li8pb_e34.in
NAMELIST
&FRESCO hcm=0.05 rmatch=40.0 jtmax=60
       thmin=5.00 thmax=160.00 thinc=2.00
       smats=2 xstabl=1
       elab= 34.404 /

&PARTITION namep='Li-8' massp=8 zp=3 namet='Pb-208'
       masst=208 zt=82 qval=0.0000 pwf=T nex=1 /
&STATES jp=2.0 bandp=1 ep=0.000 cpot=1 jt=0.0
       bandt=1 et=0.000 fexch=F /

&partition /

&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=1 itt=F p1=12.4 p2=1.3 p3=0.65 p4=60.3 p5=1.3 p6=0.65 /
&pot /
```

^{8}Li+^{208}Pb OM after fit



χ^2 fits (from F. Nunes slides)

Stategies for chi2 fitting

- Start with simplest data and simplest reaction model (for example elastic and optical model)
- Restart from any intermediate stage
- If there are ambiguities, do grid searches and look at correlations in errors
- Artificially reduce error in data points if theory is having a hard time to get close in some region
- If minimum is found near the end of the range of a parameter, this is spurious – repeat with wider range
- Constrain with other experiments
- Two correlated variables : combine into one

Progressive improvement policy

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Input examples for inelastic scattering

Summary of physical ingredients for collective excitations

The main physical ingredient of DWBA and CC calculations are the coupling potentials:

$$V_{i,f}(\mathbf{R}) = \int d\xi \phi_f(\xi)^* V(\mathbf{R},\xi)\phi_i(\xi)$$

- Coulomb excitation:

$$V_{if}(\mathbf{R}) = \sum_{\lambda>0} \frac{4\pi}{2\lambda+1}\frac{Z_t e}{R^{\lambda+1}} \langle f; I_f M_f | \mathcal{M}(E\lambda,\mu) | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

- Nuclear excitation (collective model):

$$V_{if}(\mathbf{R}) \simeq -\frac{dV_0}{dR} \sum_{\lambda} \langle f; I_f M_f | \hat{\delta}_{\lambda\mu} | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

Reduced matrix elements

Wigner-Eckart theorem (Bohr&Mottelson convention):

$$\langle I_f M_f|\hat{O}_{\lambda\mu}|I_i M_i\rangle = (2I_f+1)^{-1/2}\langle I_f M_f|\lambda\mu I_i M_i\rangle\langle I_f||\hat{O}_\lambda||I_i\rangle_{\rm BM}$$

1. Coulomb case:

$$\langle f; I_f M_f|\mathcal{M}(E\lambda,\mu)|i; I_i M_i\rangle = (2I_f+1)^{-1/2}\langle I_f M_f|\lambda\mu I_i M_i\rangle\langle f; I_f||\mathcal{M}(E\lambda)||i; I_i\rangle$$

2. Nuclear case:

$$\langle f; I_f M_f|\hat{\delta}_{\lambda\mu}|i; I_i M_i\rangle = (2I_f+1)^{-1/2}\langle f; I_f M_f|I_i M_i\lambda\mu\rangle\langle f; I_f||\hat{\delta}_\lambda||i; I_i\rangle$$

So, in general:

$$\Delta V_{if}(\mathbf{R}) \propto \mathcal{F}_\lambda(R)\langle I_f||\mathcal{T}_\lambda(\xi)||I_i\rangle$$

Relation with physical quantities

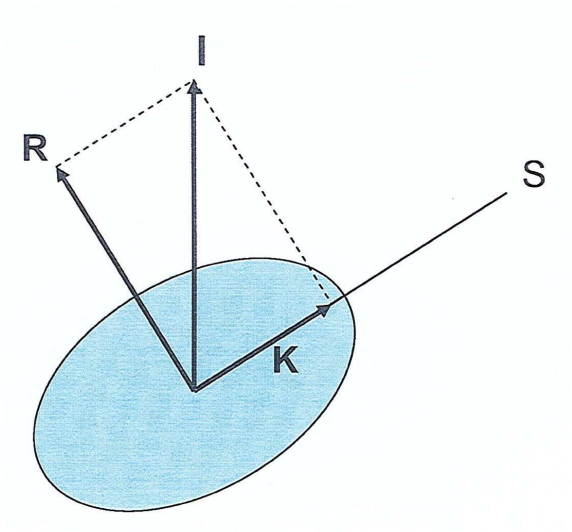
- For non-diagonal transitions ($I_i \neq I_f$)

