ISOLDE Nuclear Reaction and Nuclear Structure Course

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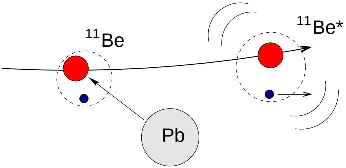
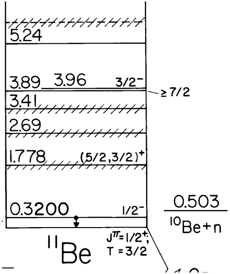
Inelastic scattering

- Nuclei are not inert or *frozen* objects; they do have an internal structure of protons and neutrons that can be modified (excited) during the collision.
- Quantum systems exhibit, in general, an energy spectrum with bound and unbound levels.



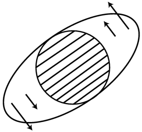


Inelastic scattering

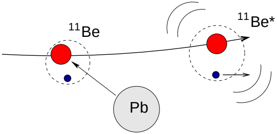
- Direct reactions → nuclei make "glancing" contact and separate immediately.
- Energy/momentum transferred from relative motion to internal motion so the projectile and/or target are left in an excited state.
- Involve small number of degrees of freedom.
- The colliding nuclei preserve their identity: $a + A \to a^* + A^*$
- Typically, they are peripheral (surface) processes.

Models for inelastic excitations

1. COLLECTIVE: Involve a collective motion of several nucleons which can be interpreted macroscopically as rotations or surface vibrations of the nucleus.



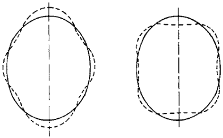
2. FEW-BODY/SIGLE-PARTICLE: Involve the excitation of a nucleon or cluster.



Types of collective excitations

The nucleons can move inside the nucleus in a coherent (collective) way.

1. Vibrations (spherical nuclei): small surface oscillations in shape.

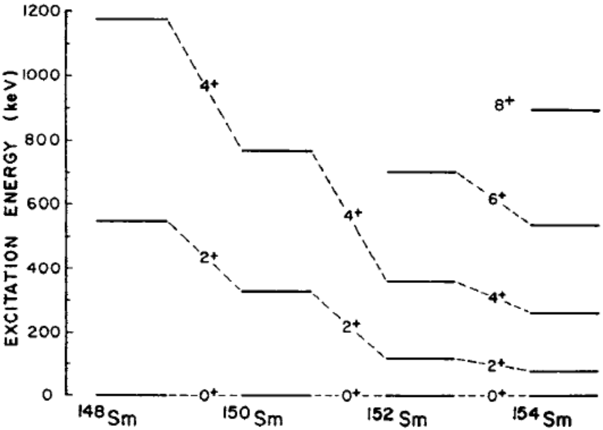


2. Rotations (non-spherical nuclei): permanent deformation.
3. Monopole (*breathing*) mode: oscillations in the size (radius).
4. Isovector excitations (protons and neutrons move out of phase) (eg. giant dipole resonance)

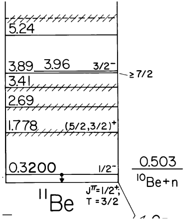
Types of collective excitations

☞ The type of collective motion is closely related to the kind of energy spectrum.

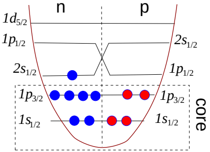
- Rotor: $E_J \propto J(J+1)$
- Vibrator: $E_J \approx n\hbar\omega$



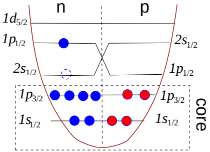
Microscopic description in the IPM: the ^{11}Be case



Ground state $(1/2^+)$

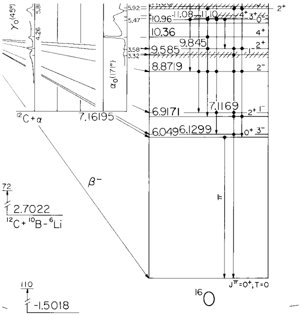


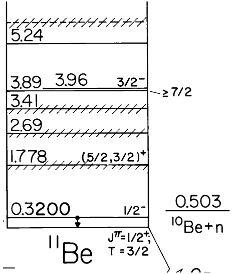
First excited state $(1/2^-)$



Models for inelastic excitations

Microscopically, what we describe in both cases are quantum transitions between discrete or continuum states:





☞*Collective excitations can be regarded as a coherent superposition of many single-particle excitations.*

- By doing inelastic scattering experiments we *measure* the *response* of the nucleus to an external field (Coulomb, nuclear). This response is related to some structure property of the nucleus.

  Example: for a Coulomb field:

$$B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle \Psi_f | \mathcal{M}(E\lambda) | \Psi_i \rangle|^2$$

  where $\mathcal{M}(E\lambda, \mu)$ is the electric multipole operator:

$$\mathcal{M}(E\lambda, \mu) \equiv e \sum_i^{Z_p} r_i^\lambda Y_{\lambda\mu}^*(\hat{r}_i)$$

- The structure $\Psi_{i,f}$ can be described in a collective, few-body or microscopic model.

Energy balance for inelastic scattering

- For projectile excitation: $a + A \longrightarrow a^* + A$

$$E^i_{\rm cm} + M_a c^2 + M_A c^2 = E^f_{\rm cm} + M^*_a c^2 + M_A c^2$$

$$M_{a^*} = M_a + E_x \qquad (E_x\text{=excitation energy})$$

- Q-value:

$$\boxed{Q = M_a c^2 + M_A c^2 - M^*_a c^2 - M^2_A c^2 = -E_x < 0}$$

$$\boxed{E^f_{\rm cm} = E^i_{\rm cm} + Q}$$

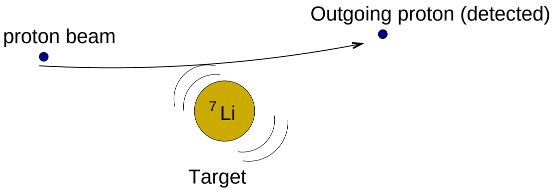
- So

$$\boxed{E_x = E^i_{cm} - E^f_{cm}}$$

What do we measure in an inelastic scattering experiment?

