ISOLDE Nuclear Reaction and Nuclear Structure Course

An experimental view of elastic and inelastic scattering: kinematics

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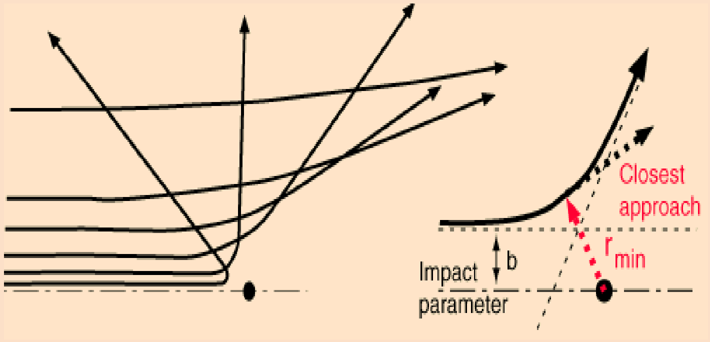
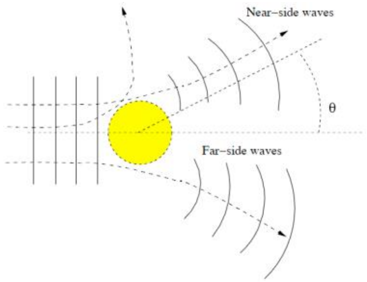
Rutherford scattering: η>>>1

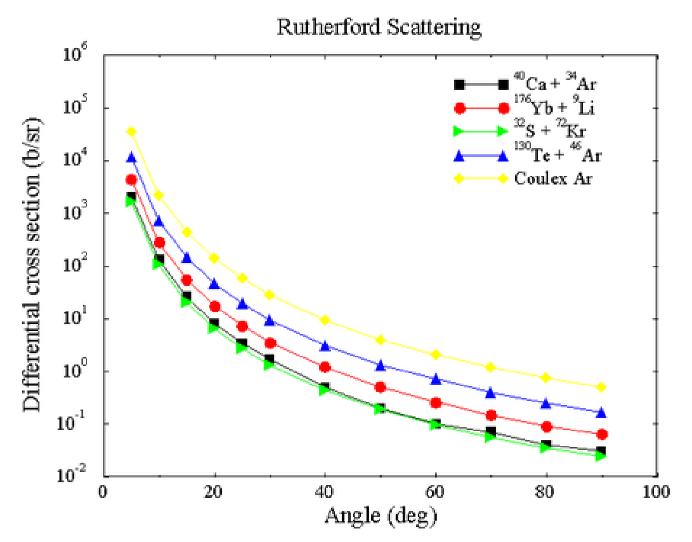
Scattering angle $\theta_{c.m.}$ related to distance of closest approach.

$$d(\theta) = \frac{2b}{\cot\left(\frac{\theta}{2}\right)}$$





- •Pure Coulomb potential
- •E<< Coulomb barrier
- **•No nuclear effects**



Rutherford Cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{zZe^2}{4E}\,\frac{1}{\sin^4(\theta/2)}$$

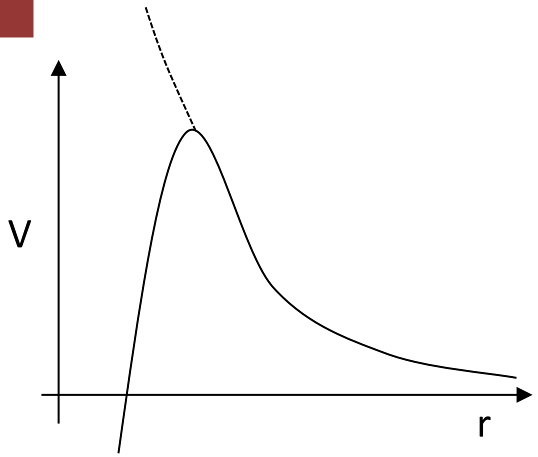
Coulomb
or Sommerfeld parameter

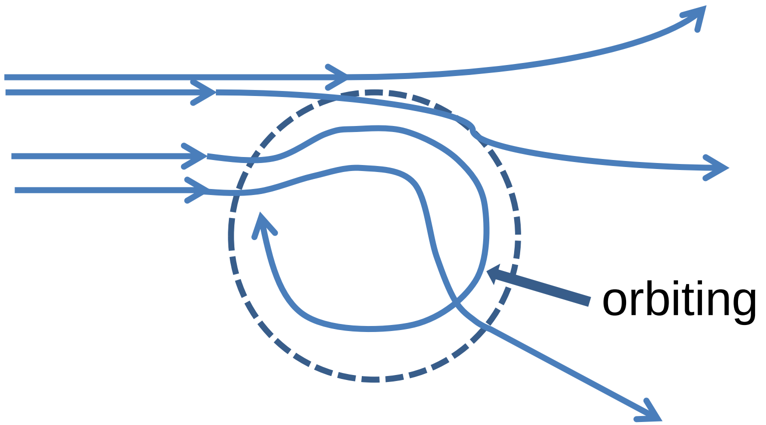
$$\eta = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar v}$$

Rutherford scattering very useful to normalise cross-sections and solid angle determination

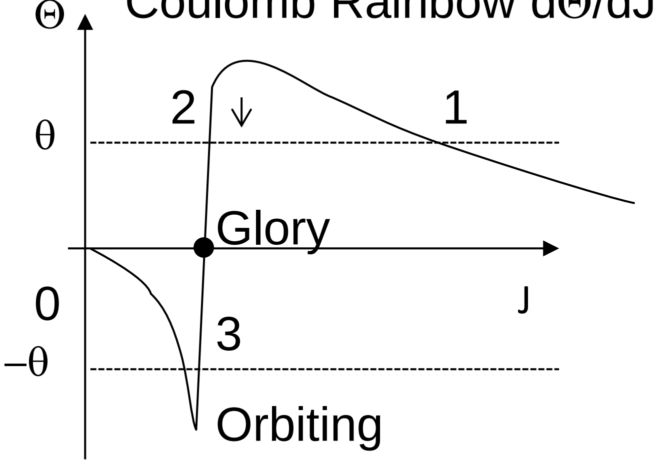
Effect of repulsive Coulomb+ attactive nuclear potential. $E_{cm}>V_C$ and NO ABSORPTION→V(r) real!

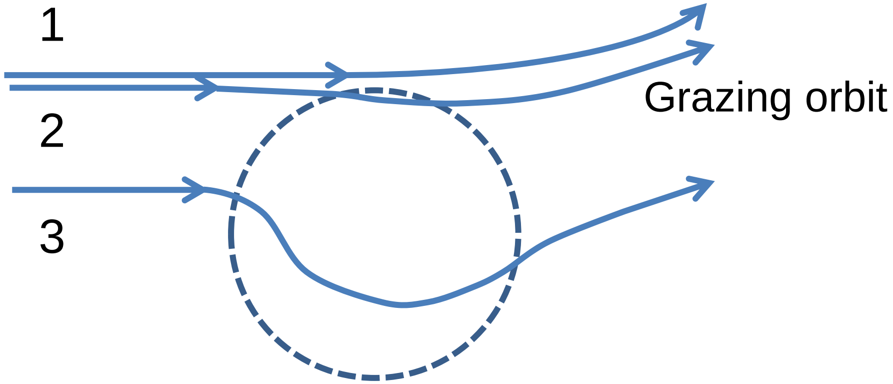
Example of classical trajectories for potential V(r) →





Coulomb Rainbow dΘ/dJ=0



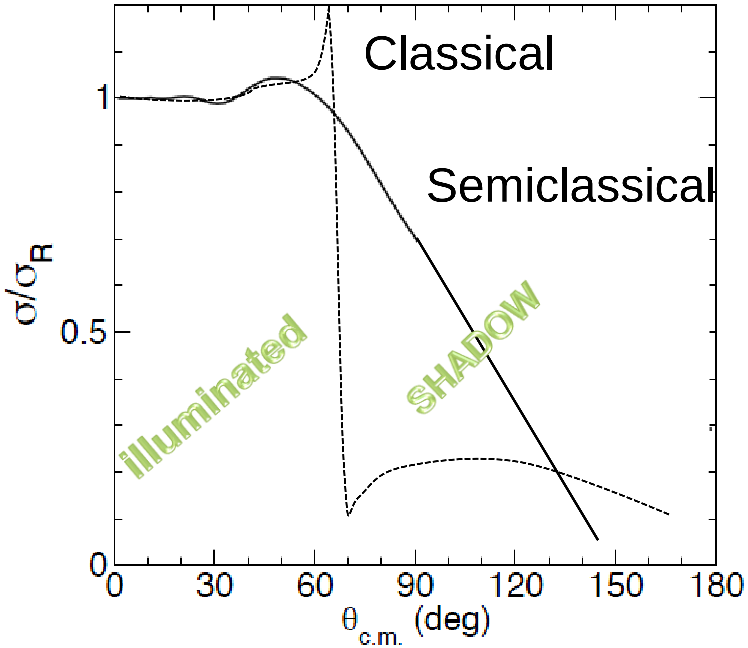


$$\frac{d\sigma}{d\Omega}=\frac{b}{\sin(\theta)\frac{d\Theta}{db}}$$

Singularities in dσ/dΩ
J=pb

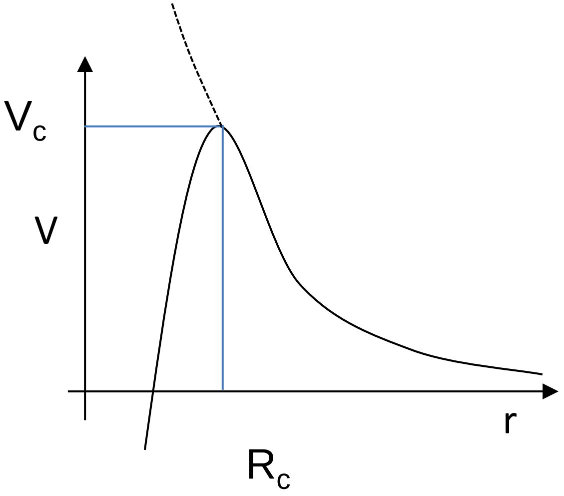
Trajectories 1,2 and 3 emerge with the same scattering angle.

Angular distribution with respect to Rutherford



The oscillations are caused by interference between the contributions from the various orbits which result in the same scattering angle

Grazing collisions



Semiclassically:

For $b>R_c$ → Coulomb trajectories (illuminated region)

For $b<R_c$ → Nuclear interaction (shadow region)

$$V_c = \frac{Z_1 Z_2 e^2}{R_c}$$

$$R_c = r_{0c}(A_1^{1/3} + A_2^{1/3})$$

In the limiting case of grazing collisions ($D=R_c$)
we obtain the corresponding Coulomb scattering angle θ_{gr}

$$\sin\frac{\theta_{gr}}{2} = \frac{\eta}{kR_c - \eta} = \frac{\varepsilon_c}{2\varepsilon - \varepsilon_c}$$

$$\varepsilon_c = \frac{V_c}{\mu}$$ μ= reduced mass

$$\varepsilon = \frac{E_{lab}}{A_1}$$

Knowing the grazing angle gives an idea about the angular region good for cross-section normalisation and measurement.

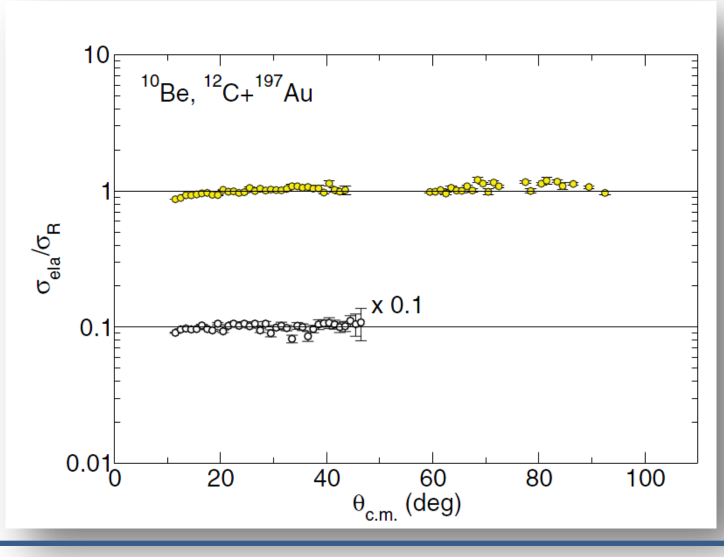
Some reading: R. Bass
Nuclear reaction with heavy ions
Springer Verlag
and
G.R. Satchler
Introduction to Nuclear reactions
Ed. Macmilar

How to use Rutherford cross-section to determine solidangles of detection set-up.

We use elastic scattering on some heavy target (e.g. Au) at sub-barrier energy where the elastic cross-section follows the Rutherford behaviour.

$$\frac{\frac{d\sigma_{ela}(\theta)}{d\Omega}}{\frac{d\sigma_{Ruth}(\theta)}{d\Omega}}=1$$

at all angles



$$\frac{d\sigma_{el}(\theta)}{d\Omega}=\frac{\boxed{I(\theta)}}{\boxed{K}\Delta\Omega}$$

Integral of elastic peak

$N_i \times N_t \times T$=normalisation constant

$$\frac{\frac{d\sigma_{el}(\theta)}{d\Omega}}{\frac{d\sigma_{Ruth}(\theta)}{d\Omega}}=\frac{I}{K\boxed{\Delta\Omega}\frac{d\sigma_{Ruth}(\theta)}{d\Omega}}$$

One can simulate the set-up and by equalising K at all angles one gets the correct detector solidangles

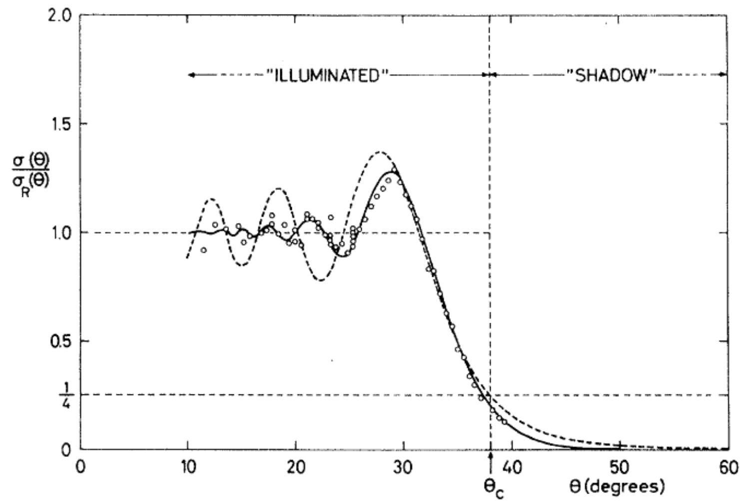
Rutherford cross-section used to normalise cross-section

If the elastic cross-section is Rutherford only in a very limited angular range by placing detectors at those angles one can get the normalisation constant *K* once the solid-angles are known.

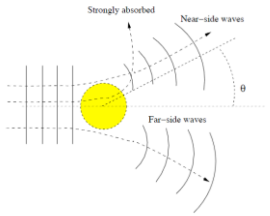
$$\frac{\dfrac{d\sigma_{ela}(\theta)}{d\Omega}}{\dfrac{d\sigma_{Ruth}(\theta)}{d\Omega}} = 1$$ for $\theta < \theta_1$ (an estimate of θ_1 can be done from the grazing angle)

$$\frac{\dfrac{d\sigma_{el}(\theta)}{d\Omega}}{\dfrac{d\sigma_{Ruth}(\theta)}{d\Omega}} = \frac{I}{\underset{\Downarrow}{\boxed{K}}\,\Delta\Omega\,\dfrac{d\sigma_{Ruth}(\theta)}{d\Omega}}$$

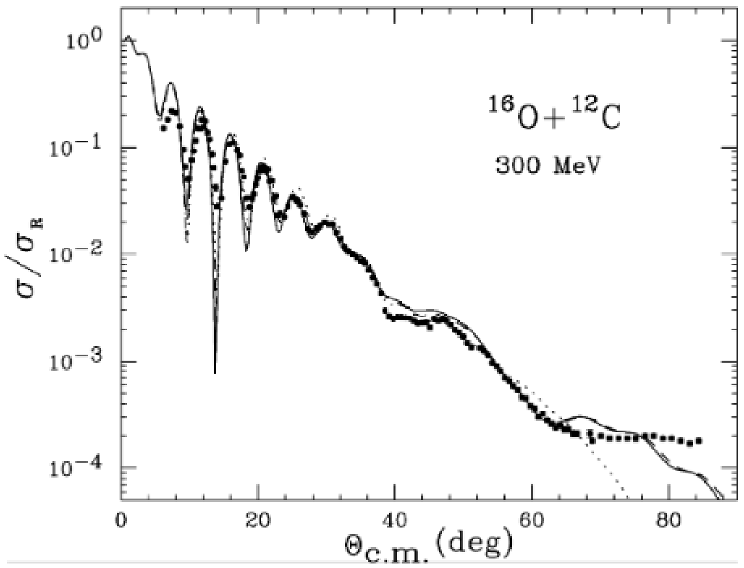
The only unknown quantity



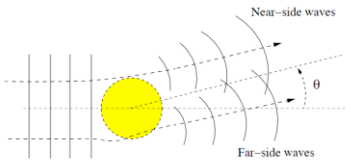
Fresnel scattering: η>>1



- Strong Coulomb potential
- E≈ Coulomb barrier
- “Illuminated” region → interference (Coulomb-nuclear)
- “Shadow region” → strong absorption



