We want to study the fusion reaction $^{28}Si+^{12}C$ @ 100MeV in inverse kinematics and measure the energy spectra of the evaporation residues identified in mass and charge. To perform the experiment you use a ΔE-E telescope coupled with a ToF spectrometer
to measure the evaporation residue angular distributions.

Assuming that:
a)the ΔE resolution of your telescope is 3%;
b) the total energy resolution we can obtain from the telescope is 1 %;
c) the energy threshold for charge identification of the telescope is ok for your purpose;

we wish to understand if the telescope you have available is suitable for the experiment and, if yes, which would be the time resolution needed of the ToF spectrometer to be coupled with the telescope knowing that the ToF base is 1.3 m.

Use PACE4 to calculate the mass and charge of evaporation residues produced in the reaction.

ER mass and charge estimate

From PACE4 statistical model calculations we estimate that our evaporation residues are in the mass range 30<A<40 and in the Z range 15<Z<20;
Therefore we need charge identification for Z<20 and mass identification for A<40.

Estimate of ER Velocity

$$\frac{v}{c} = \sqrt{\frac{2E(MeV)}{M(MeV)}} \Rightarrow v(cm/ns) = 1.39\sqrt{\frac{E(MeV)}{A}}$$

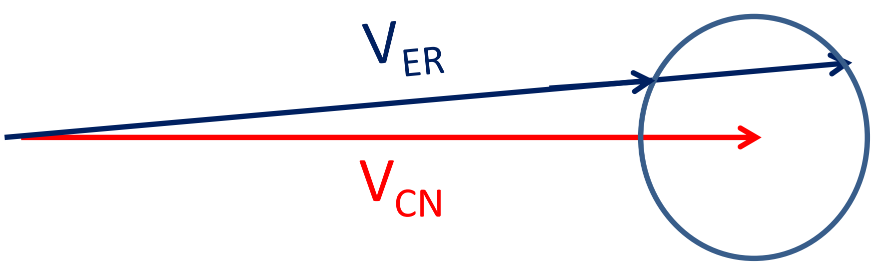
Energy and velocity associated to the CN motion

$$E_{c.m.} = E_{LAB}(M_P/(M_P+M_T))$$

$$v_{CN} = v_{c.m.} = 1.39 \bullet SQRT((E_{LAB} \bullet M_P))/(M_P+M_T)^2 = 1.84\ cm/ns$$

The ER velocity in the lab has a wide distribution centered around

$$V_{ER} \sim V_{CN} \cos \theta_{Lab} \qquad TOF_{ER} \sim 70\ ns$$

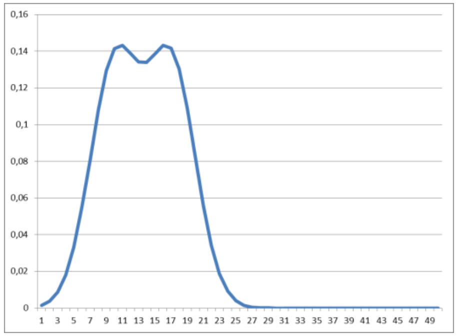


Limits for charge and mass identification

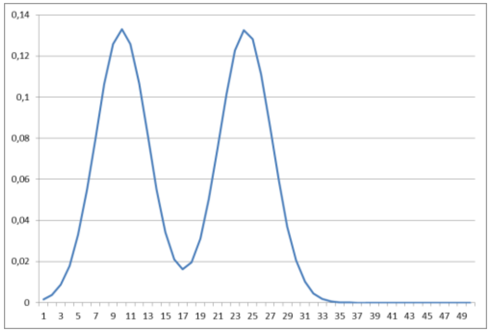
Example

sum od two Gaussian with same FWHM whose centroids are separated by

1 FWHM



2 FWHM



To have a satisfactory mass or charge identification two peaks differing by ONE mass or charge unit have to be separated by at least 2 FWHM. In other words:

$$2\underbrace{\boxed{\delta Z}}_{\text{FWHM}} < 1 \Rightarrow 2Z\frac{\delta Z}{Z} < 1$$

$$2\underbrace{\boxed{\delta M}}_{\text{FWHM}} < 1 \Rightarrow 2M\frac{\delta M}{M} < 1$$

This holds true if

$$Z < \frac{1}{2}\frac{Z}{\delta Z}$$

$$M < \frac{1}{2}\frac{M}{\delta M}$$

Charge identification

$$\Delta E \propto Z^2$$

$$\frac{\delta \Delta E}{\Delta E} = 2\frac{\delta Z}{Z}$$

$$Z < \frac{1}{2}\frac{Z}{\delta Z} \rightarrow Z < \frac{1}{2}\left(2\frac{\Delta E}{\delta \Delta E}\right) \rightarrow Z < \left(\boxed{\frac{100}{3}}\right) \Rightarrow 3\%$$

Since the charge of our ER is Z< 20 we expect satisfactory charge identification for our ER

Mass identification

$$M \propto ET^2$$

$$\left(\frac{\delta M}{M}\right)^2 = \left(\frac{\delta E}{E}\right)^2 + 4\left(\frac{\delta T}{T}\right)^2$$

$$M < \frac{1}{2}\frac{M}{\delta M} \rightarrow \frac{\delta M}{M} < \frac{1}{2\boxed{M}} \Rightarrow 40 \rightarrow \frac{\delta M}{M} < \frac{1}{80}$$

$$\frac{\delta M}{M} = \sqrt{\left(\frac{\delta E}{E}\right)^2 + 4\left(\frac{\delta T}{T}\right)^2} < \frac{1}{80}$$

Mass identification

$$\frac{\delta M}{M} = \sqrt{\left(\boxed{\frac{1}{100}}\right)^2 + 4\left(\frac{\delta T}{T}\right)^2} < \frac{1}{80}$$

1% resolution of $\Delta E + E_{residual}$

$$\frac{\delta T}{T} < 5.73 * 10^{-3}$$

$$T = 70ns$$

$$\delta T < 70 * 5.73 * 10^{-3}\, ns \rightarrow \delta T < 400\, ps$$

We need a time resolution better than 400 ps