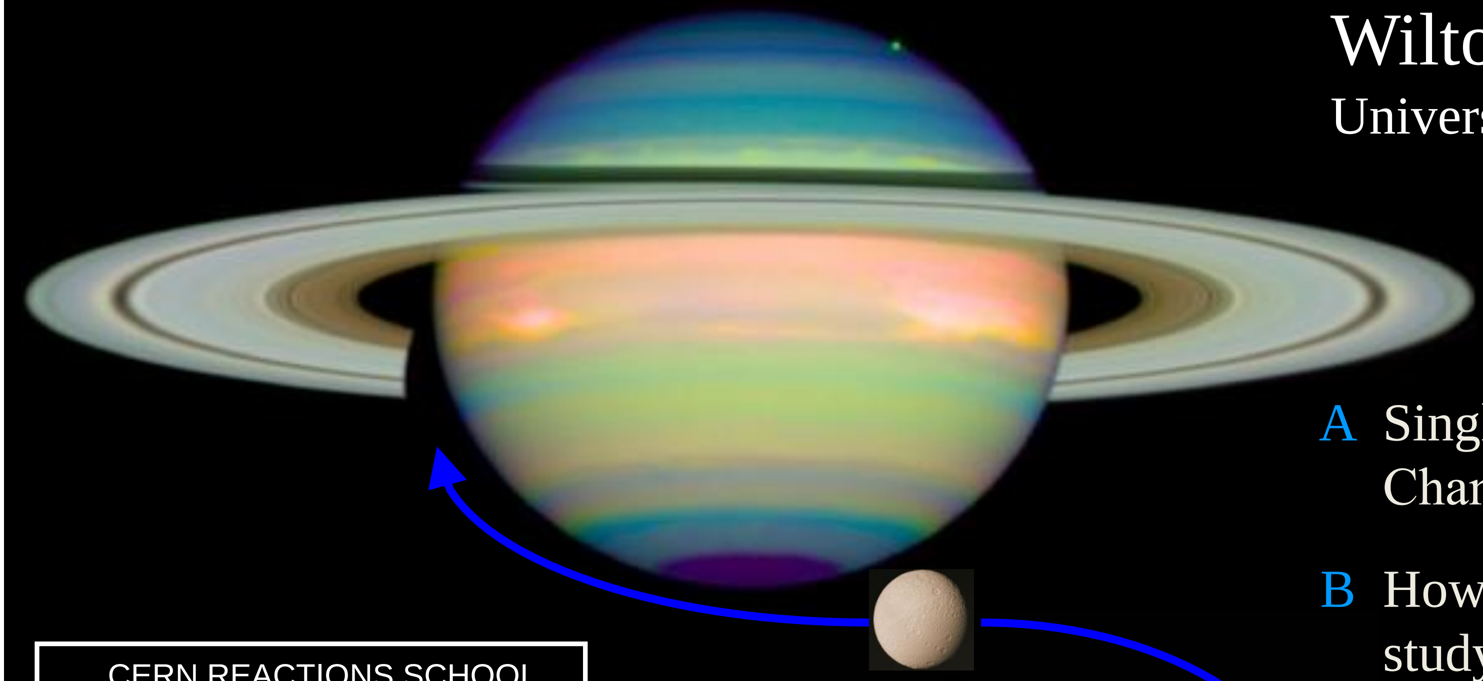
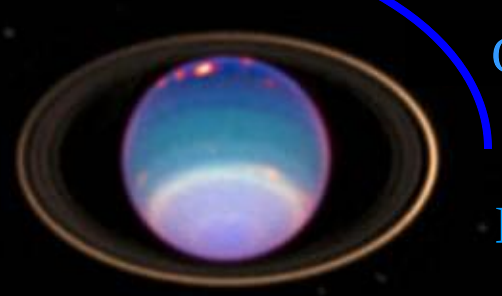
What can we learn from transfer, and how is best to do it?

Wilton Catford
University of Surrey





- A Single Particle States; Changing magic no.'s
- B How reactions can study this physics
- C Practicalities; Inverse Kinematics
- D Experimental setups
- E Results & Perspectives

CERN REACTIONS SCHOOL

CERN, Geneva ** 22-24 April 2014

SATURN HST/ IR 1998, TETHYS VOYAGER2 1981, URANUS HST/ IR 1986

- **Motivation: nuclear structure reasons for transfer**
- **What quantities we actually measure**
- **What reactions can we choose to use?**
- **What is a good beam energy to use?**
- **Inverse Kinematics**
- **Implications for Experimental approaches**
- **Example experiments and results**

CERN REACTIONS SCHOOL

CERN, Geneva ** 22-24 April 2014

LAKE GENEVA

Nucleon Transfer using Radioactive Beams: results and lessons from the TIARA and TIARA+MUST2 experiments at SPIRAL

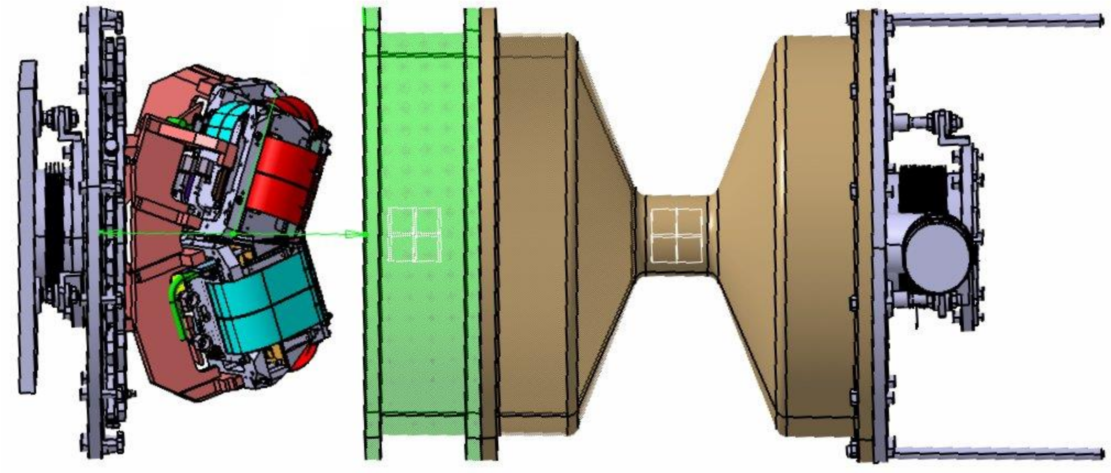
WILTON CATFORD, University of Surrey, UK

W.N. Catford, N.A. Orr, A. Matta, B. Fernandez-Dominguez, F. Delaunay, C. Timis,
M. Labiche, J.S. Thomas, G.L. Wilson, S.M. Brown, I.C. Celik, A.J. Knapton *et al.*

and the TIARA+MUST2 Collaboration
(Surrey, Liverpool, Daresbury, West of Scotland, Birmingham, Orsay, Saclay, GANIL, LPC Caen)
[1]*University of Surrey, Guildford, GU2 7SH, UK*
[2]*STFC Daresbury Laboratory, Warrington, Cheshire, WA4 4AD, UK*
[3]*University of Liverpool, Liverpool, L69 7ZE, UK*
[4]*GANIL, BP 55027, 14076 Caen Cedex 5, France*
[5]*IPN Orsay, IN2P3-CNRS, 91406 Orsay, France*

TIARA

MUST2



Results from SHARC+TIGRESS at ISAC2/TRIUMF

UNIVERSITY OF SURREY

WNC, G.L. Wilson, S.M. Brown

THE UNIVERSITY of York

C.Aa. Diget, B.R. Fulton, S.P. Fox
R. Wadsworth, M.P. Taggart

TRIUMF

G. Hackman, A.B. Garnsworthy, S. Williams
C. Pearson, C. Unsworth, M. Djongolov and
the TIGRESS collaboration

TIGRESS

LPC

N.A. Orr, F. Delaunay, N.L. Achouri

UNIVERSITY OF LIVERPOOL

Boston^2

SCHOOL OF MINES COLORADO 1874

F. Sarazin

LSU

J.C. Blackmon

UNIVERSITY of GUELPH

C. Svensson

SAINT MARY'S UNIVERSITY SINCE 1802

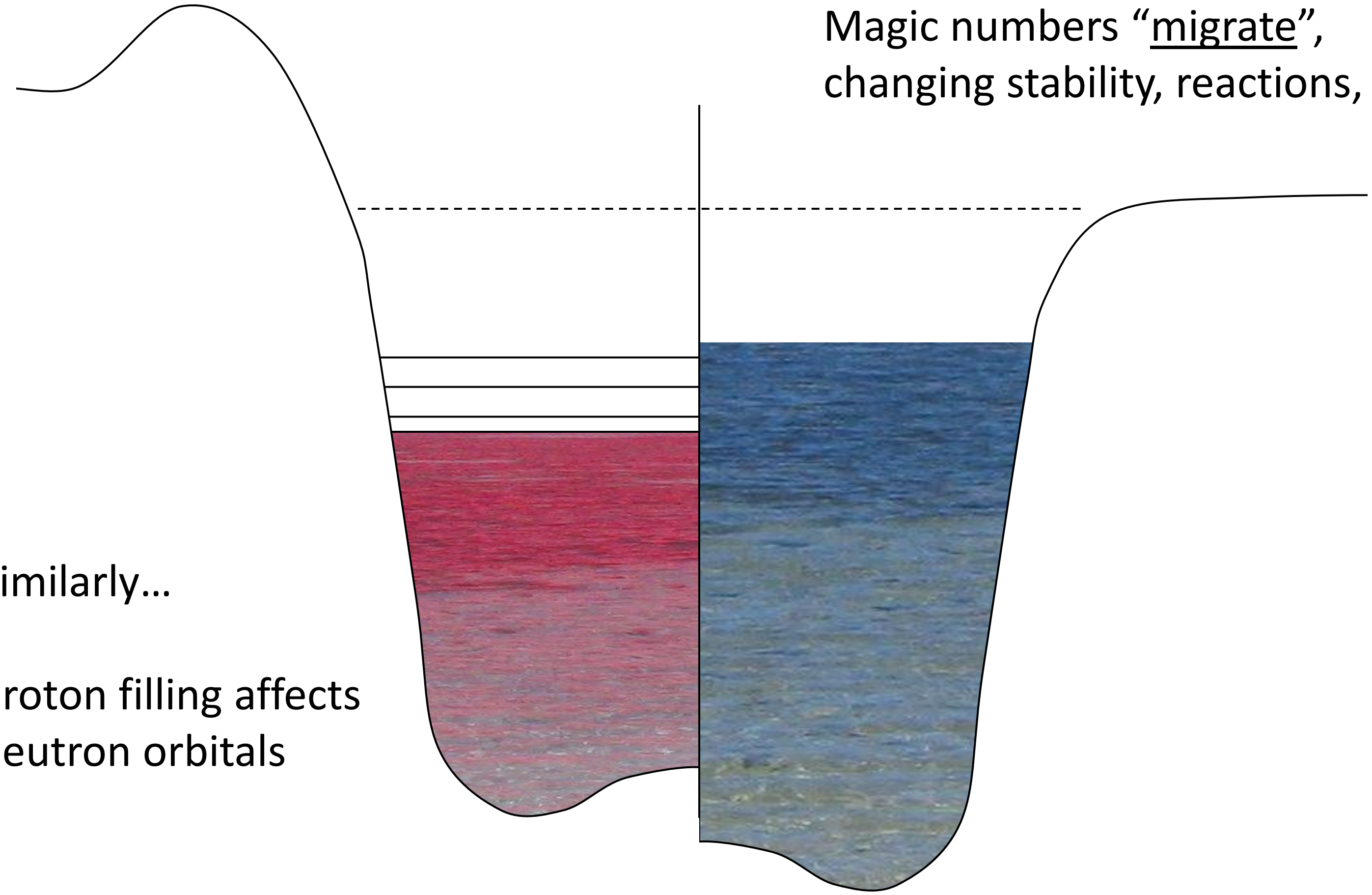
R.A.E. Austin

SINGLE PARTICLE STATES in the SHELL MODEL:

Changes – tensor force, p-n

Residual interactions <u>move</u> the
mean field levels

Magic numbers "<u>migrate</u>",
changing stability, reactions, collectivity…

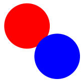


Similarly…

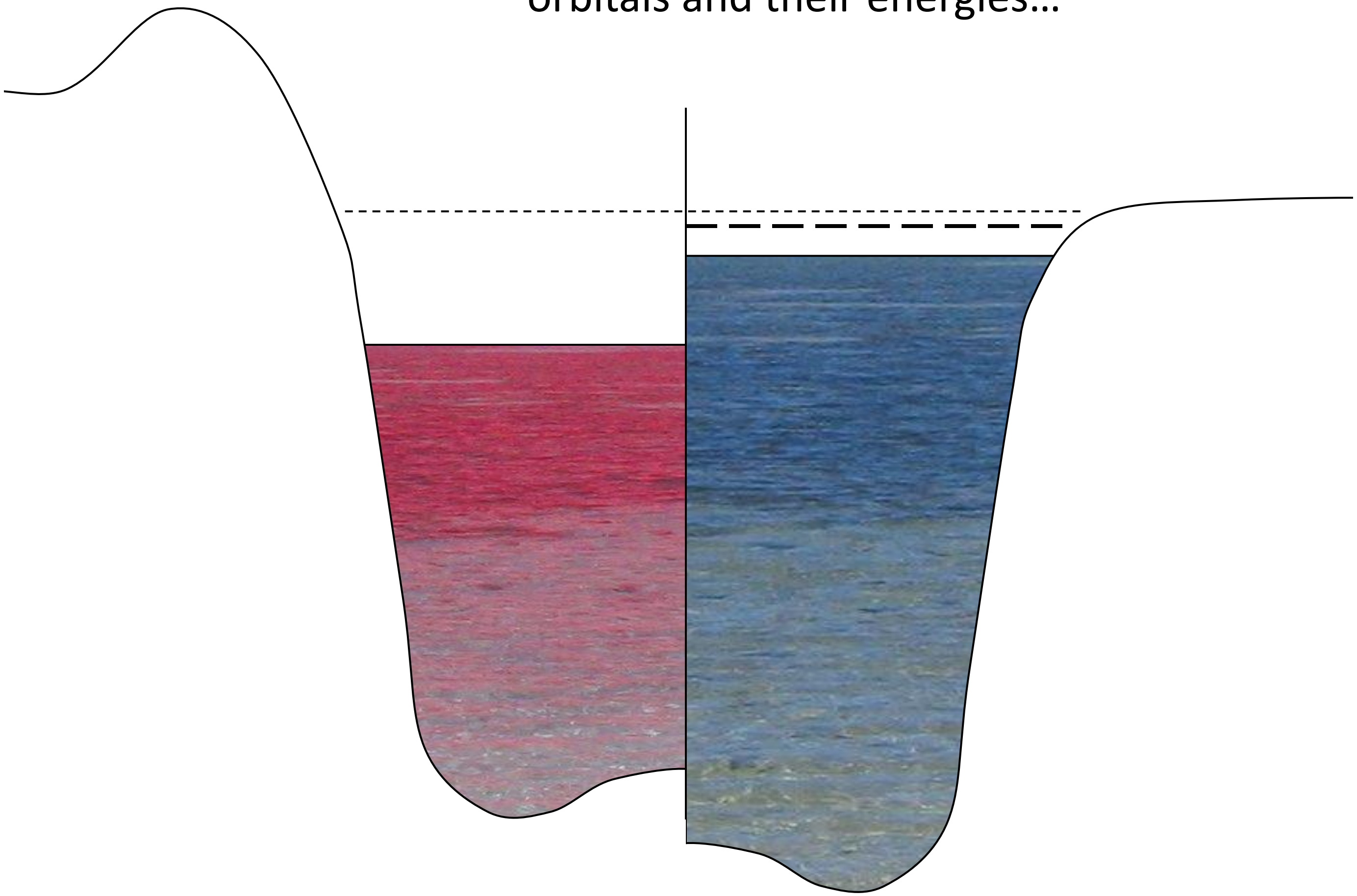
proton filling affects
neutron orbitals

SINGLE PARTICLE STATES in the SHELL MODEL:

(d, p)

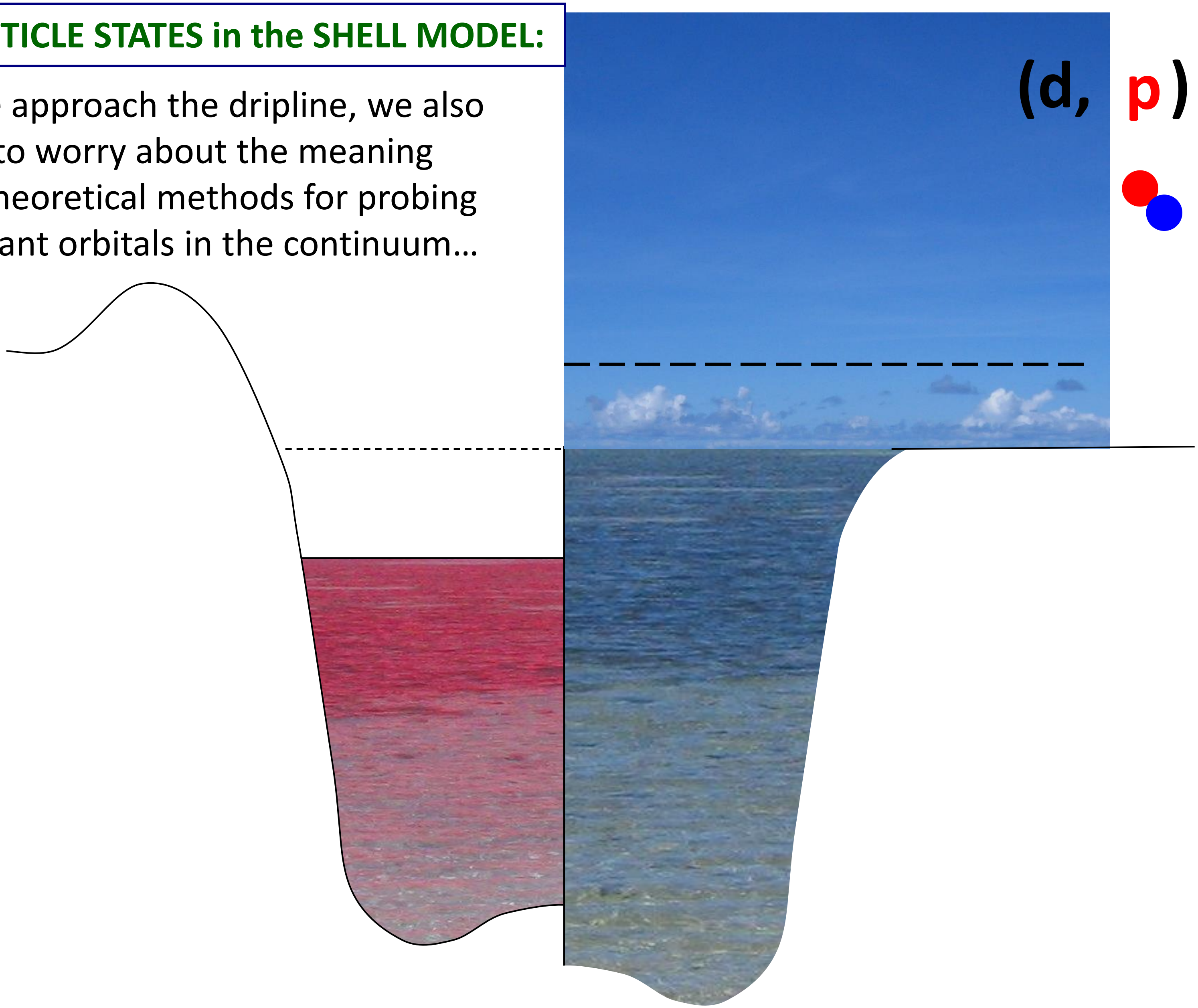


Probing the changed
orbitals and their energies…

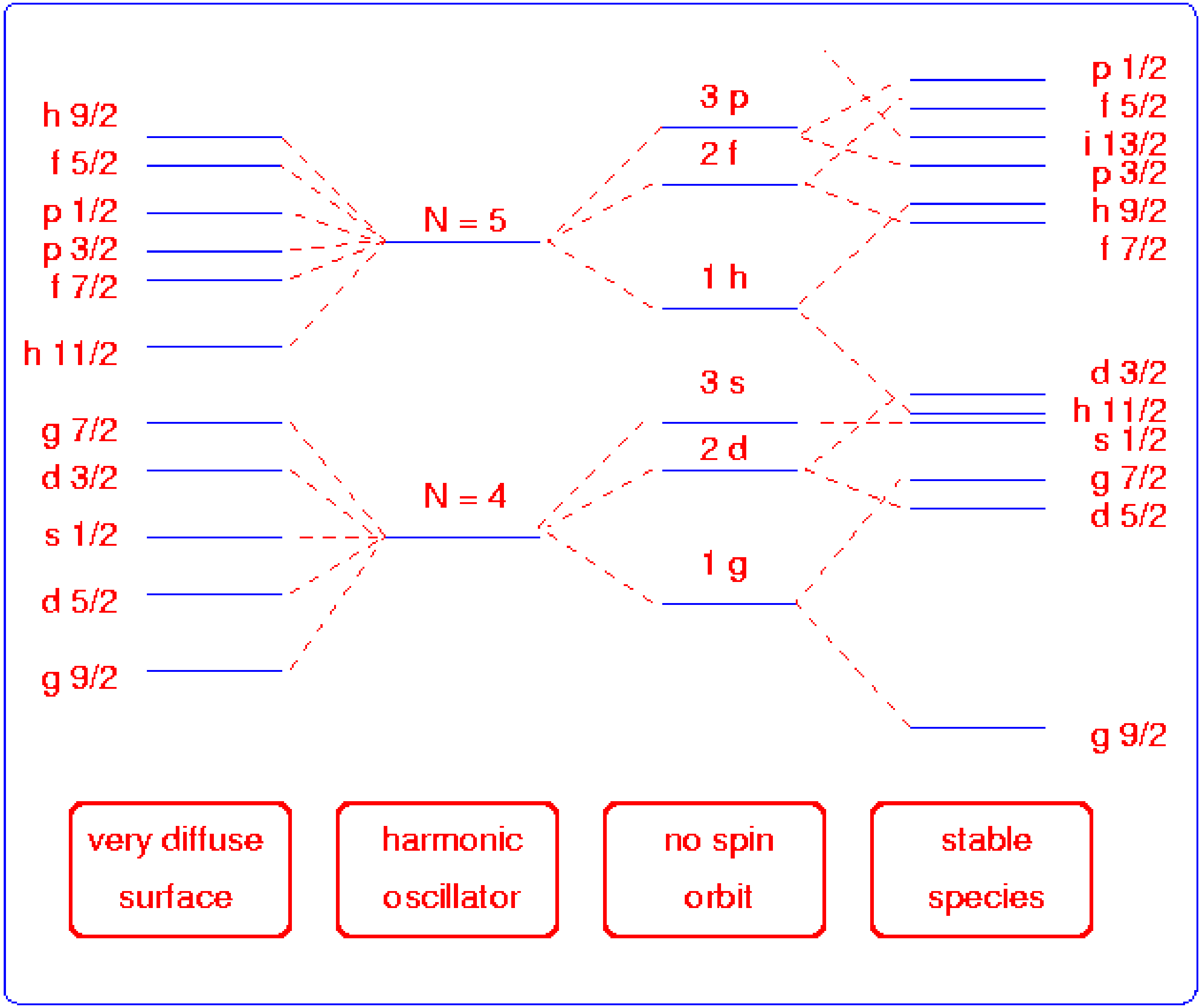


SINGLE PARTICLE STATES in the SHELL MODEL:

As we approach the dripline, we also have to worry about the meaning and theoretical methods for probing resonant orbitals in the continuum…



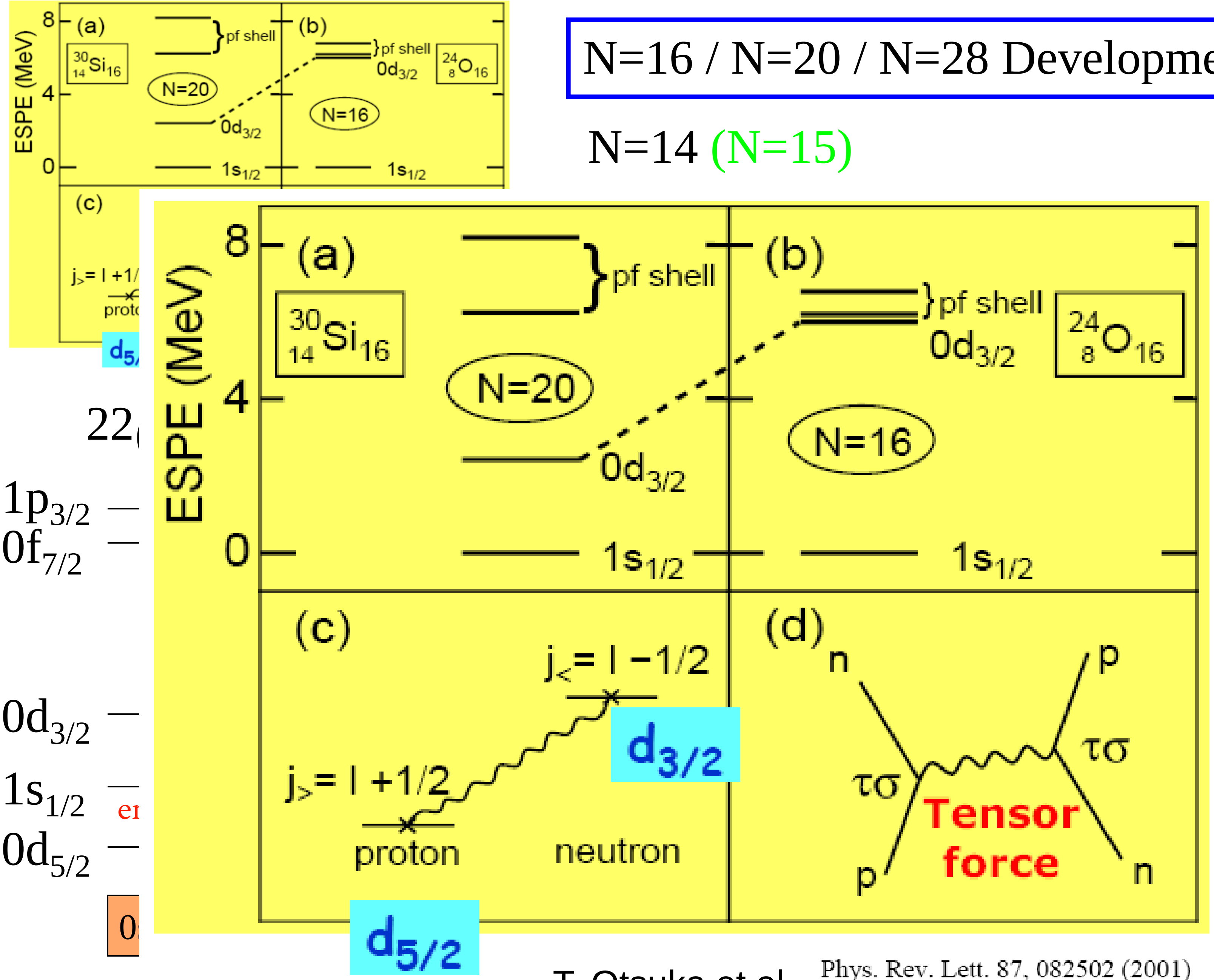
Changing shell structure and collectivity at the drip line



J.Dobaczewski et al., PRC 53 (1996) 2809

N=16 / N=20 / N=28 Development

N=14 (N=15)



directly
^{24}Ne

22(

(^{29}Si)

$1p_{3/2}$

$0f_{7/2}$

$0d_{3/2}$

$1s_{1/2}$

er

$0d_{5/2}$

0:

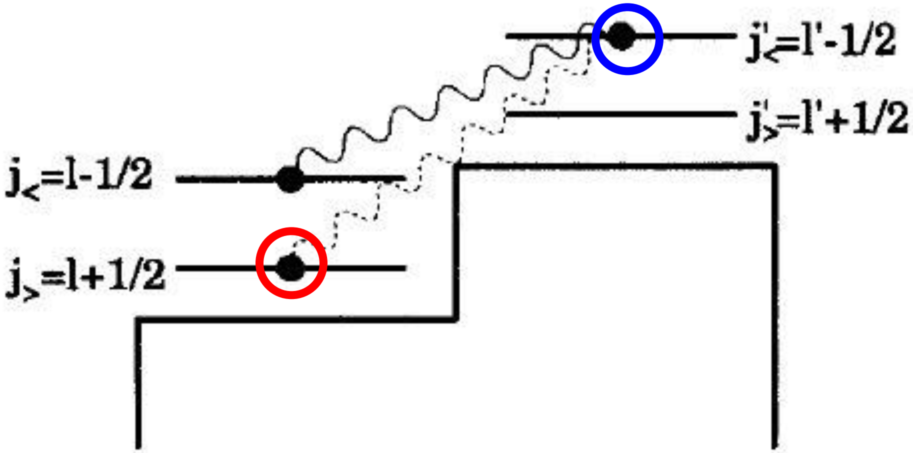
0s, 1p

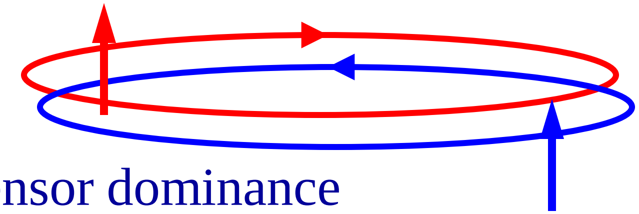
π ν | π ν ν

T. Otsuka et al.

Phys. Rev. Lett. 87, 082502 (2001)
Phys. Rev. Lett. 95, 232502 (2005)

Changing Magic Numbers





attractive p-n interaction

tensor dominance

T. Otsuka *et al.*, Phys. Rev. Lett. **97**, 162501 (2006).

T. Otsuka *et al.*, Phys. Rev. Lett. **87**, 082502 (2001).

Nuclei are quantum fluids comprising two distinguishable particle types…

They separately fill their quantum wells…

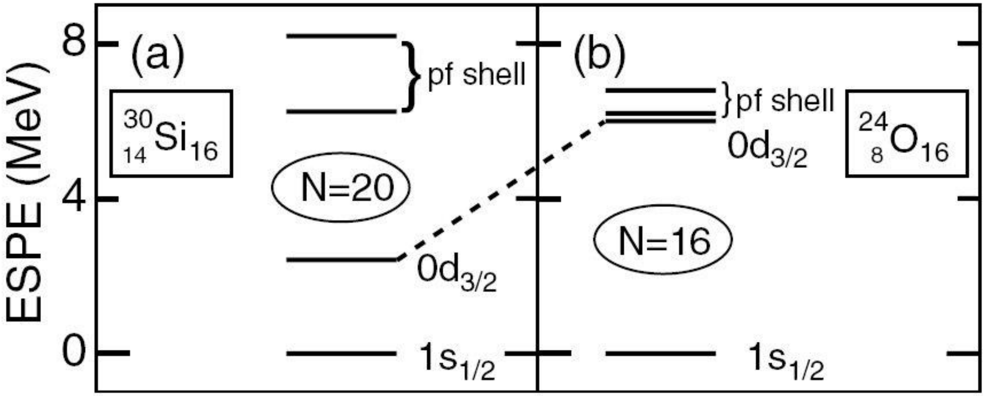
Shell structure emerges…

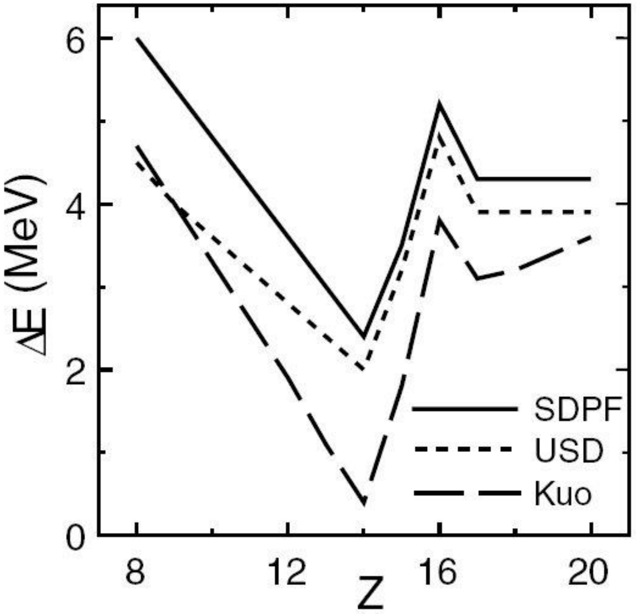
Valence nucleons interact…

This can perturb the orbital energies…

The shell magic numbers for p(n) depend on the level of filling for the n(p)

Changing Magic Numbers

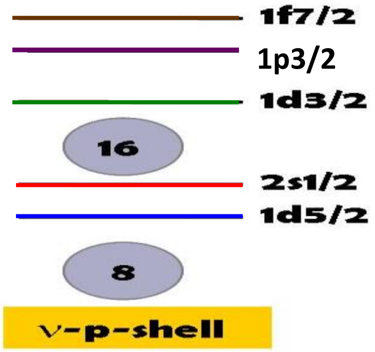




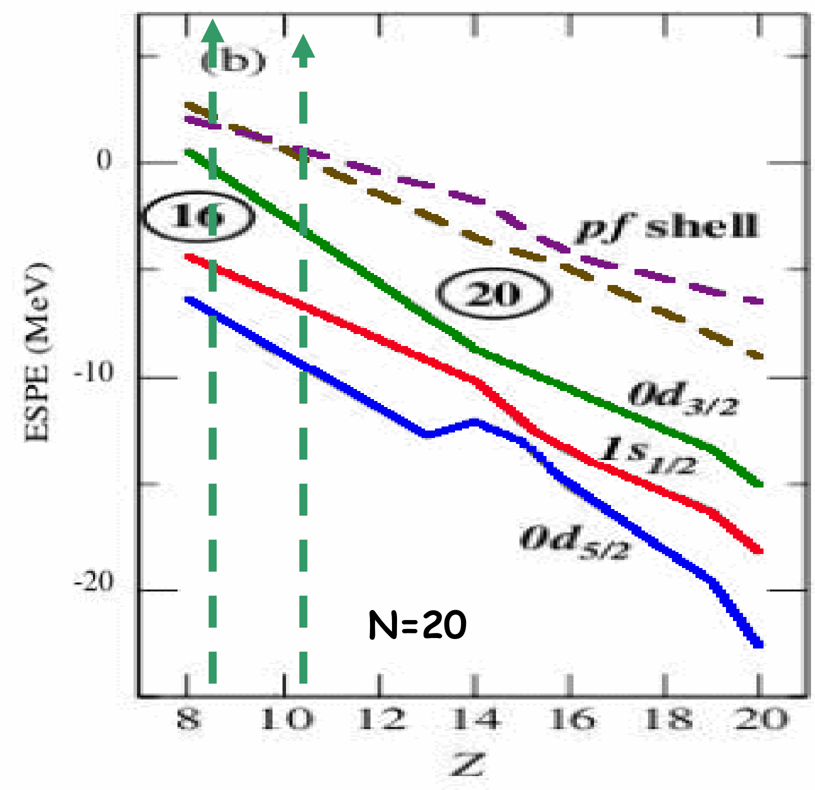
As the occupancy of the $j_>$ orbit $d_{5/2}$ is reduced in going from (a) ^{30}Si to (b) ^{24}O, then the attractive force on $j_<$ $d_{3/2}$ neutrons is reduced, and the orbital rises relatively in energy. This is shown in the final panel by the $s_{1/2}$ to $d_{3/2}$ gap, calculated using various interactions within the Monte-Carlo shell model.

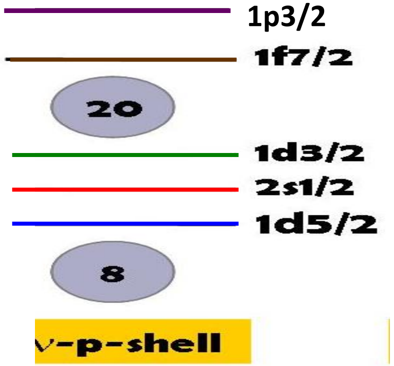
Utsuno et al., PRC,60,054315(1999)
Monte-Carlo Shell Model (SDPF-M)

Exotic ← Stable



Exotic





Stable

Removing d5/2 protons (Si →O)

←

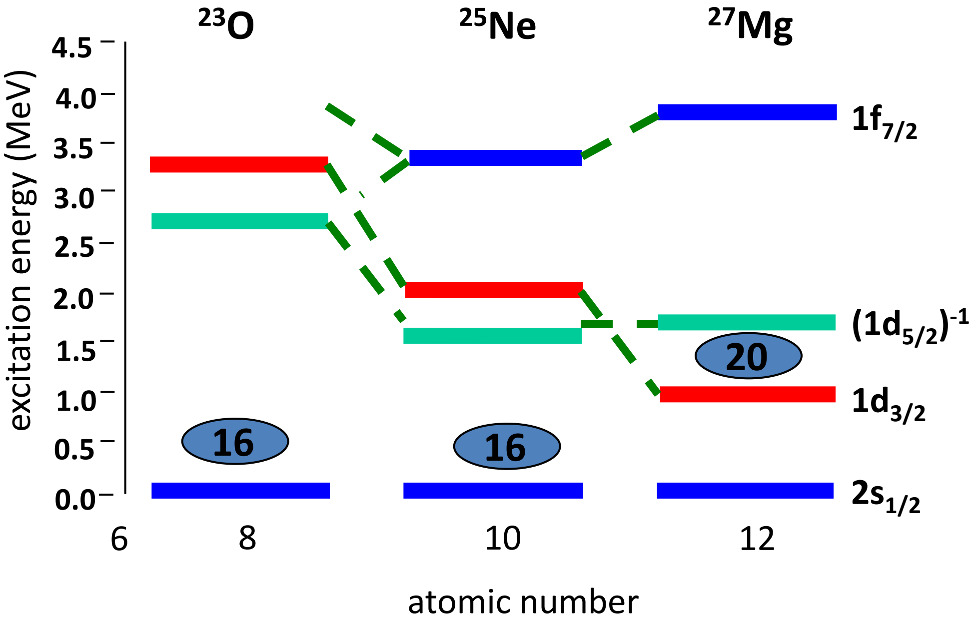
gives relative rise in ν(d3/2)

Note:
This changes collectivity, also…

SINGLE PARTICLE STATES – AN ACTUAL EXAMPLE

Systematics of the 3/2+ for N=15 isotones

removing d5/2 protons raises d3/2
and appears to lower the f7/2



Migration of the 3/2+ state creates N=16 from N=20

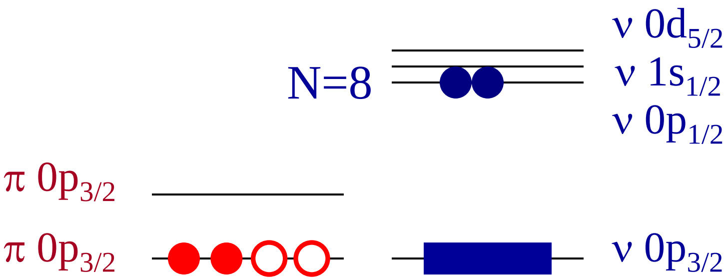
^{25}Ne TIARA → USD modified

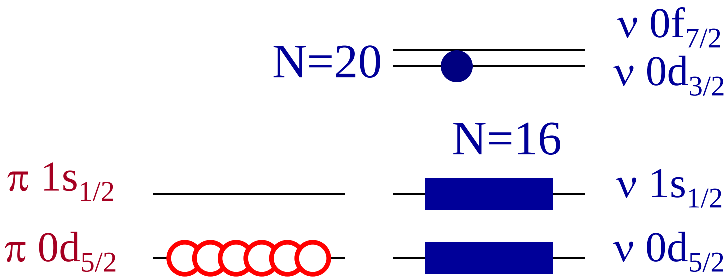
^{23,25}O raise further challenges

^{21}O has similar 3/2+-1/2+ gap (same d5/2 situation) but poses interesting question of mixing (hence recent ^{20}O(d,p)@SPIRAL)

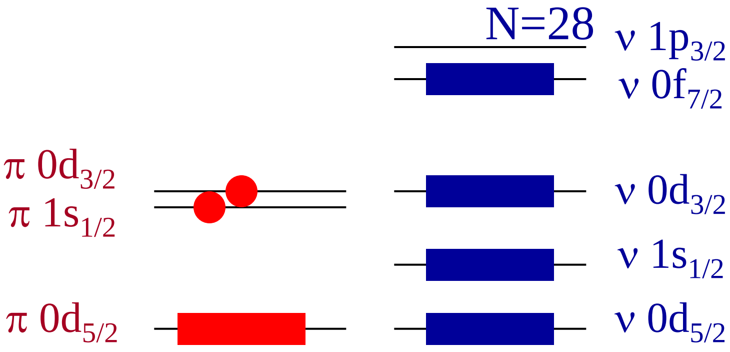
- ^{23}O from USD and Stanoiu PRC 69 (2004) 034312 and Elekes PRL 98 (2007) 102502
- ^{25}Ne from TIARA, W.N. Catford et al. Eur. Phys. J. A, 25 S1 251 (2005)

Changing Magic Numbers





In the lighter nuclei (A<50) a good place to look is near closed proton shells, since a closed shell is followed in energy by a $j_>$ orbital. For example, compared to ^{14}C the nuclei ^{12}Be and ^{11}Li (just above Z=2) have a reduced π ($0p_{3/2}$) occupancy, so the N=8 magic number is lost. Similarly, compared to ^{30}Si, the empty π ($0d_{5/2}$) in ^{24}O (Z=8) leads to the breaking of the N=20 magic number. Another possible extreme is when a particular neutron orbital is much more complete than normal.