Fraunhofer scattering: η≤1

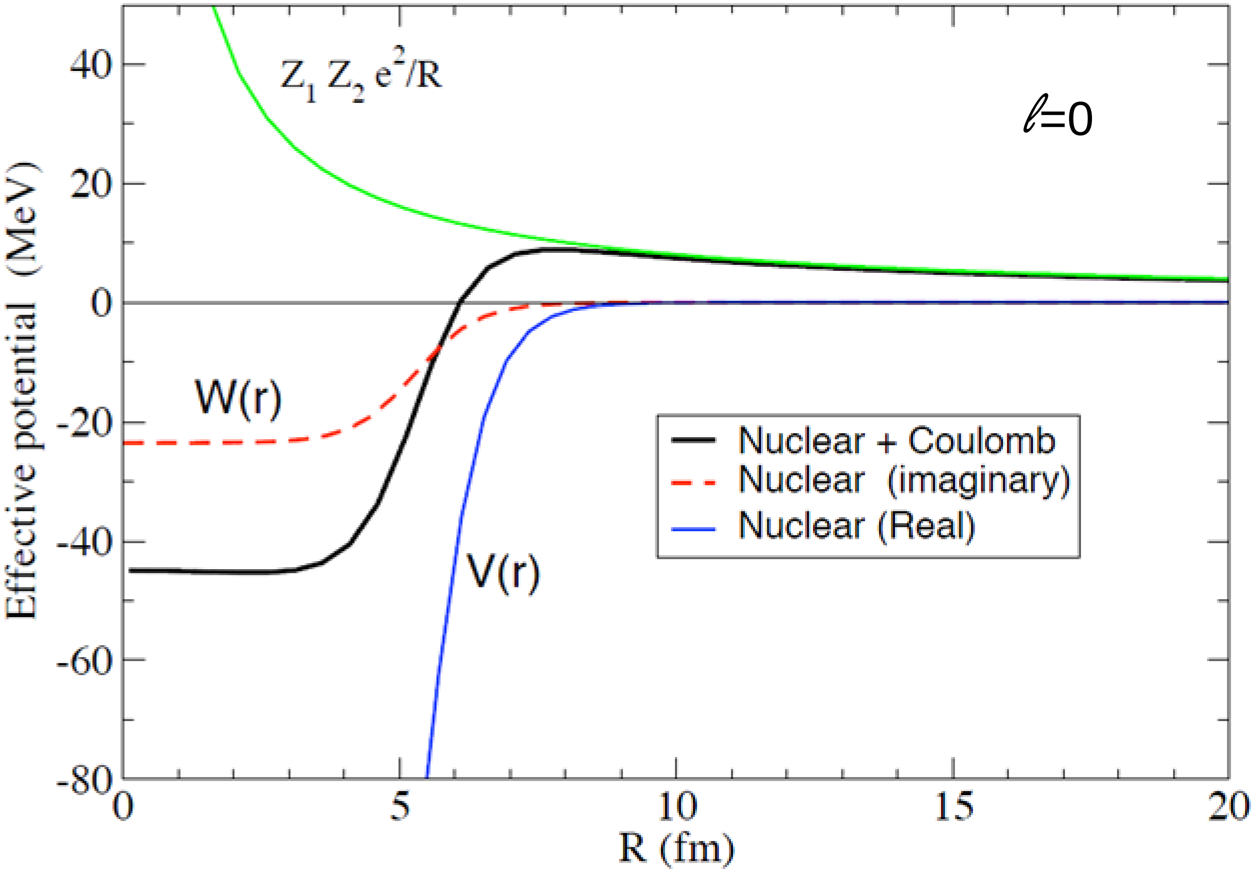


- Weak Coulomb
- E> Coulomb barrier
- Near-side/far-side interference (diffraction)

Oscillations in angular distribution→
good angular resolution required

Which information can be gathered from elastic scattering measurement?
Simple model: Optical Model→ structureless particles interacting via an effective potential (see A.M.Moro lectures).

Optical potential: $V(r)=V_C(r)+V_\ell(r)+V_N(r)+iW(r)$



from A.M.Moro

Which information can we obtain from elastic scatting meaasurement?

Total reaction cross-section:

$$\sigma_{abs} = \sum_{\beta\neq\alpha}\sigma_{\beta} = \pi\bar{\lambda}_{\alpha}^{2}\sum_{l}(2l+1)\sum_{\beta\neq\alpha}(1-\boxed{|S_{l\alpha}|^{2}})$$

Scattering matrix

Optical theorem for uncharged particles:

$$\sigma_{tot} = \sigma_{ela} + \sigma_{abs}$$

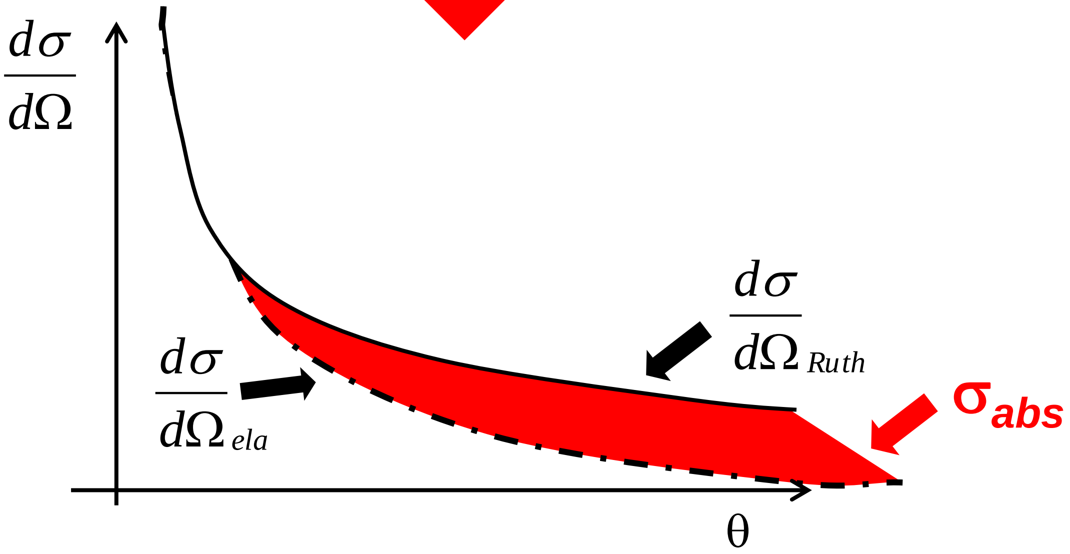
Modified optical theorem for charged particles:

$$\sigma_{tot} \rightarrow \infty \quad \text{for } \theta=0$$

$$[\sigma_{ela} - \sigma_{Ruth}] + \sigma_{abs} = 4\pi\bar{\lambda}_{\alpha}\ \mathrm{Im}\ f_{N}(\vartheta = 0)$$

In the presence of strong absorption:

$$\sigma_{abs} = \sigma_{Ruth} - \sigma_{ela}$$

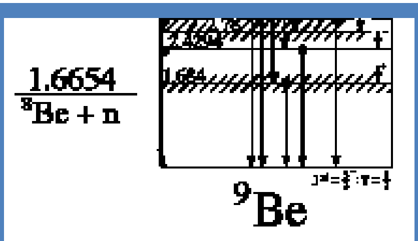


The difference between elastic and Rutherford cross-section gives the **total reaction cross-section**.

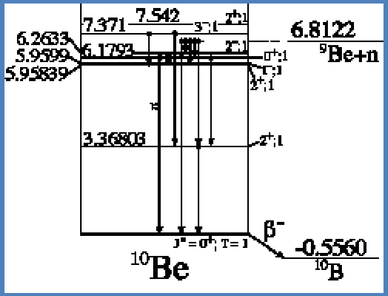
Effect of nuclear structure on elastic scattering

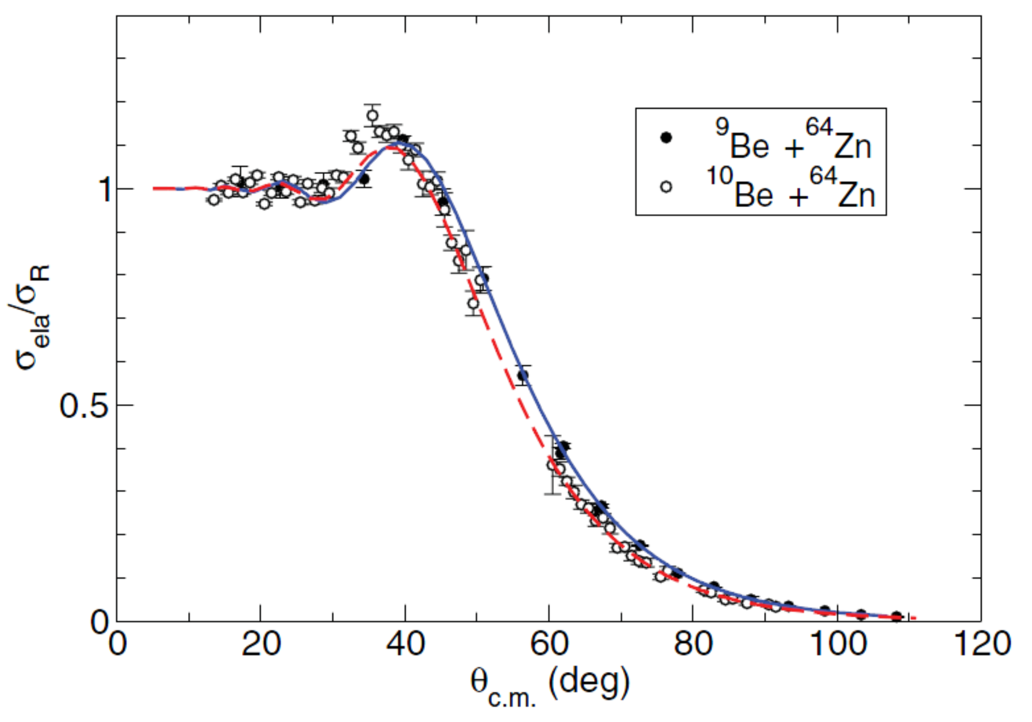
^{9,10}Be+^{64}Zn elastic scattering angular distributions @ 29MeV

^{9}Be
Sn=1.665 MeV



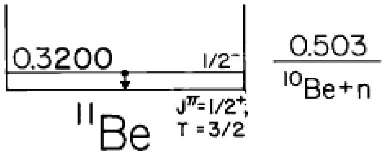
^{10}Be
Sn=6.88 MeV

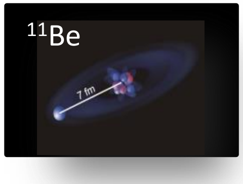




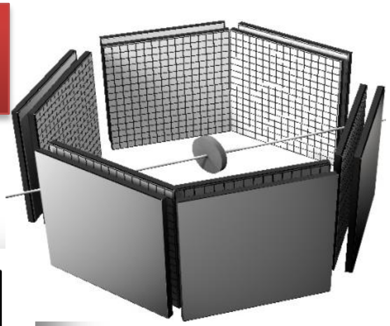
A. Di Pietro et al. Phys. Rev. Lett. 105,022701(2010)

Elastic scattering angular distributions @ 29MeV



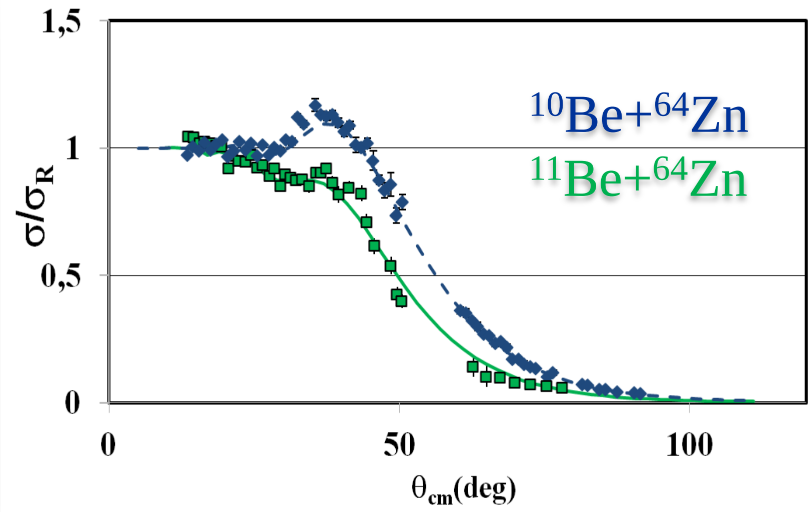


^{10,11}Be+^{64}Zn
@ Rex-Isolde, CERN



OM analysis adopted procedure:

- ➢volume potential responsible for the core-target interaction obtained from the ^{10}Be+^{64}Zn elastic scattering fit.
- ➢plus a complex surface DPP having the shape of a W-S derivative with a very large diffuseness.
- ➢Very large diffuseness: ai= 3.5 fm similar to what found in A.Bonaccrso NPA 706(2002)322



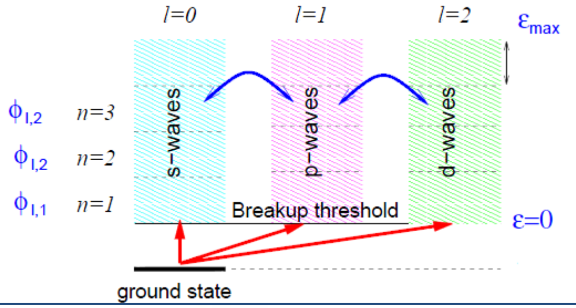
Reaction cross-sections

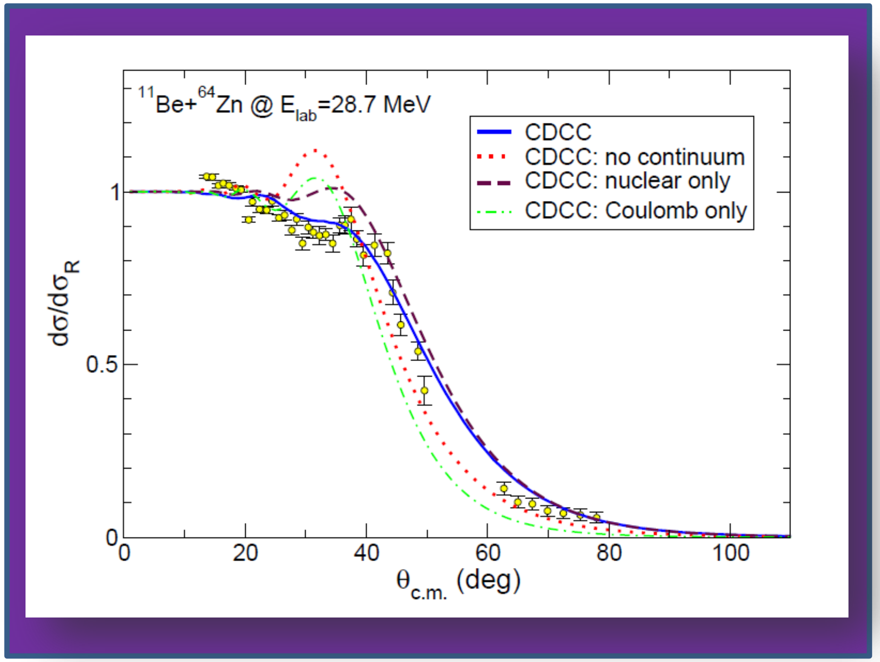
σ_R ^{9}Be≈1.1b σ_R ^{10}Be ≈1.2b **σ_R ^{11}Be ≈2.7b**

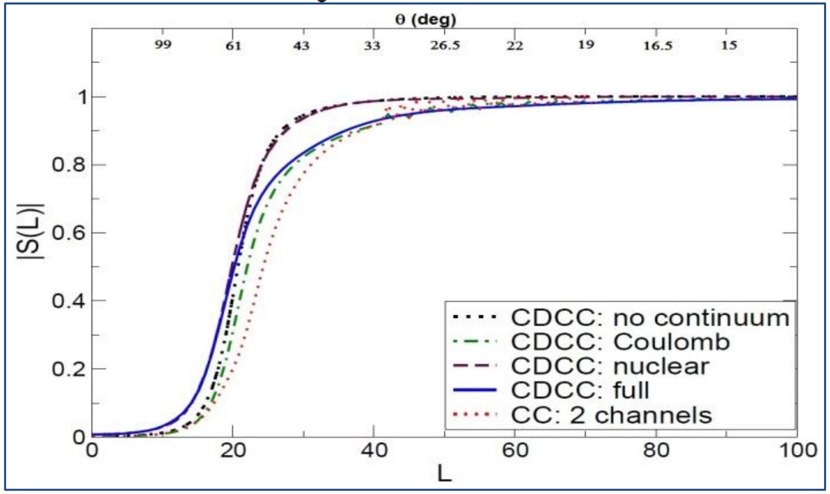
A. Di Pietro et al. Phys. Rev. Lett. 105,022701(2010)

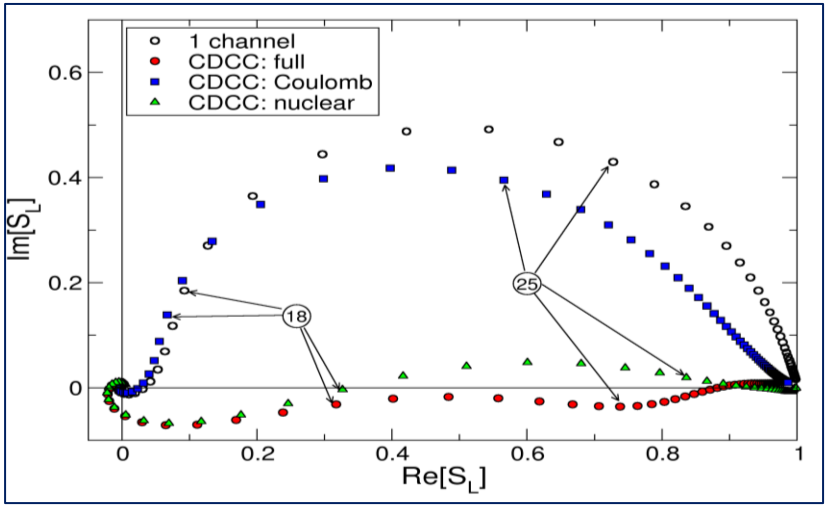
Continuum Discretized Coupled Channel Calculations (CDCC)

At low bombarding energy coupling between relative motion and intrinsic excitations important.
Halo nuclei → small binding energy, low break-up thresholds → coupling to break-up states (continuum) important → CDCC.