☞ In general, one measures the scattering angle and energy of outgoing particles.

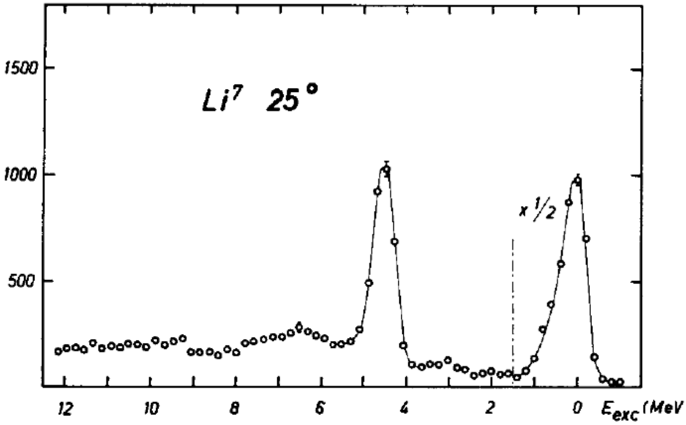
Example: p+^{7}Li → p+^{7}Li*



☞ *Eg. energy and angular distribution of the outgoing protons.*

What do we measure in an inelastic scattering experiment?

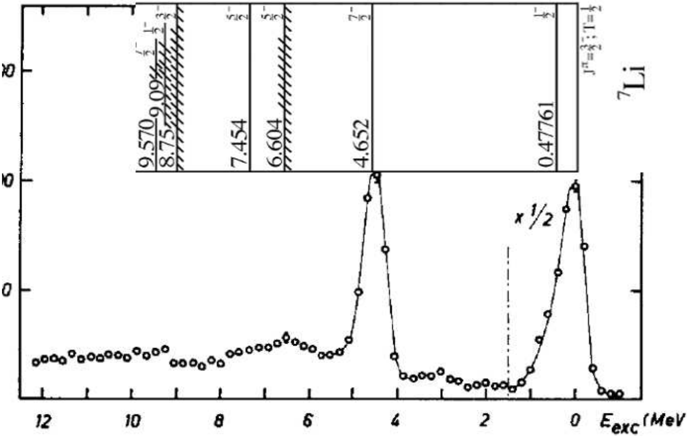
☞ The proton energy carries information on the ^{7}Li excitation spectrum.



Data from Nuclear Physics 69 (1965) 81-102

What do we measure in an inelastic scattering experiment?

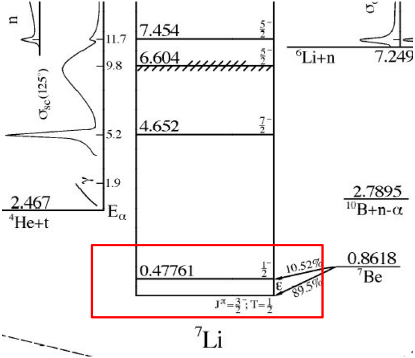
☞ The proton energy carries information on the ^{7}Li excitation spectrum.



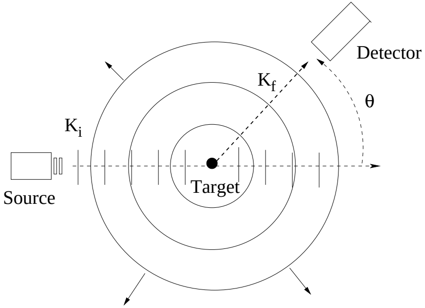
What information do we get from an inelastic scattering experiment?

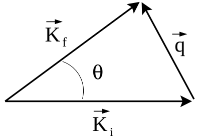
- The proton energy spectrum shows peaks which correspond to the states of the target (^{7}Li)
- The heights of peak ($\sim$ cross section) are different for each state $\Rightarrow$ not all states are populated with the same probability.
- Some peaks are narrow, other are broad. Why?...
- Above a certain excitation energy, the spectrum becomes continuous and structureless.

What information do we get from an inelastic scattering experiment?



Reminder: single-channel case





- The incident projectile is described by a plane wave $\rightarrow e^{i\mathbf{K}_i\cdot\mathbf{R}}$
- The scattered projectile is described at large distances by outgoing spherical waves: $\rightarrow \dfrac{e^{iK_f R}}{R}$

Reminder: single-channel case

- Wavefunction: $\Psi^{(+)}_{\mathbf{K}_i}(\xi, \mathbf{R}) = \phi_0(\xi)\chi^{(+)}_0(\mathbf{K}_i, \mathbf{R})$

$$\chi^{(+)}_0(\mathbf{K}_i, \mathbf{R}) \to e^{i\mathbf{K}_i \cdot \mathbf{R}} + f(\theta)\frac{e^{iK_i R}}{R}$$

  $f(\theta)$=scattering amplitude

- Cross section:

$$\boxed{\frac{d\sigma}{d\Omega}(\theta) = |f(\theta)|^2}$$

☞ *$f(\theta)$ is the coefficient of the outgoing spherical wave at large distances.*

☞ *The square of $f(\theta)$ gives the probability that the particle be scattered at an angle θ.*

Multi-channel case: the coupled-channels method

We need to incorporate explicitly in the Hamiltonian the internal structure of the nucleus being excited (eg. target).

$$H = T_R + h(\xi) + V(\mathbf{R}, \xi)$$

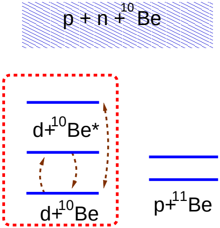
- T_R: Kinetic energy for projectile-target relative motion.
- $\{\xi\}$: Internal degrees of freedom of the target (depend on the model).
- $h(\xi)$: Internal Hamiltonian of the target.