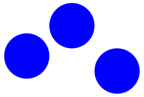
Nuclear states are not in general pure SP states, of course

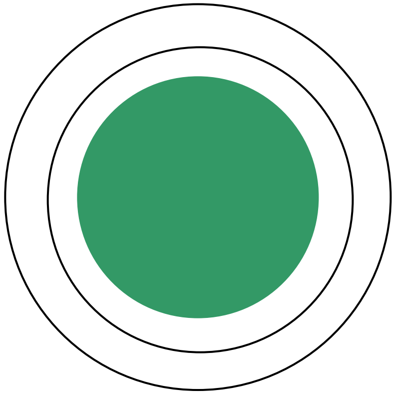
For nuclear states, we measure the spin and energy
and
the magnitude of the single-particle component for that state
(spectroscopic factor)

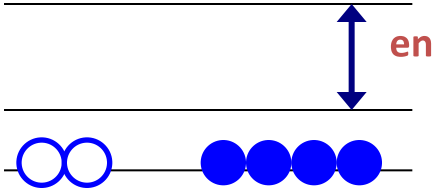
Example: (relevant to one of the experiments)… $3/2^+$ in ^{21}O

A. SINGLE PARTICLE STATES – EXAMPLE

Example of population of single particle state: ^{21}O







0d 3/2

energy of level measures this gap

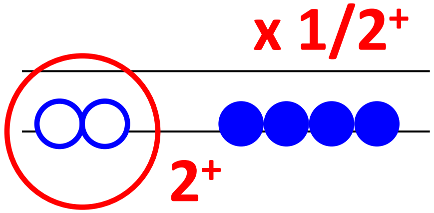
1s 1/2

0d 5/2

$J^{\pi} = 3/2^{+}$

The mean field has orbitals, many of which are filled.
We probe the energies of the orbitals by transferring a nucleon
This nucleon enters a vacant orbital
In principle, we know the orbital wavefunction and the reaction theory

But not all nuclear excited states are single particle states…



x $1/2^{+}$

2^{+}

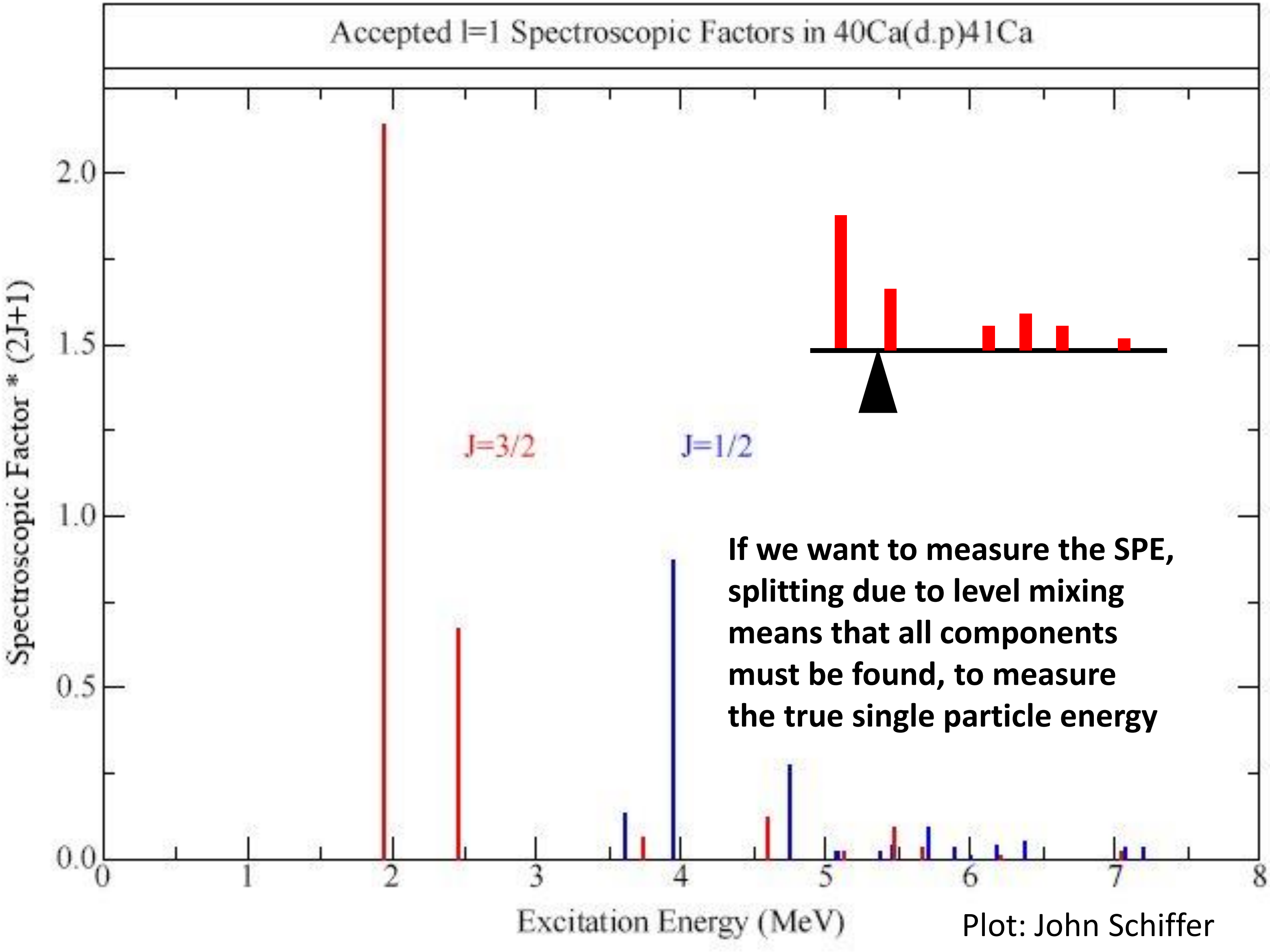
1s 1/2

0d 5/2

$J^{\pi} = 3/2^{+}$

We measure how the two $3/2^{+}$ states share the SP strength when they mix

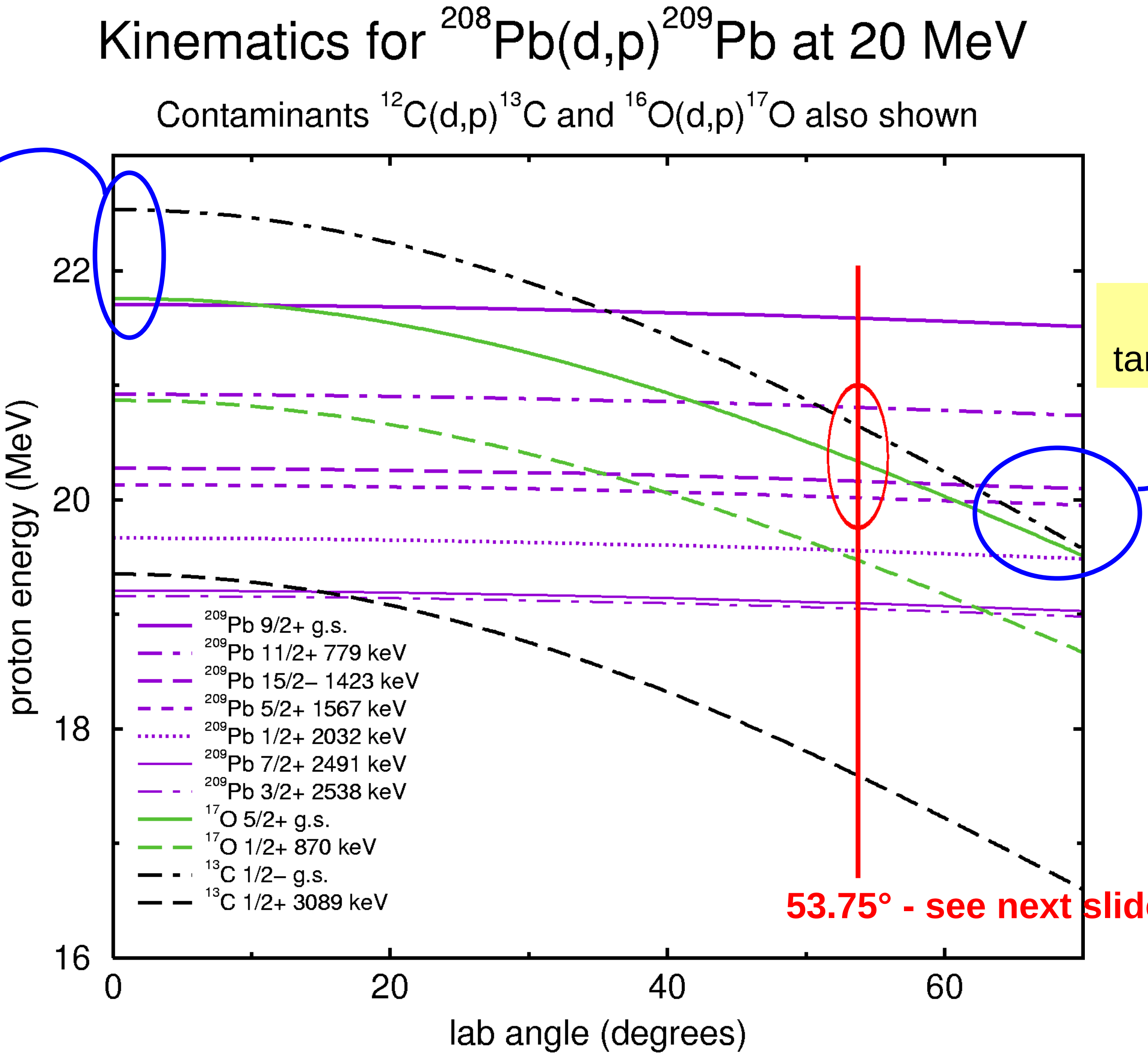
SINGLE PARTICLE STATES – SPLITTING



If we want to measure the SPE, splitting due to level mixing means that all components must be found, to measure the true single particle energy

Plot: John Schiffer

Neutron and Proton single-Particle States Built on 208 Pb



Kinematics for ^{208}Pb(d,p)^{209}Pb at 20 MeV

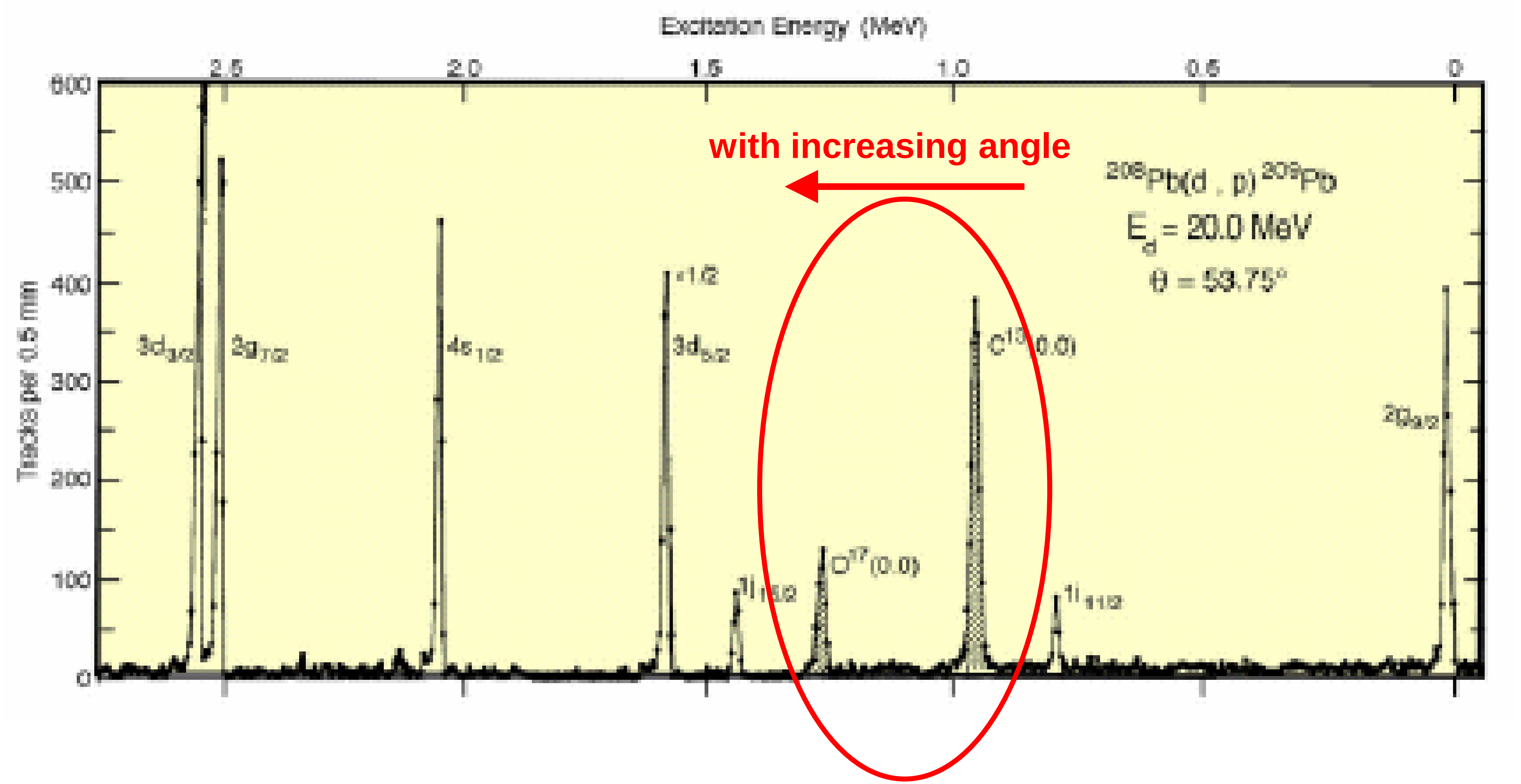
Contaminants ^{12}C(d,p)^{13}C and ^{16}O(d,p)^{17}O also shown