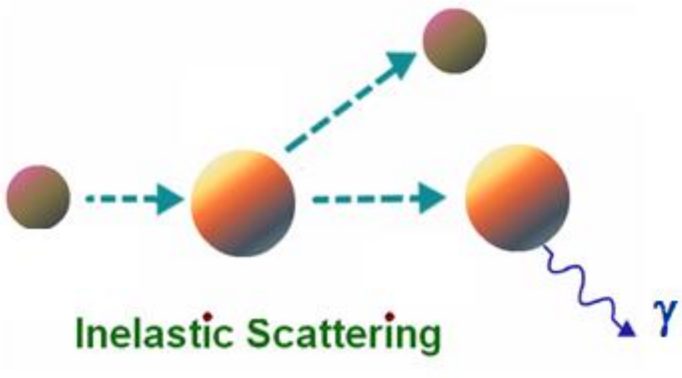


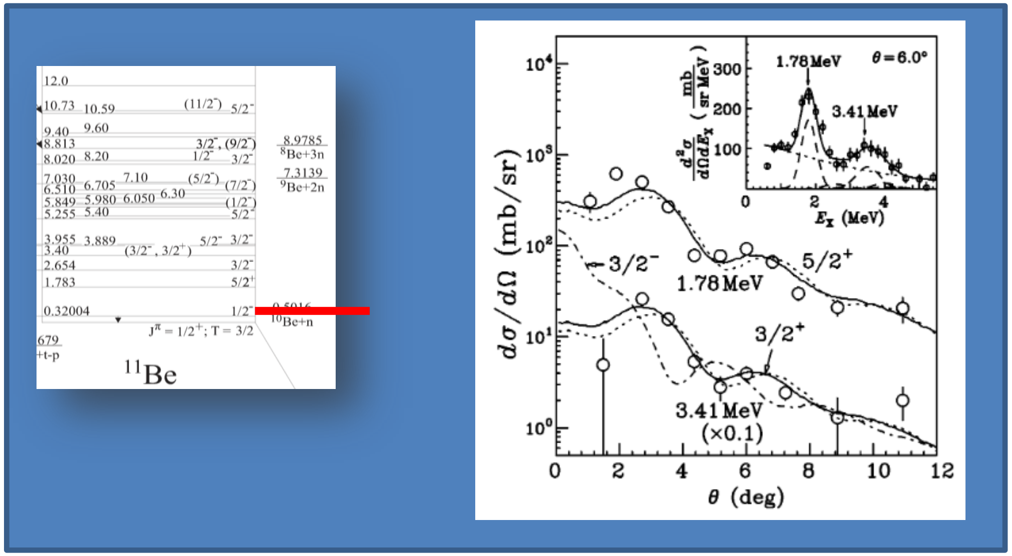


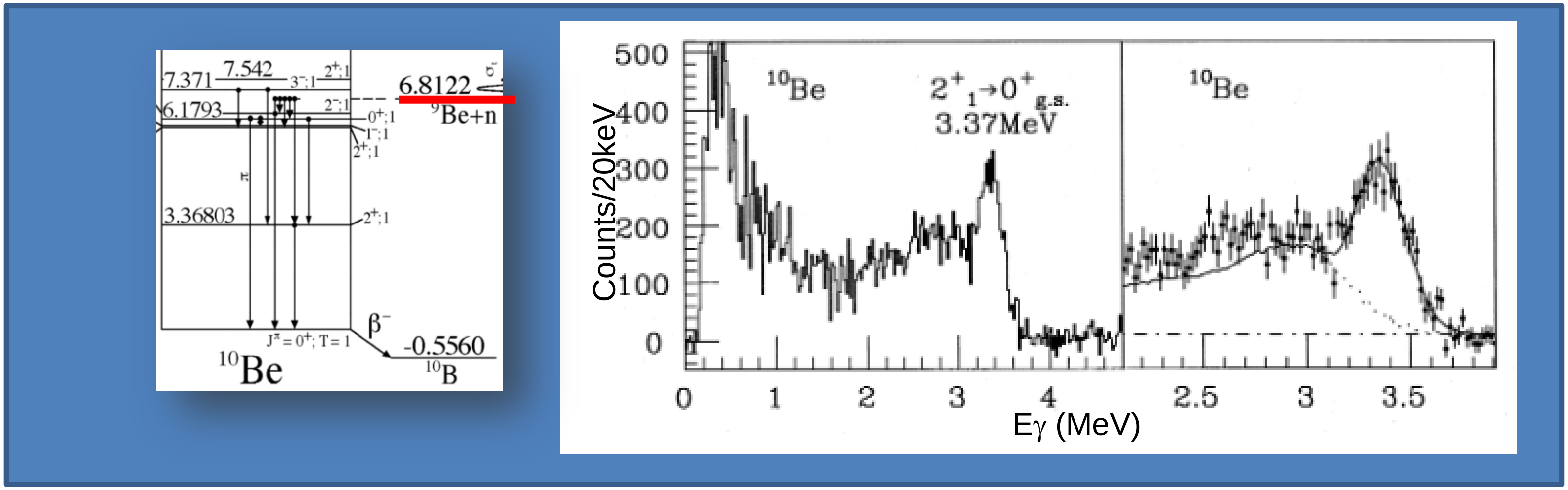




A. Di Pietro, V. Scuderi, A.M. Moro et al.
Phys. Rev. C 85, 054607 (2012)







Depending if the excited state is particle bound or unbound may change the way to identify inelastic scattering from other processes. The closer are the states the higher is the energy resolution required to discriminate them.

Supposing we have to measure an angular distribution of a given process, can you answer to the following questions?

1) where to put the detectors?
2) which solid angle do you have to cover?
3) which angular resolution do you need?
4) which energy resolution?

Before answering the following questions, do you have a clear idea about kinematics?

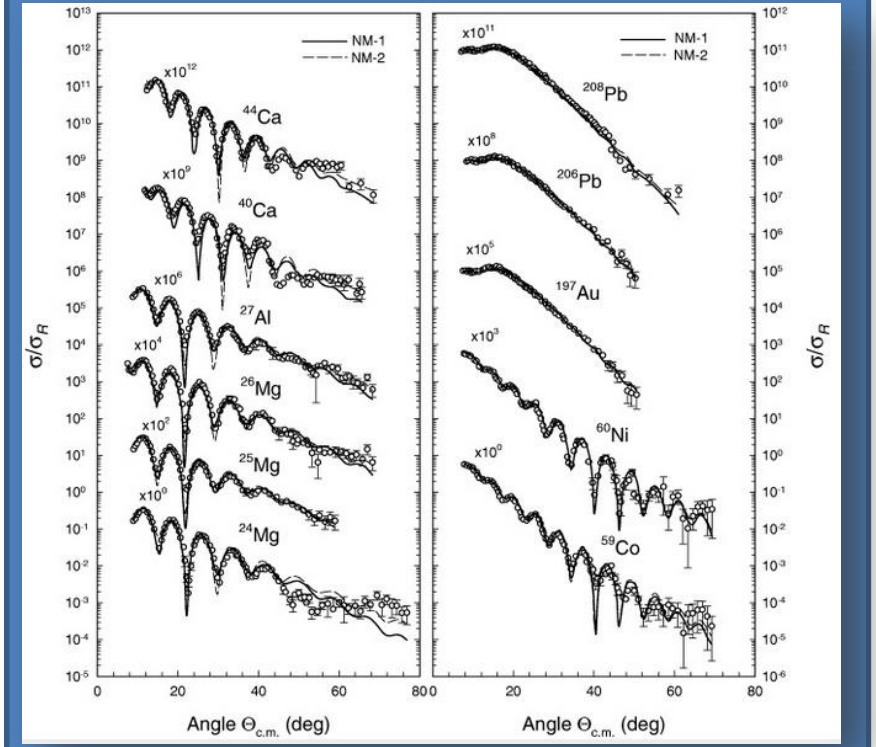
Some example of elastic scattering angular distribution

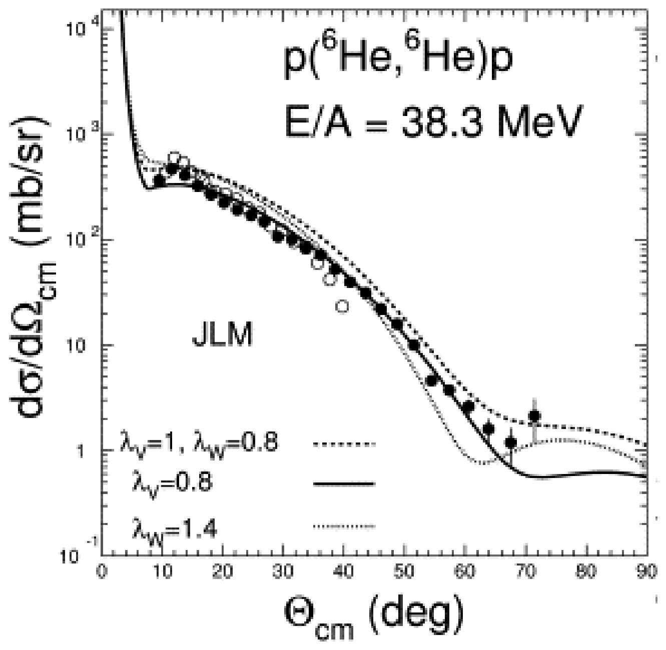
Direct kinematics

inverse kinematics scattering

^{6}Li elastic scattering @ 88 MeV

^{6}He elastic scattering on p @ 38 MeV/u

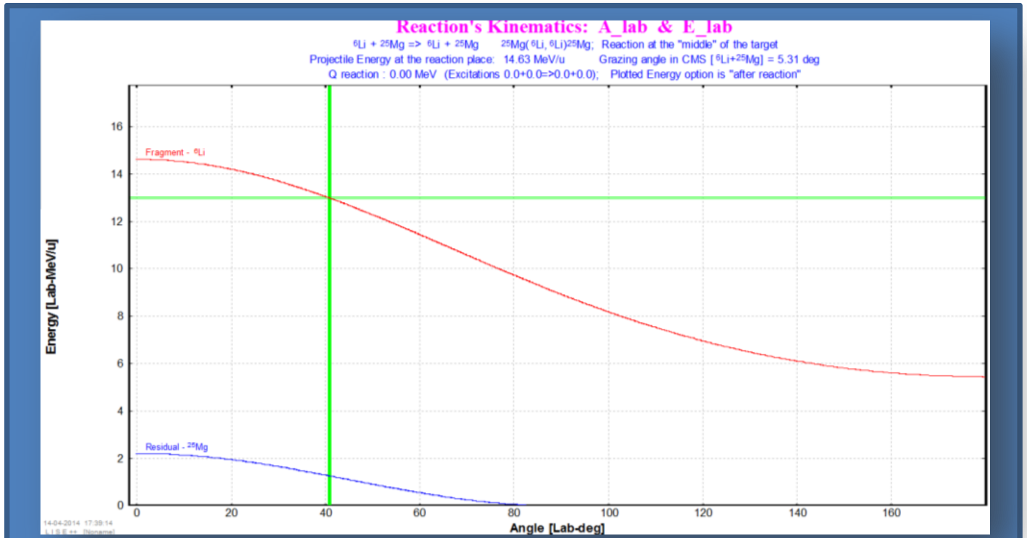




S. Hossain et al. Phys. Scr. 87(2013) 015201

V.Lapoux et al. PLB417(2001)18

Direct kinematics: e.g.elastic scattering ^{6}Li+^{25}Mg @ 88MeV

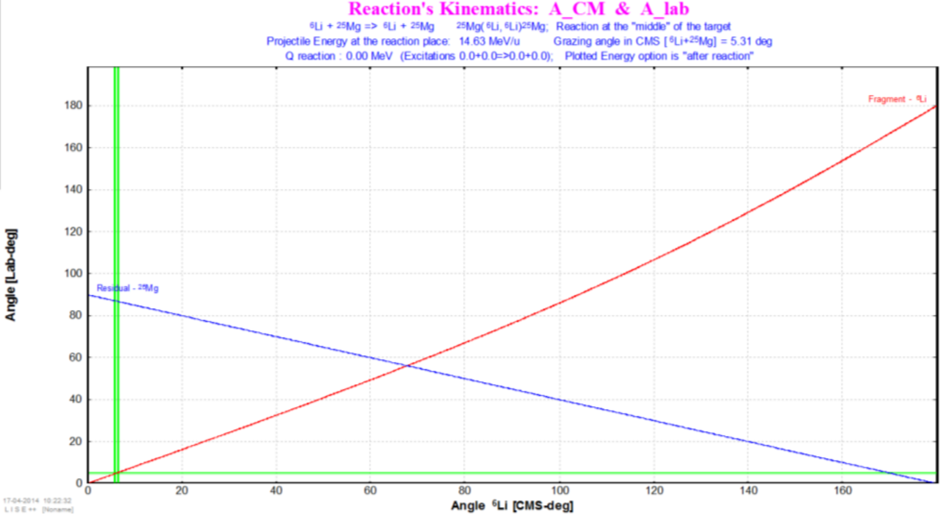


Reaction's Kinematics: A_lab & E_lab

^{6}Li + ^{25}Mg => ^{6}Li + ^{25}Mg ^{25}Mg(^{6}Li, ^{6}Li)^{25}Mg; Reaction at the "middle" of the target

Projectile Energy at the reaction place: 14.63 MeV/u Grazing angle in CMS [^{6}Li+^{25}Mg] = 5.31 deg

Q reaction : 0.00 MeV (Excitations 0.0+0.0=>0.0+0.0); Plotted Energy option is "after reaction"



Reaction's Kinematics: A_CM & A_lab

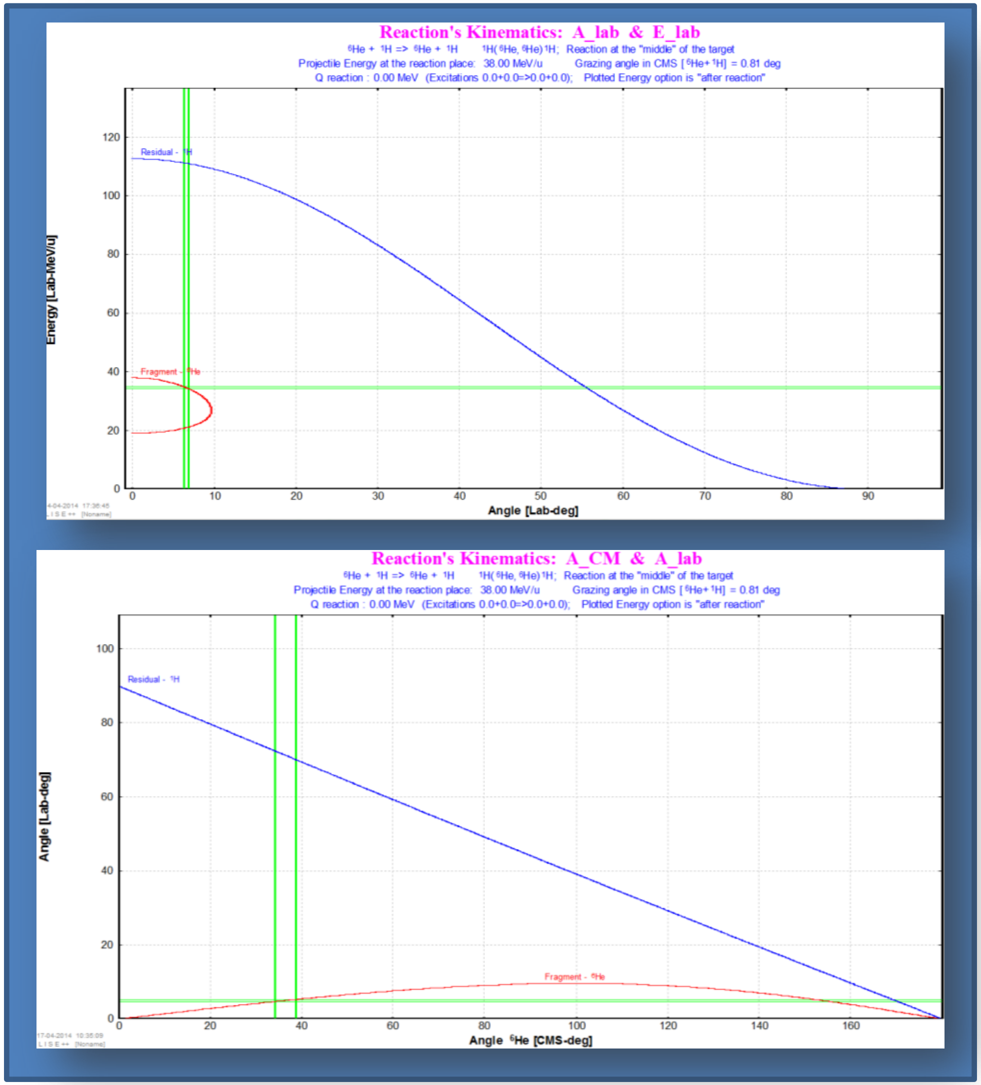
^{6}Li + ^{25}Mg => ^{6}Li + ^{25}Mg ^{25}Mg(^{6}Li, ^{6}Li)^{25}Mg; Reaction at the "middle" of the target

Projectile Energy at the reaction place: 14.63 MeV/u Grazing angle in CMS [^{6}Li+^{25}Mg] = 5.31 deg

Q reaction : 0.00 MeV (Excitations 0.0+0.0=>0.0+0.0); Plotted Energy option is "after reaction"

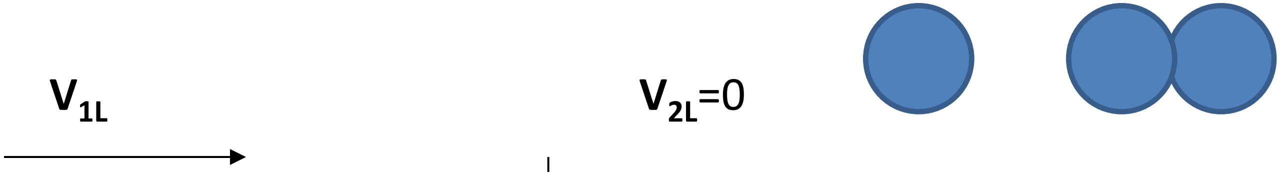
In direct kinematics we detect the projectile particle.
The difference between $\theta_{c.m.}$ and θ_{lab} depends on the mass ratio.