$$h(\xi)\phi_n(\xi) = \varepsilon_n \phi_n(\xi)$$

- $V(\mathbf{R}, \xi)$: Projectile-target interaction, eg:

$$V(\mathbf{R}, \xi) = \sum_{i=1}^{N} V_{pi}(\mathbf{r}_{pi})$$

Defining the modelspace: d+^{10}Be → d+^{10}Be* example



☞ P space composed by ground states (elastic channel) and some excited states (inelastic scattering)

Boundary conditions:

$$\Psi^{(+)}_{\mathbf{K}_0}(\mathbf{R},\xi) \xrightarrow{R\gg} \underbrace{e^{i\mathbf{K}_0\cdot\mathbf{R}}\phi_0(\xi)}_{\text{incident}} + \underbrace{f_{0,0}(\theta)\frac{e^{iK_0R}}{R}\phi_0(\xi)}_{\text{elastic}} + \underbrace{\sum_{n>0} f_{n,0}(\theta)\frac{e^{iK_nR}}{R}\phi_n(\xi)}_{\text{inelastic}}$$

Cross sections:

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{0\to n} = \frac{K_n}{K_0}|f_{n,0}(\theta)|^2$$

CC model wavefunction (target excitation)

We expand the total wave function in a subset of internal states (the P space):

$$\Psi_{\text{model}}(\mathbf{R},\xi) = \phi_0(\xi)\chi_0(\mathbf{K}_0,\mathbf{R}) + \sum_{n>0} \phi_n(\xi)\chi_n(\mathbf{K}_n,\mathbf{R})$$

Boundary conditions for the $\chi_n(\mathbf{R})$ (unknowns):

$$\chi_0^{(+)}(\mathbf{K}_0,\mathbf{R}) \to e^{i\mathbf{K}_0\cdot\mathbf{R}} + f_{0,0}(\theta)\frac{e^{iK_0R}}{R} \qquad \text{for n=0 (elastic)}$$

$$\chi_n^{(+)}(\mathbf{K}_n,\mathbf{R}) \to f_{n,0}(\theta)\frac{e^{iK_nR}}{R} \qquad \text{for n>0 (non-elastic)}$$

Calculation of $\chi_n^{(+)}(\mathbf{R})$: the coupled equations

- The model wavefunction must satisfy the Schrödinger equation:

$$[H - E]\Psi^{(+)}_{\rm model}(\mathbf{R}, \xi) = 0$$

- Multiply on the left by each $\phi_n(\xi)^*$, and integrate over $\xi \Rightarrow$ coupled channels equations for $\{\chi_n(\mathbf{R})\}$:

$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})$$

- Coupling potentials:

$$V_{n,n'}(\mathbf{R}) = \int d\xi \phi_{n'}(\xi)^* V(\mathbf{R}, \xi)\phi_n(\xi)$$

☞ *$\phi_n(\xi)$ will depend on the structure model (collective, single-particle,etc).*

Optical Model vs. Coupled-Channels method

Optical Model

- The Hamiltonian:
  $H = T_R + V(\mathbf{R})$
- Internal states: Just $\phi_0(\xi)$
- Model wavefunction:
  $\Psi_{\rm mod}(\mathbf{R}, \xi) \equiv \chi_0(\mathbf{K}, \mathbf{R})\phi_0(\xi)$
- Schrödinger equation:
  $[H - E]\chi_0(\mathbf{K}, \mathbf{R}) = 0$

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Coupled-channels method

- The Hamiltonian:
  $H = T_R + h(\xi) + V(\mathbf{R},\xi)$
- Internal states:
  $h(\xi)\phi_n(\xi) = \varepsilon_n\phi_n(\xi)$
- Model wavefunction:
  $\Psi_{\rm model}(\mathbf{R},\xi) = \phi_0(\xi)\chi_0(\mathbf{K},\mathbf{R}) + \sum_{n>0}\phi_n(\xi)\chi_n(\mathbf{K},\mathbf{R})$
- Schrödinger equation:

$$[H - E]\Psi_{\rm model}(\mathbf{R},\xi) = 0$$

$$\Downarrow$$

$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{K},\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{K},\mathbf{R})$$

Two-potential formula for $f_{\beta,\alpha}(\theta)$

Introduce auxiliary potential $U_\beta(\mathbf{R})$ and rewrite V_β as

$$V_\beta(\mathbf{R},\xi) = U_\beta(\mathbf{R}) + [V_\beta(\mathbf{R},\xi) - U_\beta(\mathbf{R})]$$

such that the scattering solution of U_β is solvable:

$$[\hat{T}_\beta + U_\beta - E_\beta]\widetilde{\chi}^{(+)}_\beta(\mathbf{R}_\beta) = 0 \qquad E_\beta = E - \varepsilon_\beta$$

Then, the exact scattering amplitude can be written as ($\beta \neq \alpha$):

$$f_{\beta,\alpha}(\theta) = -\frac{\mu}{2\pi\hbar^2}\int\int \widetilde{\chi}^{(-)*}_\beta(\mathbf{K}_\beta,\mathbf{R}_\beta)\Phi^*_\beta(\xi_\beta)[V_\beta - U_\beta]\Psi^{(+)}_{\mathbf{K}_\alpha} d\xi_\beta d\mathbf{R}_\beta$$

Deriving the DWBA approximation from the exact scattering amplitude

- Assume that we can write the p-t interaction as: $V(\mathbf{R},\xi) = V_0(\mathbf{R}) + \Delta V(\mathbf{R},\xi)$
- Apply the two-potential formula taking as auxiliary potential $U_\beta(\mathbf{R}) \equiv V_0(\mathbf{R})$:

$$f^{\text{exact}}_{i\to f}(\theta) = -\frac{\mu}{2\pi\hbar^2}\int d\mathbf{R}\,\chi_f^{(-)*}(\mathbf{K}_f,\mathbf{R})\,\Delta V(\mathbf{R},\xi)\,\Psi_i^{(+)}(\mathbf{K}_i,\mathbf{R})d\xi d\mathbf{R}$$

with

$$\left[\hat{T}_{\mathbf{R}} + V_0(\mathbf{R}) - E_f\right]\widetilde{\chi}_f^{(+)}(\mathbf{K}_f,\mathbf{R}) = 0 \qquad (E_f = E - \varepsilon_f)$$