Different Q-values

Different target masses

53.75° - see next slide

Neutron and Proton Single-Particle States Built on ^{208}Pb

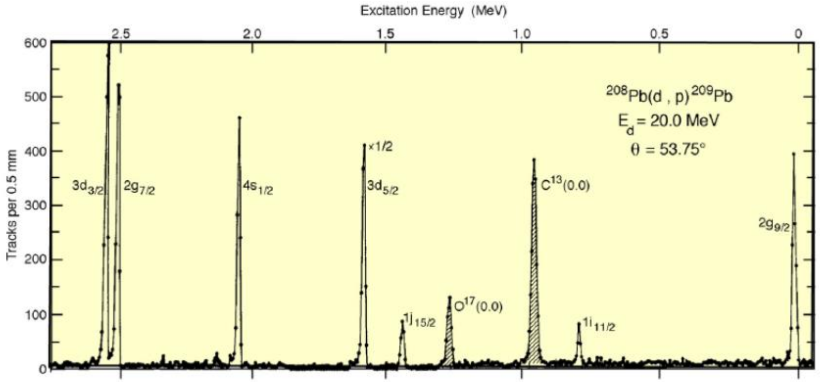


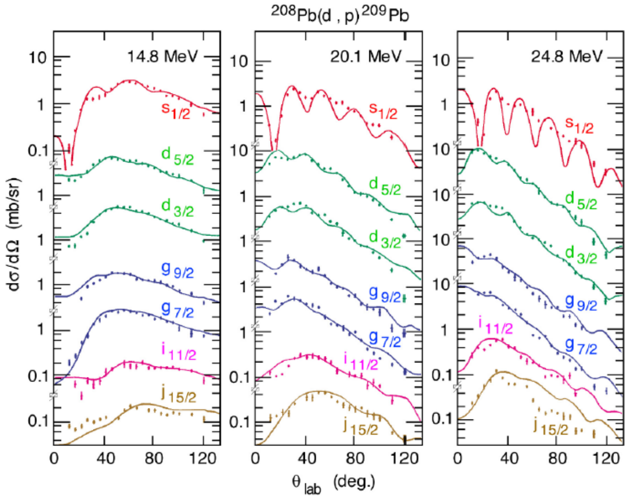
with increasing angle

John Schiffer, Argonne

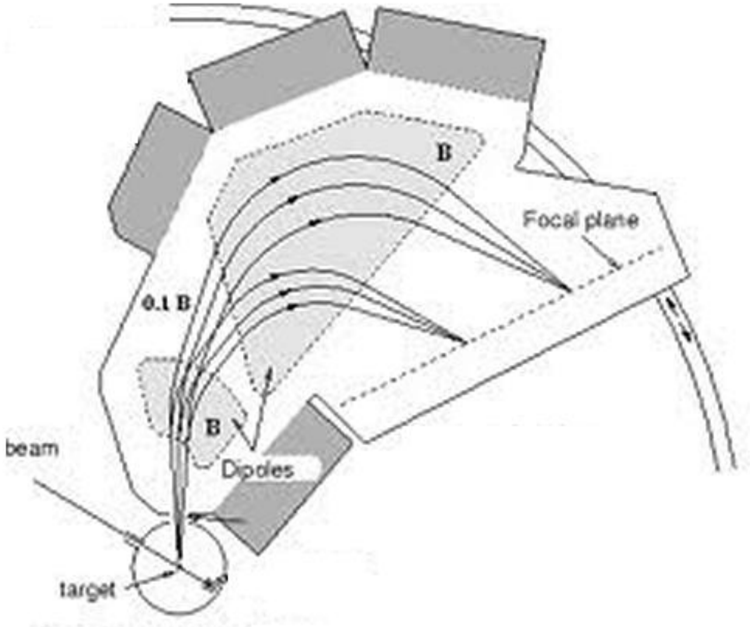
1950’s
1960’s

1967 ^{208}Pb(d,p)^{209}Pb





Muehllener et al.
Phys. Rev. 159, 1043 (1967)



Deuteron beam + target

Tandem + spectrometer

>10^{10} pps (stable) beam

Helpful graduate students

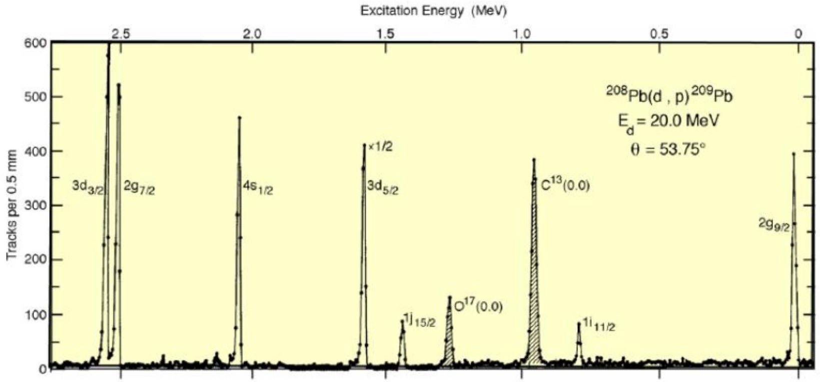
1950's
1960's

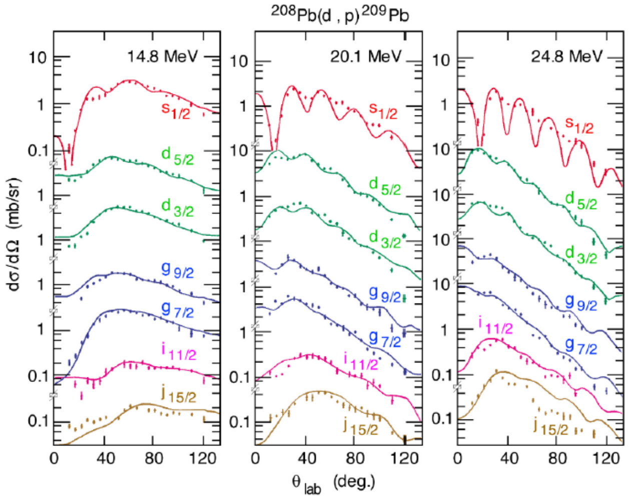
STABLE NUCLEI

RADIOACTIVE

1990's
2000's........

1967 ^{208}Pb(d,p)^{209}Pb

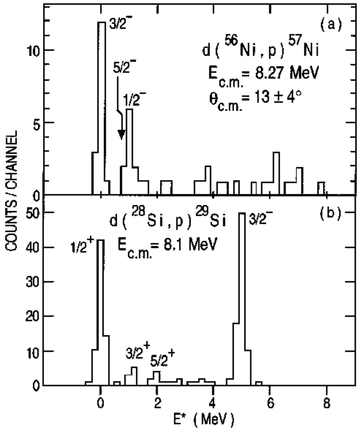


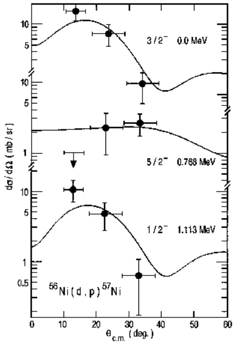


Muehllener et al.
Phys. Rev. 159, 1043 (1967)

1998 d(^{56}Ni,p)^{57}Ni

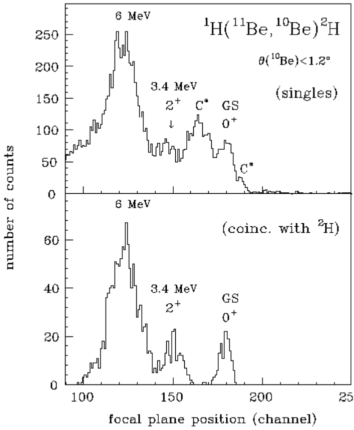
Rehm ARGONNE

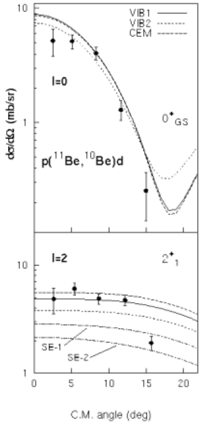




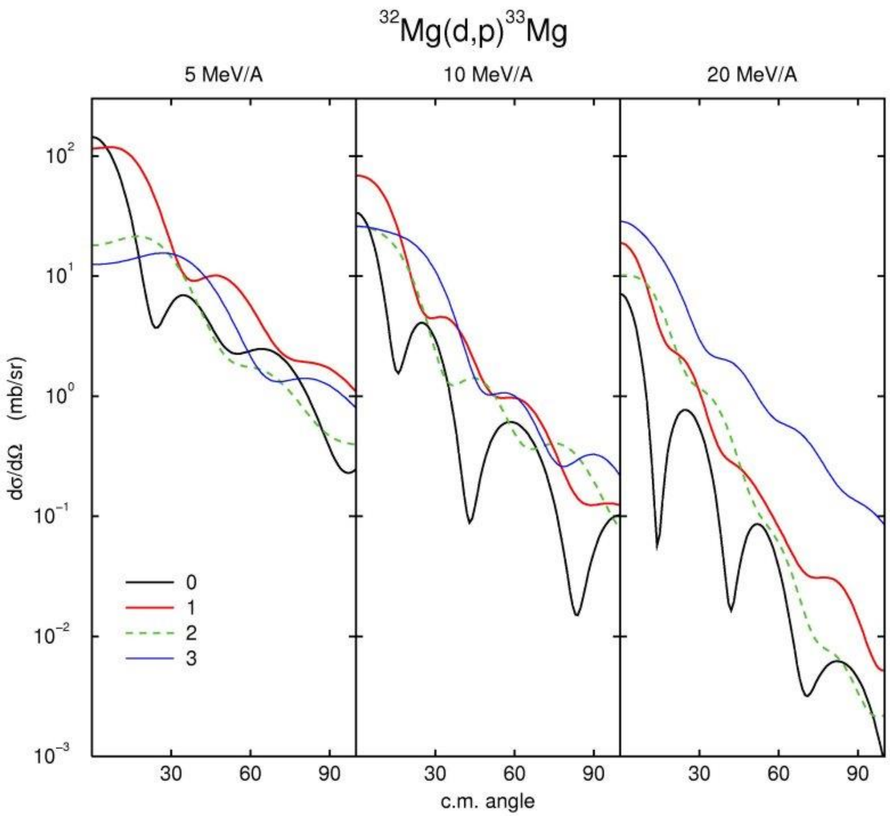
1999 p(^{11}Be,d)^{10}Be

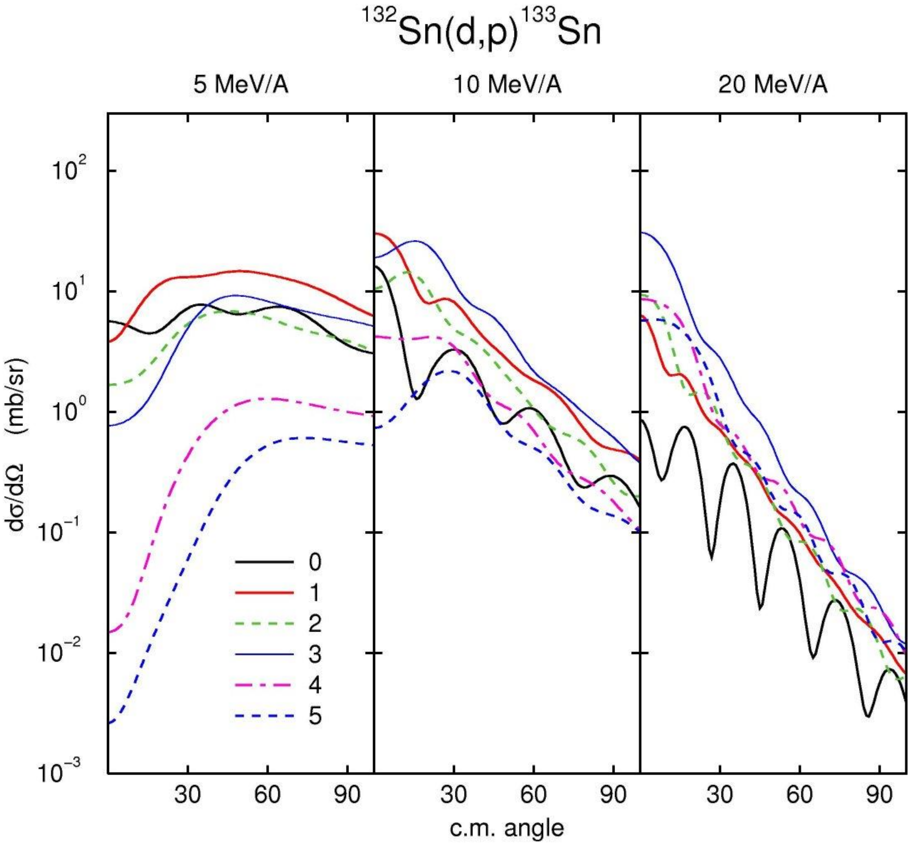
Fortier/Catford GANIL



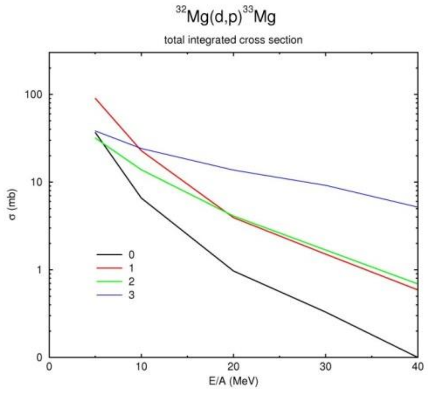


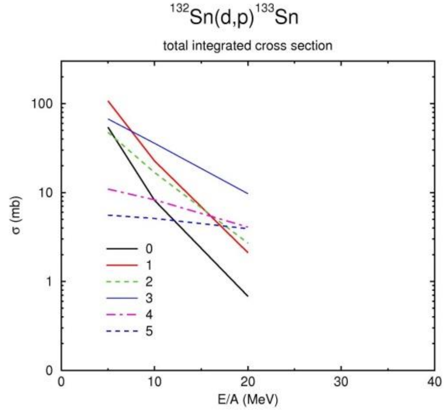
How does the differential cross section vary with beam energy ?





and the total cross section ?