Inverse kinematics: e.g. elastic scattering ^{6}He+p @ 38 MeV/u



The inverse kinematics is forward focussed in the lab system. For the projectile particle there are two kinematical solutions and small $\Delta\theta_{lab}$ corresponds to large $\Delta\theta_{c.m.}$

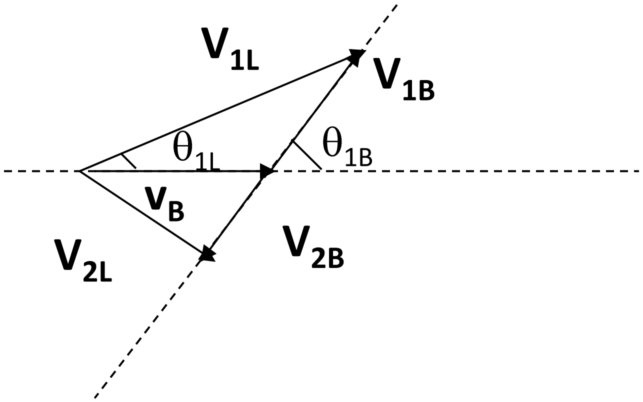
Two body kinematics for elastic scattering



$$V_{BL} = \frac{m1 V_{1L} + \cancel{m2 V_{2L}}}{m1+m2}$$ velocity of the c.m. in the Lab system

$$V_{1L} = \frac{m1+m2}{m1} V_{BL}$$

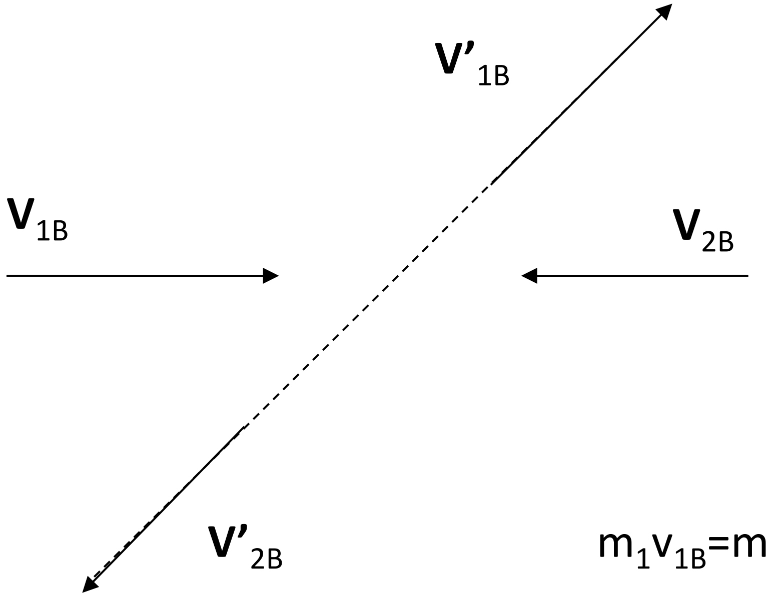
we will use this later



Two body kinematics

Elastic scattering

c.m. system



$m_1 v_{1B} = m_2 v_{2B}$

In the c.m. system before and after the collision the velocities are the same and the c.m. is at rest.

$v_{1B} = v'_{1B}$

$v_{2B} = v'_{2B}$

$$m_1V_{1B}=m_2V_{2B}$$

Momentum conservation in the c.m.

we will use this → $V_{2B}=\frac{m_1}{m_2}V_{1B}$

Energy in the c.m. system

$$\boxed{E_{cm}} = E_L - E_{BL} = \frac{1}{2}m_1V_{1L}^{\,2} - \frac{1}{2}(m_1+m_2)V_{BL}^{\,2} = \frac{1}{2}m_1V_{1L}^{\,2} - \frac{1}{2}(m_1+m_2)\frac{m_1^{\,2}}{(m_1+m_2)^2}V_{1L}^{\,2} =$$

$$= \frac{1}{2}\frac{m_1m_2V_{1L}^{\,2}}{m_1+m_2} = \boxed{\frac{m_2}{m_1+m_2}E_{1L}}$$

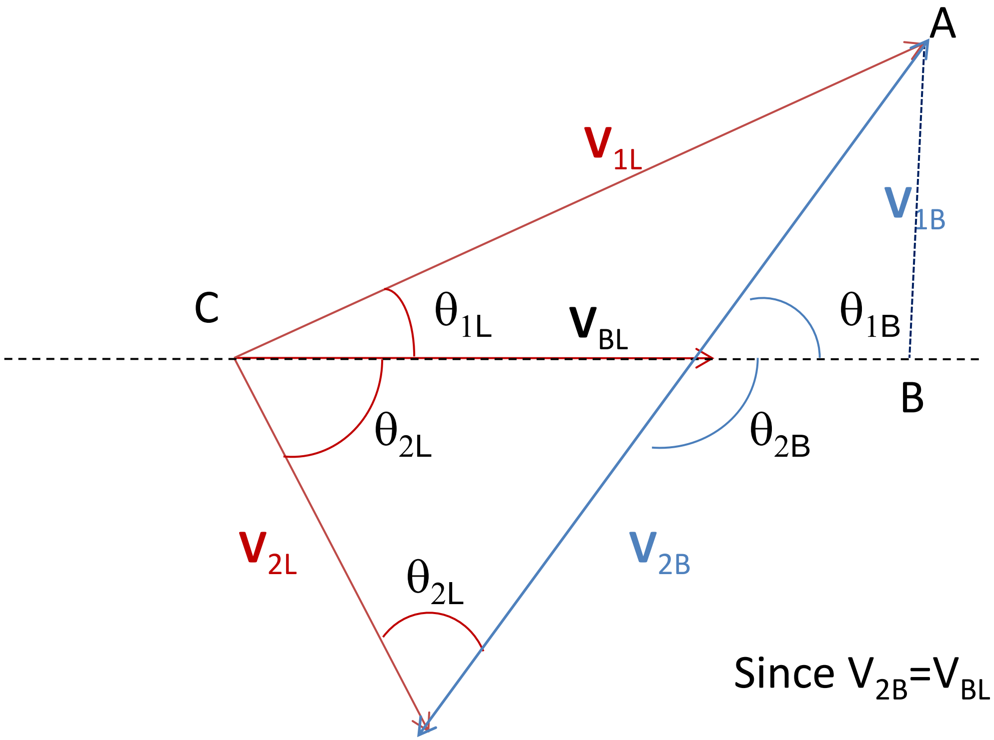
IMPORTANT! **$V_{2B}=V_{BL}$**

We prove this true ....

$$Ecm = \frac{1}{2}\frac{m_1m_2V_{1L}^{\,2}}{m_1+m_2} = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}\left(\frac{(m_1+m_2)}{m_1}\right)^2 V_{BL}^{\,2} = \boxed{\frac{1}{2}\frac{m_2}{m_1}(m_1+m_2)V_{BL}^{\,2}}$$

$$Ecm = \frac{1}{2}m_1V_{1B}^{\,2} + \frac{1}{2}m_2V_{2B}^{\,2} = \frac{1}{2}m_1\left(\frac{m_2}{m_1}\right)^2 V_{2B}^{\,2} + \frac{1}{2}m_2V_{2B}^{\,2} = \boxed{\frac{1}{2}\frac{m_2}{m_1}(m_2+m_1)V_{2B}^{\,2}}$$

Some relations between angles



Since $V_{2B}=V_{BL}$ → $\theta_{2L}+\theta_{2L}=\mathbf{2\theta_{2L}=\theta_{2B}=\pi-\theta_{1B}}$

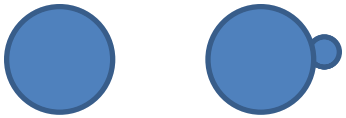
$AB=V_{1B}\sin\theta_{1B}$

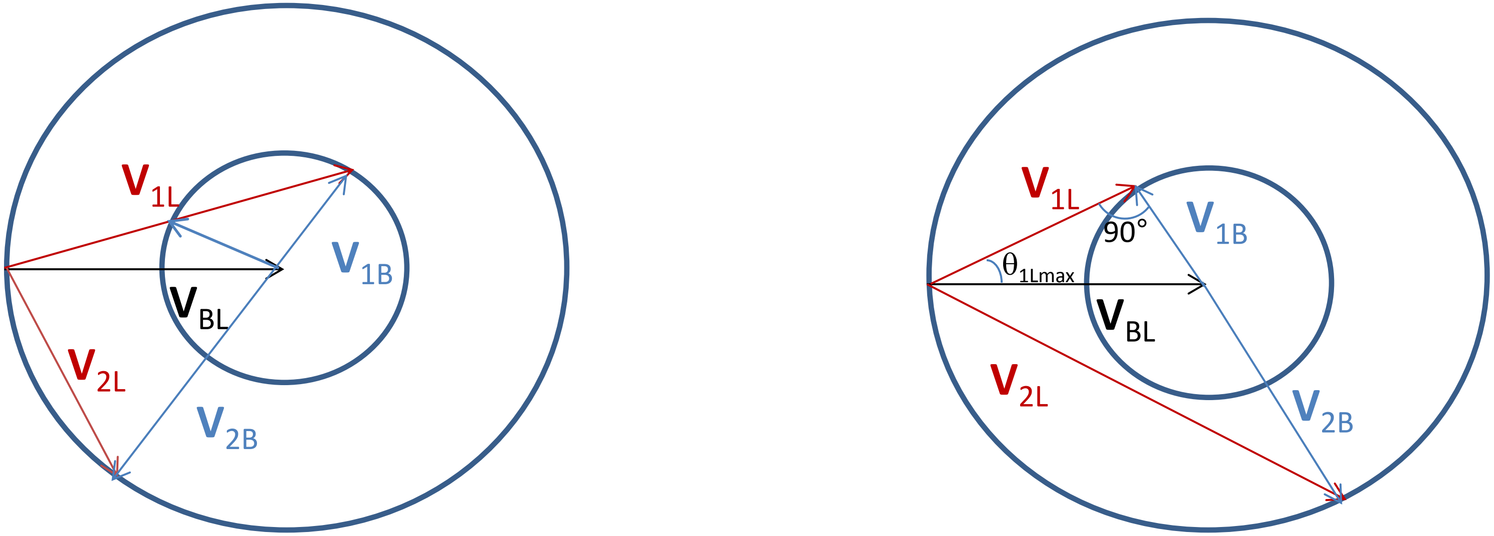
$BC=V_{BL}+V_{1B}\cos\theta_{1B}$

$$tg\theta_{1L}=\frac{V_{1B}\sin\theta_{1B}}{V_{BL}+V_{1B}\cos\theta_{1B}}=\frac{\sin\theta_{1B}}{\frac{m_1}{m_2}+\cos\theta_{1B}}$$

We draw two cyrcles having radii: $R_1=V_{1B}$ and $R_2=V_{2B}=V_{BL}$

$m_1>m_2$ (inverse kinematics)





In inverse kinematics there is a maximum angle at which particle m1 and m2 are scattered in the lab system.

θ_{1Lmax} → V_{1L} tangent to the inner circle

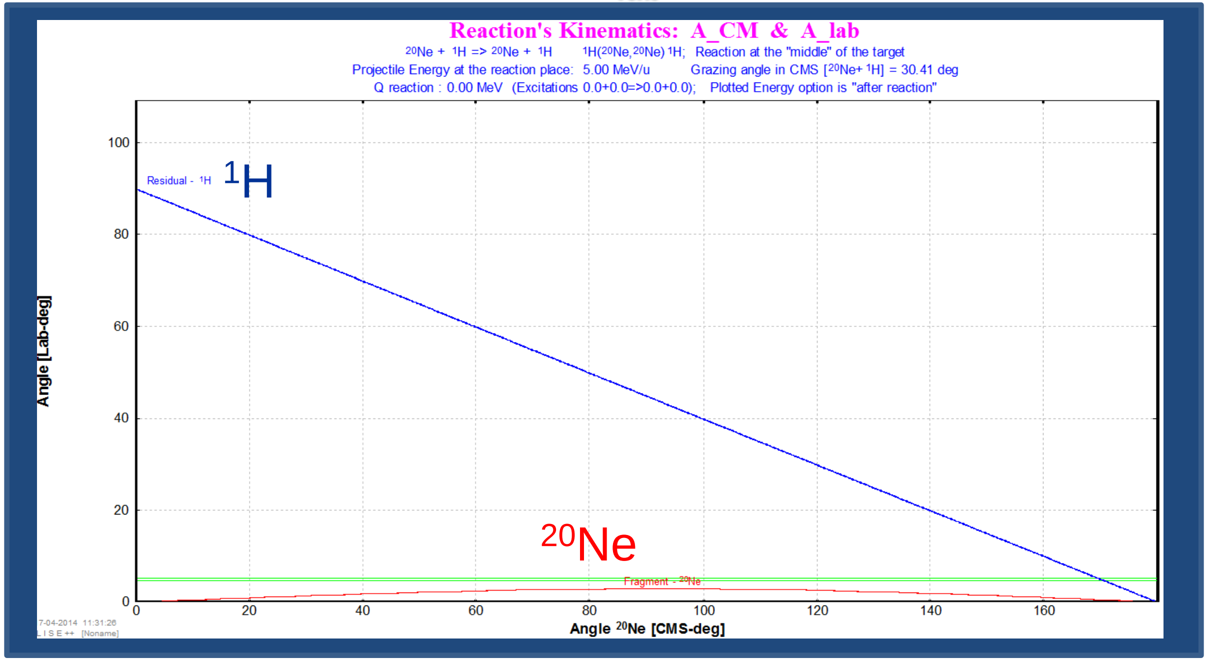
$$\sin\theta_{1Lmax}=\frac{V_{1B}}{V_{BL}}=\frac{m_2}{m_1}$$

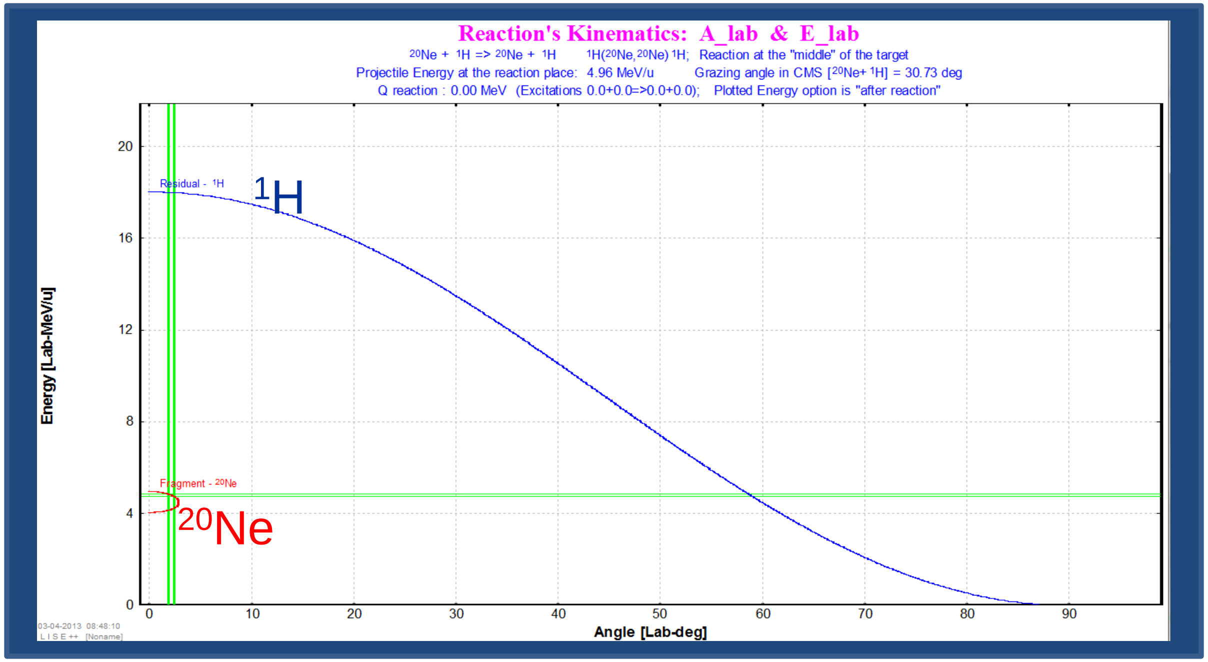
since→ $2\theta_{2L}=\theta_{2B}=\pi-\theta_{1B}$