$$B(E\lambda; i \rightarrow f) = \frac{1}{2I_i + 1} |\langle f\ I_f || \mathcal{M}(E\lambda) || i\ I_i \rangle_{\rm BM}|^2$$

- For diagonal transitions ($I_i = I_f \equiv I$) and $\lambda = 2$:

$$Q_2 = \sqrt{16\pi/5}\ (2I+1)^{-1/2}\ \langle II20|II\rangle\ \langle I||M(E2||I\rangle$$

Rotor model



- I=total spin (angular momentum) of the nucleus
- K=projection of I along symmetry axis

- The nucleus is described by a permanent deformation of matter and charge.
- The charge deformation for a multipole λ is characterized by the Coulomb intrinsic deformation: $M_n(E\lambda)$
- The matter deformation for a multipole λ is characterized by the deformation parameter (β_2) or the deformation length parameter (δ_λ)
- Transitions occur among states with the same value of K.

Reduced matrix elements in the strict rotor model

Coulomb excitation:

$$\langle K\ I_f||\mathcal{M}(E\lambda)||K\ I_i\rangle_{\rm BM} = \sqrt{2I_i+1}\langle I_i K\lambda 0|I_f K\rangle M_n(E\lambda)$$

⇨ $M_n(E\lambda)$=reduced matrix element of the charge deformation in intrinsic frame.

For nuclear excitation:

$$\langle f; K\ I_f||\hat{\delta}_\lambda||i; K\ I_i\rangle_{\rm BM} = \sqrt{2I_i+1}\langle I_i K\lambda 0|I_f K\rangle \beta_\lambda R_0$$

- β_λ= deformation parameter
- $\delta_\lambda = \beta_\lambda R_0$= deformation length parameter

⇨ $M_n(E\lambda)$ and β_λ represent the charge and matter deformation in the intrinsic frame

$$Mn(E\lambda) = \frac{3\ Z\ \beta_\lambda\ R^\lambda}{4\pi}$$

Reduced matrix elements for inelastic couplings

Coupling	TYPE	P(k)	STR
Coulomb (rotational)	10,11	$M_n(k)$	
Coulomb (general)	12,13	$\neq 0$ (arbitrary)	$M(Ek) = (-1)^{\frac{I-I'+\|I-I'\|}{2}} \langle I' \| M(Ek) \| I \rangle)$
Nuclear (rotational)	10,11	$DEF(k) = R\beta_k$	-
Nuclear (general)	12,13	$DEF(k) = R\beta_k$	$RDEF(k) = (-1)^{\frac{I-I'+\|I-I'\|}{2}} \langle I' \| \delta_k \| I \rangle$

When running inelastic (DWBA, CC) calculations with FRESCO...

- Do not confuse δ_λ (deformation length parameter in intrinsic frase) with $\hat{\delta}_{\lambda\mu}$ (deformation length operator in LAB frame):
  - δ_λ is a c-number and is the same for all transitions within the same rotational band
  - $\hat{\delta}_{\lambda\mu}$ is characterized by its matrix elements, which are different for each transition.
- When using $\langle I_f||\hat{\delta}_\lambda||I_i\rangle$ from literature, make sure what convention is used for the Wigner-Eckart theorem.
- FRESCO uses Bohr-Mottelson convention so

$$\langle I_f||\hat{\delta}_\lambda||I_i\rangle_{\rm BM} = \langle I_i||\hat{\delta}_\lambda||I_f\rangle_{\rm BM}$$

Inelastic scattering with general reduced matrix elements ^{10}Be(p,p')

^{10}Be(p,p') with general reduced matrix elements