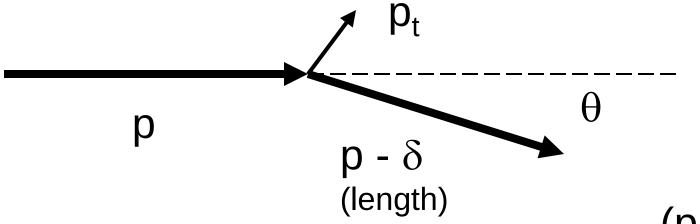




TIARA ★★★

W.N. CATFORD SURREY

Angular Momentum transfer



p_t

p

θ

$p - \delta$
(length)

Cosine rule, 2nd order:
$$\theta^2 = \frac{(p_t/p)^2 - (\delta/p)^2}{1 - (\delta/p)}$$

But $p_t \times R \geq \sqrt{\ell(\ell+1)}\ \hbar$ (R = max radius)

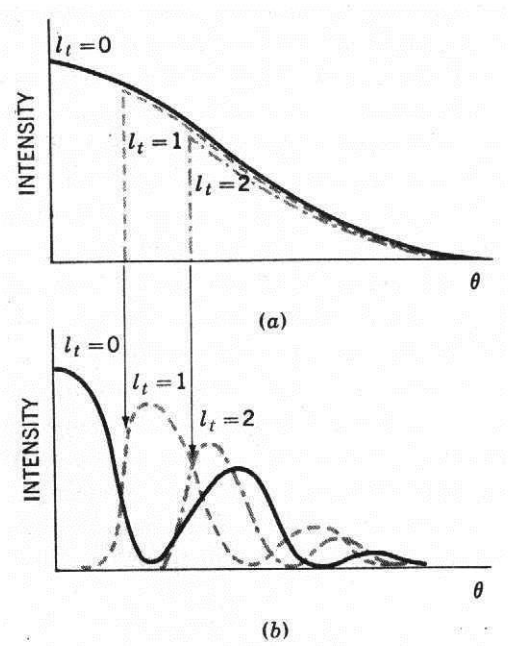
So
$$\theta^2 \geq \frac{\ell(\ell+1)\,\hbar^2 / p^2R^2 \quad - (\delta/p)^2}{1 - (\delta/p)}$$

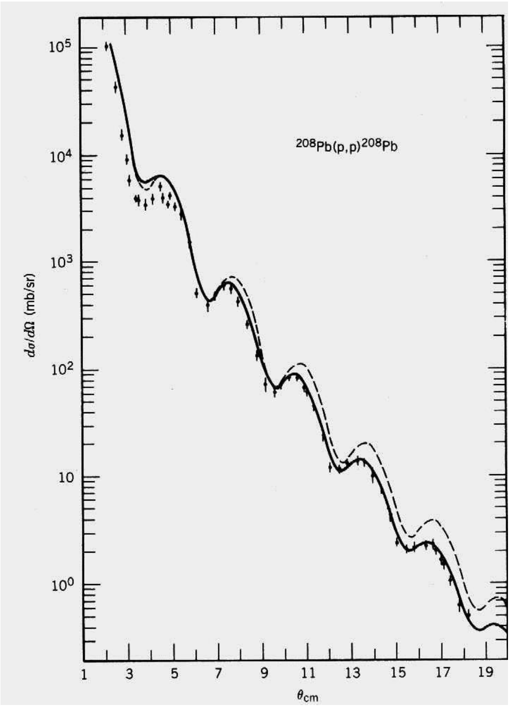
or $\theta \geq \text{const} \times \sqrt{\ell(\ell+1)}$

$$\boxed{\theta_{\min} \approx \text{const} \times \ell}$$

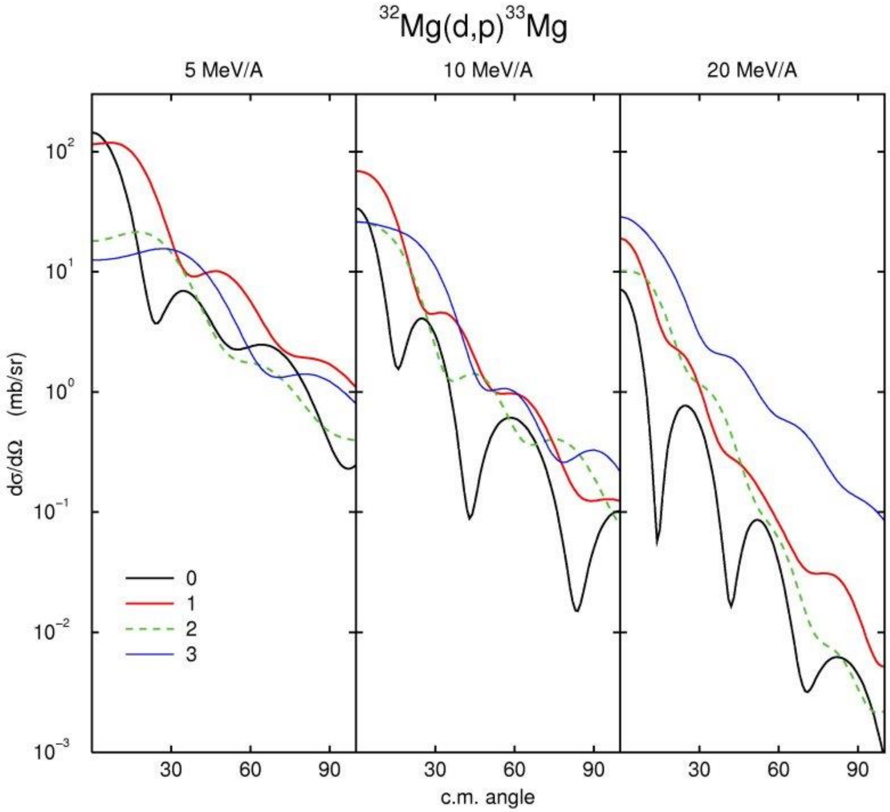
Diffraction structure also expected (cf. Elastics)

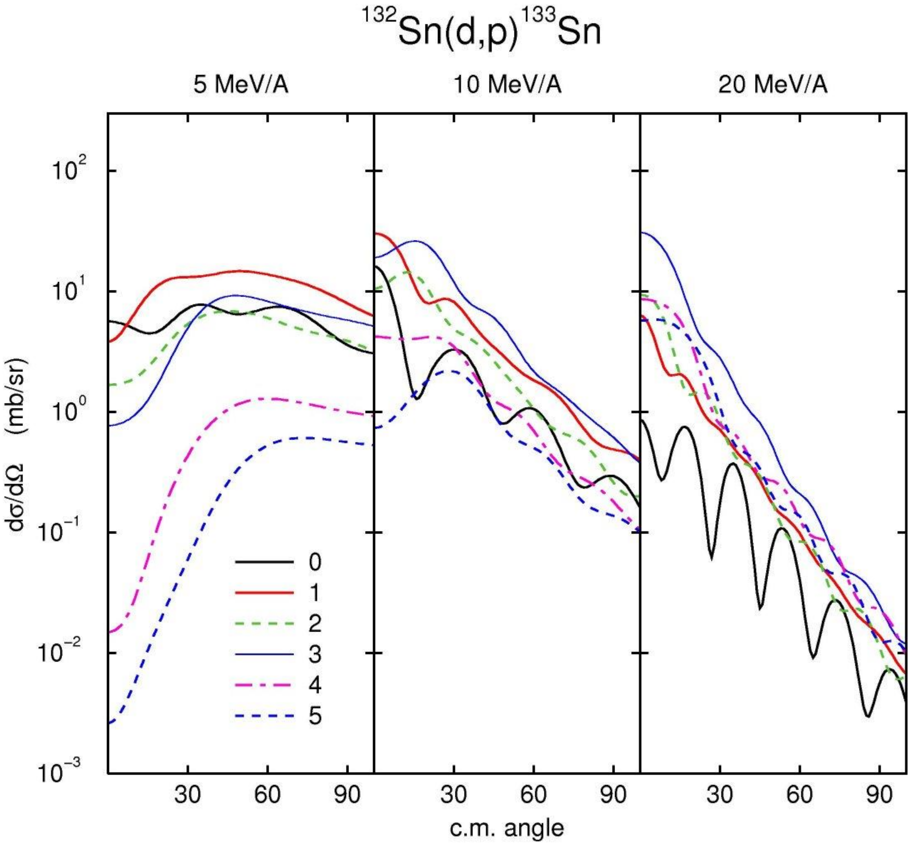
PWBA ⇒ spherical Bessel function, $\theta_{\text{peak}} \approx 1.4\ \sqrt{\ell(\ell+1)}$



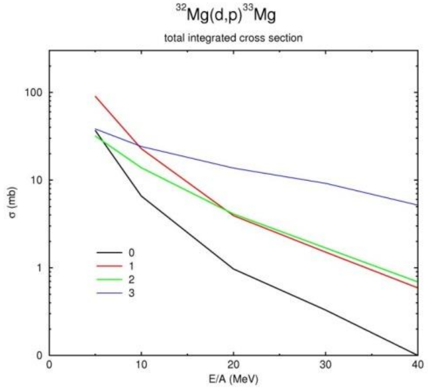


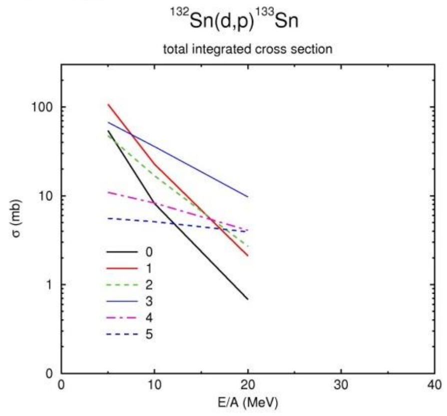
How does the differential cross section vary with beam energy ?





and the total cross section ?

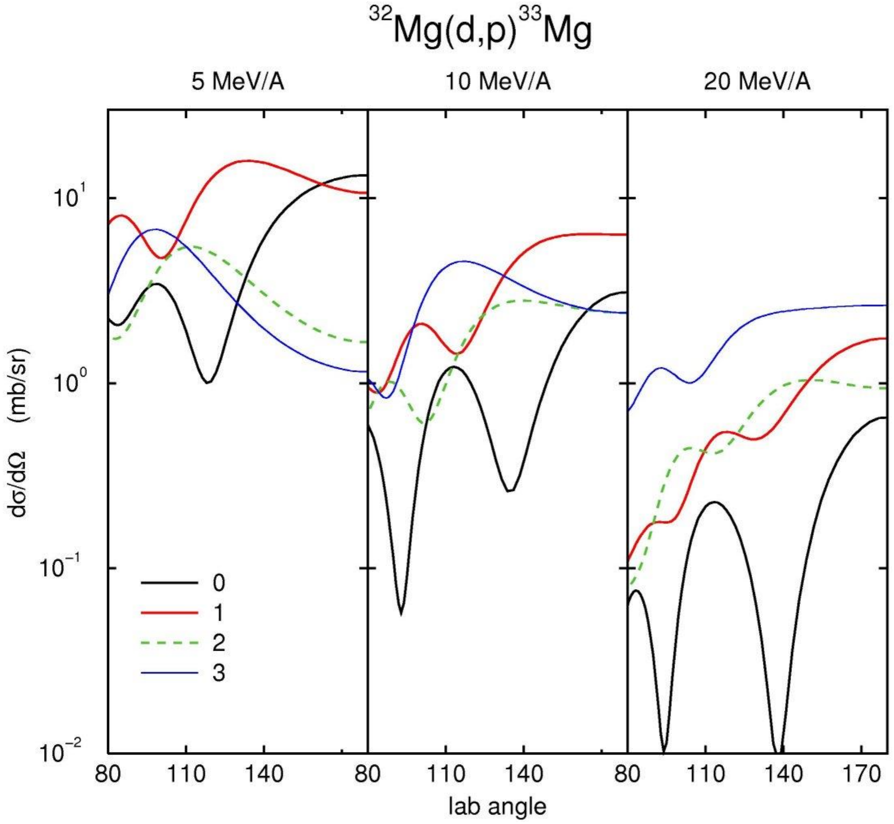


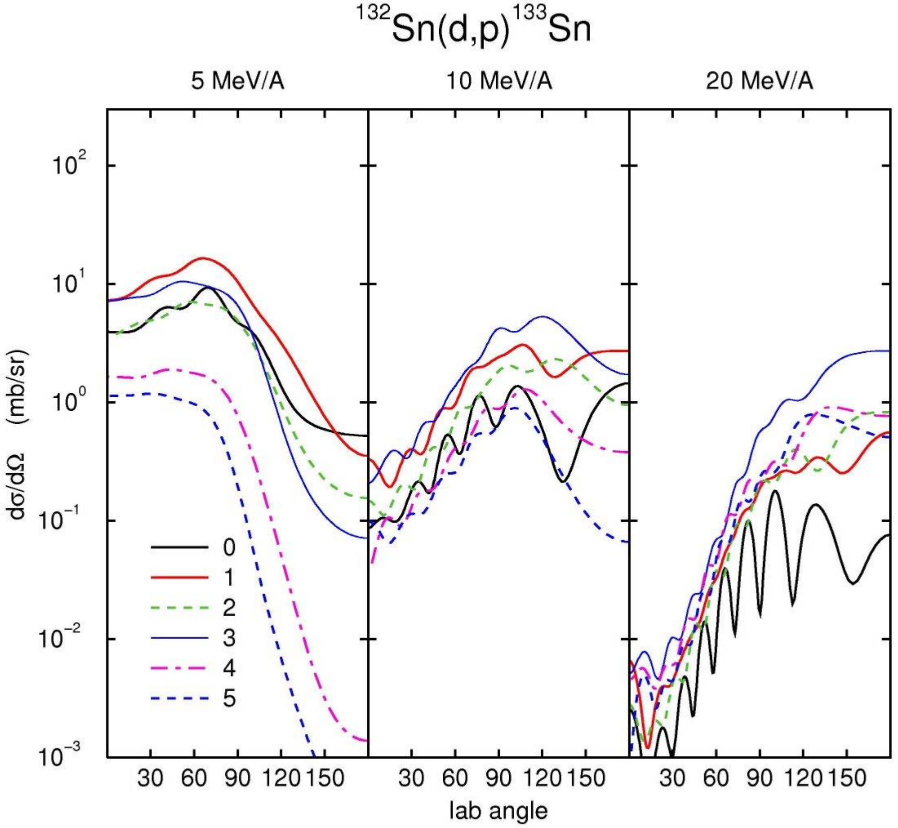


TIARA ★★★

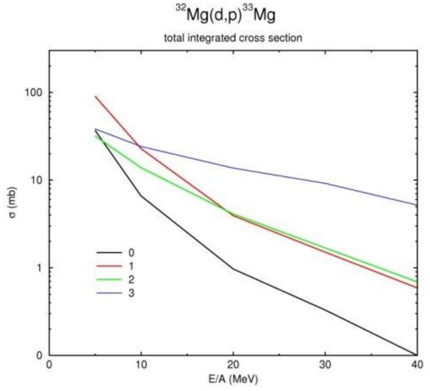
W.N. CATFORD SURREY

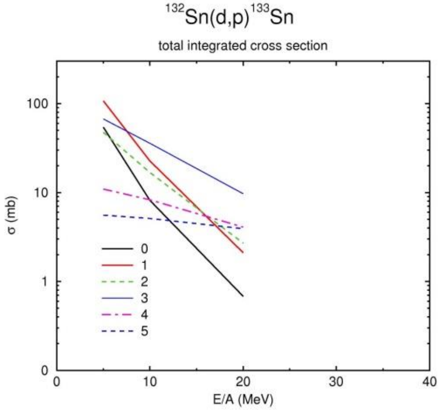
How does the differential cross section vary with beam energy ?





and the total cross section ?