$\theta_{2Lmax}= 90°$ for $\theta_{2B}=180°$

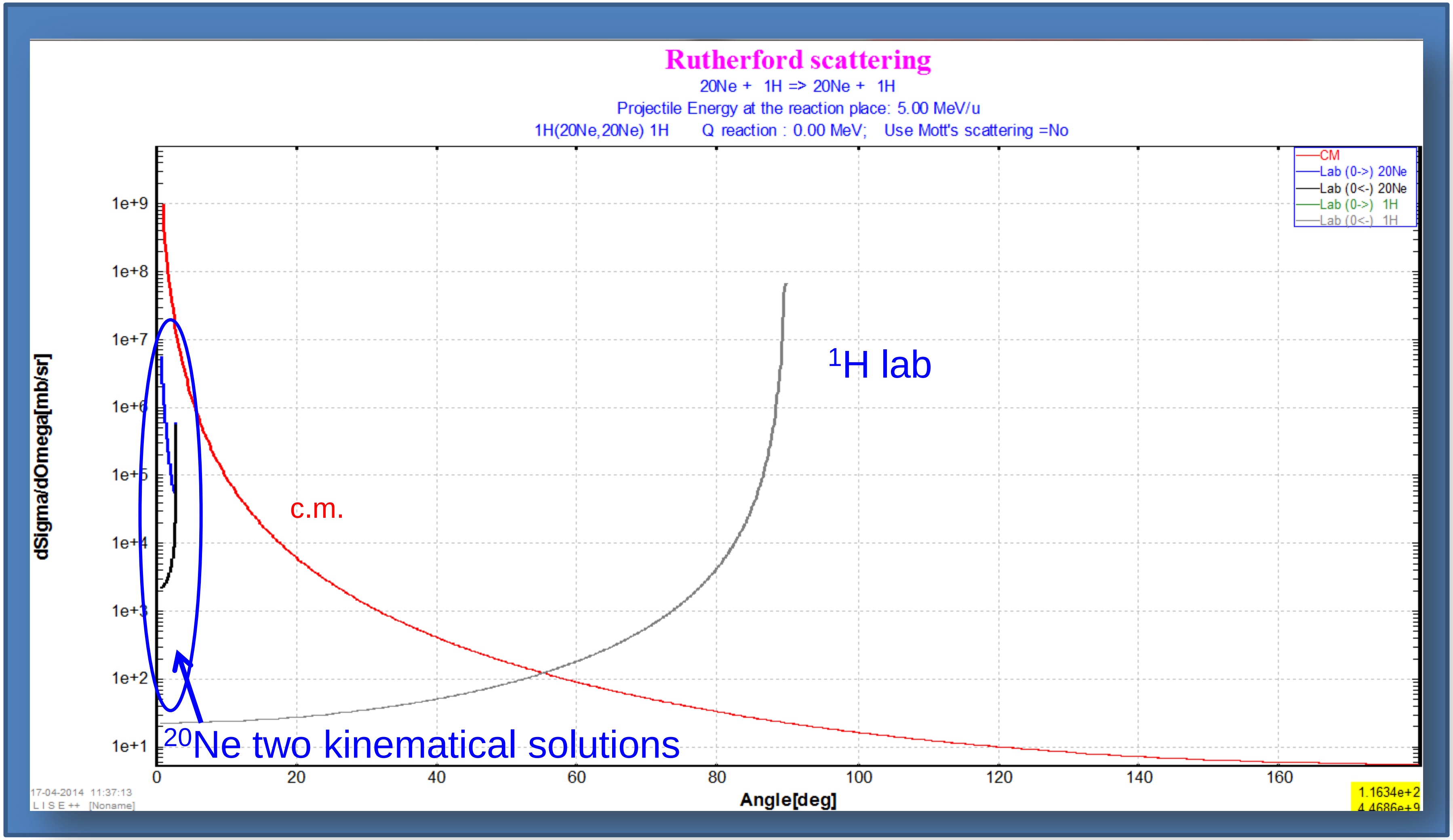
The inverse kinematics is forward focussed.

E.g. ^{20}Ne+p E_{lab}=100 MeV





Rutherford cross-section



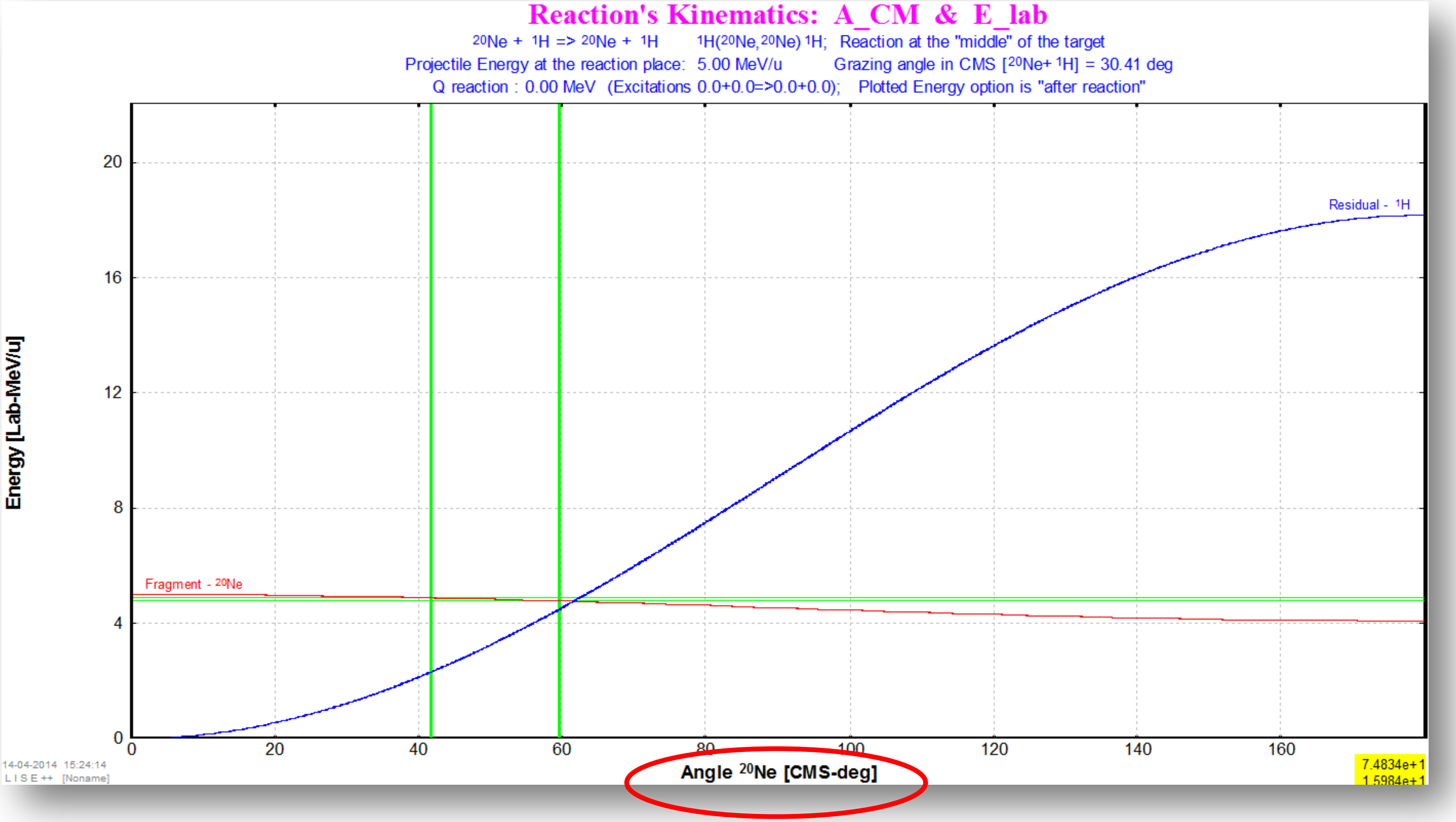
Rutherford scattering

20Ne + 1H => 20Ne + 1H

Projectile Energy at the reaction place: 5.00 MeV/u

1H(20Ne,20Ne) 1H   Q reaction : 0.00 MeV;   Use Mott's scattering =No

E.g. ^{20}Ne+p E_{lab}=100 MeV



Reaction's Kinematics: A_CM & E_lab

^{20}Ne + ^{1}H => ^{20}Ne + ^{1}H ^{1}H(^{20}Ne,^{20}Ne) ^{1}H; Reaction at the "middle" of the target

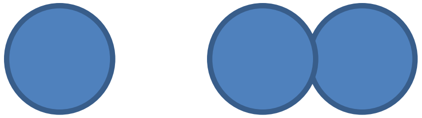
Projectile Energy at the reaction place: 5.00 MeV/u   Grazing angle in CMS [^{20}Ne+ ^{1}H] = 30.41 deg

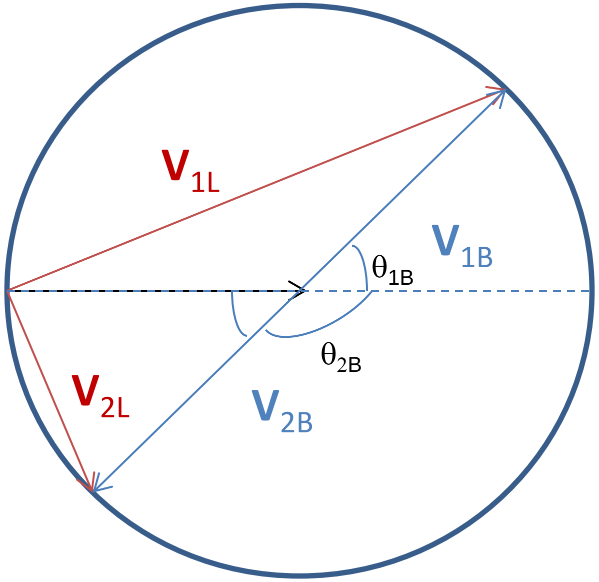
Q reaction : 0.00 MeV (Excitations 0.0+0.0=>0.0+0.0); Plotted Energy option is "after reaction"

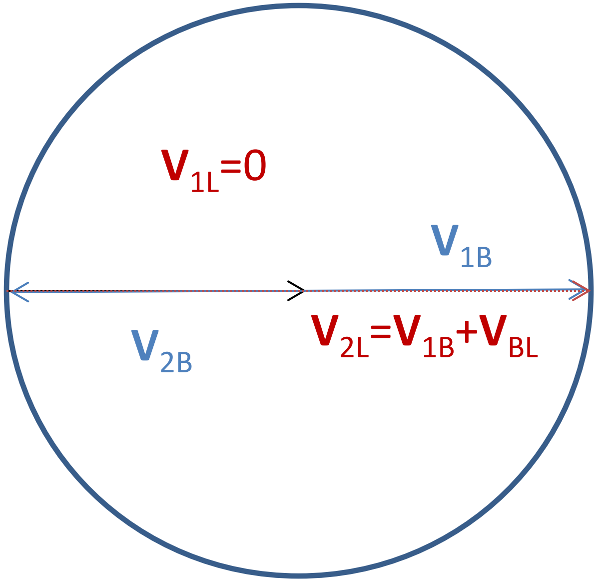
NOTE:In inverse kinematic scattering the c.m. angle is not the one of the light particle that one generally detects.

Identical particle scattering

m1=m2





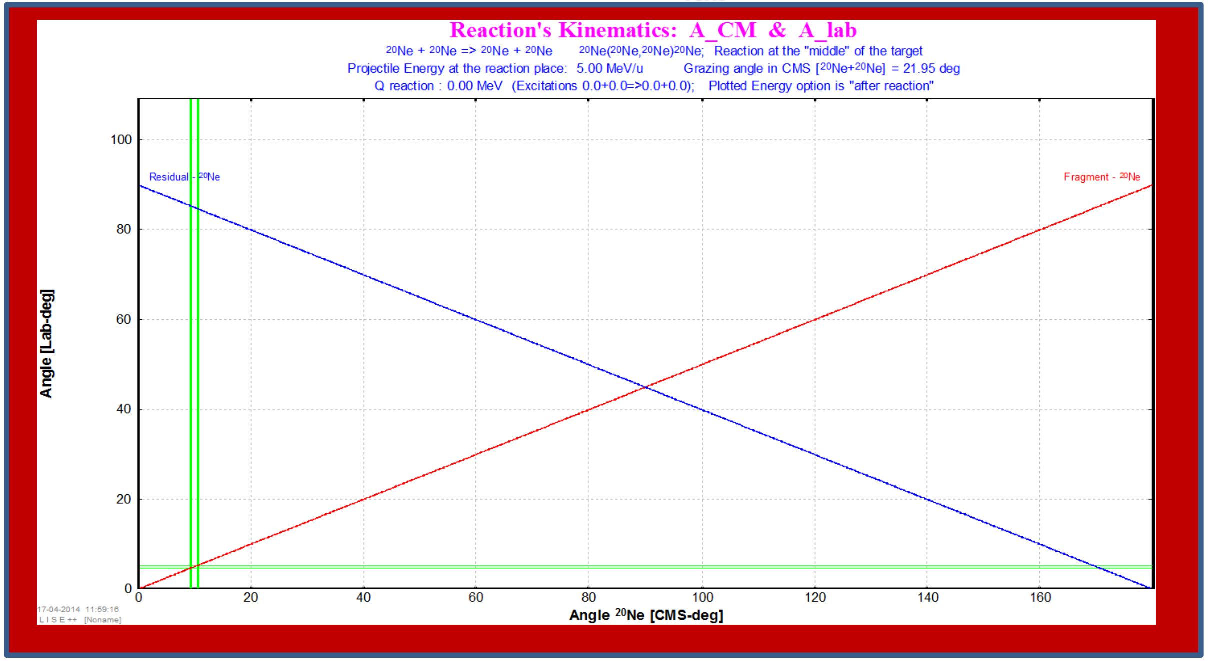


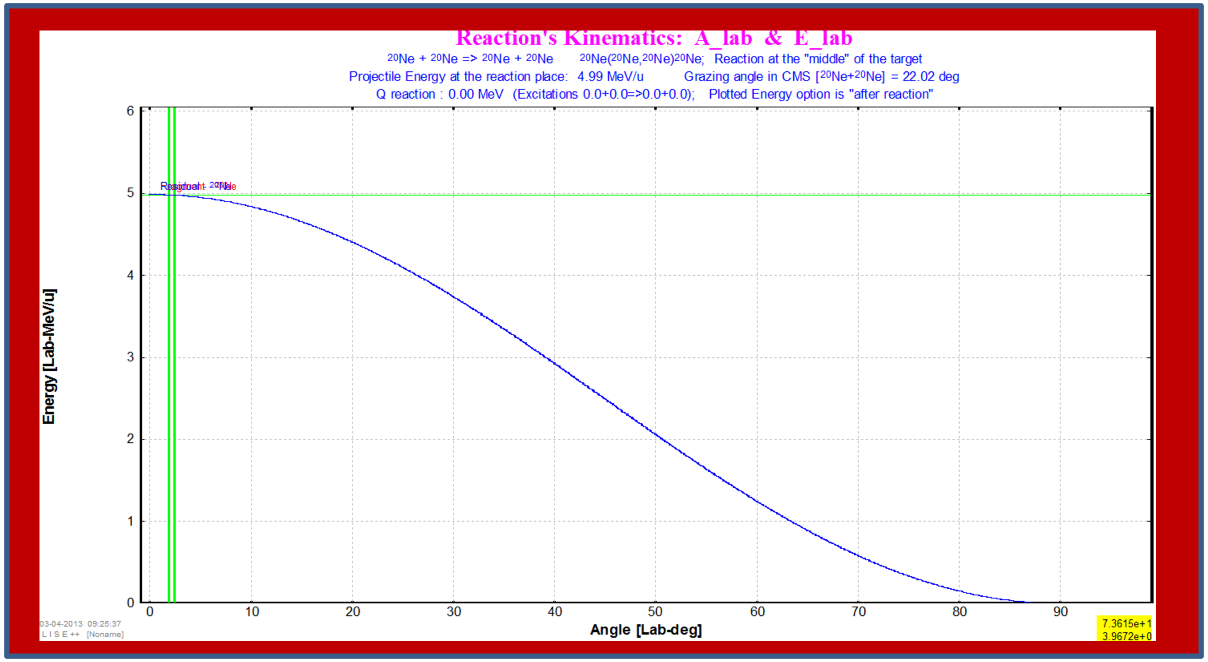
$\theta_{1L}+\theta_{2L}=90°$ $\theta_{2B}=\pi-\theta_{1B}$ → $\sin\theta_{1B}=\cos\theta_{2B}$

$\theta_{1Lmax}=\theta_{2Lmax=}90°$

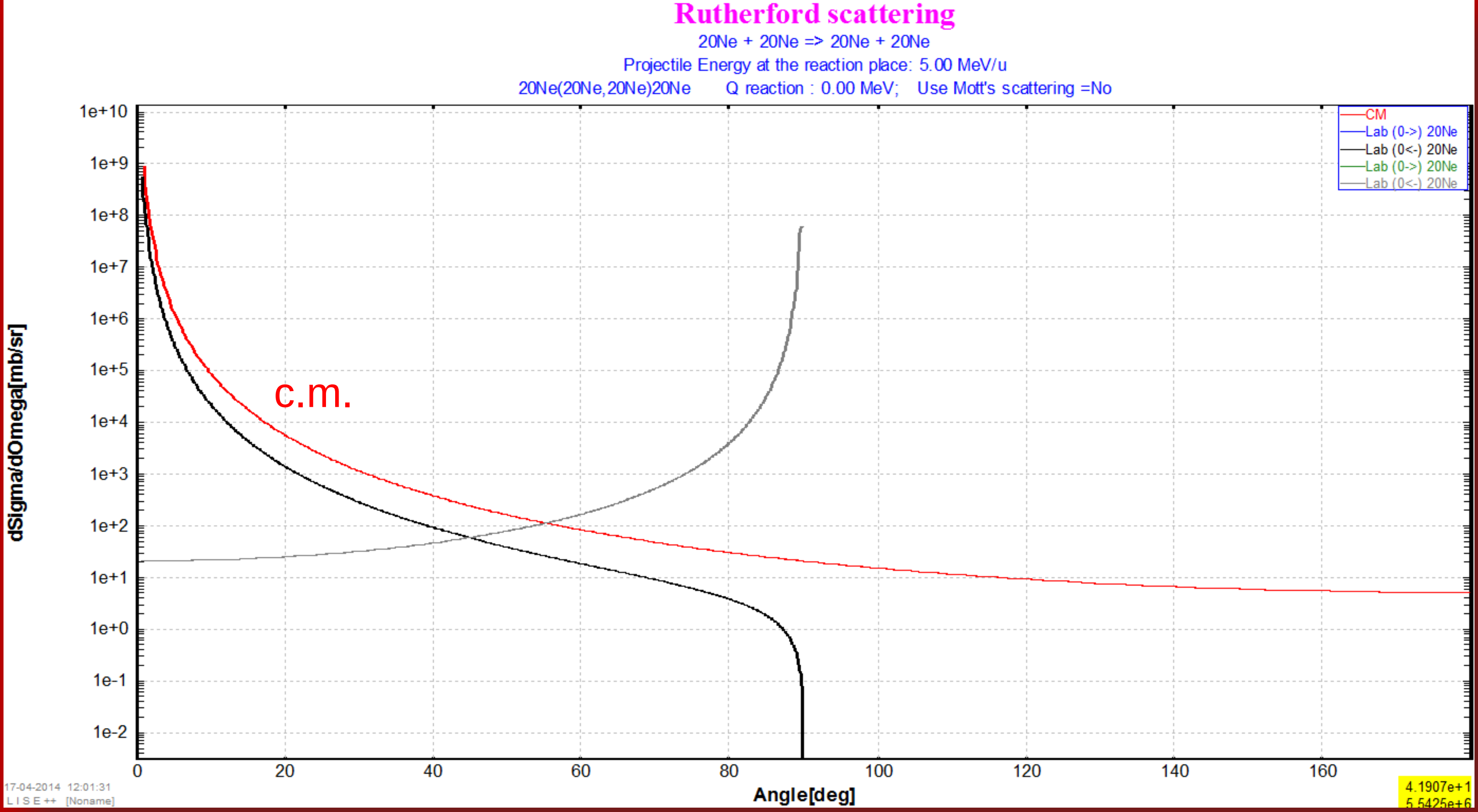
The maximum angle for both, projectile and recoil is 90°