```
0Be+p @ 63.7 MeV/u DWBA
AMELIST
FRESCO hcm=0.05 rmatch=40.0
    jtmin=0.0 jtmax=50 absend=-1
    thmin=0.1 thmax=90.00
    iter=1 nnu=24
    smats=2
      xstabl=1
      elab=637.7 /

PARTITION namep='10Be' massp=10.0113 zp=4
        namet='p' masst=1.0078 zt=1 nex=2 /
STATES jp=0.0 bandp=1 ep=0.000 cpot=1 ep=0.0 jt=0.0 /
STATES jp=2.0 bandp=1 ep=3.368 cpot=1 copyt=1 /
partition /

POT kp=1 ap=10 at=0.0 rc=1.0 /
POT kp=1 type=12 shape=10 p2=1 /
STEP ib=2 ia=1 k=2 str=7.8 /
STEP/

POT kp=1 type=1 p1=31.64 p2=1.145 p3=0.69
                  p4=8.78 p5=1.134 p6=0.69 /
POT kp=1 type=12 shape=10 p2=1 /
STEP ib=2 ia=1 k=2 str=1.97 /
STEP /
pot /

overlap /

coupling /
```

- `iter`=1: Solve CC to 1st order (DWBA)
- For the excited pair: `copyt=1`
- Coulomb deformation:
  - `type=12`: read reduced matrix elements
  - `shape=10`: radial dependence usual deformed charge distribution
  - `STR=7.8`: Coulomb r.m.e. (e fm^2)
- Nuclear deformation:
  - `type=12`: read reduced matrix elements
  - `shape=10`: derivative of undeformed potential
  - `STR=1.97`: nuclear reduced matrix element.

^{10}Be(p,p') with general reduced matrix elements

FRESCO namelist:

```
&FRESCO ITER=1 hcm=0.05 rmatch=40.0 elab=637.7
     jtmin=0.0 jtmax=50 absend=-1 thmin=0.1 thmax=90.00 smats=2 xstabl=1 /
```

ITER=1: Inelastic couplings to first order (DWBA)

PARTITIONS/STATES namelists:

```
&PARTITION namep='10Be' massp=10.0113 zp=4
          namet='p' masst=1.0078 zt=1 nex=2 /
&STATES jp=0.0 bandp=1 ep=0.000 cpot=1 ep=0.0 jt=0.0 /
&STATES jp=2.0 bandp=1 ep=3.368 cpot=1 copyt=1 /
```

NEX=2: Define two pairs of states

COPYT=1: The second target state is just a copy of the first one

ep,jp,bandp: Energy, spin, parity.

^{10}Be(p,p') with general reduced matrix elements

Coulomb potential:

```
&POT kp=1 ap=10 at=0.0 rc=1.0 /
&POT kp=1 type=12 shape=10 p2=1 /
```

- type=12: couple projectile states by deforming previous potential and read r.m.e
- p1,...,p6: consider couplings for multipoles k with pk$\neq 0$
- shape=10: usual deformed charge sphere: $V^k_{nm}(R) \propto M(Ek)/R^{k+1}$

```
&STEP ib=2 ia=1 k=2 str=7.8 /
&STEP/
```

Couple from state IA=1 to state IB=2 with multipole K=2 and r.m.e. 7.8 e fm^2

IB=2: final state (2nd state within partition)

IB=1: initial state (1st state within partition)

k=2 : multipole

str=7.8: reduced matrix element $\langle \text{IB}||\text{M(Ek}||\text{IA}\rangle = \sqrt{(2I_A+1)B(E\lambda; IA \rightarrow IB)}$

^{10}Be(p,p') with general reduced matrix elements

NUCLEAR potential:

```
&POT kp=1 type=1 p1=31.64 p2=1.145 p3=0.69 p4=8.78 p5=1.134 p6=0.69 /
&POT kp=1 type=12 shape=10 p2=1 /
```

- type=12: couple projectile states by deforming previous potential and read r.m.e.
- shape=10: standard derivative radial formfactor
- p2=1: quadrupole deformation

```
&STEP ib=2 ia=1 k=2 str=1.664 /
&STEP /
```

- k=2: quadrupole coupling
- ia, ib: indexes for states coupled.
- STR=1.97=$\langle IB||\delta_k||IA\rangle$ (fm)

Inelastic scattering within the rotor model: ^{10}Be(p,p')

^{10}Be(p,p') within rotor model

```
0Be+p @ 63.7 MeV/u DWBA (rotor model)
AMELIST
FRESCO hcm=0.05 rmatch=40.0
    jtmin=0.0 jtmax=50 absend=-1
    thmin=0.1 thmax=90.00
    iter=1 nnu=24
    smats=2
      xstabl=1
      elab=637.7 /

PARTITION namep='10Be' massp=10.0113 zp=4
         namet='p' masst=1.0078 zt=1 nex=2 /
STATES jp=0.0 bandp=1 ep=0.000 cpot=1 ep=0.0 jt=0.0 /
STATES jp=2.0 bandp=1 ep=3.368 cpot=1 copyt=1 /
partition /