- Make the Born approximation: $\Psi^{(+)}_{\mathbf{K}_i}(\mathbf{R},\xi) \simeq \widetilde{\chi}_i^{(+)}(\mathbf{K}_i,\mathbf{R})\phi_i(\xi)$, with

$$\left[\hat{T}_{\mathbf{R}} + V_0(\mathbf{R}) - E_i\right]\widetilde{\chi}_i^{(+)}(\mathbf{K}_i,\mathbf{R}) = 0$$

- The scattering amplitude becomes (DWBA):

$$f^{\text{DWBA}}_{i\to f}(\theta) = -\frac{\mu}{2\pi\hbar^2}\int d\mathbf{R}\chi_f^{(-)*}(\mathbf{K}_f,\mathbf{R})\,\Delta V_{if}(\mathbf{R})\,\chi_i^{(+)}(\mathbf{K}_i,\mathbf{R})$$

with the transition potential:

$$\Delta V_{if}(\mathbf{R}) \equiv \int d\xi \phi_f^*(\xi)\,\Delta V(\mathbf{R},\xi)\,\phi_i(\xi)$$

Physical interpretation of the DWBA method

- DWBA can be interpreted as a first-order approximation of a full coupled-channels calculation:

- The auxiliary potential U_β generating the entrance and exit distorted waves is usually chosen in order to reproduce the elastic scattering at the corresponding c.m. energy.

Multipole expansion of the interaction: reduced matrix elements

- In actual calculations, the internal states will have definite spin/parity:

$$\phi_i(\xi) = |I_i M_i\rangle \qquad \text{and} \qquad \phi_f(\xi) = |I_f M_f\rangle$$

- The projectile-target interaction can be expanded in multipoles:

$$V(\mathbf{R},\xi) = \sqrt{4\pi}\sum_{\lambda,\mu} V_{\lambda\mu}(R,\xi) Y_{\lambda\mu}(\hat{R})$$

- CC and DWBA calculations require the transition potentials:

$$\langle I_f M_f|V(\mathbf{R},\xi)|I_i M_i\rangle = \sqrt{4\pi}\sum_{\lambda,\mu} \langle I_f M_f|V_{\lambda\mu}(R,\xi)|I_i M_i\rangle Y_{\lambda\mu}(\hat{R})$$

- Wigner-Eckart theorem → reduced matrix elements:

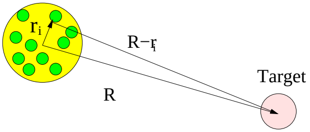
$$\langle I_f M_f|V_{\lambda\mu}(R,\xi)|I_i M_i\rangle = (2I_f+1)^{-1/2}\langle I_f M_f|I_i M_i \lambda\mu\rangle \underbrace{\langle I_f\|V_\lambda(R,\xi)\|I_i\rangle}_{\text{r.m.e}}$$

Models for inelastic scattering

Inelastic scattering: Coulomb excitation

- Projectile-target Coulomb interaction:

$$V_C(\mathbf{R},\xi) = \sum_i^{Z_p} \frac{Z_t e^2}{|\mathbf{R}-\mathbf{r}_i|}; \qquad \xi \equiv \{\mathbf{r}_i\}$$



- Multipolar expansion:

$$\frac{1}{|\mathbf{R}-\mathbf{r}_i|} = \sum_{\lambda\mu} \frac{4\pi}{2\lambda+1} \frac{r_i^\lambda}{R^{\lambda+1}} Y^*_{\lambda\mu}(\hat{r}_i) Y_{\lambda\mu}(\hat{R}) \qquad (R > r_i)$$

- Electric multipole operator: $\mathcal{M}(E\lambda,\mu) \equiv e \sum_i^{Z_p} r_i^\lambda Y^*_{\lambda\mu}(\hat{r}_i)$

$$V_C(\mathbf{R},\xi) = \frac{Z_t Z_p e^2}{R} + \sum_{\lambda>0,\mu} \frac{4\pi}{2\lambda+1} \frac{Z_t e}{R^{\lambda+1}} \mathcal{M}(E\lambda,\mu) Y_{\lambda\mu}(\hat{R}) \equiv \frac{Z_t Z_p e^2}{R} + \Delta V(\mathbf{R},\xi)$$

Coupling potentials for Coulomb excitation

- Transition potentials:

$$\Delta V_{if}(\mathbf{R}) = \sum_{\lambda>0,\mu} \frac{4\pi}{2\lambda+1} \frac{Z_t e}{R^{\lambda+1}} \langle f; I_f M_f | \mathcal{M}(E\lambda,\mu) | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

- Wigner-Eckart theorem⇒ reduced matrix elements:

$$\langle f; I_f M_f | \mathcal{M}(E\lambda,\mu) | i; I_i M_i \rangle = (2I_f+1)^{-1/2} \langle I_i M_i \lambda\mu | I_f M_f \rangle \langle f; I_f \| \mathcal{M}(E\lambda,\mu) \| i; I_i \rangle_{\mathrm{BM}}$$

- Relation to physical quantities

$$\langle f; I_f \| \mathcal{M}(E\lambda,\mu) \| i; I_i \rangle_{\mathrm{BM}} = \sqrt{2I_i+1 B(E\lambda; I_i \rightarrow I_f)}$$