E.g. ^{20}Ne+^{20}Ne E_{lab}=100 MeV

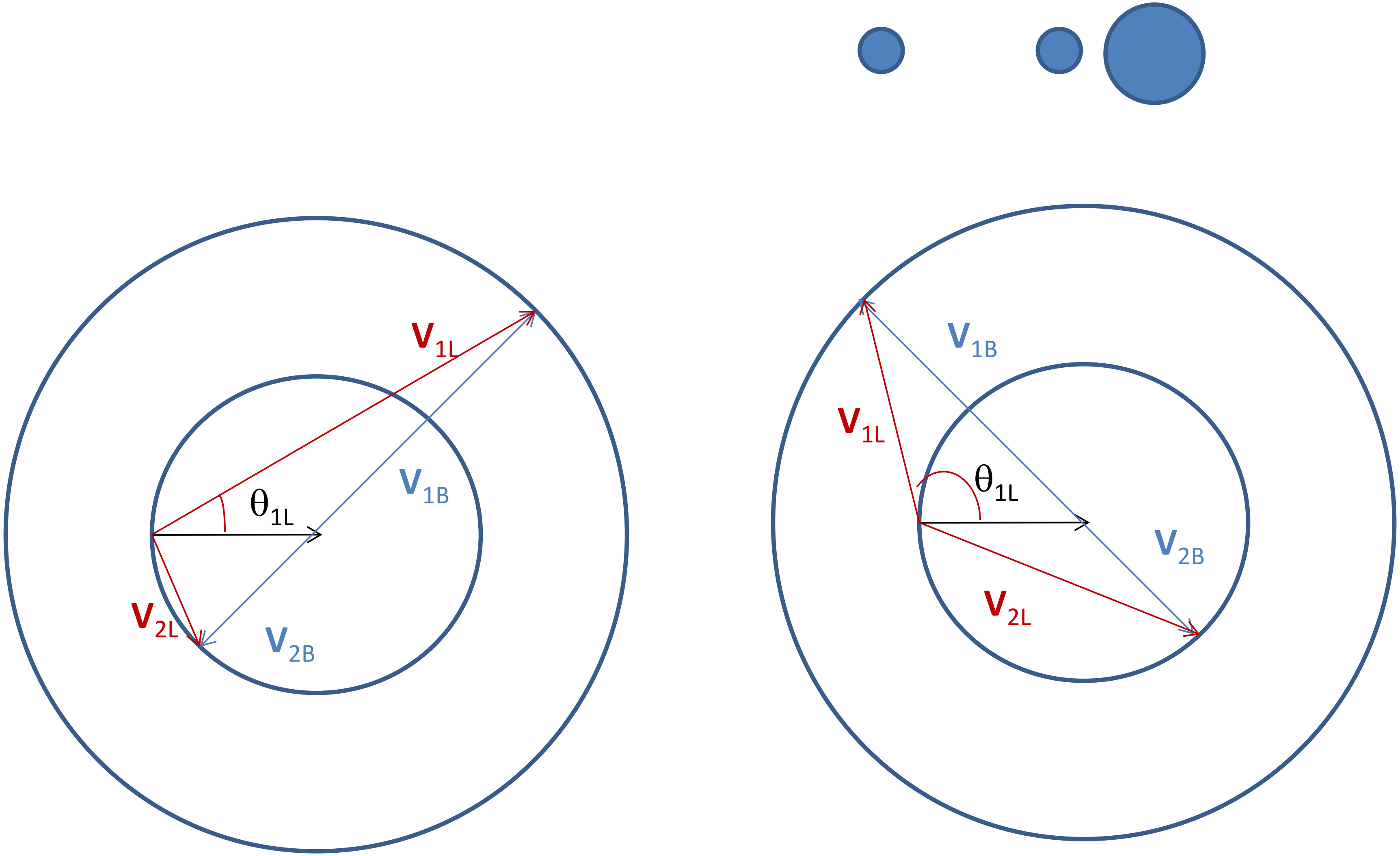




Rutherford cross-section

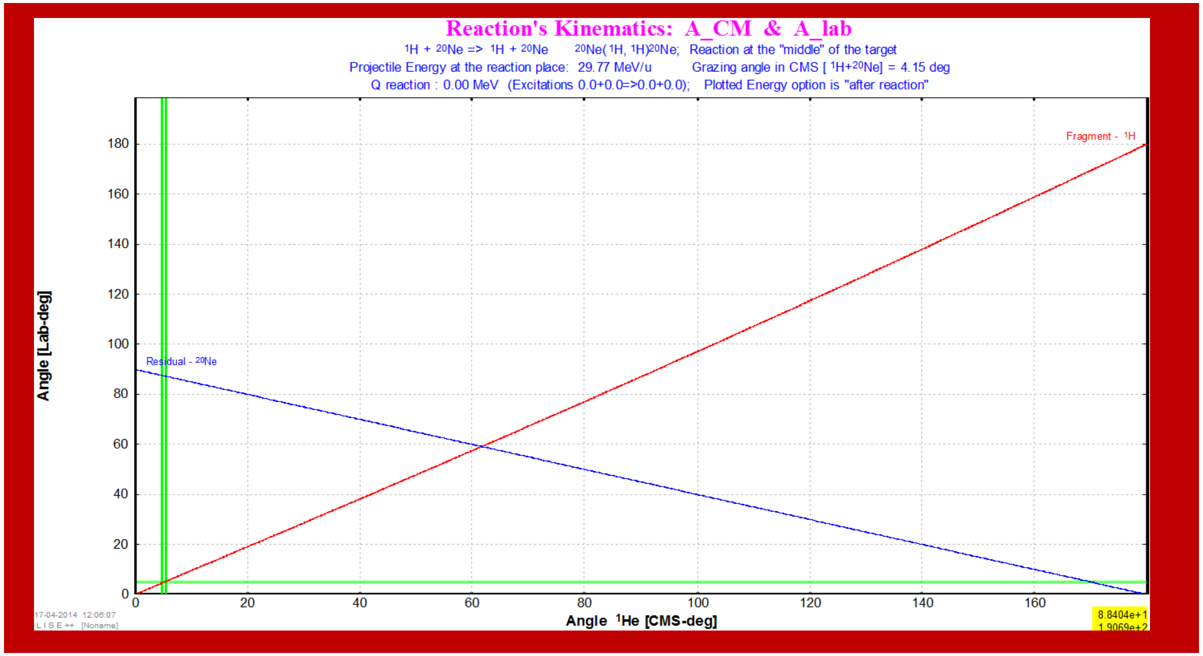


m1<m2 (direct kinematics)



For the projectile particle all angles are allowed both in the c.m. and laboratory system.

E.g. p+ ^{20}Ne E_{lab}=100 MeV

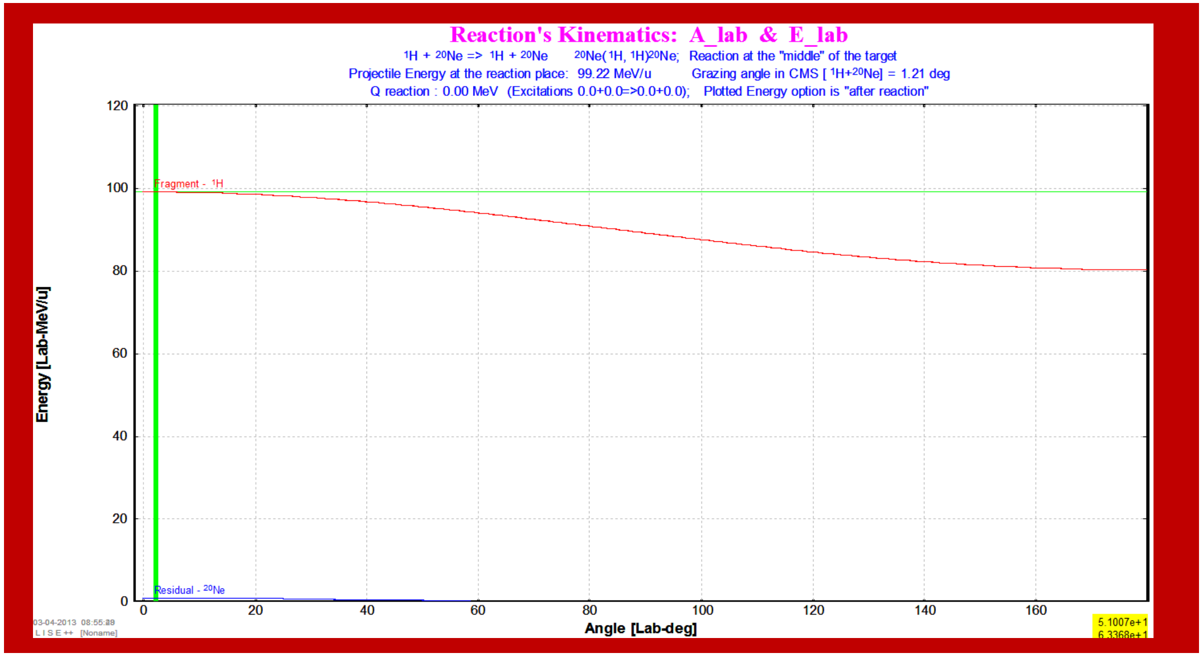


Reaction's Kinematics: A_CM & A_lab

^{1}H + ^{20}Ne => ^{1}H + ^{20}Ne ^{20}Ne(^{1}H, ^{1}H)^{20}Ne; Reaction at the "middle" of the target

Projectile Energy at the reaction place: 29.77 MeV/u Grazing angle in CMS [^{1}H+^{20}Ne] = 4.15 deg

Q reaction : 0.00 MeV (Excitations 0.0+0.0=>0.0+0.0); Plotted Energy option is "after reaction"



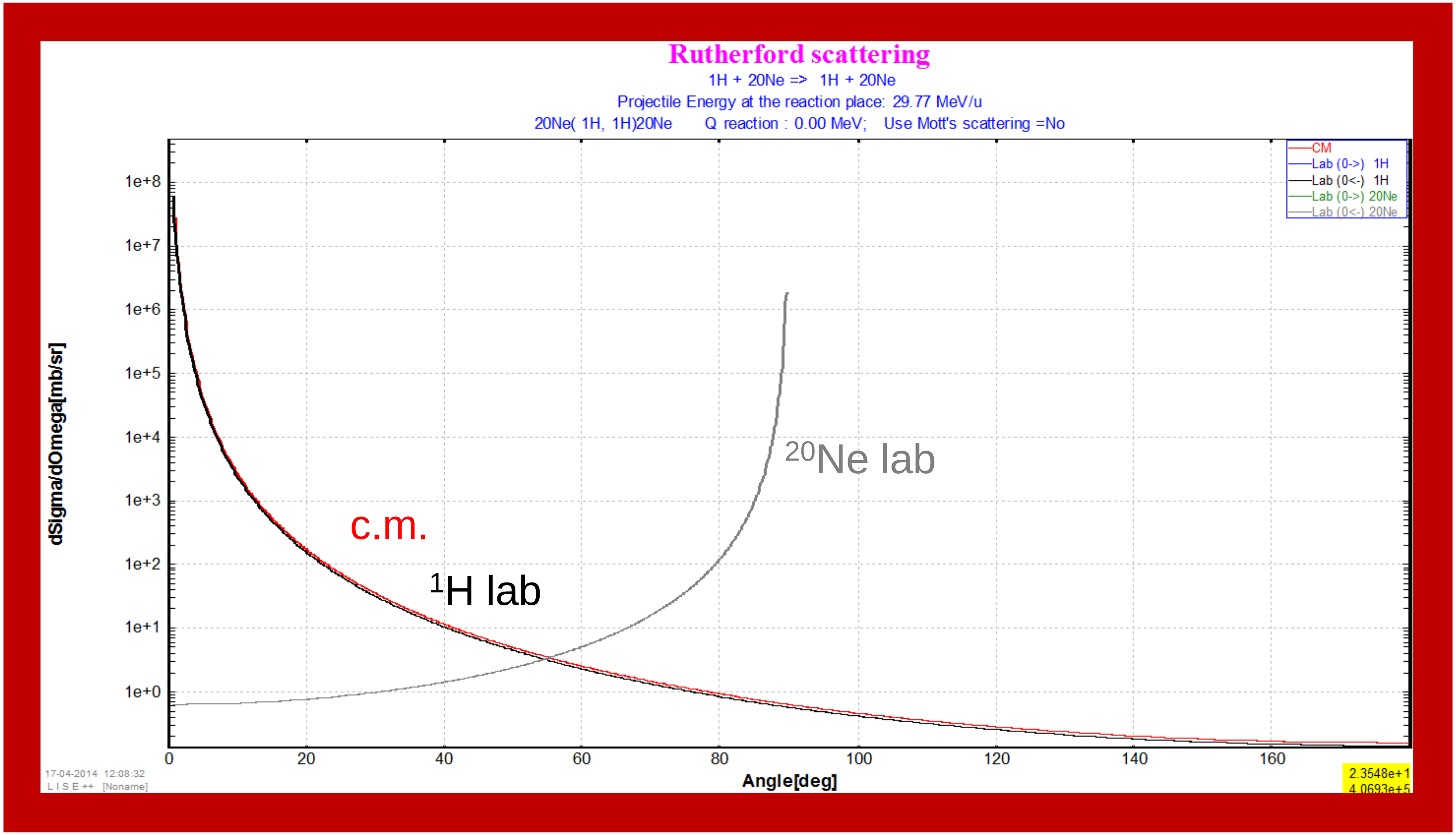
Reaction's Kinematics: A_lab & E_lab

^{1}H + ^{20}Ne => ^{1}H + ^{20}Ne ^{20}Ne(^{1}H, ^{1}H)^{20}Ne; Reaction at the "middle" of the target

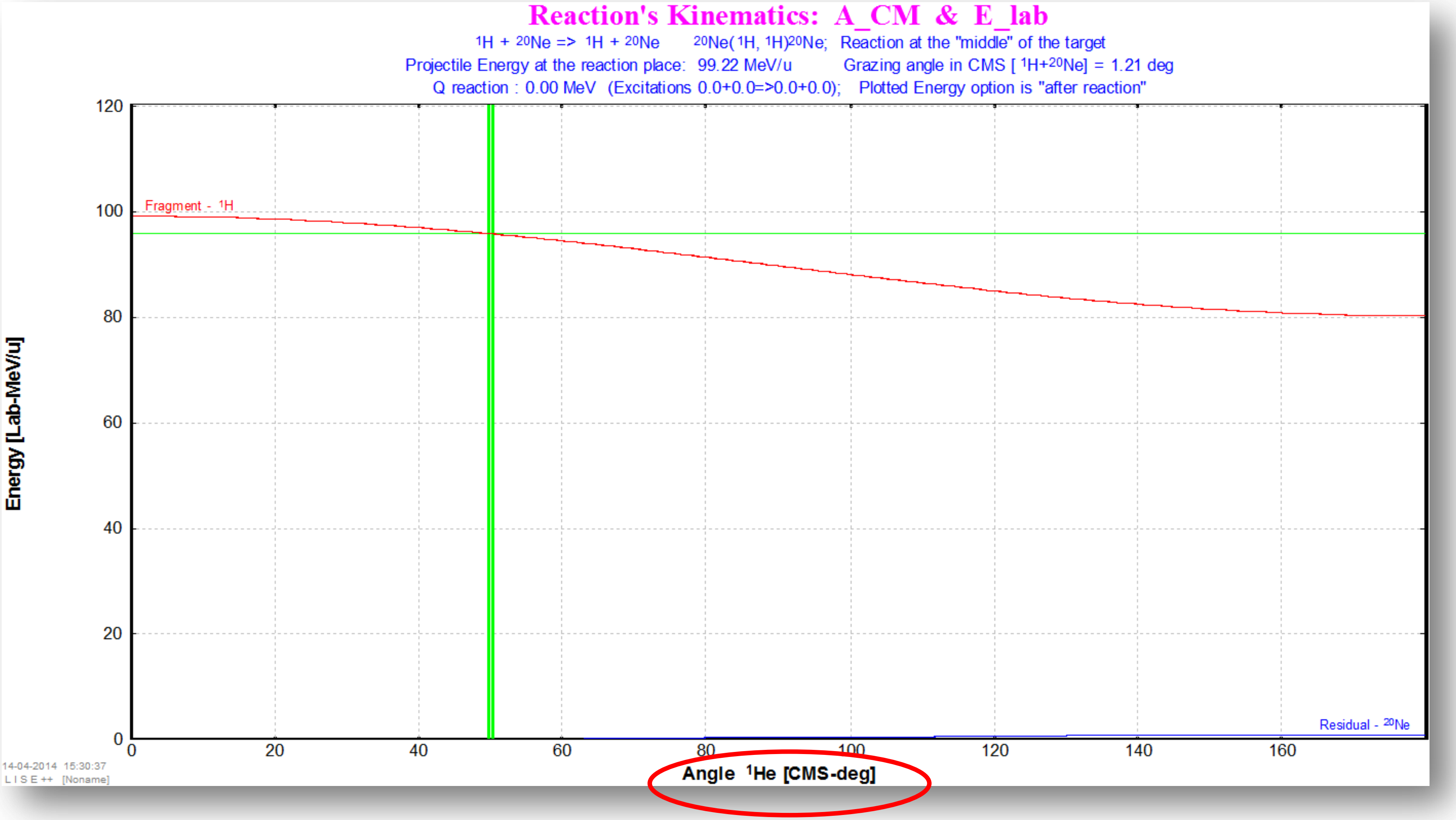
Projectile Energy at the reaction place: 99.22 MeV/u Grazing angle in CMS [^{1}H+^{20}Ne] = 1.21 deg