POT kp=1 ap=10 at=0.0 rc=1.0 /
POT kp=1 type=10 shape=10 p2=7.8 /
POT kp=1 type=1 p1=31.64 p2=1.145 p3=0.69
                    p4=8.78 p5=1.134 p6=0.69 /
POT kp=1 type=10 shape=10 p2=1.97 /

pot /

overlap /

coupling /
```

- `iter`=1: Solve CC to 1st order (DWBA)
- For the excited pair: `copyt=1`
- Coulomb deformation:

  `type`=10 (rotor)
  `shape`=10 (usual deformed charge distribution)
  `Mn`=7.8 e fm^2
- Nuclear deformation:

  `type`=10 (rotor)
  `shape`=10 (derivative of undeformed potential)
  δ_2=1.97 fm.

^{10}Be(p,p') within rotor model

FRESCO namelist:

```
&FRESCO ITER=1 hcm=0.05 rmatch=40.0 elab=637.7
     jtmin=0.0 jtmax=50 absend=-1 thmin=0.1 thmax=90.00 smats=2 xstabl=1 /
```

ITER=1: Inelastic couplings to first order (DWBA)

PARTITIONS/STATES namelists:

```
&PARTITION namep='10Be' massp=10.0113 zp=4
          namet='p' masst=1.0078 zt=1 nex=2 /
&STATES jp=0.0 bandp=1 ep=0.000 cpot=1 ep=0.0 jt=0.0 /
&STATES jp=2.0 bandp=1 ep=3.368 cpot=1 copyt=1 /
```

- NEX=2: number of pairs of states
- COPYT=1: The second target state is just a copy of the first one
- ep,jp,bandp: Energy, spin, parity.

^{10}Be(p,p') within rotor model

Coulomb potential:

```
&POT kp=1 ap=10 at=0.0 rc=1.0 /
&POT kp=1 type=10 shape=10 p2=7.8 /
```

- type=10: rotor model
- shape=10: usual deformed charge distribution
- P2=7.8: intrinsic Mn value (e fm^2)

NUCLEAR potential:

```
&POT kp=1 type=1 p1=31.64 p2=1.145 p3=0.69 p4=8.78 p5=1.134 p6=0.69 /
&POT kp=1 type=10 shape=10 p2=1.97 /
```

- type=10: rotor model
- shape=10: derivative formfactor
- p2=1.97: deformation length

Input examples for transfer (DWBA)

Transfer example: ^{56}Fe(d,p)^{57}Fe

```
6Fe(d,p)57Fe @ Ed=12 MeV;
AMELIST
FRESCO hcm=0.1 rmatch=20.000
      rintp=0.20 hnl=0.100 rnl=4 centre=-0.45
      jtmax=15
      thmin=1.00 thmax=180.00 thinc=1.00
      it0=1 iter=1
      chans=1 smats=2 xstabl=1
      elab= 12 /

PARTITION namep='d' massp=2.014 zp=1 namet='56Fe'
      masst=55.934 zt=26 nex=1 /
STATES jp=1.0 bandp=1 ep=0.0 cpot=1 jt=0.0
      bandt=1 et=0.0 /

PARTITION namep='p' massp=1.0078 zp=1 namet='57Fe'
      masst=56.935 zt=26 qval=5.421 pwf=F nex=1 /
STATES jp=0.5 bandp=1 ep=0.0 cpot=2 jt=0.5
      bandt=-1 et=0.0 /
partition /

POT kp=1 itt=F at=56 rc=1.15 /
POT kp=1 type=1 itt=F p1=90 p2=1.15 p3=0.81 /
POT kp=1 type=2 itt=F p4=21 p5=1.34 p6=0.68 /

POT kp=2 itt=F at=57 rc=1.15 /
POT kp=2 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
POT kp=2 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

```
POT kp=3 itt=F at=56 rc=1.00 /
POT kp=3 type=1 itt=F p1=65.0 p2=1.25 p3=0.65 /
POT kp=4 itt=F ap=1.0000 at=0.0000 rc=1.0000 /
POT kp=4 type=1 shape=2 itt=F p1=72.1500
      p2=0.0000 p3=1.4840 /
POT kp=5 itt=F at=56 rc=1.15 /
POT kp=5 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
POT kp=5 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
pot /

OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=1 sn=0.5
      j=0.5 kbpot=4 be=2.2250 isc=1 /
OVERLAP kn1=2 ic1=1 ic2=2 in=2 nn=2 l=1 sn=0.5
      j=0.5 kbpot=3 be=7.646 isc=1 /
overlap /

COUPLING icto=2 icfrom=1 kind=7 ip2=-1 ip3=5 /
CFP in=1 ib=1 ia=1 kn=1 a=1.0000 /
CFP in=2 ib=1 ia=1 kn=2 a=1.0000 /
cfp /

coupling /
```