DWBA expression for Coulomb excitation

- Projectile–target interaction:

$$V(\mathbf{R},\xi) = U_{\rm nuc}(R) + V_C(\mathbf{R},\xi) = \underbrace{U_{\rm nuc}(R) + \frac{Z_t Z_p e^2}{R}}_{V_0(R)} + \Delta V(\mathbf{R},\xi)$$

- Use $V_0(R)$ as auxiliary potential for entrance and exit channels:

$$\left[\hat{T}_{\mathbf{R}} + V_0(\mathbf{R}) - E_i\right]\widetilde{\chi}_i^{(+)}(\mathbf{K}_f,\mathbf{R}) = 0 \qquad (E_i = E - \varepsilon_i)$$

$$\left[\hat{T}_{\mathbf{R}} + V_0(\mathbf{R}) - E_f\right]\widetilde{\chi}_f^{(+)}(\mathbf{K}_f,\mathbf{R}) = 0 \qquad (E_f = E - \varepsilon_f)$$

- DWBA scattering amplitude for a transition of multipolarity λ:

$$f(\theta)_{iM_i\to fM_f} = -\frac{\mu}{2\pi\hbar^2}\frac{4\pi Z_t e}{2\lambda+1}\int d\mathbf{R}\,\widetilde{\chi}_f^{(-)*}(\mathbf{K}_f,\mathbf{R})\,\Delta V_{if}^{(\lambda)}(\mathbf{R})\,\widetilde{\chi}_i^{(+)}(\mathbf{K}_i,\mathbf{R})$$

Scattering amplitude and cross sections

DWBA scattering amplitude for a transition of multipolarity λ:

$$f(\theta)_{iM_i\to fM_f} = -\frac{\mu}{2\pi\hbar^2}\frac{4\pi Z_t e}{2\lambda+1}\langle f; I_f M_f|\mathcal{M}(E\lambda,\mu)|i; I_i M_i\rangle \int d\mathbf{R}\,\widetilde{\chi}_f^{(-)*}(\mathbf{K}_f,\mathbf{R})\frac{Y_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}\widetilde{\chi}_i^{(+)}(\mathbf{K}_i,\mathbf{R})$$

Cross sections:

$$\left(\frac{d\sigma}{d\Omega}\right)_{iM_i\to fM_f} = \frac{K_f}{K_i}\left|f(\theta)_{iM_i\to fM_f}\right|^2$$

Unpolarized cross section:,

$$\left(\frac{d\sigma}{d\Omega}\right)_{I_i\to I_f} = \frac{1}{(2I_i+1)}\frac{K_f}{K_i}\sum_{M_i,M_f}\left|f(\theta)_{iM_i\to fM_f}\right|^2$$

What can we learn measuring Coulomb excitation?

- For a inelastic excitation $i \to f$ of multipolarity λ the differential cross section is proportional to the electric transition probability $B(E\lambda; I_i \to I_f)$ because

$$B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle f\ I_f || \mathcal{M}(E\lambda) || i\ I_i \rangle_{\text{BM}}|^2$$

$$\Downarrow$$

$$\frac{d\sigma}{d\Omega} \propto |\langle f\ I_f || \mathcal{M}(E\lambda) || i\ I_i \rangle|^2 \propto B(E\lambda; I_i \to I_f)$$

- *If the approximations involved in the derivation of the DWBA approximation are valid, the transition probabilities $B(E\lambda; I_f \to I_f)$ can be obtained comparing the magnitude of the inelastic cross sections with DWBA calculations.*

Nuclear collective excitations

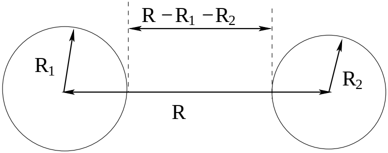
- The nuclear interaction is of short-range, so it depends on the distance between the surfaces of the projectile and targets:

$$U_{\rm nuc}(\mathbf{R}) = V(R - R_0), \quad R_0 = R_1 + R_2$$

E.g.: Woods-Saxon parametrization

$$U_{\rm nuc}(R) = -\frac{V_0}{1 + \exp\left(\frac{R-R_0}{a_r}\right)} - i\frac{W_0}{1 + \exp\left(\frac{R-R_i}{a_i}\right)}$$

- For spherical nuclei, $U(\mathbf{R})$ will not depend on the orientation of the nuclei.



Deformed potential

- Deformed surface:

$$r(\theta,\phi) = R_0 + \sum_{\lambda,\mu} \hat{\delta}_{\lambda\mu}(\xi) Y_{\lambda\mu}(\theta,\phi)$$

$\hat{\delta}_{\lambda\mu}(\xi)$ =deformation length operator

- Deformed potential: $V(R - R_0) \to V(R - r(\theta,\phi)) \equiv V(\mathbf{R},\xi)$
- Multipole expansion of the potential:

$$V(\mathbf{R},\xi) = V(R - R_0) - \sum_{\lambda,\mu} \hat{\delta}_{\lambda\mu}(\xi) \frac{dV(R-R_0)}{dR} Y_{\lambda\mu}(\theta,\phi) + \ldots \equiv V_0(R) + \Delta V(\mathbf{R},\xi)$$

- Transition potentials for a multipole λ:

$$\Delta V^{(\lambda)}_{if}(\mathbf{R}) \equiv \langle f|\Delta V^{(\lambda)}|i\rangle = -\frac{dV(R-R_0)}{dR}\langle f; I_f M_f|\hat{\delta}_{\lambda\mu}|i; I_i M_i\rangle Y_{\lambda\mu}(\hat{R})$$

DWBA amplitude

DWBA scattering amplitude:

$$f(\theta)_{I_f M_f \to I_f M_f} = -\frac{\mu}{2\pi\hbar^2} \langle f; I_f M_f | \hat{\delta}_{\lambda\mu} | i; I_i M_i \rangle \int d\mathbf{R}\, \widetilde{\chi}_f^{(-)*}(\mathbf{K}', \mathbf{R}) \frac{dV}{dR} Y_{\lambda\mu}(\hat{\mathbf{R}}) \widetilde{\chi}_i^{(+)}(\mathbf{K}, \mathbf{R})$$