Distorted Wave Born Approximation - Outline

e.g. (*d*,*p*) with a deuteron beam (following H.A.Enge Chap.13 with ref. also to N.Austern)

$H_{tot} = \Sigma T + \Sigma V$ ... in either entrance/exit ch

Entrance: $H_{tot} = T_{aA} + T_{xb} + V_{xb} + V_{xA} + V_{bA}$

Exit: $H_{tot} = T_{bB} + T_{xA} + V_{xb} + V_{xA} + V_{bA}$

Same in each case

→ But the final scattering state can be written approximately as an outgoing DW using the optical potential for the exit channel:

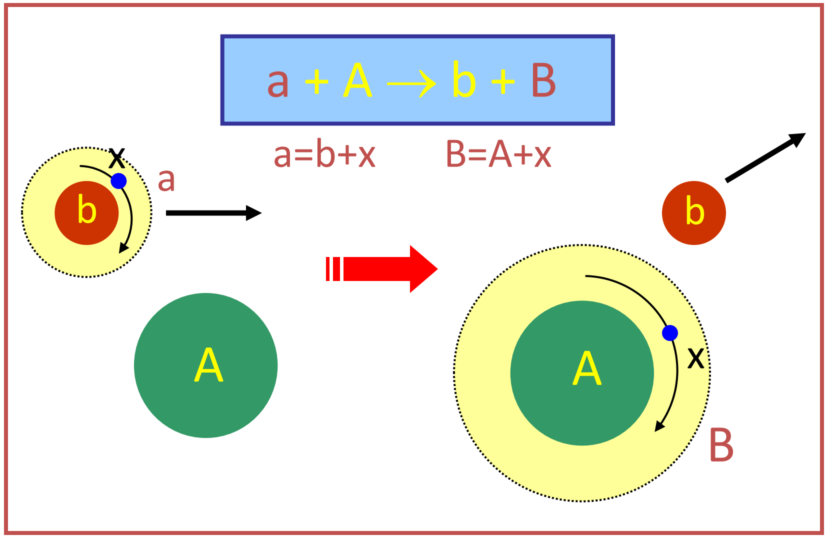
$$|\psi_f\rangle \approx \underbrace{|\phi_b\rangle \quad |\phi_B\rangle}_{\text{Internal wave functions}} \underbrace{\chi^{-}_{bB}}_{\text{outgoing distorted wave}}$$

In the optical model picture, $V_{xb} + V_{bA} \approx U_{bB}$ ($= V^{opt}_{bB} + i\, W^{opt}_{bB}$, the optical potential)

And the final state, we have said, can be approximated by an eigenstate of U_{bB}

Thus, the transition is induced by the interaction $V_{int} = H_{entrance} - H_{exit} = V_{xb} + \underbrace{V_{bA} - U_{bB}}_{\text{Remnant term}}$

≈ 0 if $x << A$



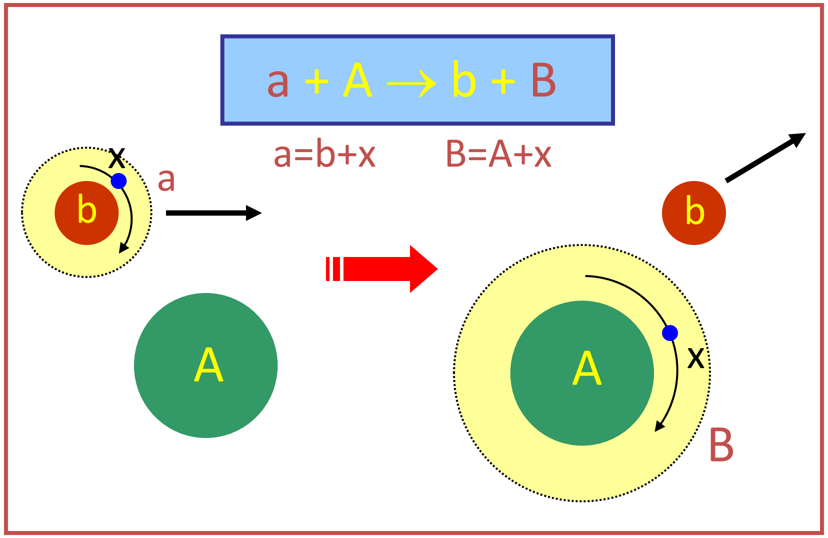
i.e. $V_{int} \approx V_{xb}$ which we can estimate reasonably well

$$T_{i,f}^{\,DWBA} = \langle\, \phi_b\, \phi_B\, \chi_{bB}^{-} \,|\, V_{xb} \,|\, \chi_{aA}^{+}\, \phi_a\, \phi_A \,\rangle$$

$= \phi_x\, \phi_A\, \psi_{rel,Ax}$ $\qquad$ $= \phi_x\, \phi_b\, \psi_{rel,bx}$

$$\text{so}\quad T_{i,f}^{\,DWBA} = \langle\, \underbrace{\psi_{rel,Ax}}_{\text{known as radial form factor for the transferred nucleon}}\, \chi_{bB}^{-} \,|\, V_{xb} \,|\, \chi_{aA}^{+}\, \underbrace{\psi_{rel,bx}}_{\text{often simple, e.g. if a = d}} \,\rangle$$

Distorted Wave Born Approximation - Outline 2



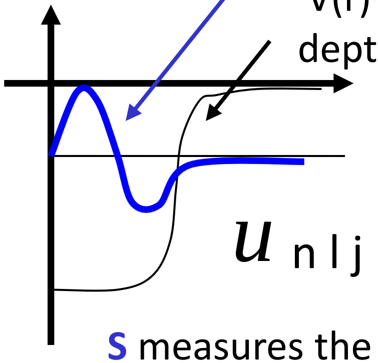
$$\text{so } T_{i,f}^{\text{DWBA}} = \langle \underbrace{\psi_{rel,Ax}\,\chi_{bB}^{-}} \,|\, V_{xb} \,|\, \chi_{aA}^{+}\,\psi_{rel,bx} \rangle$$

(compare Enge eq. 13-60)

The wave function of the transferred nucleon, orbiting A, inside of B

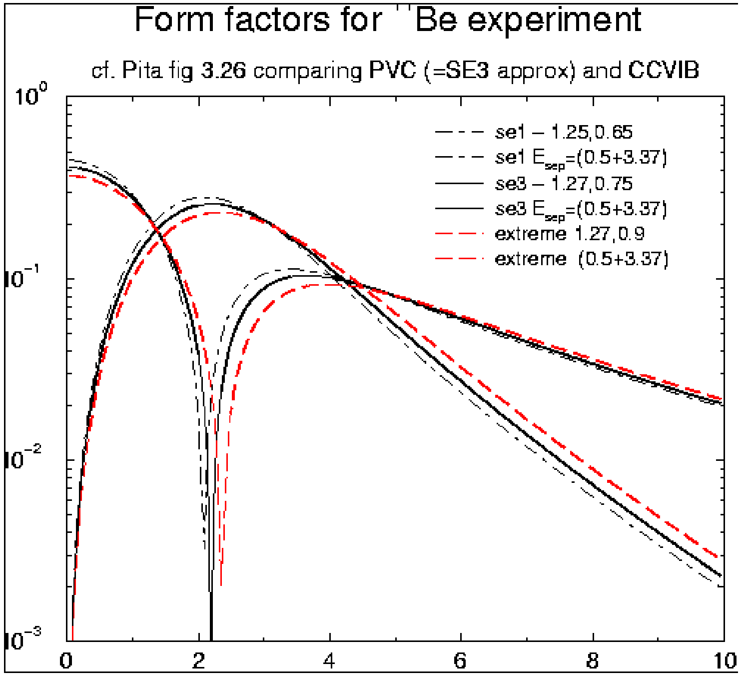
radial wave function u(r) given by $\psi(r) = u(r)/r$

V(r) given by Woods-Saxon; depth determined by known binding energy



$$u_{n\,l\,j} * \underbrace{S^{1/2}_{n\,l\,j}}$$

S measures the occupancy of the shell model orbital… the ***spectroscopic factor***



Radial wave functions in Woods-saxon potential with various geometries

Woods-Saxon:

$$V(r) = \frac{-V_0}{1 + e^{(r - r_0 A^{1/3})/a}}$$

Photographs of Distorted Waves

N. Austern
Direct Reactions

Beam of α’s on ^{40}Ca
18 MeV from left

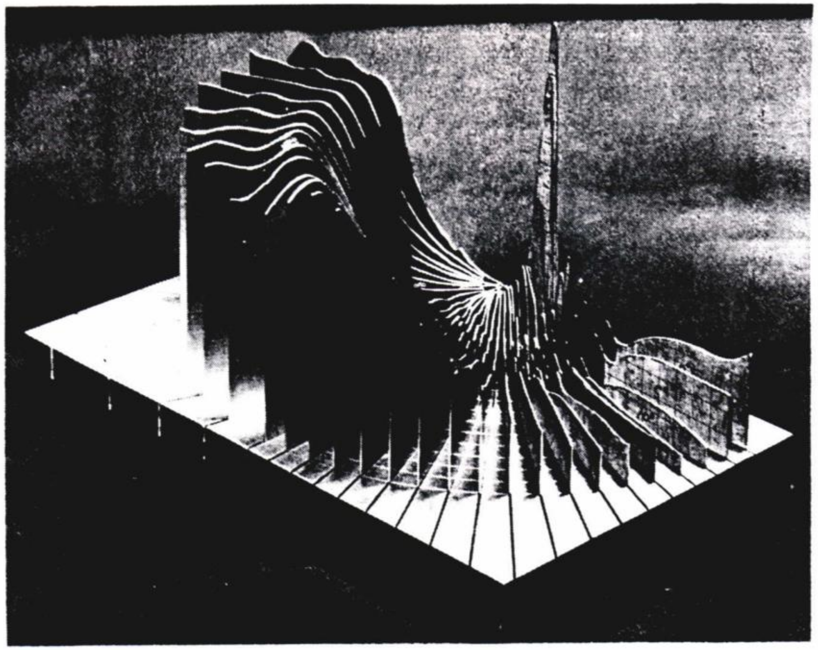


Fig. 7.1 Three-dimensional model of $|\chi^{(+)}|$, the modulus of the optical model wavefunction, for 18 MeV alpha particles bombarding Ca^{40}. The beam is incident from the left. The dark zone is the 10%–90% region of the optical potential. (Computed by R. M. Drisko and N. Austern, unpublished.)

Beam of p’s on ^{40}Ca
40 MeV from left

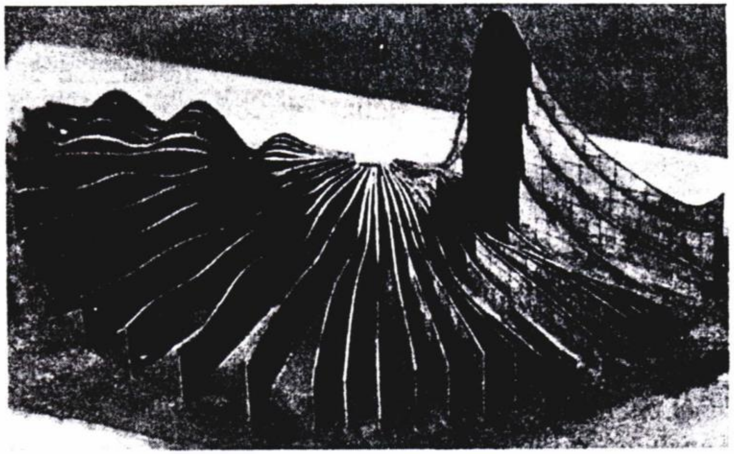


Fig. 7.2 This figure is the same as Fig. 7.1, for 40 MeV protons bombarding Ca^{40}.

SUM RULES FOR 1N TRANSFER

For adding a nucleon to a given j-shell the sum rule gives the vacancy in the shell

$$Number\ of\ Holes = \sum_i \left(\frac{2T_f^i+1}{2T_0+1}\right)\left(\frac{2J_f^i+1}{2J_0+1}\right)S_i$$

for removing a nucleon from a given j-shell it gives the occupancy of the shell, with the sum running over all final states *i*.

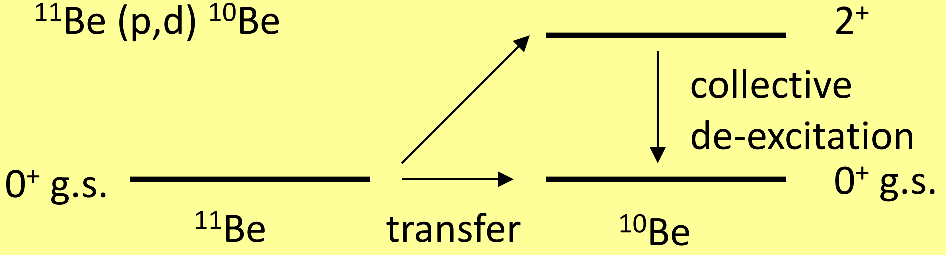
$$Number\ of\ Particles = \sum_i \left(\frac{2T_f^i+1}{2T_0+1}\right)S_i$$

Note that only one value of isospin $T_f(=T_0+1/2)$ is allowed for neutron adding or proton removing reactions, and two values $T_f(=T_0\pm1/2)$ for neutron removal or proton adding.