Transfer example: ^{56}Fe(d,p)^{57}Fe

General variables:

```
56Fe(d,p)57Fe @ Ed=12 MeV;
NAMELIST
&FRESCO hcm=0.1 rmatch=20.000
       rintp=0.20 hnl=0.100 rnl=4 centre=-0.45
       jtmax=15
       thmin=1.00 thmax=180.00 thinc=1.00
       it0=1 iter=1
       chans=1 smats=2 xstabl=1
       elab= 12 /
```

- `rnl`: range of non-locality
- `centre,rintp,hnl`: parameters for numerical integration (see `fresco` manual)
- `iter`: Number of iterations so, for DWBA, `iter=1`

Transfer example: ^{56}Fe(d,p)^{57}Fe

Partitions and states:

- Incoming (initial) partition: d+^{56}Fe

```
&PARTITION namep='d' massp=2.014 zp=1
          namet='56Fe' masst=55.934 zt=26 nex=1 /
&STATES jp=1.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
```

- Outgoing (final) partition: p+^{57}Fe

```
&PARTITION namep='p' massp=1.0078 zp=1
          namet='57Fe'masst=56.935 zt=26
          qval=5.421 nex=1 /
&STATES jp=0.5 bandp=1 ep=0.0 cpot=2 jt=0.5 bandt=-1 et=0.0 /
```

- `qval`: Q-value for gs-gs transfer

Transfer example: ^{56}Fe(d,p)^{57}Fe

Interactions:

- Entrance channel distorted potential: d+^{56}Fe

```
&POT kp=1 itt=F at=56 rc=1.15 /
&POT kp=1 type=1 itt=F p1=90 p2=1.15 p3=0.81 /
&POT kp=1 type=2 itt=F p4=21 p5=1.34 p6=0.68 /
```

- Exit channel distorted potential: p+^{57}Fe

```
&POT kp=2 itt=F at=57 rc=1.15 /
&POT kp=2 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
&POT kp=2 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

- Core-core potential: p+^{56}Fe

```
&POT kp=5 itt=F at=56 rc=1.15 /
&POT kp=5 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
&POT kp=5 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

Transfer example: ^{56}Fe(d,p)^{57}Fe

Interactions (continued)

Binding potentials:

- n+ ^{56}Fe: Woods-Saxon

```
&POT kp=3 at=56 rc=1.0 /
&POT kp=3 type=1 p1=65.0 p2=1.25 p3=0.65 /
```

- n+p: Gaussian

```
&POT kp=4 ap=1.0 at=0.0 /
&POT kp=4 type=1 shape=2 p1=72.15 p2=0.00 p3=1.484 /
```

Transfer example: ^{56}Fe(d,p)^{57}Fe

Bound wavefunctions (overlaps):

- d=p+n: simple 1S model

```
&OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=1 l=0 sn=0.5 j=0.5
        kbpot=4 be=2.2250 isc=1 /
```

- ^{57}Fe=^{56}Fe+n: assume $2p_{1/2}$ configuration

```
 &OVERLAP kn1=2 ic1=1 ic2=2 in=2 nn=2 l=1 sn=0.5 j=0.5
         kbpot=3 be=7.646 isc=1 /
```

  - in=1: projectile
    in=2: target
  - nn,l,sn,j: quantum numbers: $\vec{l} + \vec{sn} = \vec{j}$
  - be: binding (separation) energy
  - kbpot: potential index

Transfer example: ^{56}Fe(d,p)^{57}Fe

Transfer coupling between the two partitions:

```
&COUPLING icfrom=1 icto=2 kind=7 ip1=0 ip2=-1 ip3=5 /
```

- `icfrom`: index for partition of initial state
- `icto`: index for partition of final state
- `kind`: kind of coupling. kind=7 means finite-range transfer.
- `ip1=0`: post representation
  `ip1=1`: prior
- `ip2=-1`: include full remnant
- `ip3`: index for core-core potential (p+^{56}Fe)

Transfer example: ^{56}Fe(d,p)^{57}Fe

Spectroscopic amplitudes:

```
&CFP in=1 ib=1 ia=1 kn=1 a=1.0 /
&CFP in=2 ib=1 ia=1 kn=2 a=1.0 /
```

- `in=1`: projectile state
  `in=2`: target state
- `ib`: index for state of composite
  `ia`: index for state of core
- `a`: spectroscopic amplitude