Cross sections:

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{i\to f} = \frac{K_f}{K_i}\left(\frac{\mu}{2\pi\hbar^2}\right)^2 \left|\langle f; I_f M_f | \hat{\delta}_{\lambda\mu} | i; I_i M_i \rangle\right|^2 \times \left|\int d\mathbf{R} \widetilde{\chi}_f^{(-)*}(\mathbf{K}', \mathbf{R}) \frac{dV}{dR} Y_{\lambda\mu}(\hat{\mathbf{R}}) \widetilde{\chi}_i^{(+)}(\mathbf{K}, \mathbf{R})\right|^2$$

- *The differential cross section is proportional to the deformation parameters*
- *If the approximations are valid, the deformation parameters can be obtained comparing the magnitude of the inelastic cross sections with DWBA calculations.*

Summary of physical ingredients for collective excitations

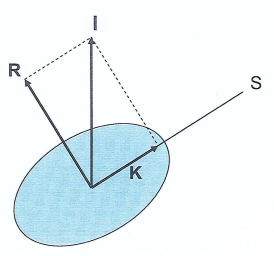
- Coulomb excitation → electric reduced matrix elements

$$\Delta V_{if}(\mathbf{R}) = \sum_{\lambda>0} \frac{4\pi}{2\lambda+1} \frac{Z_t e}{R^{\lambda+1}} \langle f; I_f M_f | \mathcal{M}(E\lambda,\mu) | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

- Nuclear excitation (collective model) → deformation lengths

$$\Delta V_{if}(\mathbf{R}) = -\frac{dV_0}{dR} \sum_{\lambda} \langle f; I_f M_f | \hat{\delta}_{\lambda\mu} | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

Strict rotor model

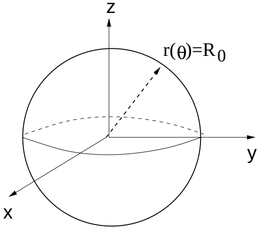


- I=total spin (angular momentum) of the nucleus
- K=projection of I along symmetry axis

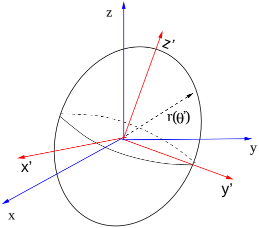
- The nucleus is described by a permanent deformation of matter and charge.
- The charge deformation for a multipole λ is characterized by the Coulomb intrinsic deformation: $M_n(E\lambda)$
- The matter deformation for a multipole λ is characterized by the deformation parameter (β_2) or the deformation length parameter (δ_λ)
- Transitions occur among states with the same value of K.

Description of a deformed surface

Spherical nucleus ($\beta = 0$)



Deformed nucleus ($\beta \neq 0$)



$$r(\theta') = R_0 \left[1 + \beta_2 Y_{20}(\theta', 0)\right]$$

☞ Axial deformed nucleus characterized by β_λ (deformation parameters)

Surface of a permanently deformed nucleus (rotor model)

- Deformed nucleus with axial symmetry: $r(\theta') = R_0\,[1 + \beta_2 Y_{20}(\theta', 0)]$

  $\delta_2 = \beta_2 R_0$ = (quadrupole) deformation length

- $Y_{20}(\theta', \phi')$ can be transformed to the laboratory frame:

$$Y_{\lambda 0}(\theta', 0) = \sum_{\mu} \sqrt{\frac{4\pi}{2\lambda + 1}} Y_{\lambda\mu}(\hat{S}) Y_{\lambda\mu}(\theta, \phi)$$

($\hat{S} \equiv \{\theta_0, \phi_0\}$ gives the orientation of the symmetry axis in the lab frame)

- Define **deformation length operator:**

$$\boxed{\hat{\delta}_{2\mu}(\xi) \equiv \beta_2 R_0 \sqrt{\frac{4\pi}{2\lambda + 1}} Y_{\lambda\mu}(\hat{S})} \qquad \{\xi\} = \hat{S}$$

- Deformed surface in LAB frame:

$$r(\theta, \phi) = R_0 + \sum_{\mu} \hat{\delta}_{2\mu}(\xi) Y_{2\mu}(\theta, \phi)$$

Reduced matrix elements in the strict rotor model

Coulomb excitation:

$$\langle K\, I_f \| \mathcal{M}(E\lambda) \| K\, I_i \rangle_{\rm BM} = \sqrt{2I_i+1}\langle I_i K \lambda 0 | I_f K\rangle M_n(E\lambda)$$

⇨ $M_n(E\lambda)$=reduced matrix element of the charge deformation in intrinsic frame.

For nuclear excitation:

$$\langle f; K\, I_f \| \hat{\delta}_\lambda \| i; K\, I_i \rangle_{\rm BM} = \sqrt{2I_i+1}\langle I_i K \lambda 0 | I_f K\rangle \beta_\lambda R_0$$

- β_λ= deformation parameter
- $\delta_\lambda = \beta_\lambda R_0$= deformation length parameter

⇨ $M_n(E\lambda)$ and β_λ represent the charge and matter deformation in the intrinsic frame

$$M_n(E\lambda) = \frac{3\, Z\, \beta_\lambda\, R^\lambda}{4\pi}$$

Coulomb + nuclear potential

- We expect the Coulomb excitation to be more important when:
  - The projectile and/or target charges are large (i.e. large $Z_1 Z_2 \gg 1$)
  - At energies below the Coulomb barrier (where nuclear effects are less important).
  - At very forward angles (large impact parameters).