John Schiffer, Argonne

Some Illustrations of Complications in Transfer Calculations

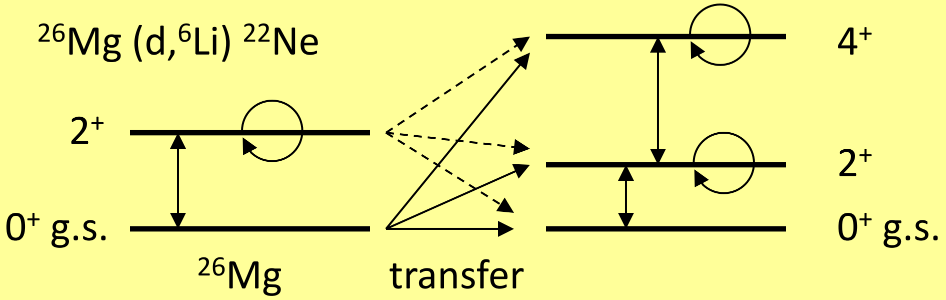
^{11}Be (p,d) ^{10}Be



Example of two-step

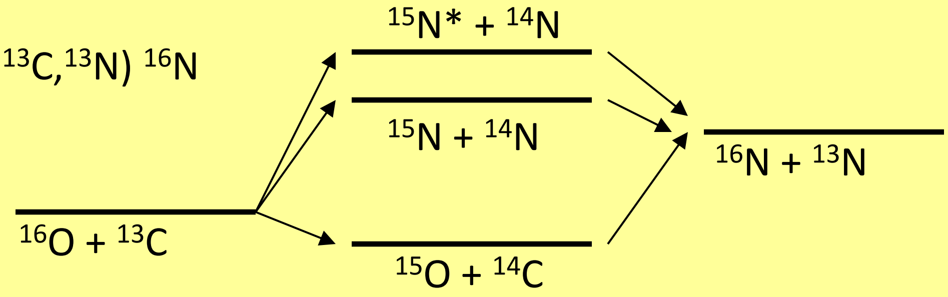
the two paths will interfere

^{26}Mg (d,^{6}Li) ^{22}Ne



Example of coupled channels

^{16}O (^{13}C,^{13}N) ^{16}N



Example of coupled reaction channels

REACTION MODEL FOR (d,p) TRANSFER – the ADWA

Johnson-Soper Model: an **alternative to DWBA** that gives a simple prescription for taking into account coherent *entangled* **effects of deuteron break-up** on (d,p) reactions [1,2]

- does not use deuteron optical potential – uses ***nucleon-nucleus optical potentials*** only
- formulated in terms of adiabatic approximation, which is sufficient but not necessary [3]
- uses parameters (overlap functions, **spectroscopic factors**, ANC's) just as in DWBA

[1] Johnson and Soper, PRC 1 (1970) 976

[2] Harvey and Johnson, PRC 3 (1971) 636; Wales and Johnson, NPA 274 (1976) 168

[3] Johnson and Tandy NPA 235 (1974) 56; Laid, Tostevin and Johnson, PRC 48 (1993) 1307

Spectroscopic Factor

Shell Model: overlap of **$|\psi\,(N+1)\,\rangle$ with $|\psi\,(N)\,\rangle_{core} \otimes\ n\,(\ell\ j)$**

Reaction: the observed yield is not just proportional to this S, because in T the **overlap integral** has a radial-dependent weighting or sampling

Many-body theory of $d + A(N,Z) \rightarrow B(N+1,Z) + p$

overlap integral

$$\phi_n^{BA}(\vec{r}_n) = \sqrt{N+1}\int d\xi_A \phi_B^*(\xi_A,\vec{r}_n)\phi_A(\xi_A)$$

spectroscopic factor

$$S^{AB} = \int d\vec{r}_n \mid \phi_n^{AB}(\vec{r}_n) \mid^2$$

$$T_{d,p} = \langle \chi_p^{(-)} \phi_n^{BA} \mid V_{np} \mid \Psi_{\vec{K}_d} \rangle$$

Hence the yield, and hence deduced spectroscopic factor, depends on the radial wave function and thus the geometry of the assumed potential well for the transferred nucleon, or details of some other structure model

REACTION MODEL FOR (d,p) TRANSFER – the ADWA

A **CONSISTENT** application of ADWA gives 20% agreement with large basis SM

JENNY LEE, M. B. TSANG, AND W. G. LYNCH

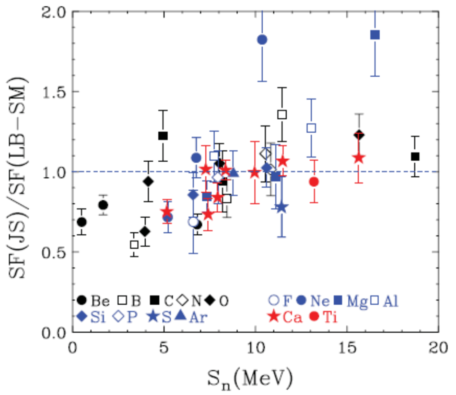


FIG. 12. (Color online) Ratios of SF(JS) values and the LBSM predicted SF values as a function of neutron separation energy (S_n). Open and closed symbols denote elements with odd and even Z, respectively. Only data with an overall uncertainties of less than 25% are included.

PHYSICAL REVIEW C 75, 064320 (2007)

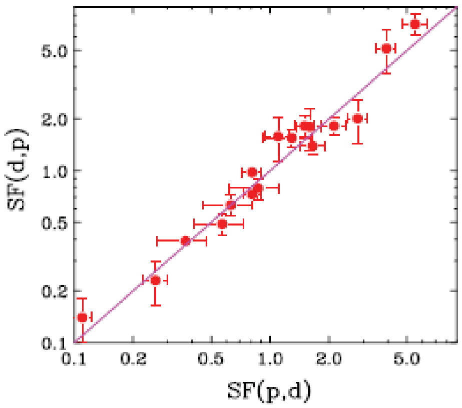


FIG. 8. (Color online) Comparison of spectroscopic factors obtained from (p, d) and (d, p) reactions as listed in Table II. Line indicates perfect agreement between the two values.

80 spectroscopic factors
Z = 3 to 24
Jenny Lee et al.

Tsang et al
PRL 95 (2005) 222501

Lee et al
PRC 75 (2007) 064320

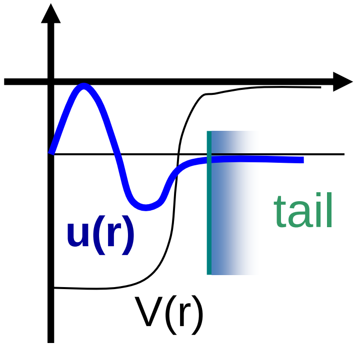
Delaunay at al
PRC 72 (2005) 014610

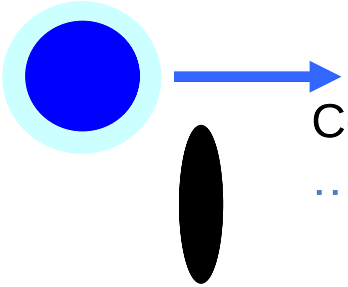
REACTION MODEL FOR (d,p) TRANSFER

Given what we have seen, **is transfer the BEST way** to isolate and study single particle structure and its evolution in exotic nuclei?

Transfer – decades of (positive) experience

Removal – high cross section, similar outputs, requires full orbitals





Complementary to (d,p)

…to be validated with (d,t)

(e,e’p) – a bit ambitious for general RIB application

(p,p’p) – more practical than (e,e’p) for RIB now, does have problems

YES

Also:

Heavy Ion transfer (^{9}Be),

^{3,4}He-induced reactions

Some Common Codes in Transfer Reaction Work

DWUCK - can be zero range or finite range

CHUCK - a coupled channels, zero range code

FRESCO - full finite range, non-locality, coupled channels,
coupled reaction channels, you-name-it code
(Ian Thompson, University of Surrey)

TWOFNR - includes an implementation of ADWA which is
very well suited to (d,p)… plus other options
(J. Tostevin, University of Surrey, on-line version)
(A. Moro, University of Seville, examples in this School)

Summary of single-nucleon transfer and knockout

Each of these processes can probe single-particle structure:
- measure the occupancy of single-particle (shell model) orbitals *(spectroscopic factors)*
- identify the angular momentum of the relevant nucleon.

Knockout has recently been developed specifically for radioactive beams (initially for haloes) and the nucleus being studied is the projectile. The removed nucleon may go anywhere.

Transfer was developed in the 1950's for stable beams (initially for p, d, t, ^{3}He, …) and the nucleus being studied was the target. The removed nucleon must transfer and "stick". With radioactive beams, the p, d, …etc., becomes the target, known as *inverse kinematics*

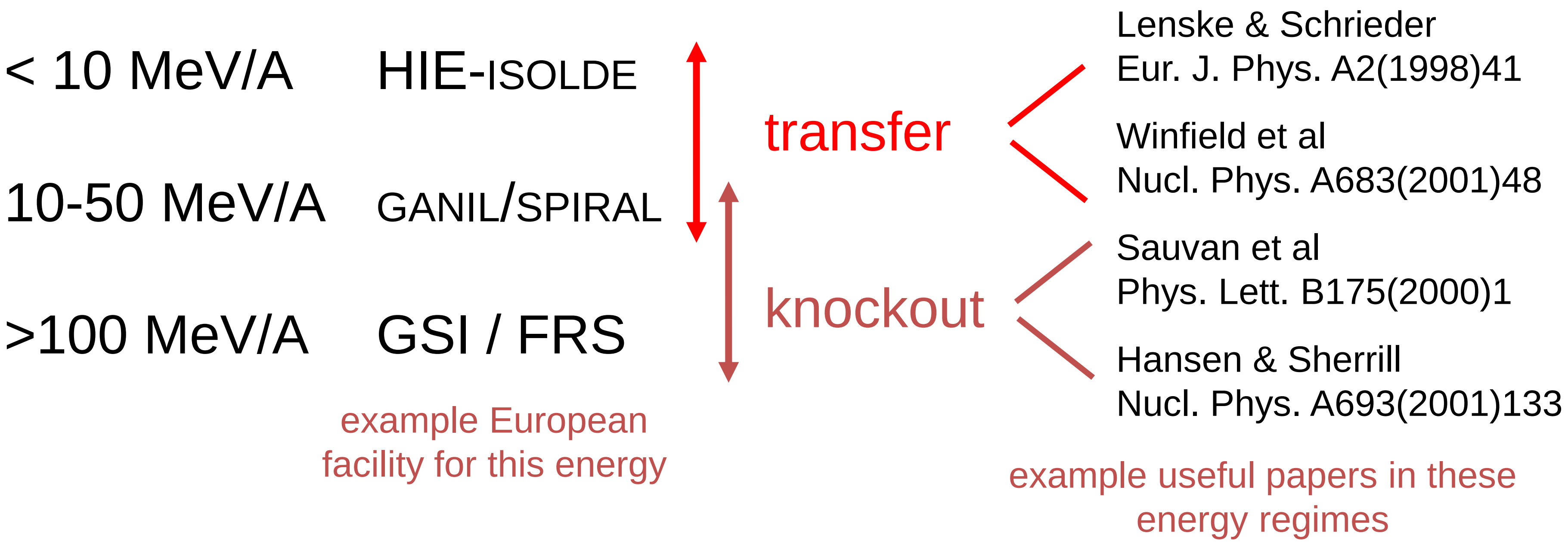
With knockout we can probe:
- occupancy of single-particle (shell model) orbitals in the projectile ground state
- identify the angular momentum of the removed nucleon
- hence, identify the s.p. level energies in odd-A nuclei produced from even-even projectiles

and the projectile-like particle is detected essentially at zero degrees

With transfer we can probe:
- occupancy of single-particle (shell model) orbitals in the original nucleus A ground state
  or distribution of s.p. strength in all final states of A–1 or A+1 nucleus
  that is, can add a nucleon to the original nucleus, e.g. by (d,p)
- identify the angular momentum of the transferred nucleon
- hence, identify the s.p. level energies in A–1 or A+1 nuclei produced from even-even nuclei
- identify the s.p. purity of coupled states in A–1 or A+1 nuclei produced from odd nuclei

and the scattered particle is detected, with most yield being at small centre-of-mass angles

Energy regimes of single-nucleon transfer and knockout



< 10 MeV/A HIE-ISOLDE

10-50 MeV/A GANIL/SPIRAL

>100 MeV/A GSI / FRS

example European facility for this energy

transfer

knockout

Lenske & Schrieder
Eur. J. Phys. A2(1998)41

Winfield et al
Nucl. Phys. A683(2001)48

Sauvan et al
Phys. Lett. B175(2000)1

Hansen & Sherrill
Nucl. Phys. A693(2001)133

example useful papers in these energy regimes

Intensity regimes of single-nucleon transfer and knockout

knockout

> 1 pps near drip-lines; $>10^3$ pps for more-bound projectiles
~ 100 mb near drip-lines, closer to 1 mb for more-bound

transfer

$>10^4$ pps is essentially the minimum possible
~ 1 mb cross sections typical

Some general observations for transfer reactions

The nucleon having to “stick” places kinematic restrictions on the population of states:

- the reaction Q-value is important (for Q large and negative, higher ℓ values are favoured)
- the degree (ℓ-dependent) to which the kinematics favour a transfer is known as ***matching***

Various types of transfer are employed typically, and using different mass probe-particles:

- **light-ion transfer reactions**: (probe$\leq \alpha$ say) … (d,p) (p,d) (d,t) (d,^{3}He) also (^{3}He,α) etc.
- **heavy-ion transfer reactions**: e.g. (^{13}C,^{12}C) (^{13}C,^{14}C) (^{17}O,^{16}O) (^{9}Be,^{8}Be)
- **two-nucleon transfer**: e.g. (p,t) (t,p) (^{9}Be,^{7}Be) (^{12}C,^{14}C) (d,α)
- **alpha-particle transfer** (or α-transfer): e.g. (^{6}Li,d), (^{7}Li,t), (d,^{6}Li), (^{12}C,^{8}Be)

Light-ion transfer reactions with Radioactive Beams

Light-ion induced reactions give the clearest measure of the transferred ℓ,
and have a long history of application in experiment and refinement of the theory

Thus, they are attractive to employ as an essentially reliable tool, as soon as radioactive beams of sufficient intensity become available (i.e. NOW)

To the **theorist**, there are some new aspects to address, near the drip lines.

To the **experimentalist**, the transformation of reference frames is a much bigger problem!

The new experiments need a hydrogen (or He) nucleus as target the beam is much heavier.
This is inverse kinematics, and the energy-angle systematics are completely different.

A PLAN for how to STUDY STRUCTURE

- Use **transfer reactions** to identify strong single-particle states, measuring their spins and strengths

- Use the energies of these states to compare with theory

- Refine the theory

- Improve the extrapolation to very exotic nuclei

- **Hence learn the structure of very exotic nuclei**

N.B. The **shell model** is arguably the best theoretical approach
for us to confront with our results, but it’s **not the only one**.
The experiments are needed, no matter which theory we use.

N.B. Transfer (as opposed to knockout) allows us to study orbitals
that are empty, so **we don’t need** quite such exotic beams.