Q reaction : 0.00 MeV (Excitations 0.0+0.0=>0.0+0.0); Plotted Energy option is "after reaction"

Rutherford cross-section

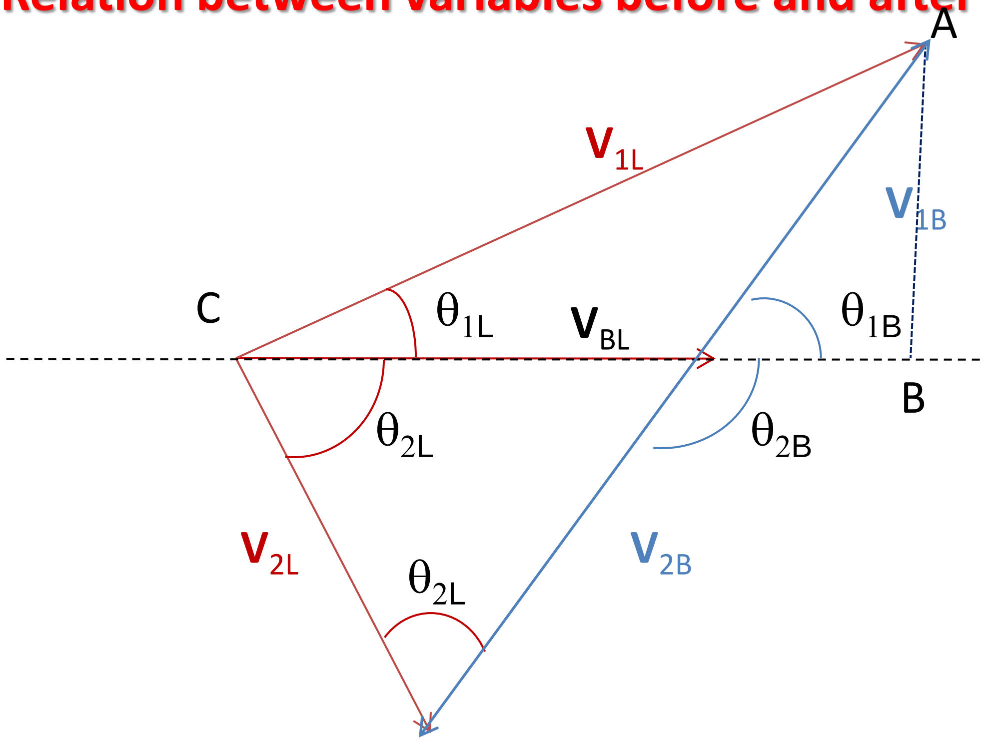


E.g. p+ ^{20}Ne E_{lab}=100 MeV



NOTE:In direct kinematics the c.m. angle is the one of the light particle which is also the projectile particle.

Relation between variables before and after the collision



$$V_{1L}^{\ 2} = V_{1B}^{\ 2} + V_{BL}^{\ 2} - 2COS(180^\circ - \theta_{1B}) = V_{1B}^{\ 2} + V_{BL}^{\ 2} + 2COS\theta_{1B}$$

We remind that :

v1B=V1B;   V2B=v2B=VBL; $\frac{V_{2B}}{V_{1B}} = \frac{v_{2B}}{v_{1B}} = \frac{m_1}{m_2};$ $V_{BL} = \frac{m_1}{m_1 + m_2} v_{1L};$

E_{1L} ,**v1L**   before collision E'_{1L} ,**V1L** after collision

By combining these equations we can express the energy after the collision as a function of the energy before:

$$E_{1L} = \frac{1}{2} m_1 v_{1L}^{2}$$

$$E_{1L}' = \frac{1}{2} m_1 V_{1L}^{2} = \frac{1}{2}\left[v_{1L}^{2} \frac{m_1^{2}}{(m_1+m_2)^2} + v_{1L}^{2} \frac{m_2^{2}}{(m_1+m_2)} + v_{1L}^{2} \frac{2m_1 m_2\cos\theta_{1B}}{(m_1+m_2)^2}\right] =$$

$$= E_{1L}\left[\frac{m_1^{2}+m_2^{2}+2m_1 m_2\cos\theta_{1B}}{(m_1+m_2)^2}\right]$$

Eq.1

For particles having the same masses **m1=m2=m**:

$$\boxed{E_{1L}'} = E_{1L}\left[\frac{2m^{2}+2m^{2}\cos\theta_{1B}}{(2m)^2}\right] = E_{1L}\left(\frac{1+\cos\theta_{1B}}{2}\right) = \boxed{E_{1L}\cos^2\theta_{1L}}$$

$$E_{1L} = E_{1L}' + E_{2L}'$$

$$E_{1L} = E_{1L}(\cos^2\theta_{1L} + \sin^2\theta_{1L}) = E_{1L}' + E_{2L}'$$

$$\boxed{E_{2L}'} = E_{1L}\sin^2\theta_{1L} = \boxed{E_{1L}\cos^2\theta_{2L}}$$

We know that $\theta_{1L}+\theta_{2L}=90°$

By measuring energy and angle of one particle we can completely reconstruct the kinematics.

Now we calculate the relative energy trasferred in one collision:

$$\frac{\Delta E_{1L}}{E_{1L}} = \frac{E_{1L} - E'_{1L}}{E_{1L}} 1 - \frac{E'_{1L}}{E_{1L}} = 1 - \left[\frac{m_1{}^2 + m_2{}^2 + 2m_1 m_2 \cos\theta_{1B}}{(m_1 + m_2)^2} \right] =$$

$$= \frac{2m_1 m_2}{(m_1 + m_2)^2}(1 - \cos\theta_{1B})$$

The maximum energy is transferred in a collision between two identicle particles

$$\boxed{\left(\frac{\Delta E_{1L}}{E_{1L}}\right)_{\max}} \Rightarrow \frac{d}{dm_1}\left[\frac{m_1 m_2}{(m_1 + m_2)^2}\right] = \frac{m_2(m_1 + m_2)^2 - 2(m_1 + m_2)m_1 m_2}{(m_1 + m_2)^4}$$

$$m_2(m_1 + m_2)^2 - 2(m_1 + m_2)m_1 m_2 = 0$$

If m1=m2

Relative energies.

$$E_{1B}=\frac{1}{2}m_1V_{1B}{}^2=\frac{1}{2}m_1\left(\frac{m_2}{m_1+m_2}\right)^2V_{1L}{}^2$$

$$E_{2B}=\frac{1}{2}m_2V_{2B}{}^2=\frac{1}{2}m_2\left(\frac{m_1}{m_1+m_2}\right)^2V_{1L}{}^2$$

$$\boxed{E_{1B}+E_{2B}}=\frac{1}{2}V_{1L}{}^2\underbrace{\left(\frac{m_1m_2}{m_1+m_2}\right)}_{\mu}{}^2\underbrace{\left[\frac{m_2}{m_1+m_2}+\frac{m_1}{m_1+m_2}\right]}_{1}=\boxed{\frac{1}{2}\mu V_{1L}{}^2}\langle\frac{1}{2}m_1V_{1L}{}^2=E_{1L}$$

The total energy in the cm is less than the incident energy in the lab system owing to the kinetic energy used for the cm motion in the lab.

$E_{1B}+E_{2B}+E_{BL}=E_{1L}=E'_{1L}+E'_{2L}$

$\frac{1}{2}\mu V_{1L}{}^2+E_{BL}=E_{1L}$

$\mathbf{V}_{rel}=\mathbf{V}_{1B}-\mathbf{V}_{2B}=\mathbf{V}_{1L}-\mathbf{V}_{2L}$

$V^2_{rel}=V^2_{1B}+V^2_{2B}+2V_{1B}V_{2B}=V^2_{1L}+V^2_{2L}+2V_{1L}V_{2L}\cos\theta_{rel}$

$m_1V_{1B}-m_2V_{2B}=0$ From momentum conservation

$$(m_1V_{1B}-m_2V_{2B})^2=0=m_1{}^2V_{1B}{}^2+m_2{}^2V_{2B}{}^2-2m_1V_{1B}m_2V_{2B}$$

$$2V_{1B}V_{2B}=\frac{m_1{}^2V_{1B}{}^2+m_2{}^2V_{2B}{}^2}{m_1m_2}$$

$$\boxed{\frac{1}{2}\mu V_{rel}{}^2}=\frac{1}{2}\frac{m_1m_2}{m_1+m_2}\left[V_{1B}{}^2+V_{2B}{}^2+2V_{1B}V_{2B}\right]=$$

$$=\frac{1}{2}\frac{m_1m_2}{m_1+m_2}\left[V_{1B}{}^2(1+\frac{m_1}{m_2})+V_{2B}{}^2(1+\frac{m_2}{m_1})\right]=$$

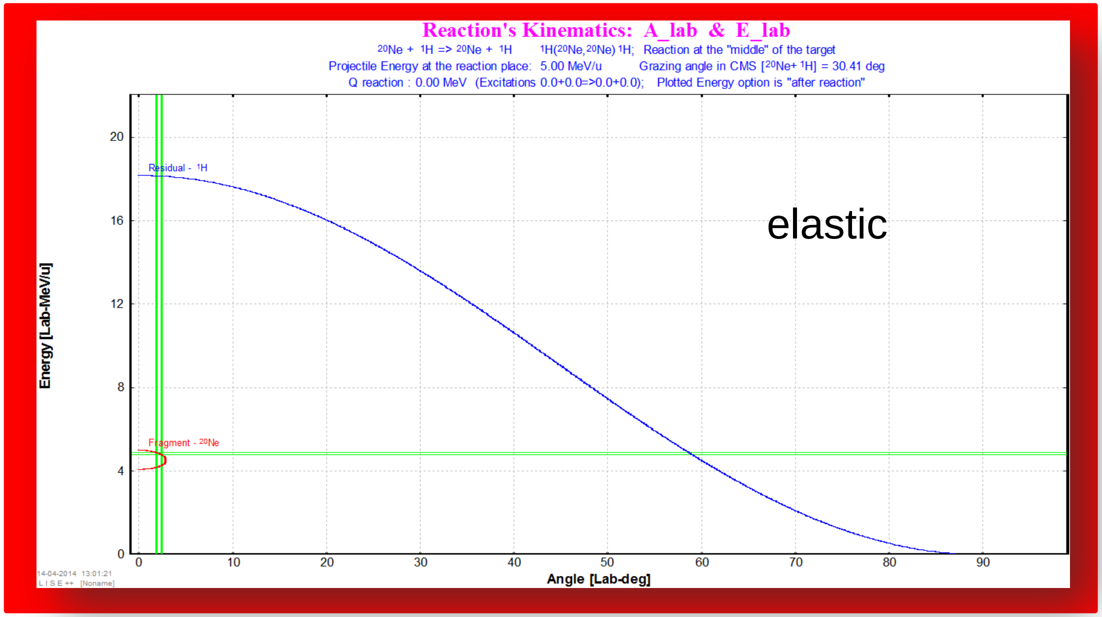
$$=\boxed{\frac{1}{2}m_1V_{1B}{}^2+\frac{1}{2}m_2V_{2B}{}^2=E_{cm}}$$

$$\frac{1}{2}\mu V_{1L}{}^2=E_{1B}+E_{2B}=E_{cm}$$

$$\boxed{E_{cm}=\frac{m_2}{m_1+m_2}E_{1L}}$$

For $m_1<<m_2$ $E_{cm}\approx E_{1L}$

We now consider the case of Q≠0
a) Inelastic scattering ^{20}Ne+p



Reaction's Kinematics: A_lab & E_lab

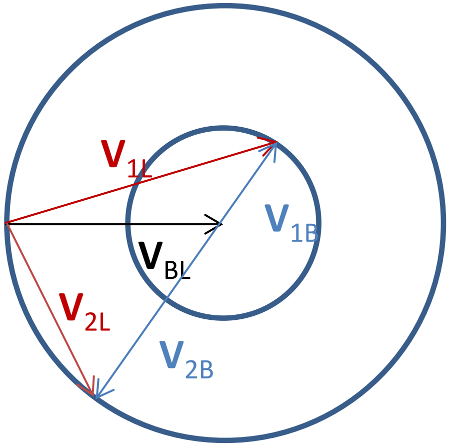
^{20}Ne + ^{1}H => ^{20}Ne + ^{1}H ^{1}H(^{20}Ne,^{20}Ne) ^{1}H; Reaction at the "middle" of the target

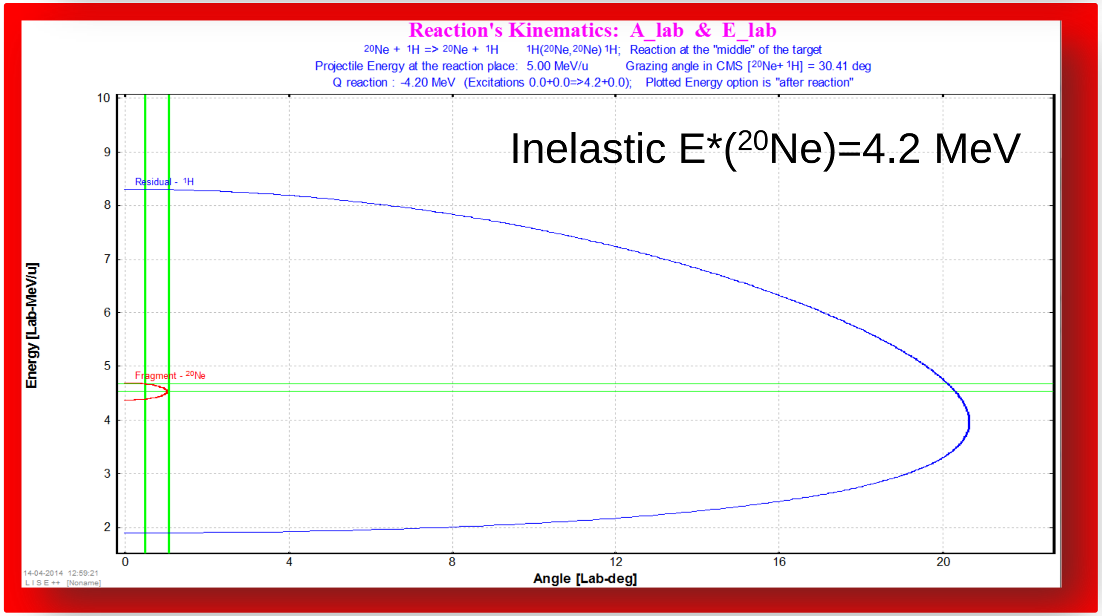
Projectile Energy at the reaction place: 5.00 MeV/u   Grazing angle in CMS [^{20}Ne+ ^{1}H] = 30.41 deg

Q reaction : 0.00 MeV (Excitations 0.0+0.0=>0.0+0.0);   Plotted Energy option is "after reaction"

elastic

Two solutions for projectile fragment





Reaction's Kinematics: A_lab & E_lab

^{20}Ne + ^{1}H => ^{20}Ne + ^{1}H ^{1}H(^{20}Ne,^{20}Ne) ^{1}H; Reaction at the "middle" of the target

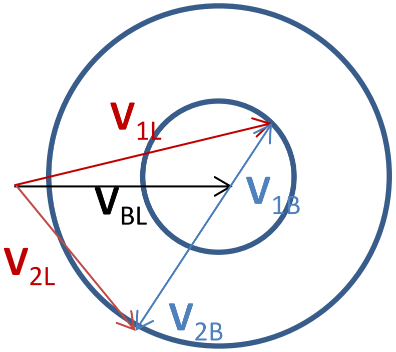
Projectile Energy at the reaction place: 5.00 MeV/u   Grazing angle in CMS [^{20}Ne+ ^{1}H] = 30.41 deg

Q reaction : -4.20 MeV (Excitations 0.0+0.0=>4.2+0.0);   Plotted Energy option is "after reaction"

Inelastic E*(^{20}Ne)=4.2 MeV

Two solutions for both fragments depending upon excitation energy

V_{2L}<V_{2B}



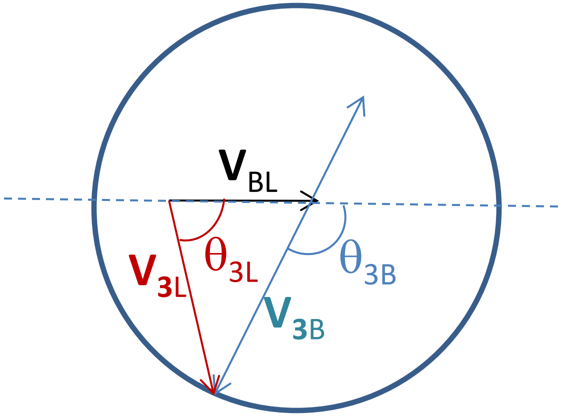
b) reaction

1+2→3+4   3=light   4=heavy

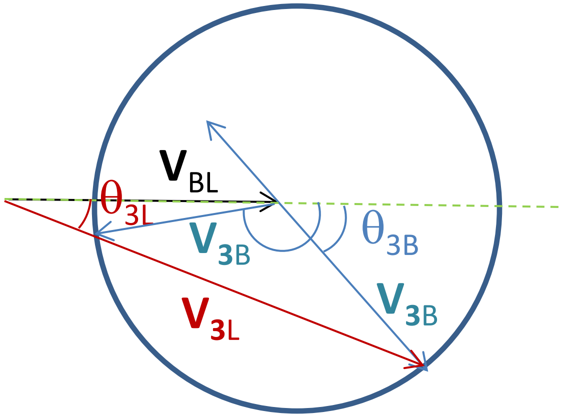
$$Q=(E_{3L}+E_{4L})-(E_{1L}+E_{2L})=[(m_1+m_2)-(m_3+m_4)]c^2$$