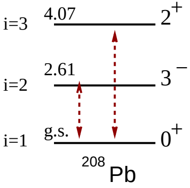
- If both Coulomb and nuclear contributions are important the scattering *amplitudes* for both processes should be added:

$$\left(\frac{d\sigma}{d\Omega}\right)_{i\to f} = \frac{K_f}{K_i} \left| f_{if}^{\text{coul}} + f_{if}^{\text{nucl}} \right|^2$$

☞ *In this case, interferences effects will appear!*

Collective excitations: example

Physical example: $^{16}O + {}^{208}Pb \rightarrow {}^{16}O + {}^{208}Pb(3^-, 2^+)$



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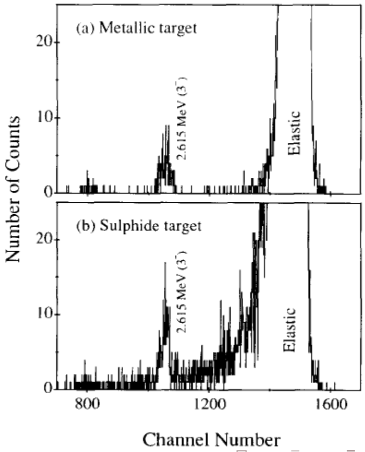
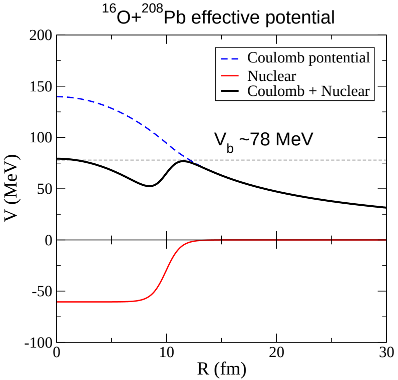
Outgoing ^{16}O energy:



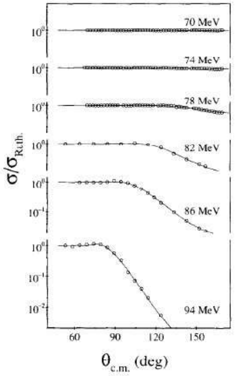
$^{16}O+^{208}Pb$ effective interaction

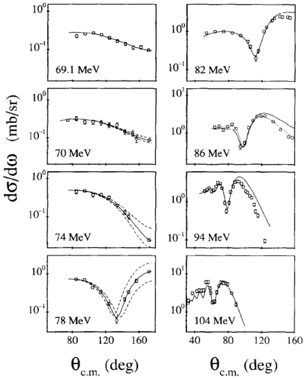


☞ Coulomb barrier:

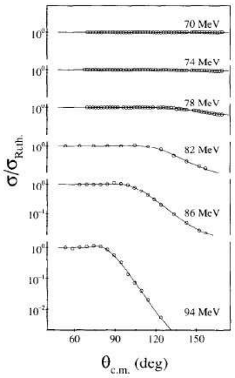
$$V_{\text{barrier}} \approx \frac{Z_p Z_t e^2}{1.44(A_p^{1/3} + A_t^{1/3})} \simeq 78 \text{ MeV}$$

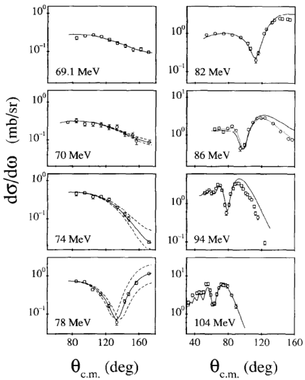
Collective excitations: example





Collective excitations: example



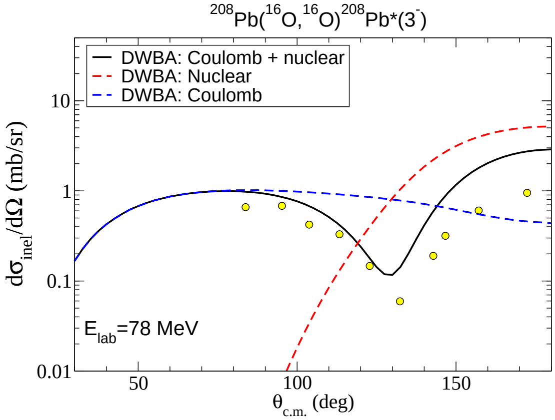


☞ Coulomb barrier:

$$V_{\text{barrier}} \approx \frac{Z_p Z_t e^2}{1.44(A_p^{1/3} + A_t^{1/3})} \simeq 78 \text{ MeV}$$

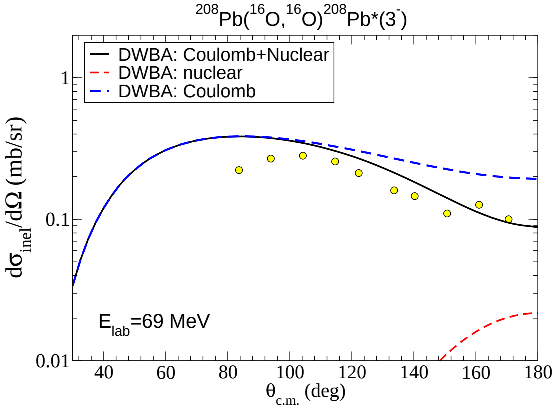
$^{208}Pb(^{16}O,^{16}O)^{208}Pb$ inelastic scattering

Coulomb and Nuclear excitations can produce constructive or destructive interference:



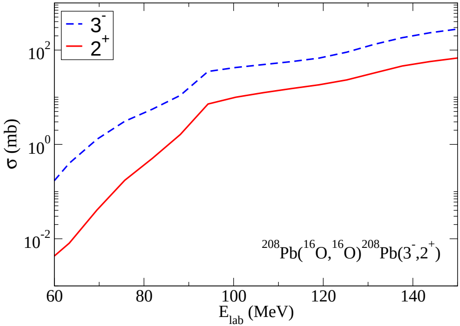
$^{208}Pb(^{16}O,^{16}O)^{208}Pb$ inelastic scattering

Below the barrier, the Coulomb excitation is dominant, and the interference is smaller:



^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Effect of the incident energy:



Extra stuff...

DWBA approximation as 1st order CC

- Two-states model $n = 0, 1$:

$$\Psi(\mathbf{R},\xi) = \underbrace{\phi_0(\xi)\chi_0(\mathbf{R})}_{\text{elastic}} + \underbrace{\phi_1(\xi)\chi_1(\mathbf{R})}_{\text{inelastic}}$$

- Coupled-channels equations:

$$[E - \varepsilon_0 - T_0 - V_{00}(\mathbf{R})]\chi_0(\mathbf{R}) = V_{01}(\mathbf{R})\chi_1(\mathbf{R})$$
$$[E - \varepsilon_1 - T_1 - V_{11}(\mathbf{R})]\chi_1(\mathbf{R}) = V_{10}(\mathbf{R})\chi_0(\mathbf{R})$$

- Iterative solution of the CC equations (DWBA):

$$[E - \varepsilon_0 - T_0 - V_{00}(\mathbf{R})]\chi_0(\mathbf{R}) \approx 0$$
$$[E - \varepsilon_1 - T_1 - V_{11}(\mathbf{R})]\chi_1(\mathbf{R}) \approx V_{10}(\mathbf{R})\chi_0(\mathbf{R})$$

DWBA approximation as 1st order CC

- Asymptotically:

$$\chi_1^{(+)}(\mathbf{R}) \rightarrow f_{10}(\theta)\frac{e^{iK_1 R}}{R}$$

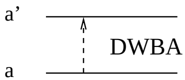
with (not proven here!)

$$f_{10}(\theta) = -\frac{\mu}{2\pi\hbar^2}\int d\mathbf{R}\,\widetilde{\chi}_1^{(-)*}(\mathbf{K}_1,\mathbf{R})V_{10}(\mathbf{R})\widetilde{\chi}_0^{(+)}(\mathbf{K}_0,\mathbf{R})$$

where $\widetilde{\chi}_0(\mathbf{K}_0,\mathbf{R}), \widetilde{\chi}_1(\mathbf{K}_1,\mathbf{R})$ are solutions of:

$$[E-\varepsilon_0-T_0-V_{00}(\mathbf{R})]\widetilde{\chi}_0(\mathbf{K}_0,\mathbf{R}) = 0$$
$$[E-\varepsilon_1-T_1-V_{11}(\mathbf{R})]\widetilde{\chi}_1(\mathbf{K}_1,\mathbf{R}) = 0$$

☞ *The DWBA approximation amounts at solving the CC equations to 1st order (Born approximation)*



Reminder of Wigner-Eckart theorem: reduced matrix elements

$$\langle I_f M_f | \hat{O}_{\lambda\mu} | I_i M_i \rangle = C(I_i, I_f, \lambda) \langle I_f M_f | \lambda \mu I_i M_i \rangle \underbrace{\langle I_f \| \hat{O}_\lambda \| I_i \rangle}_{\text{r.m.e}}$$

Two popular conventions in Nuclear Physics:

1. Bohr-Mottelson (BM) convention: $C(I_i, I_f, \lambda) = (2I_f + 1)^{-1/2}$

$$\langle I_f M_f | \hat{O}_{\lambda\mu} | I_i M_i \rangle = (2I_f + 1)^{-1/2} \langle I_f M_f | \lambda \mu I_i M_i \rangle \langle I_f \| \hat{O}_\lambda \| I_i \rangle_{\rm BM}$$

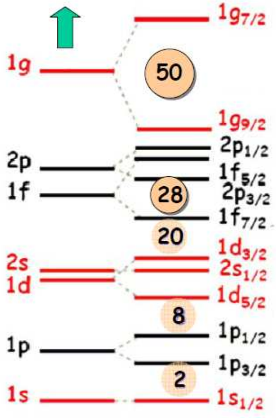
2. Brink-Satchler (BS) convention: $C(I_i, I_f, \lambda) = (-1)^{2\lambda}$

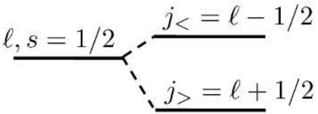
$$\langle I_f M_f | \hat{O}_{\lambda\mu} | I_i M_i \rangle = (-1)^{2\lambda} \langle I_f M_f | \lambda \mu I_i M_i \rangle \langle I_f \| \hat{O}_\lambda \| I_i \rangle_{\rm BS}$$

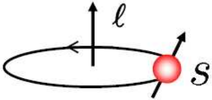
So, the r.m.e. are related by:

$$\langle I_f \| \hat{O}_\lambda \| I_i \rangle_{\rm BM} = \sqrt{2I_f + 1} \langle I_f \| \hat{O}_\lambda \| I_i \rangle_{\rm BS}$$

Reminder of the independent particle model (IPM)





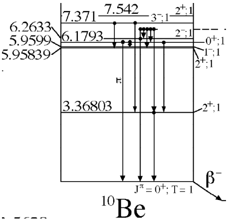


$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

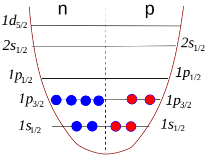
$$V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$
$$V_{so}(r) < 0$$

Microscopic description in the IPM: the ^{10}Be case

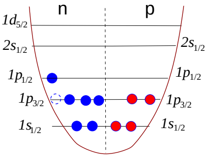
Extreme IPM model:



Ground state (0^+)



First excited state (2^+)

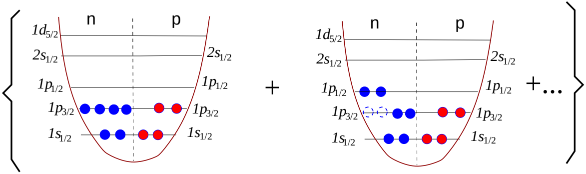


Microscopic description in the IPM: the ^{10}Be case

But life is not that simple…

Ground state (0^+)

$|^{10}Be(gs;0+)\rangle =$



First excited state (2^+)

$|^{10}Be^*(2^+)\rangle =$