The total energy $\sum mc^2$+ E is conserved: $E_T=E_{1L}+Q=E_{3L}+E_{4L}$

$$(m_1c^2+E_1)+(m_2c^2+E_2)=(m_3c^2+E_3)+(m_4c^2+E_4)$$



$V_{BL}<V_{3B}$→ one solution

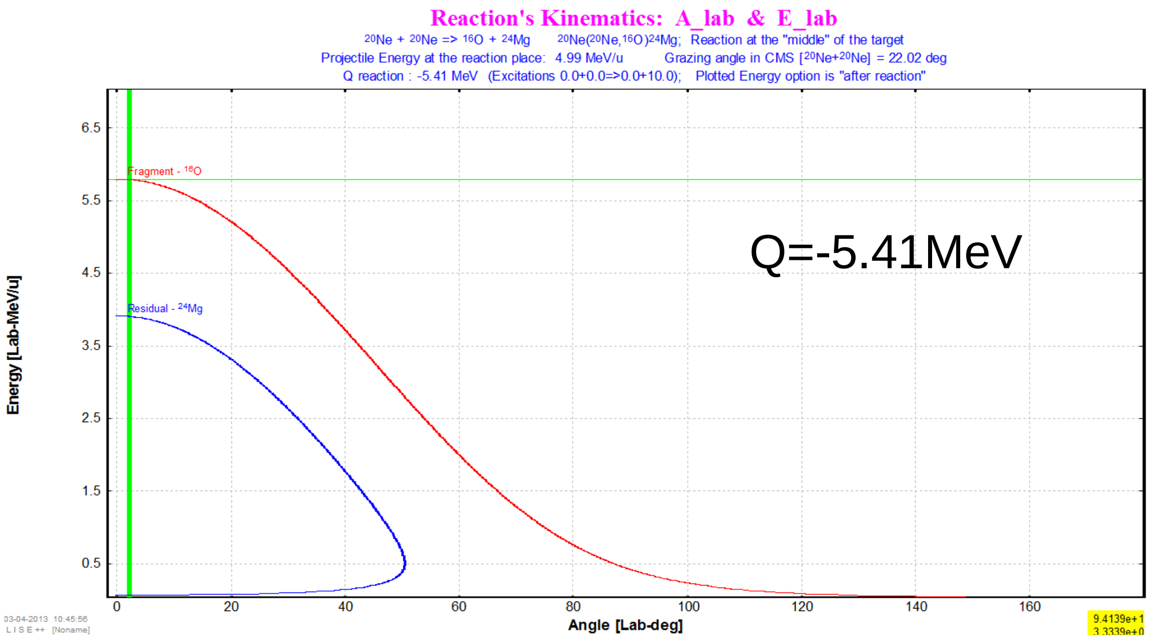


$V_{BL}<V_{3B}$→ two solutions

The number of solutions depends upon Q

$^{20}Ne+^{20}Ne \rightarrow ^{16}O^*+^{24}Mg$

$$Q=Q_{gg}-E^*_1-E^*_2$$



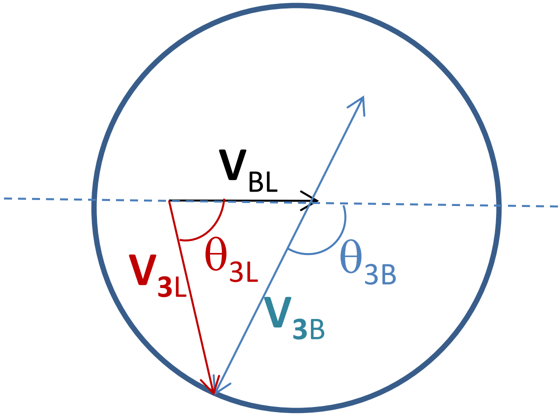
Reaction's Kinematics: A_lab & E_lab

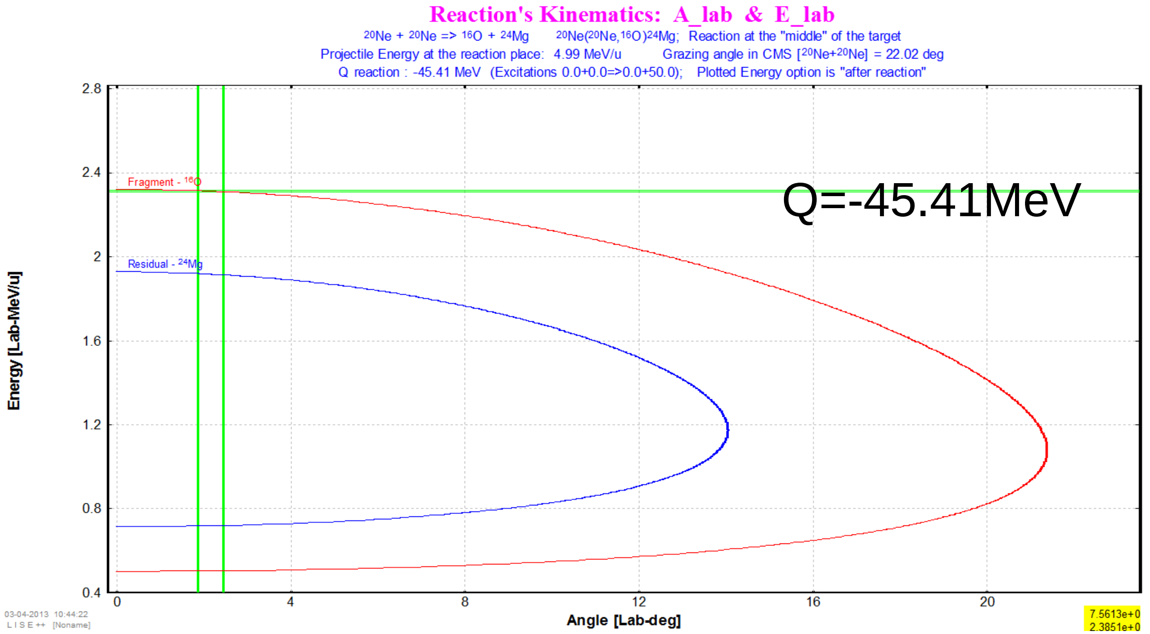
$^{20}Ne + ^{20}Ne => ^{16}O + ^{24}Mg$ $^{20}Ne(^{20}Ne,^{16}O)^{24}Mg$; Reaction at the "middle" of the target

Projectile Energy at the reaction place: 4.99 MeV/u Grazing angle in CMS [$^{20}Ne+^{20}Ne$] = 22.02 deg

Q reaction : -5.41 MeV (Excitations 0.0+0.0=>0.0+10.0); Plotted Energy option is "after reaction"

Q=-5.41MeV





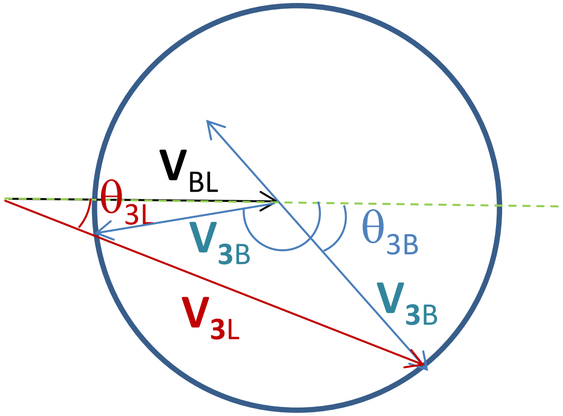
Reaction's Kinematics: A_lab & E_lab

$^{20}Ne + ^{20}Ne => ^{16}O + ^{24}Mg$ $^{20}Ne(^{20}Ne,^{16}O)^{24}Mg$; Reaction at the "middle" of the target

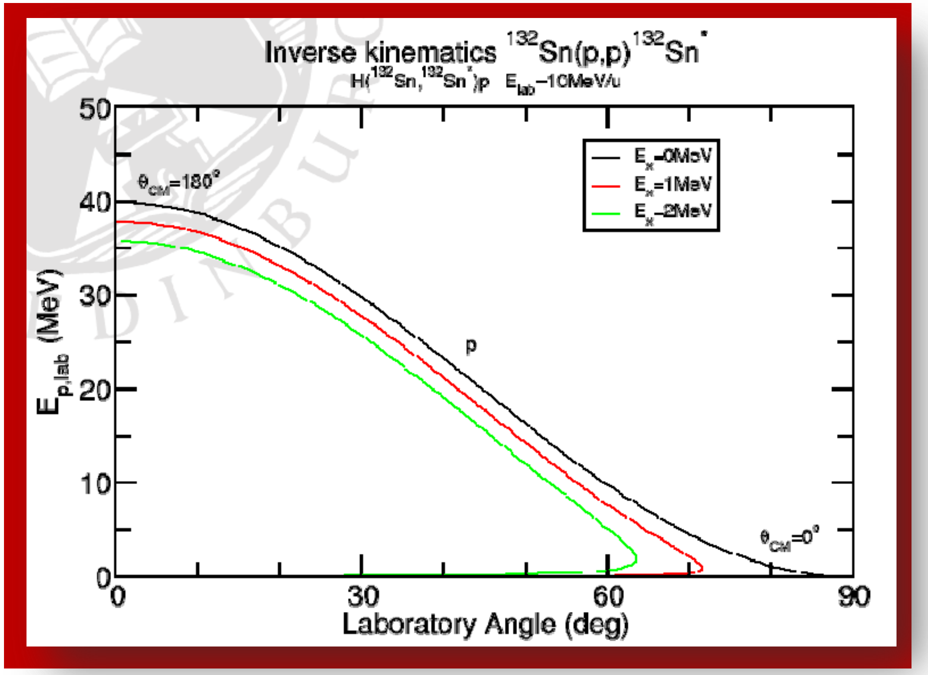
Projectile Energy at the reaction place: 4.99 MeV/u Grazing angle in CMS [$^{20}Ne+^{20}Ne$] = 22.02 deg

Q reaction : -45.41 MeV (Excitations 0.0+0.0=>0.0+50.0); Plotted Energy option is "after reaction"

Q=-45.41MeV

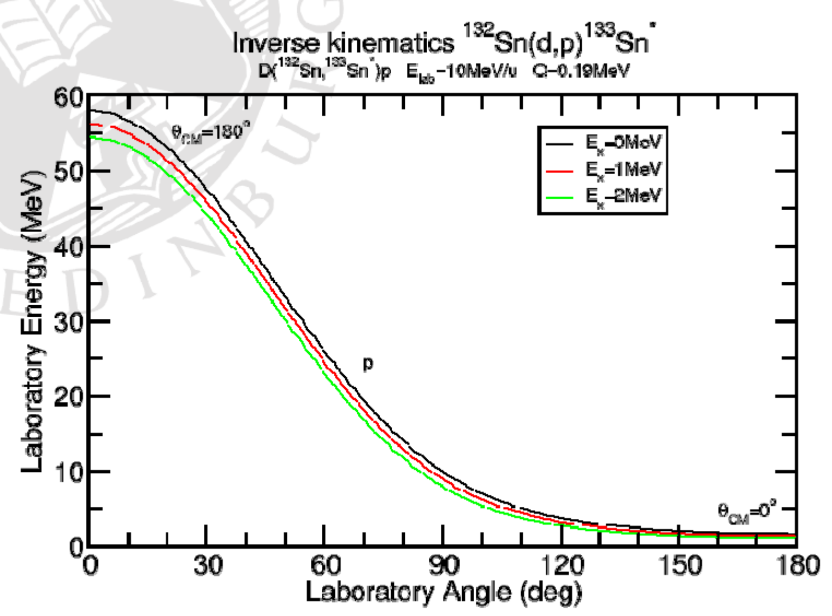


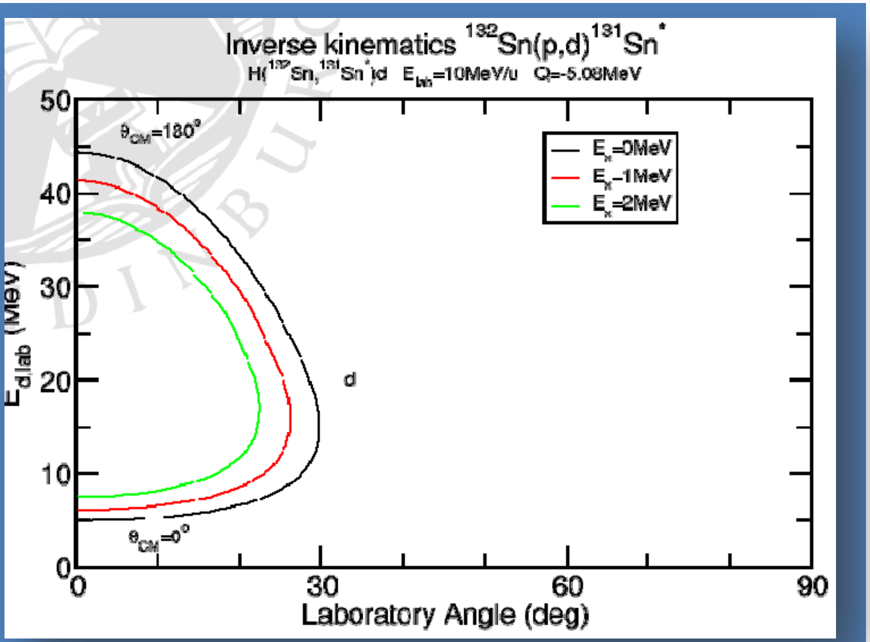
Inelastic scattering



From T.Davinson

Transfer reactions





Threshold energy for a reaction to occur:

$$\boxed{Eth = |Q|\left(\frac{m_1 + m_2}{m_2}\right)}$$

$$A = \frac{m_1 m_4 (E_{1L}/E_T)}{(m_1 + m_2)(m_3 + m_4)}$$

$$B = \frac{m_1 m_3 (E_{1L}/E_T)}{(m_1 + m_2)(m_3 + m_4)}$$

$$C = \frac{m_2 m_3}{(m_1 + m_2)(m_3 + m_4)}(1 + \frac{m_1 Q}{m_1 E_T}) = \frac{E_{4B}}{E_T}$$

$$D = \frac{m_2 m_4}{(m_1 + m_2)(m_3 + m_4)}(1 + \frac{m_1 Q}{m_1 E_T}) = \frac{E_{3B}}{E_T}$$

A+B+C+D=1

AC=BD

If one or both the emitted particles are excited the $Q=Q_{gg}-E^*_1-E^*_2$

$$\frac{E_{3L}}{E_T} = B + D + 2\sqrt{AC}\cos\theta_{3B} = B\left[(\cos\theta_{3L}) \pm \sqrt{(D/B - \sin^2\theta_{3L})}\right]^2$$

Use only sign + (one solution) unless B>D (two solutions), in this case there is a maximum angle for the heavy particle in the Lab: θ_{Lmax}=sin-^{1}(D/B)$^{1/2}$

$$\frac{E_{4L}}{E_T} = A + C + 2\sqrt{AC}\cos\theta_{4B} = A\left[(\cos\theta_{4L}) \pm \sqrt{(C/A - \sin^2\theta_{4L})}\right]^2$$

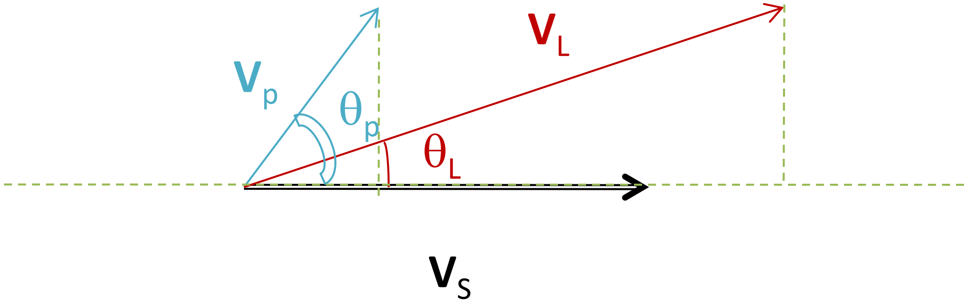
Use only sign + (one solution) unless A>C (two solutions), in this case there is a maximum angle for the heavy particle in the Lab: θ_{4Lmax}=sin-^{1}(C/A)$^{1/2}$

$$\sin\theta_{4B} = \sqrt{\left(\frac{m_3 E_{3L}}{m_2 E_{2L}}\right)}\sin\theta_{3L}$$

$$\sin\theta_{3B} = \left(\frac{E_{3L}/E_T}{D}\right)\sin\theta_{3L}$$

We suppose now that the two particles form a compound system S.
The velocity of S equals the cm velocity after the collision: $V_S = V_{BL}$

If from S is emitting a particle with velocity Vp in the cm system, we have:



$V_L \sin\theta_L = V_p \sin\theta_p$
$V_L \cos\theta_L = V_S + V_p \cos\theta_p$

$$\boxed{tg\theta_L = \frac{\sin\theta_p}{\frac{V_S}{V_p} + \cos\theta_p}}$$

This equation completely determines c.m. angles once Lab angles are measured.